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The Strength of the Iceland Plume: A Geodynamical Scaling Approach

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Abstract

1
2 An important measure of the strength of a mantle plume is its buoyancy
3 flux B , defined as the integral over a horizontal plane of the product of the
4 vertical velocity and the density deficit within the plume. In the case of
5 the Iceland plume, which currently rises directly beneath the mid-Atlantic
6 ridge, published estimates of B cover a range of a factor of 37. To reconcile
7 these diverse estimates, we study a simple fluid mechanical model of a ridge-
8 centered plume in which plume fluid with spreadability σ (buoyancy over
9 viscosity) is supplied at a volumetric rate Q from a plume conduit located
10 directly beneath a ridge with a half spreading rate U . The plume fluid spreads
11 laterally to form a thin pool beneath a lithosphere whose thickness increases
12 as the square root of age. Application of scaling and dimensional analysis
13 to this model leads to a general scaling law for the ‘waist width’ W_w , the
14 length of the plume-induced elevation anomaly along the ridge. The law has
15 the form $W_w/W_0 = f_2(\Pi_b, \Pi_s)$, where $W_0 = (\sigma Q^4/U^5)^{1/6}$ is the fundamental
16 length scale for plume-ridge interaction, $\Pi_b = (\sigma Q/U^2)^{1/3}$ is of the order
17 of the aspect ratio (width/thickness) of the plume pool, $\Pi_s = (\kappa^2 \sigma/U^3)^{1/4}$
18 measures the effect on the pool of the sloping base of the lithosphere (κ is the
19 thermal diffusivity), and f_2 is an unknown function. We determine f_2 using
20 a suite of 32 numerical solutions of a three-dimensional thermomechanical
21 model implemented in the code StagYY (Tackley, 2008). To apply our scaling
22 law to Iceland, we invert it to estimate the buoyancy flux B required to
23 produce a waist width $W_w = 2300 \pm 300$ km. After correction for the effect

24 of ridge migration, we find $B = 2.3 \pm 0.6 \text{ Mg s}^{-1}$. This is comparable within
25 uncertainty to the buoyancy flux $B = 3.0 \pm 0.8 \text{ Mg s}^{-1}$ of the Hawaiian plume
26 estimated using a 3-D dynamical model by Ribe and Christensen (1999).

27 **keywords:** mantle plumes; plume-ridge interaction; Iceland; lubrication the-
28 ory

29 1 Introduction

30 The mantle plume that creates Iceland is, together with the Hawaiian plume,
31 one of the two best known and most intensively studied of all mantle plumes.
32 By a nice coincidence, these two plumes happen to represent the two end-
33 member extremes of the phenomenon of plume-ridge interaction (PRI), whereby
34 a mantle plume influences the bathymetry, geochemistry and crustal struc-
35 ture along a portion of a nearby mid-ocean ridge. In the Hawaiian case,
36 the distance between the plume and the nearest ridge is so great that PRI
37 does not occur at all. At the opposite extreme, the Iceland plume is lo-
38 cated directly below the mid-Atlantic ridge (MAR), making this system the
39 paradigmatic example of PRI.

40 Intuition tells us that the intensity of PRI should be proportional to some
41 measure of the ‘strength’ of the mantle plume involved. Since the pioneering
42 work of Sleep (1990), the most commonly used measure of plume strength is
43 the ‘buoyancy flux’ B . It is defined as

$$B = \int_S w \delta \rho dS, \quad (1)$$

44 where w is the (laterally variable) vertical velocity within the plume, $\delta \rho > 0$
45 is the density deficit of the plume material, and the surface integral is taken

46 over a horizontal cross-section of the plume. The SI units of B are kg s^{-1} ,
 47 but we shall use the more customary units $\text{Mg s}^{-1} = 10^3 \text{ kg s}^{-1}$. If the
 48 plume is in steady-state, then B will be independent of the chosen cross-
 49 section. If the plume’s buoyancy is due to temperature differences alone,
 50 then $\delta\rho = \rho_0\alpha(T - T_0)$ where T is the temperature, T_0 is the temperature
 51 outside the plume, ρ_0 is the reference density at the temperature T_0 and
 52 α is the coefficient of thermal expansion. The buoyancy flux of a mantle
 53 plume is important because it is proportional to the heat flux that the plume
 54 carries. Because many of Earth’s largest mantle plumes are likely to arise at
 55 the core-mantle boundary, estimating their combined buoyancy flux provides
 56 an estimate of the heat flux coming out of the core (Davies, 1988; Hoggard
 57 et al., 2020).

58 Two general methods have been used to estimate the buoyancy fluxes of
 59 mantle plumes interacting with mid-ocean ridges. The first, which we shall
 60 call the ‘flux balance’ method, is based on a balance between the vertical
 61 buoyancy flux in the plume conduit and the horizontal flux of buoyancy in the
 62 elevated topography of the hotspot swell carried by the moving plate (Sleep,
 63 1990; Schilling, 1991). In its simplest form, the equation for this method is
 64 $B = c_1(\rho_0 - \rho_w)EUW_w$, where E is the maximum excess elevation, U is the
 65 half-spreading rate, W_w is the width of the elevation anomaly along the ridge
 66 itself, ρ_w is the density of seawater, and c_1 is a model-dependent constant of
 67 proportionality of order unity. Implicit in this method is the assumption that
 68 the negative buoyancy of the hotspot swell is in isostatic equilibrium with the
 69 positive buoyancy of the underlying low-density plume material that com-
 70 pensates it. An alternative version of the flux balance method (Sleep, 1990;

71 Schilling, 1991) equates the volumetric flux Q (rather than the buoyancy
72 flux) of the plume with the rate at which new lithosphere of thickness H
73 and width W_w is carried away from the ridge at the rate U . This gives
74 $Q = c_2 H U W_w$, where c_2 is a model-dependent constant of proportionality.
75 The volume flux thus estimated can be transformed into a buoyancy flux
76 $B = \rho_0 \alpha \Delta T_m Q$, where ΔT_m is an assumed maximum temperature anomaly
77 of the plume.

