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Abstract. The development of absorbing or reflecting panels is of interest for underwater acoustic stealth. Most of the time, losses are not considered in the structures studied, specifically in the propagation number determination, despite the fact that most materials used, such as polymer matrices, have non negligible viscous losses. So, for a better understanding of the acoustic properties of these structures and to allow their optimization, simulations should consider the material properties, including losses. In order to obtain more realistic results from simulation, two numerical tools based on the finite element method (FEM) are proposed, with the help of the *ATILA* software. One is based on a differential method, the other on the transfer matrix. The two methods are first validated in the lossless case, then when losses are taken into account. Both methods give results in good agreement and give the propagation number where losses are taken into account.

1. Introduction

Since the Second World War, acoustic control is a priority in naval stealth. Ships need absorbing or reflecting panels. Periodic materials are a potential solution for this problem. These structures behave as mechanical filters which do not allow wave propagation in certain frequency ranges. Periodic structure characterization is done by resolving the dispersion relation where the angular frequency ω is a function of the wave number k that is $\omega = f(k)$. This computation usually does not consider damping terms in materials. However, acoustic panels are made of materials with non negligible viscous losses in order to increase absorption.

There are several ways to compute the dispersion curve considering periodic structures. Gaunaurd [1] presented a one-dimensional analysis of cylindrical cavities, Audoly [2] compared his work with the Waterman-Truell theory about random distribution of inclusions. Wei [3] worked on the influence of viscosity and he determined band gaps with the plane wave expansion method. Here the finite element method (FEM) provided by the *ATILA* software [4, 5] is used. Thanks to this tool, the modal analysis of a meshed structure gives real solutions for the dispersion relation [6, 7]. This computation considers materials without losses. However, the software is able to consider viscosity during the harmonic analysis, transmission and reflection coefficients can be calculated for a slab with a finite thickness. In this paper, two different methods are presented : Bianco & Parodi method, which is a differential method, and transfer matrix method. They have been adapted to retrieve the dispersion relation considering losses in the materials. Both methods are able to take into account damping due to the materials used.

An example of a periodic bi-layer structure is studied. In a first step, no damping is considered to validate the methods. Next, a small part of viscous losses is added into the material in order to observe the damping influence on the dispersion curve.

2. Transfer matrix method

This method proposes to compute all complex solutions for the wave number at a given frequency. It has first been developed by D. J. Mead [8, 9] and applied by M. L. Accorsi [10] in periodic structures. Lately, it was used by M. Bavencoffe [11].

For a given frequency ω_0, finite element equation defines a relation between the nodal displacement $\tilde{U}$ and the nodal applied force $\tilde{F}$ with the help of the stiffness matrix K and the mass matrix M. This relation connecting $\tilde{U}$ and $\tilde{F}$ can be summarised as written in equation 1,

$$\left([K] - \omega_0^2[M]\right)\tilde{U} = [D]\tilde{U} = \tilde{F}. \tag{1}$$

Moreover, positions can be discriminated between the left side nodes (l), right side nodes (r) and inner nodes (i) of the elementary cell, as shown on figure 1 which is an example of a meshed multi-layer cell. Equation 2 considers this distinction, written

$$\begin{bmatrix} D^{ll} & D^{li} & D^{lr} \\ D^{il} & D^{ii} & D^{ir} \\ D^{rl} & D^{ri} & D^{rr} \end{bmatrix} \begin{pmatrix} \tilde{U}^l \\ \tilde{U}^i \\ \tilde{U}^r \end{pmatrix} = \begin{pmatrix} \tilde{F}^l \\ \tilde{F}^i \\ \tilde{F}^r \end{pmatrix}. \tag{2}$$

