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Performance assessment of a Duffing-type vibration energy harvester

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Abstract — Non-linear mechanical oscillators combined with piezoelectric transducers are widely used in vibration energy harvesting applications for their broadband behavior. However, the complexity of their behavior for a given excitation and especially the existence of multiple orbits needs to be addressed. In this paper, a new approach has been developed to predict the probability of occurrence of each orbit. Combined with the power harvestable by each one, it allowed us to calculate an expected power at each drive frequency. This approach was used to demonstrate that our generator has a constant expected harvested power between $f_d = 23$ Hz and 55 Hz.

1 Introduction

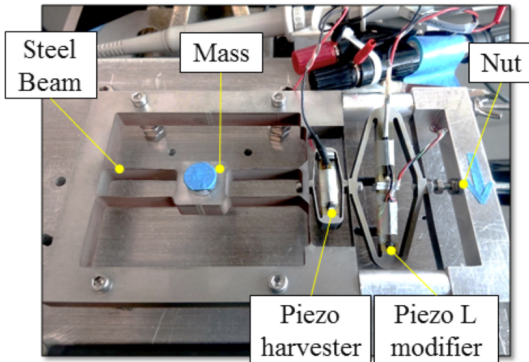


Figure 1: Experimental setup from [1] used in this article.

A vibration energy harvester usually involves 3 sub-systems : a mechanical oscillator, an extraction circuit and a storage unit. In this article, we focus only on the mechanical oscillator and its extraction circuit. Linear mechanical oscillators are ideal if the available vibrational energy is concentrated in a narrow frequency band. In the case where the energy is distributed over a wider frequency band, non-linear oscillators such as the Duffing oscillator allow for increased performance. However, the behavior of nonlinear oscillators is much more complex than that of their linear counterparts. For a given excitation, they can oscillate on several orbits, often very different. These orbits can be periodic but also chaotic. In particular, the output power of each of them can differ drastically. The ability to predict these orbits but also their probability of occurrence is an essential challenge in the development of vibration energy harvesting. In this paper, we present a new method to predict the probability of occurrence of each orbit to derive an expected harvested power, as a function of the driving frequency.

2 Materials and methods

2.1 Experimental setup

A complete experimental set-up including a frame, a mechanical Duffing bistable oscillator, its extraction circuit and associated power measurement has been developed [1] as represented on Fig 1. The oscillator consists of a buckled steel beam of length L on which is fixed a mass M . Two amplified piezoelectric actuators are used. The smaller one has a force factor α and a blocked capacity C_p and is used to extract energy from the mechanical oscillator. The second and stiffer one is used to adjust the level of buckling. It is therefore not directly taken into account in the model. The voltage in the electrical circuit is noted v . The harvested power is dissipated in a resistor of constant resistive load $R = 7.83$ k Ω .

2.2 Numerical model

The mechanical oscillator and its extraction circuit have been modeled as follows according to [2].

$$\begin{cases} \ddot{x} = -\frac{\omega_0^2}{2} \left(\frac{x^2}{x_0^2} - 1 \right) x - \frac{\omega_0}{Q} \dot{x} - 2 \frac{\alpha}{ML} x v + A \sin(2\pi f_d t) \\ \dot{v} = 2 \frac{\alpha}{LC_p} x \dot{x} - \frac{1}{RC_p} v \end{cases} \quad (1)$$

Where x represents the position of the mass M . The oscillator has two stable positions at $x = \pm x_0$. The linear behavior for small oscillations around $x \approx x_0$ has a natural angular frequency ω_0 and a mechanical quality factor Q as defined in [3]. The oscillator is submitted to a sinusoidal ambient acceleration of amplitude $A = 2.5$ m.s⁻² and frequency f_d .

The non-linear ODE Eq. 1 were solved numerically using the Python package [Scipy](https://docs.scipy.org/doc/scipy/). A representative grid of initial conditions $X_0 = (x(0) \ \dot{x}(0) \ v(0))^T$ with 400 points was determined. For each frequency, a simulation was performed from each X_0 until convergence to a harmonic or chaotic regime. The stable orbits were deduced and classified by type (harmonic or chaotic) and if necessary by sub-harmonic, i.e. those for which we oscillate at a frequency which is sub-multiple of the driving frequency. An orbit is high when it often crosses the saddle point at $x = 0$ and goes from one equilibrium to the other. It is considered low when it oscillates around one of the two equilibrium points $\pm x_0$. We take the following notations: H1H (resp. H1L) for a first order high (resp. low) harmonic and SH3H for a three order high sub-harmonic. The proportion of oscillators converging to a given type of orbit is noted r and is called