78 The second method is based on the width W_w of the excess elevation alone
79 (Feighner and Richards, 1995; Ribe and Delattre, 1998). It simply asks how
80 large the volume flux Q must be to generate an elevation anomaly with
81 the observed width W_w . In view of laboratory observations that the width
82 of a pool of buoyant plume material centered on a ridge increases in both
83 directions away from the ridge, Feighner and Richards (1995) called W_w the
84 ‘waist width’. We shall therefore call the second method for estimating B the
85 ‘waist width’ method. This method requires a dynamical model (numerical
86 or experimental) to predict the relation between W_w and Q .

87 The aforementioned methods have been applied by several authors to esti-
88 mate the buoyancy flux of the Iceland plume. Sleep (1990) estimated $B = 1.4$
89 Mg s^{-1} using a modified version of the second flux balance method described
90 above. Schilling (1991) used both versions of the flux balance method to
91 estimate $Q \approx 45 \text{ m}^3 \text{ s}^{-1}$, which corresponds to $B = 1.4 \text{ Mg s}^{-1}$ for his es-
92 timated excess plume temperature $\Delta T_m = 263 \text{ K}$. He used a reduced waist
93 width $W_w = 920 \text{ km}$ corresponding to the width of geochemical (rather than
94 elevation) anomalies along the ridge around Iceland. Feighner and Richards
95 (1995) used the waist width method in conjunction with laboratory experi-

96 ments. They proceeded indirectly by verifying that the volume flux $Q \approx 45$
 97 $\text{m}^3 \text{s}^{-1}$ estimated by Schilling (1991) corresponded to a waist width of 850
 98 km according to their experimentally-based scaling law, reasonably close to
 99 the value (920 km) of Schilling (1991). Ribe and Delattre (1998) used the
 100 waist width method together with a dynamical model based on lubrication
 101 theory to conclude that a volume flux $Q = 30 \text{ m}^3 \text{ s}^{-1}$ (corresponding to
 102 $B = 0.7 \pm 0.17 \text{ Mg s}^{-1}$ for $\Delta T_m = 200 \pm 50 \text{ K}$) was required to explain a
 103 waist width $W_w = 920 \text{ km}$. King and Adam (2014) used a geometric flux
 104 balance approach and two versions of the MiFil (minimization and filtering)
 105 method of Adam et al. (2005) to recalculate buoyancy fluxes for 54 terres-
 106 trial hotspots. Their three estimates for Iceland were 1.40 Mg s^{-1} , 1.61 Mg
 107 s^{-1} and 1.52 Mg s^{-1} . Parnell-Turner et al. (2014) used three independent
 108 flux-balance arguments to estimate $B = 18 \pm 7 \text{ Mg s}^{-1}$, $B = 26 \pm 9 \text{ Mg}$
 109 s^{-1} and $B = 17 \pm 5 \text{ Mg s}^{-1}$, respectively. Finally, Hoggard et al. (2020)
 110 estimated $B = 4.0 \pm 1.0 \text{ Mg s}^{-1}$ using a flux balance argument in the context
 111 of a model in which the plume pool comprises discrete ‘fingers’ that spread
 112 radially away from the hotspot.

113 The most striking aspect of the foregoing list of estimates is the enor-
 114 mous range they cover, encompassing a factor of 37 from the lowest (0.7 Mg
 115 s^{-1}) to the highest (26 Mg s^{-1}). The aim of the present study is to explain
 116 and reconcile this disagreement using a geodynamical modeling approach.
 117 We begin (in § 2) by applying dimensional and scaling analysis to a simple
 118 fluid mechanical model for a ridge-centered plume, including the effects of
 119 a lithosphere that thickens with age and migration of the ridge relative to
 120 the plume. The result is a general scaling law for the waist width W_w that

121 involves an undetermined function of the key dimensionless parameters of
122 the problem. Next (§ 3), we use three-dimensional thin-layer and thermome-
123 chanical numerical models to characterize the undetermined function from
124 § 2. Finally (§ 4), we invert our scaling law for W_w to estimate the buoyancy
125 flux ($B = 2.3 \pm 0.6 \text{ Mg s}^{-1}$) required to explain the observed waist width
126 ($W_w = 2300 \pm 300 \text{ km}$) of the Iceland plume. The paper concludes (§ 5) with
127 a discussion of our new estimate in light of previous estimates of the same
128 quantity.

129 **2 Dimensional and scaling analysis**

130 To begin our study, we use a combination of dimensional and scaling analy-
131 sis to determine as much as we can about how the waist width W_w depends
132 on the various input parameters of the problem. Dimensional analysis is a
133 generally applicable method based on the fact that a mathematical relation
134 among a number of dimensional model parameters is equivalent to a rela-
135 tion among a smaller number of dimensionless combinations (‘groups’) of
136 those parameters. The basic theorem of dimensional analysis, called Buck-
137 ingham’s Π -theorem (Buckingham, 1914), is a recipe for determining how
138 many dimensionless groups are both necessary and sufficient for the problem
139 at hand. Scaling analysis, by contrast, is a more specific method that starts
140 from the differential equation(s) that govern the phenomenon of interest.
141 By requiring the dominant terms in the equation to be of the same order
142 of magnitude, one can determine the fundamental length and (in unsteady
143 problems) time scales that characterize the phenomenon, and that cannot be
144 determined by dimensional analysis alone.