The free waves condition implies that no force is applied on inner nodes, i.e. $\tilde{F}_i = 0$. This condition allows to retrieve the expression for inside nodes displacement as follows :

$$\tilde{U}_i = -[D^{ii}]^{-1}(D^{il}\tilde{U}^l + D^{ir}\tilde{U}^r). \tag{3}$$

If $\tilde{U}_i$ is replaced in equation 2, it can be written as

$$\begin{bmatrix} D^{ll} - D^{li}[D^{ii}]^{-1}D^{il} & D^{lr} - D^{li}[D^{ii}]^{-1}D^{ir} \\ D^{rl} - D^{ri}[D^{ii}]^{-1}D^{il} & D^{rr} - D^{ri}[D^{ii}]^{-1}D^{ir} \end{bmatrix} \begin{pmatrix} \tilde{U}^l \\ \tilde{U}^r \end{pmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{pmatrix} \tilde{U}^l \\ \tilde{U}^r \end{pmatrix} = \begin{pmatrix} \tilde{F}^l \\ \tilde{F}^r \end{pmatrix}, \tag{4}$$

where the left matrix is summarized by A, B, C and D terms for a tractable equation. As a periodic structure, the $\jmath^{\text{th}}$ cell is considered. Also, $\jmath^{\text{th}}$ cell's right side is $(\jmath+1)^{\text{th}}$ cell's left side. In other terms, if $U^l = U^\jmath$, $U^r = U^{\jmath+1}$. Using this notation, equation 4 is equivalent to

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{pmatrix} \tilde{U}^\jmath \\ \tilde{U}^{\jmath+1} \end{pmatrix} = \begin{pmatrix} \tilde{F}^\jmath \\ -\tilde{F}^{\jmath+1} \end{pmatrix}. \tag{5}$$

Now, a relation can be defined between the quantities of interest at the right hand side of the cell and those at the other side,

$$\begin{pmatrix} \tilde{U}^{\jmath+1} \\ \tilde{F}^{\jmath+1} \end{pmatrix} = \begin{bmatrix} -B^{-1}A & B^{-1} \\ DB^{-1}A - C & -DB^{-1} \end{bmatrix} \begin{pmatrix} \tilde{U}^\jmath \\ \tilde{F}^\jmath \end{pmatrix} = [T] \begin{pmatrix} \tilde{U}^\jmath \\ \tilde{F}^\jmath \end{pmatrix}. \tag{6}$$

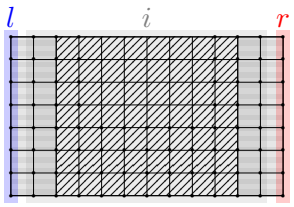

Figure 1. Mesh for an elementary cell of symetric multi-layer structure - Plain : layer 1 ; stripped lines : layer 2

A similar relation exists in periodic medium. It is the Bloch-Floquet relation, using the wave number k and periodic distance a,

$$\begin{pmatrix} \tilde{U}^{\mathrm{J}+1} \\ \tilde{F}^{\mathrm{J}+1} \end{pmatrix} = \mathrm{e}^{jka} \begin{pmatrix} \tilde{U}^{\mathrm{J}} \\ \tilde{F}^{\mathrm{J}} \end{pmatrix}. \tag{7}$$

Both equations 6 and 7 are combined to get a dispersion relation :

$$[T - e^{jka} I] \begin{pmatrix} \tilde{U}^{\mathrm{J}} \\ \tilde{F}^{\mathrm{J}} \end{pmatrix} = 0. \tag{8}$$

3. Bianco & Parodi method

This method was firstly developed to find out electromagnetic waves in micro-strip. This task is difficult due to interface effects and evanescent waves. B. Bianco and M. Parodi [12] came with the idea of comparing measurements from two lines with different lengths. Both lines are long enough to allow only the first order mode in the middle part. The aim is to retrieve the wave number in the material core. Lately, this method was applied by C. Croenne [13] in metamaterials.

On figure 2, two slabs made of the same medium but with some extra-material Δl added in the second case, between points 5 and 6, are represented.