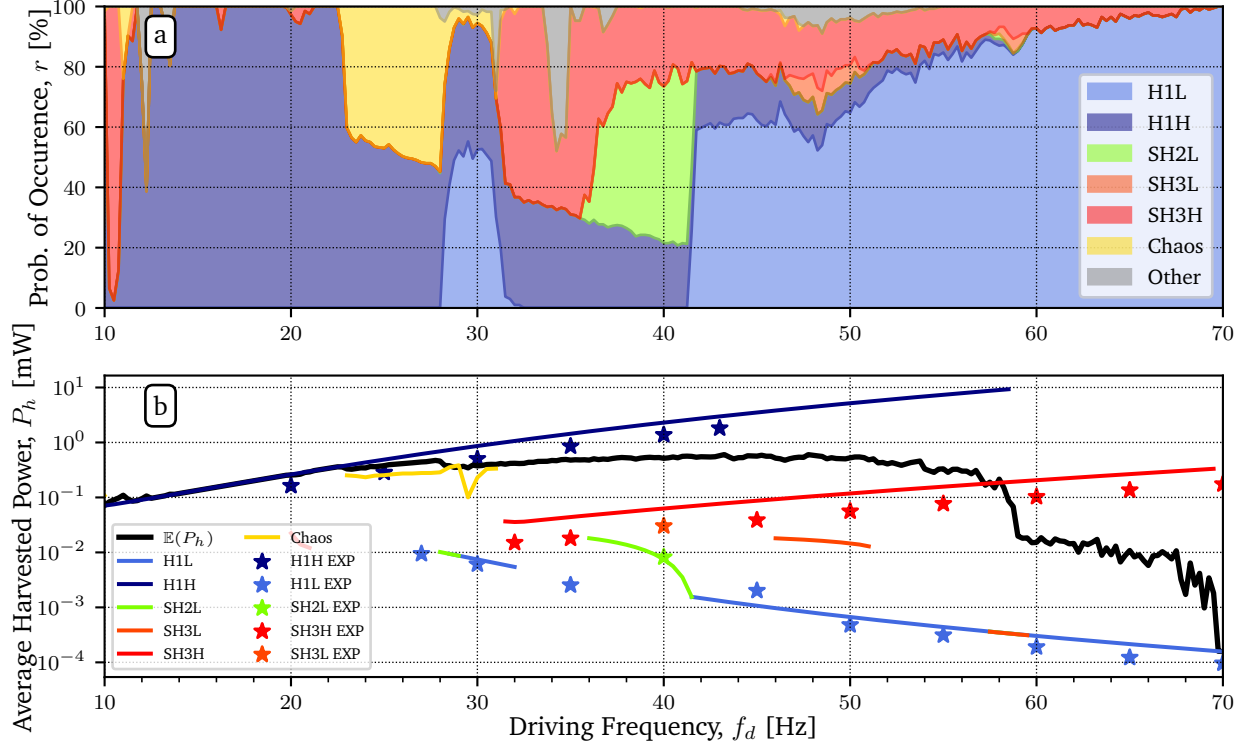


Figure 2: (a) Probability of occurrence. (b) Orbital average harvested power (from simulation and experiments) and expected power as a function of the driving frequency f_d .

the probability of occurrence in what follows. The average power dissipated in the resistor $P_{h,k}$ was also calculated for each orbit in steady operation. The expected power $\mathbb{E}[P_h]$ is calculated as the sum of the average powers $P_{h,k}$ associated with each orbit weighted by their probability of occurrence for each frequency as follows:

$$\mathbb{E}[P_h] = \sum_k r_k P_{h,k} \quad (2)$$

3 Results and discussion

Fig 2-(a) shows the probability of occurrence r of each orbit. It illustrates in particular that the more energetic a high orbit becomes with the increase of f_d , the smaller its r is. Fig 2-(b) shows that the expected power $\mathbb{E}[P_h]$ does not follow a specific orbital harvested power for all drive frequencies f_d . In particular, it is close to the high orbit 1 (H1H) at low frequency but moves away from it as its r decreases. It also indicates that the expected power is almost constant between $f_d = 23$ Hz and 55 Hz. Finally, it proves that the decrease in power beyond 55 Hz is related to the increasing value of r of the H1L orbit. This approach allows us to know which orbits one should try to modify to increase the power expectation.

4 Conclusion

In this paper, we have presented a tool predicting the expected power that can be harvested with a bistable piezoelectric generator. Each orbit accessible at a given driving frequency corresponds to an available power that can be ex-

tracted from the mechanical oscillator. Indeed, this tool allowed us to see that the more a high orbit sees its energy increasing according to the driving frequency f_d , the more its probability of occurrence r tends to decrease. Considering the presented results, an orbit jumping strategy will be established in order to stay on the most energetic orbit as long as possible regardless of the excitation frequency.

Acknowledgements

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