145 To help build intuition, the following discussion proceeds step by step,
 146 considering first the idealized case of a vanishingly thin lithosphere, then a
 147 more realistic lithosphere whose thickness increases as the square root of age,
 148 and finally the effect of migration of the ridge relative to the plume.

149 **2.1 Vanishingly thin lithosphere**

150 Fig. 1 shows the idealized model upon which we shall base our scaling anal-
 151 ysis. Plume material with constant viscosity η_p and density deficit $\delta\rho$ is
 152 supplied at a volumetric rate Q by a plume conduit located directly beneath
 153 a stationary ridge with half spreading rate U . The steady-state thickness of
 154 the plume material beneath the lithosphere is $h(x, y)$, and the lithosphere is
 155 assumed to have a vanishingly small thickness. The width of the plume pool
 156 is $W(x)$, and the waist width is $W_w = W(0)$.

157 Our starting point is the partial differential equation that governs the
 158 steady-state thickness $h(x, y)$ of the pool of buoyant plume material shown
 159 in Fig. 1. That equation can be derived using the theory of viscous flow in
 160 thin layers (lubrication theory), and is (Ribe et al., 1995; Ribe, 2018)

$$\frac{\partial}{\partial x}\psi(x, h) = \sigma\nabla^2(h^4) + \frac{Q}{\pi a^2} \exp\left(-\frac{r^2}{a^2}\right), \quad (2)$$

161 where

$$\psi(x, z) = \frac{2U}{\pi} z \tan^{-1}\left(\frac{x}{z}\right), \quad (3)$$

162 is the ‘corner flow’ streamfunction of the ridge-generated mantle flow (Batch-
 163 elor, 1967; Ribe, 2018),

$$\sigma = \frac{g\delta\rho}{12\eta_p} \quad (4)$$

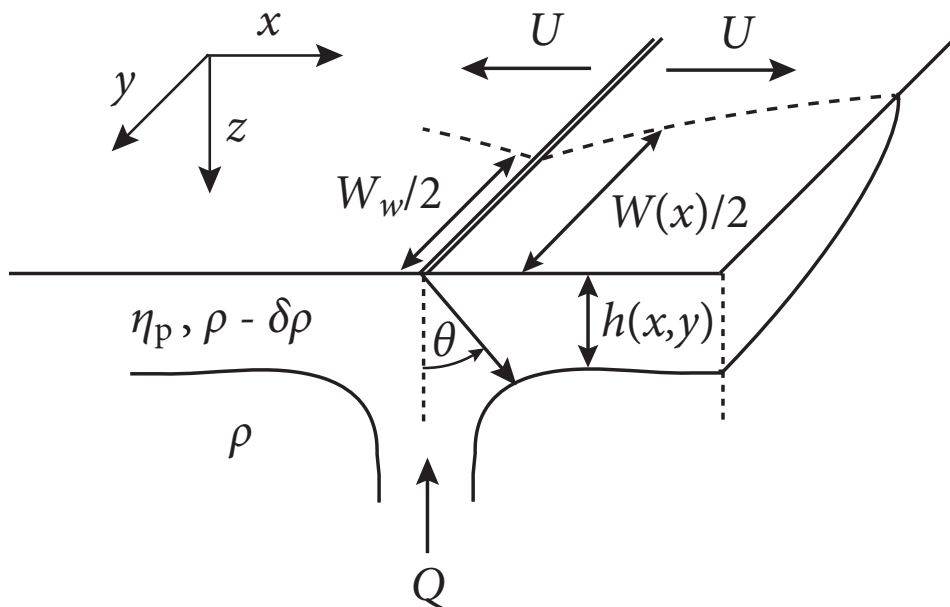


Figure 1: Lubrication model for a ridge-centered plume. Plume material with viscosity η_p and density deficit $\delta\rho$ is supplied at a volumetric rate Q beneath a stationary (non-migrating) ridge with half spreading rate U . The steady-state thickness of the pool of plume material beneath the lithosphere is $h(x, y)$, and the lithosphere is assumed to have vanishingly small thickness. The width of the pool is $W(x)$, and the waist width is $W_w = W(0)$.

164 is the ‘spreadability’ of the plume material, ∇ is the horizontal gradient
 165 operator, and r is the horizontal radial distance from the center of the plume
 166 conduit of radius a . From left to right, the terms in (2) represent advection of
 167 the plume material by the ambient flow, gravity-driven spreading of plume
 168 material, and injection of plume material into the pool, respectively. The
 169 definition (4) of σ assumes that the shear stress on the lower surface of the
 170 plume pool is zero. This boundary condition is more appropriate than a
 171 no-slip condition when the viscosity ratio (exterior/pool) is smaller than the
 172 pool’s width/thickness ratio (Ribe, 2018).

173 The characteristic scales for the thickness $h(x, y)$ and the width $W(x)$
 174 of the plume pool can now be determined by a scaling analysis of (2). To
 175 keep the notation simple, we shall denote these characteristic scales by the
 176 symbols h and W , respectively. There are two distinct pairs of length scales,
 177 depending on the angle θ shown in fig. 1.