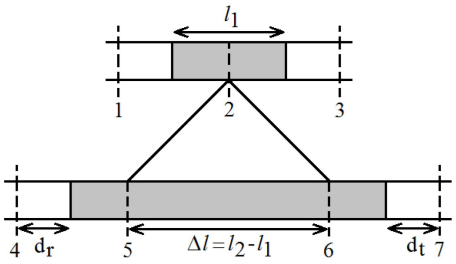

Figure 2. Drawing of two different slabs (grey) with a variation of thickness Δl surrounded by fluid (white)

The matrix representation is considered for the case presented on figure 2. M_{ij} is the propagation matrix between points i and j. Some relations appear :

$$M_{13} = M_{12} M_{23} \quad ; \quad M_{47} = M_{45} M_{56} M_{67} = M_{12} M_{56} M_{23}. \tag{9}$$

Now, the product of the second slab matrix with the inverse matrix of the first slab,

$$M_{47} M_{13}^{-1} = (M_{12} M_{56} M_{23}) M_{23}^{-1} M_{12}^{-1} = M_{12} M_{56} M_{12}^{-1} \tag{10}$$

is computed. One can readily conclude that $M_{47} M_{13}^{-1}$ and M_{56} are similar matrices. So traces of these matrices are identical, i.e.

$$tr(M_{47} M_{13}^{-1}) = tr(M_{56}). \tag{11}$$

M_{56} can be expressed using the expression from transmission lines theory for Δl length,

$$M_{56} = \begin{bmatrix} \cos(k\Delta l) & jZ_c \sin(k\Delta l) \\ jZ_c^{-1} \sin(k\Delta l) & \cos(k\Delta l) \end{bmatrix}. \tag{12}$$

The wave number k appears. Then the first part of equation 11 is expressed using D. A. Frickey's work on S parameters [14]. Frickey shows that the propagation matrix is expressed

$$M = \frac{1}{2S_{21}} \begin{bmatrix} (1+S_{11})(1-S_{22}) + S_{12}S_{21} & (1+S_{11})(1+S_{22}) - S_{12}S_{21} \\ (1-S_{11})(1-S_{22}) + S_{12}S_{21} & (1-S_{11})(1+S_{22}) + S_{12}S_{21} \end{bmatrix}, \tag{13}$$

which can be simplified. If slabs are reciprocal, the reflection coefficient is noted $r = S_{11} = S_{22}$ and transmission $t = S_{12} = S_{21}$. Coefficients are independent of the propagation way, *e.g.* with a symetrical disposition. The propagation matrix can be expressed with these coefficients as follows :

$$M_{13} = \frac{1}{2t_1} \begin{bmatrix} 1 - r_1^2 + t_1^2 & (1+r_1)^2 - t_1^2 \\ (1-r_1)^2 - t_1^2 & 1 - r_1^2 + t_1^2 \end{bmatrix}; \tag{14a}$$

$$M_{47} = \frac{1}{2t_2} \begin{bmatrix} 1 - r_2^2 + t_2^2 & (1+r_2)^2 - t_2^2 \\ (1-r_2)^2 - t_2^2 & 1 - r_2^2 + t_2^2 \end{bmatrix}, \tag{14b}$$

where subscripts indicate the thin slab (1) or the thick slab (2). Each term r_i and t_i is associated to a delay term $\mathrm{e}^{jk_0 d}$ due to propagation in the fluid distance d_r or d_t (figure 2) with k_0 as the wave number in the surrounding fluid. All delay terms are summarized in a unique exponential term in equation 15, which is the pseudo dispersion relation from equation 11,

$$2\cos(k\Delta l) = \frac{t_1^2 + t_2^2 - (r_1 - r_2)^2 \mathrm{e}^{2jk_0(d_t - d_r)}}{t_1 t_2}. \tag{15}$$