178 The first pair of length scales applies at distances far from the ridge
 179 where $\theta \approx \pi/2$. As a preliminary, we note that the derivatives $\partial/\partial x$ and
 180 $\partial/\partial y$ both scale as W^{-1} , because the geometry of the problem does not
 181 impose different length scales in the x - and y -directions. Now far from the
 182 ridge $\tan^{-1}(x/z) \approx \pi/2$ and $(\partial/\partial x)\psi(x, h) \approx U\partial h/\partial x \sim Uh/W$, where the
 183 symbol \sim means ‘scales as’, i.e. ‘is proportional to and of the same order
 184 of magnitude as’. This term must balance the gravitational spreading term
 185 $\sigma\nabla^2(h^4)$, implying $Uh/W \sim \sigma h^4/W^2$. In addition, conservation of volume
 186 flux requires $Q \sim U_dWh$, where U_d is the downstream (x -direction) velocity
 187 of the plume pool. Because the rate of gravitational spreading in the x -
 188 direction is much smaller than the plate speed U (Ribe and Christensen,

189 1994), we can set $U_d = U$. Solving the two foregoing relations for h and W
 190 we find

$$W \sim \left(\frac{\sigma Q^3}{U^4} \right)^{1/4} \equiv W_1, \quad h \sim \left(\frac{Q}{\sigma} \right)^{1/4} \equiv h_1. \quad (5)$$

191 The scales (5) are identical to those that apply for a Hawaii-type plume rising
 192 beneath an intact (unrifted) lithosphere moving at speed U relative to the
 193 plume (Ribe and Christensen, 1994).

194 The second pair of lengthscales applies when $\theta \ll 1$ and $y > a$ (i.e., out-
 195 side the plume conduit), in which case the ridge-generated flow is primarily
 196 vertical. Then $\tan^{-1}(x/z) \approx x/z$, and (3) implies $(\partial/\partial x)\psi(x, h) \approx U$. The
 197 balance of the advection and gravitational spreading terms in (2) then gives
 198 $U \sim \sigma h^4/W^2$. As before, conservation of volume flux requires $Q \sim UWh$.
 199 Solving these two relations for h and W we obtain

$$W \sim \left(\frac{\sigma Q^4}{U^5} \right)^{1/6} \equiv W_0, \quad h \sim \left(\frac{Q^2}{\sigma U} \right)^{1/6} \equiv h_0. \quad (6)$$

200 These scales were first found by Ribe (2018). Unlike the scales (5) they
 201 are specific to plume-ridge interaction. The scale W_0 corrects the erroneous
 202 lateral length scale $(Q/U)^{1/2}$ proposed by Ribe et al. (1995) and adopted
 203 by a number of subsequent authors (Feighner and Richards, 1995; Feighner
 204 et al., 1995; Ribe, 1996; Ito et al., 1997; Ribe and Delattre, 1998).

205 **2.2 Thickening lithosphere**

206 The simple model described in the previous subsection assumes that the litho-
 207 sphere has a vanishingly small thickness. In reality, of course, the thickness
 208 of the lithosphere increases as the square root of its age. This increase has
 209 two competing dynamical effects that were not accounted for in the model

210 of § 2.1. First, the slope of the lithosphere-asthenosphere boundary (LAB)
211 provides an additional gravitational driving force for upslope flow of buoyant
212 plume material towards the ridge (Kincaid et al., 1995, 1996). This effect
213 tends to increase the waist width. Second, and less obviously, the LAB is
214 not a material surface. Instead, material continuously flows across it because
215 the lithosphere’s vector velocity, which is horizontal at the sloping LAB, has
216 a small component normal to that surface. This normal component of ve-
217 locity corresponds to a sink of plume material, which gets transformed into
218 effectively rigid lithosphere as it moves across the LAB. This effect tends to
219 decrease the waist width. (Ribe, 1996).

220 To quantify the physics just described, we use dimensional analysis to
221 determine how the waist width W_w depends on the input parameters of the
222 problem. Inspection of the lubrication equation (2) and the expression (3) for
223 the streamfunction shows that W_w depends on σ , Q and U ; the dependence
224 on a can be neglected as long as $a \ll W_w$. In addition, W_w must depend on
225 the thermal diffusivity κ , which together with U controls the thickening of
226 the lithosphere. There exists therefore a functional relationship among the
227 $M = 5$ quantities W_w , σ , Q , U and κ . Of these five quantities, $K = 2$ have
228 independent dimensions. According to Buckingham’s Π -theorem, $M - K =$
229 3 independent dimensionless groups can be formed from these quantities.
230 While the groups can be chosen in an infinite number of ways, it is generally
231 good practice to use physically meaningful definitions. The obvious choice
232 for the first group is W_w/W_0 , where W_0 is the fundamental length scale for
233 PRI found by scaling analysis in § 2.1. As the second group, we choose the

234 ‘buoyancy number’

$$\Pi_b = \frac{W_0}{h_0} = \left(\frac{\sigma Q}{U^2} \right)^{1/3}, \quad (7)$$

235 which is of the order of the aspect ratio (width/thickness) of the plume pool.

236 As the third group, we choose the ratio of the slope of the lithosphere at
237 $x = W_0$ to the slope in the x -direction of the plume pool itself. According to
238 the half-space cooling model the former slope is $\sim (\kappa/UW_0)^{1/2}$. The slope of
239 the plume pool is $\sim h_0/W_0$. Taking the ratio of these slopes and using the
240 definitions (6), we obtain the ‘slope number’

$$\Pi_s = \left(\frac{\kappa^2 \sigma}{U^3} \right)^{1/4}. \quad (8)$$

241 Putting everything together, we expect the waist width W_w of a ridge-
242 centered plume beneath a thickening lithosphere to obey a scaling law having
243 the general form

$$\frac{W_w}{W_0} = f_2(\Pi_b, \Pi_s) \quad (9)$$

244 where f_2 is an undetermined function that remains to be found.

245 **2.3 Effect of ridge migration**

246 The last physical factor influencing the waist width is the speed U_m at which
247 the ridge is migrating relative to the hotspot. Ridge migration of course im-
248 plies that the distance between the plume conduit and the ridge is constantly
249 changing; here we are interested only in the instant in time when the plume
250 conduit is directly beneath the ridge.

251 The most obvious dimensionless group to use to characterize the effect
252 of ridge migration is the ratio Π_m of the ridge migration speed to the half

253 spreading rate, which we call the ‘migration number’:

$$\Pi_m = \frac{U_m}{U}. \quad (10)$$

254 Therefore when the ridge is migrating, the waist width of a ridge-centered
255 plume must follow a scaling law having the general form

$$\frac{W_w}{W_0} = f_3(\Pi_b, \Pi_s, \Pi_m) \quad (11)$$

256 where f_3 is an unknown function. It is related to the function f_2 previously
257 introduced by $f_3(\Pi_b, \Pi_s, 0) = f_2(\Pi_b, \Pi_s)$.

258 **3 Numerical models**

259 The general scaling laws for the waist width derived in the previous section
260 involve undetermined functions f_2 and f_3 of the dimensionless groups Π_b , Π_s
261 and Π_m . The appearance of an undetermined function is typical whenever a
262 scaling law involves more than a single dimensionless group, as in our case. In
263 general, the form of the function is not simple and cannot be determined by
264 dimensional or scaling analysis; instead, laboratory experiments or explicit
265 solutions of a numerical model are necessary. Accordingly, this section is
266 devoted to characterizing the functions f_2 and f_3 numerically. For clarity, the
267 exposition is organized under the same headings as in the previous section.
268 However, a section entitled ‘Vanishly thin lithosphere’ is absent because we
269 jump directly to the more realistic case of a thickening lithosphere.

270 **3.1 Thickening lithosphere**

271 As shown in § 2.2, the scaling law for the waist width in the presence of
272 a thickening lithosphere involves the undetermined function $f_2(\Pi_b, \Pi_s)$. The

273 definition (8) of the dimensionless group Π_s contains the thermal diffusivity
 274 κ . This suggests that we should use a 3-D thermomechanical model to deter-
 275 mine the function f_2 . The model we use here builds on a series of previous
 276 3-D convection models of ridge-centered plumes and of the Iceland plume
 277 more specifically (Ribe et al., 1995; Ito et al., 1996, 1999; Albers and Chris-
 278 tensen, 2001; Ruedas et al., 2004; Marquart et al., 2007; Gallego et al., 2013).
 279 However, our approach differs from most of these earlier ones in that it uses
 280 a large suite of numerical solutions to determine a quantitative scaling law.

281 Accordingly, in this subsection we study a 3-D convection model of a
 282 ridge-centered plume implemented in the code StagYY (Tackley, 2008). The
 283 details of the numerical implementation are outlined in Appendix A. Briefly,
 284 the domain of the solution is a Cartesian box 400 km deep and of vari-
 285 able length and width. The ridge flow is generated by a spreading velocity
 286 $U_{\text{spread}}(x) = U \tanh(x/b)$ imposed on the upper surface, where $b = 25$ km is a
 287 small transition width. The ridge-centered plume is generated by a Gaussian
 288 temperature anomaly $\Delta T \exp[-(x^2 + y^2)/a^2]$ imposed on the bottom of the
 289 box. In most of the solutions presented below, the rheology is Newtonian
 290 and the viscosity obeys a standard Arrhenius law with realistic activation
 291 parameters. For each steady-state solution obtained, the spreadability σ and
 292 the volume flux Q are calculated as described in Appendix A.

293 Fig. 2 shows an example of a solution for reference values of the tempera-
 294 ture anomaly $\Delta T = 225$ K, plume radius $a = 55$ km, and half spreading rate
 295 $U = 3.1$ cm yr⁻¹. The buoyancy flux of the plume is $B = 1.17$ Mg s⁻¹. Fig.
 296 2a shows the temperature field in the symmetry plane $y = 0$, and fig. 2b
 297 shows the ‘isostatic topography’ $\zeta(x, y)$, which is proportional at each point

298 to the vertical integral over the depth of the box of the temperature anomaly
299 associated with the plume (Appendix A).

300 Note that both Π_s and Π_b depend on the spreadability σ , which in turn
301 depends on the characteristic plume viscosity η_p . We identify η_p with η_m , the
302 minimum viscosity over the whole model domain. In practice this definition
303 picks out the viscosity in the hottest central part of the plume beneath the
304 ‘hotspot’ $(x, y) = (0, 0)$.

305 To test the scaling law (9), we obtained 32 steady-state numerical solu-
306 tions with different values of ΔT , a and U . The results are collected in fig.
307 3, which shows the normalized waist width $W_w/W_0 \equiv f_2$ (colored squares)
308 as a function of Π_b and Π_s . The points (Π_b, Π_s) are irregularly distributed
309 because Π_b and Π_s are model outputs, not inputs. The colors vary smoothly
310 over the Π_b - Π_s plane, showing that all the data points collapse onto a single
311 two-dimensional surface even though three parameters (ΔT , a and U) were
312 varied to obtain them. Fig. 3 thus confirms the validity of the scaling law
313 (9).

314 Another important feature of fig. 3 is that the scaled waist width W_s/W_0
315 decreases as the slope number Π_s increases. To understand why, recall (§ 2.2)
316 that the sloping base of the lithosphere has two competing dynamical effects:
317 it enhances upslope flow of plume material, which tends to increase the waist
318 width, and it represents a sink of plume material, which tends to decrease
319 the waist width. The results of fig. 3 show that the second of these effects is
320 the dominant one.