4. Results

4.1. Without losses

In order to validate the tools developed, a periodic multi-layer material is numerically studied. The elementary cell of the structure is made of 60 % steel and 40 % epoxy with a periodic distance a equal to 10mm. The disposition is symetrical, as shown in figure 1, to satisfy equations 14 conditions. This case is simple enough to allow an analytical resolution in order to compare results. Analytical solutions for longitudinal waves and transverse waves are developed in Langlet's work [6]. Dispersion relation is

$$\cos(ak) = \cos(\frac{\omega d_1}{c_1})\cos(\frac{\omega d_2}{c_2}) - \frac{(\rho_1 c_1)^2 + (\rho_2 c_2)^2}{2\rho_1\rho_2 c_1 c_2}\sin(\frac{\omega d_1}{c_1})\sin(\frac{\omega d_2}{c_2}) \tag{16}$$

where different materials are subscripted 1 or 2, with ρ the volumic density, c the wave celerity (longitudinal or tranverse) and d the material thickness where $d_1 + d_2 = a$. In a first step, the quality of results need to be ensured. So computations are considered without losses. Results are presented on figure 3. To allow easy reading, analytical solutions are represented by lines on both figures. Longitudinal waves colored in red are distinguished from transverse waves colored in blue. Results are represented by dots, computed by the transfer matrix method in figure 3a and computed by the Bianco & Parodi method in figure 3b. Both methods give results in good agreement with analytical solution. Here the wave number gets an imaginary part, contrary to the modal analysis by FEM that only gives the real part [7]. It helps the dispersion curve reading by following the modes branches. One can notice that there is less information with Bianco & Parodi results. The reflection and transmission coefficients used are computed for a normal incident wave in a fluid environment. In these conditions, only information about longitudinal modes can be retrieved. This particularity allows to identify clearly which curve corresponds to a longitudinal or a transverse mode.

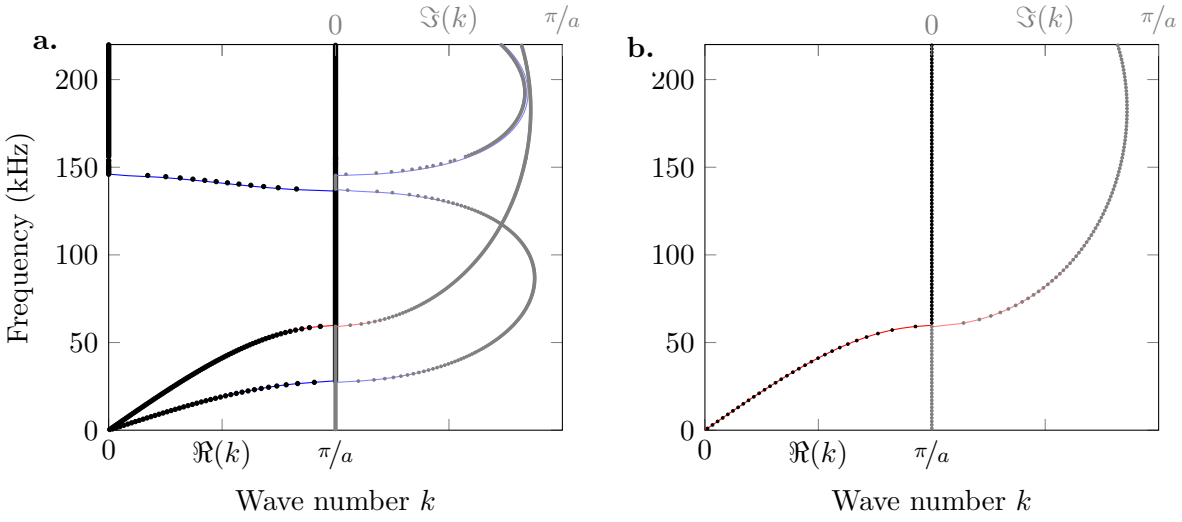

Figure 3. Dispersion curve of multilayer periodic structure with propagation at normal incidence without considering viscous losses. **a.** Transfer matrix method **b.** Bianco & Parodi method - black dots = real part of the wave number ; grey dots = imaginary part - On both figures, the analytical solution is represented by lines : longitudinal part (red), transverse part (blue)