321 For the convenience of readers who may wish to use the scaling law shown
322 in fig. 3, we provide in Appendix C the coefficients of a bicubic polynomial

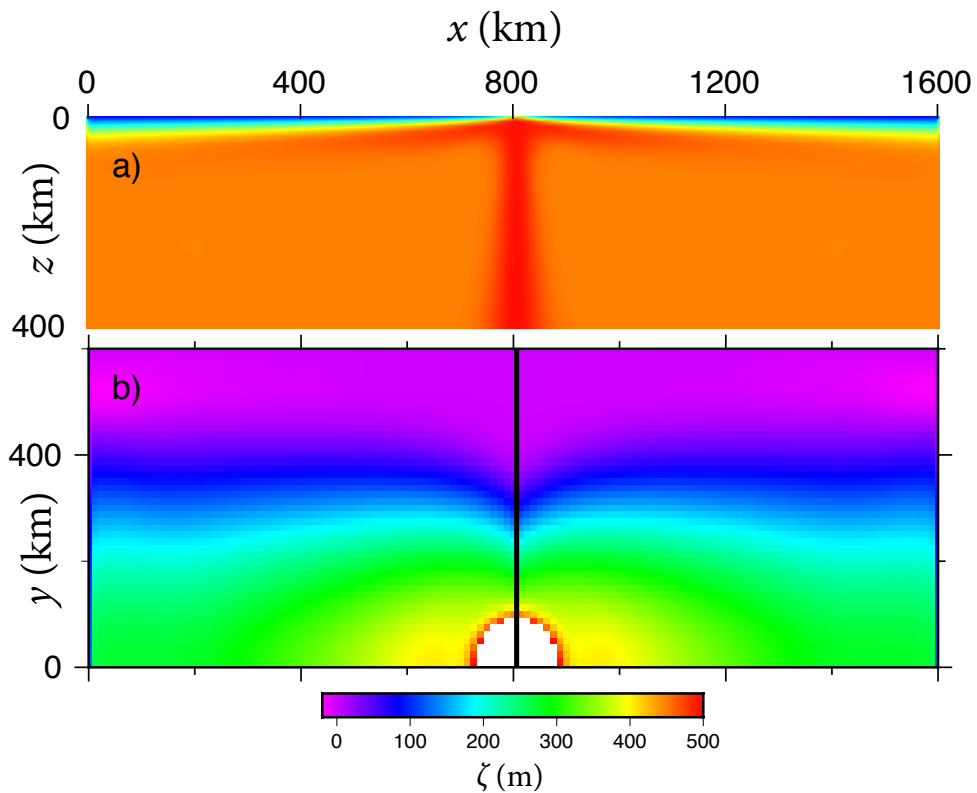


Figure 2: Three-dimensional steady-state numerical solution for a ridge-centered plume with reference values of the temperature anomaly $\Delta T = 225$ K, plume radius $a = 55$ km, and half spreading rate $U = 3.1$ cm yr $^{-1}$. The buoyancy flux of the plume is $B = 1.17$ Mg s $^{-1}$. (a) Temperature anomaly in the vertical symmetry plane $y = 0$. (b) Isostatic topography $\zeta(x, y)$ (see Appendix A for definition). The vertical black line shows the location of the ridge. White indicates a region where $\zeta > 500$ m.

323 fit of the function $f_2(\Pi_s, \Pi_b)$.

324 It is of some interest to examine how non-Newtonian (dislocation creep)
325 rheology influences the scaling law we have determined. The composite
326 Newtonian/non-Newtonian rheological law we used is specified by (20) and
327 (21). It involves a reference stress τ_0 at which the diffusion creep and disloca-
328 tion creep viscosities are the same. For the parameters of the solution shown
329 in fig. 2, we found that the addition of non-Newtonian rheology has essen-
330 tially no effect on the waist width for $\tau_0 = 10$ MPa, and increases it by only
331 7.5% for $\tau_0 = 0.3$ MPa. The smallness of this effect is somewhat surprising,
332 because the flow just beneath the ridge should be controlled primarily by the
333 non-Newtonian viscosity due to the high strain rates there. The smallness
334 of the effect is probably due to the smoothing of the velocity singularity at
335 the ridge by the finite transition width b , and by the fact that we impose
336 a minimum cutoff viscosity $\eta_{\min} = 10^{17}$ Pa s. In any case, the small effect
337 of non-Newtonian rheology justifies our decision to consider only Newtonian
338 rheology in the solutions used to construct fig. 3.

339 **3.2 Effect of ridge migration**

340 As shown in § 2.3, the scaling law for the waist width in the presence of
341 ridge migration involves an undetermined function $f_3(\Pi_b, \Pi_s, \Pi_m)$. Because
342 it is impractical to characterize completely a function of three arguments,
343 we opted for the simpler approach of estimating the magnitude of the ridge
344 migration effect for an Iceland-type plume. Our first attempt was to use
345 StagYY with $a = 68$ km, $\Delta T = 275$ K and $U = 3.96 \times 10^{-10}$ m s⁻¹. The
346 code was first run for 60 Ma with a fixed ridge to give the plume time to

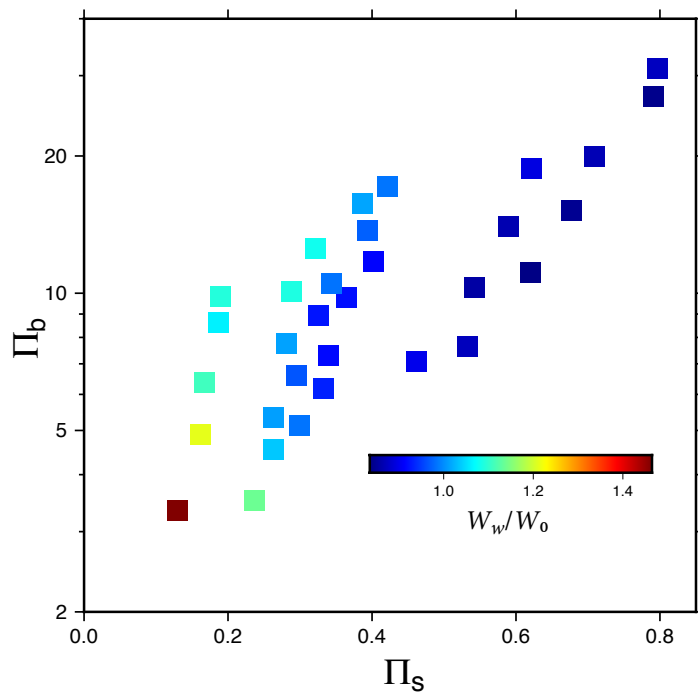


Figure 3: Normalized waist width $W_w/W_0 \equiv f_2$ as a function of the slope number Π_s and buoyancy number Π_b for 32 numerical solutions for a steady ridge-centered plume with different values of the half spreading rate U , the temperature anomaly ΔT , and the plume radius a .

347 develop 1200 km away from the ridge, after which the ridge migration speed
 348 was ramped up during 20 Ma to $U_m = 1.5U$. However, we found that the
 349 buoyancy flux of the plume diminished by more than a factor of ten before
 350 increasing again when the ridge was close to the plume, an unrealistic result
 351 that is probably an artefact of the model boundary conditions. We therefore
 352 decided to use a simpler model that contains the basic physics, namely an
 353 extended version of the lubrication model of § 2.1 with ridge migration added.
 354 The extended lubrication equation is

$$\frac{\partial h}{\partial t} + \frac{\partial}{\partial x}\psi(x, h) = \sigma\nabla^2(h^4) + \frac{Q}{\pi a^2} \exp\left(-\frac{r^2}{a^2}\right), \quad (12a)$$

355

$$r^2 = (x - x_0 + U_m t)^2 + y^2. \quad (12b)$$

356 Equation (12) differs from the simpler lubrication equation (2) by the addi-
 357 tion of an unsteady term $\partial h/\partial t$ and by a new expression for r that includes
 358 relative motion between the plume and the ridge. Equation (12) with the ex-
 359 pression (3) for $\psi(x, z)$ is valid for a lithosphere of vanishing thickness, which
 360 is of course unrealistic. However, we anticipate that this should not matter
 361 much because we are only interested here in a relative effect, i.e. the factor
 362 by which ridge migration changes the waist width. For convenience, (12) is
 363 solved in the reference frame of the ridge, in which the plume moves toward
 364 the ridge with velocity $-U_m \mathbf{e}_x$, where \mathbf{e}_x is a unit vector in the x -direction.
 365 The position of the plume at the beginning of the simulation $t = 0$ is $x = x_0$.

366 Fig. 4 shows the results for $\Pi_b = 25$, a typical buoyancy number for
 367 an Iceland-sized plume. Fig. 4a shows the steady-state thickness $h(x, y)$ of
 368 the plume pool for a non-migrating ridge directly above the plume source,
 369 while Fig. 4b shows $h(x, y, t_0)$ with a migrating ridge with $\Pi_m = 1.23$, at

370 the instant $t = t_0$ when the ridge is directly above the plume source. The
371 waist width for the latter case is smaller than that for a steady ridge-centered
372 plume by 17%. This decrease of the waist width due to ridge migration has a
373 simple physical explanation. During the time when the ridge is approaching
374 the plume, the lithosphere above the plume is moving relative to it not at
375 speed U , but rather at the enhanced speed $U + U_m$. This greater relative
376 speed corresponds to stronger advection of the plume pool, which counteracts
377 gravitational spreading more effectively and makes the pool narrower than
378 it would be in the absence of ridge migration (Ribe and Delattre, 1998).

379 4 Strength of the Iceland plume

380 We now use the ‘waist width’ method to obtain a new estimate of the
381 buoyancy flux of the Iceland plume by inverting our scaling law $W_w/W_0 =$
382 $f_2(\Pi_s, \Pi_b)$ for a ridge-centered plume. Our first task is to choose a range
383 of values of W_w appropriate for the Iceland plume. Fig. 5 shows the ax-
384 ial elevation of the Mid-Atlantic Ridge as function of distance from Iceland.
385 Based on these data we choose $W_w = 2300 \pm 300$ km as the representative
386 width of the ridge elevation anomaly associated with Iceland. The maximum
387 and minimum values in this range are shown in fig. 5 by horizontal arrows.
388 Our chosen range is consistent with the value $W_w = 2400$ km assumed by
389 Parnell-Turner et al. (2014). Moreover, the lower end of the range (= 2000
390 km) corresponds to twice the distance from Iceland of the transition between
391 smooth and rough crust along the Reykjanes ridge, which is a possible proxy
392 for the waist width (N. White, personal communication).