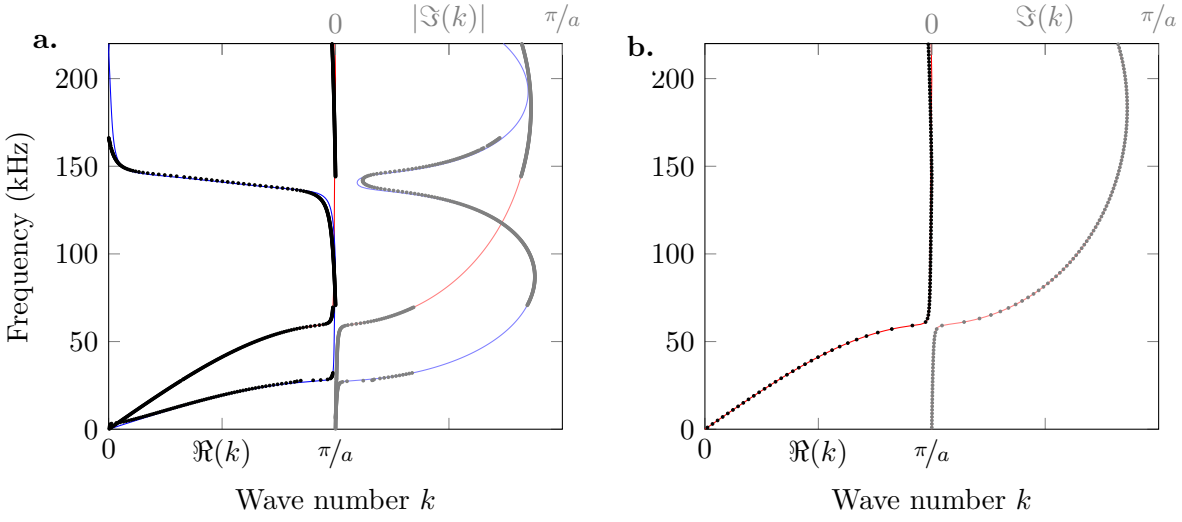

Figure 4. Dispersion curve of multilayer periodic structure with propagation at normal incidence considering viscous losses. **a.** Transfer matrix method **b.** Bianco & Parodi method - black dots = real part of the wave number ; grey dots = imaginary part - On both figures, the analytical solution is represented by lines : longitudinal part (red), transverse part (blue)

4.2. With losses

The purpose of this paper is to consider viscous losses in dispersion curves when considering dissipative materials. Both methods use the harmonic analysis. This computation in the *ATILA FEM* software allows complex values for material properties. An imaginary part for the wave number implies that these waves are evanescent. The same bi-layer structure used in the previous part is kept, but a very small amount of losses in epoxy, i.e. an imaginary part of the elasticity moduli equal to 2% of its real part, is considered.

Both methods are presented on figures 4a and 4b. Transfer matrix solutions are sometimes missing, due to the computing selection during resolution, but both methods give results in good agreement. One can see that curves are smoothed and stop-band boundaries are not strictly defined. Moreover, even in previous pass bands, an imaginary part is obtained : it means that the waves are partially evanescent due to viscous losses in the materials.

5. Conclusion

Two methods have been presented to retrieve the dispersion curve when materials have losses. The Bianco & Parodi method only gives information about the longitudinal propagation mode. To use this method, the slabs studied must be thick enough to allow a periodic behaviour. According to previous studies [15], slabs should be composed of 4 layers at least to obtain periodic effects. The transfer matrix method gives information about all propagative modes. However, the computation time is considerably heavier, especially in the 3D case. Moreover, the resolution is approximate and some results are missing due to the computing selection. Both methods give an imaginary part for the wave number, to get a readable representation. By taking into account losses, results show that previous pass bands have an imaginary part, and stop band boundaries are not strictly defined.

These tools can be used to optimize the dispersion of a periodic material in realistic conditions (i.e. with losses). By providing an unusual dispersion curve, simulations describe more accurate behaviour for periodic structures. Using both methods is also a good way to identify clearly which curve corresponds to a longitudinal or a transverse mode. Furthermore, whereas the transfer matrix method detects all propagative modes, Bianco & Parodi method is able to consider oblique propagation. Together, these methods appear to be a powerful way to put insight on physical phenomena encountered for both propagative and evanescent waves in periodic media.

By using the tools developed, it will allow to study new absorbing acoustic panels before experimental validation.

Acknowledgement

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