393 Turning next to our scaling law, we first note that the relevant portion

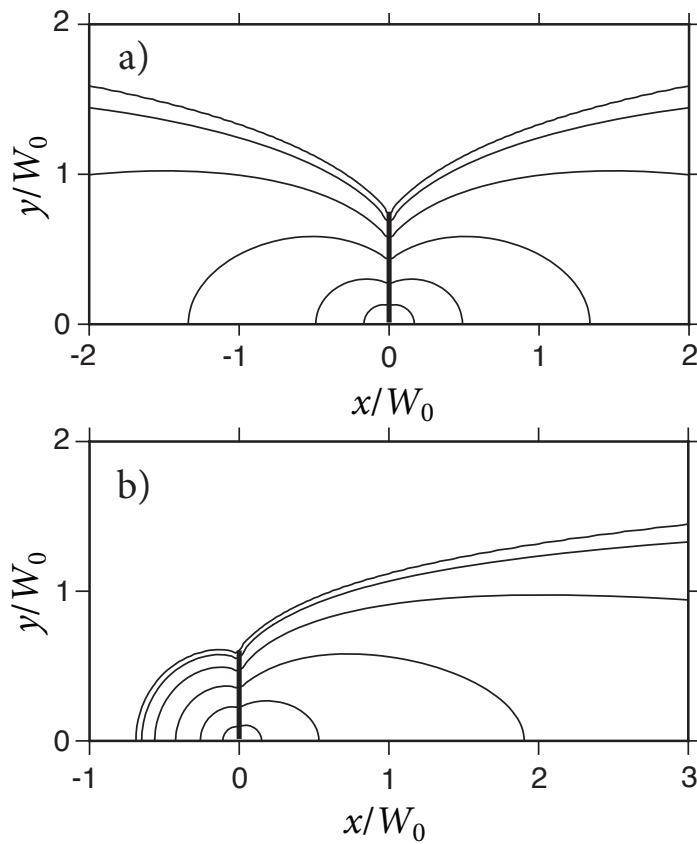


Figure 4: Effect of ridge migration on the waist width W_w for a plume directly beneath a ridge, as predicted by the lubrication model with $\Pi_b = 25$ and a lithosphere of vanishing thickness. (a) Contour plot of steady-state plume pool thickness $h(x, y)$ for a plume source directly beneath a stationary ridge. The contour interval is $0.05h_0$ and the outermost contour is $0.05h_0$. The position of the ridge is $x = 0$, and the thick vertical line has length $W_w/2$. (b) Same as (a), but with ridge migration at speed $1.23U$ to the right relative to the plume stem.

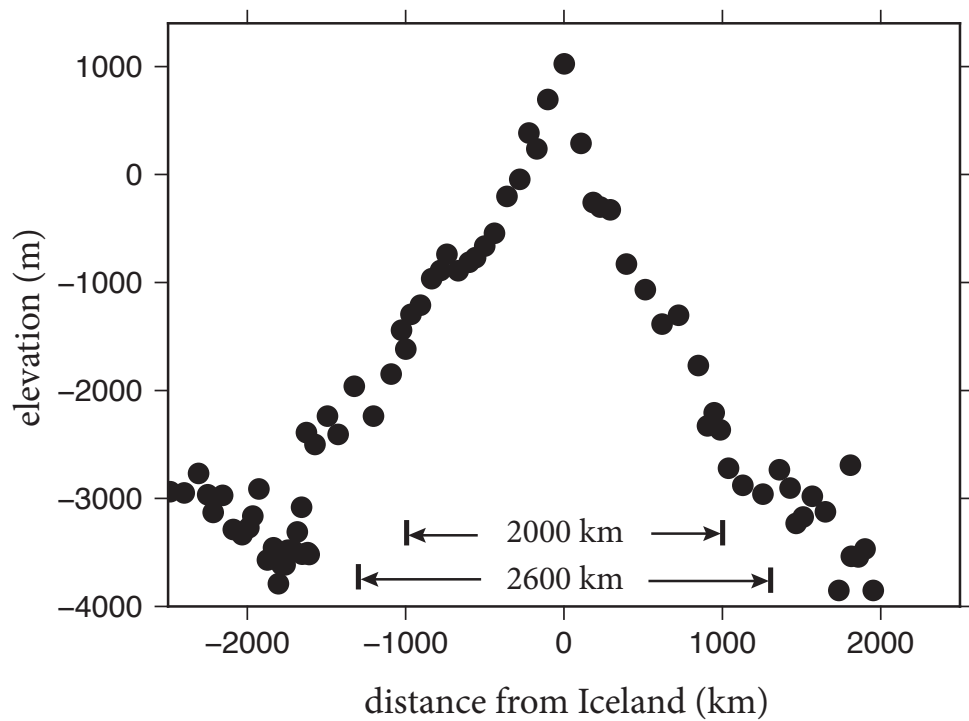


Figure 5: Axial elevation of the mid-Atlantic Ridge as a function of distance from Iceland. Negative distances are south of Iceland. Data courtesy of C. Dalton (personal communication).

394 of the Π_s - Π_b space for the Iceland plume is the upper right corner of fig. 3,
 395 as we shall verify *a posteriori*. Using the average of the values of W_w/W_0 for
 396 the two points closest to that corner, we obtain the scaling law

$$W_w = 0.86W_0. \quad (13)$$

397 Solving (13) for $B \equiv \alpha\rho_0\Delta T_m Q$, we obtain

$$B = 1.25\alpha\rho_0\Delta T_m \left(\frac{W_w^6 U^5}{\sigma} \right)^{1/4}. \quad (14)$$

398 We use a value $U = 2.85 \times 10^{-10} \text{ m s}^{-1}$ (0.90 cm yr^{-1}) calculated in Appendix
 399 B from the plate rotation vectors of model HS3-NUVEL1A of Gripp and
 400 Gordon (2002). Next, we calculate σ at the depth z_m where the viscosity
 401 within the plume is a minimum, and where the plume's excess temperature
 402 is ΔT_m . Using (21), we find

$$\sigma = \frac{g\alpha\rho_0\Delta T_m}{12\eta_m}, \quad \eta_m = \eta_0 \exp \left[\frac{E_1 + g\rho_0 z_m V_1}{R(T_0 + \Delta T_m)} - \frac{E_1}{RT_0} \right] \quad (15)$$

403 In (15), the quantities g , ρ_0 , α , η_0 , E_1 , V_1 , R and T_0 are all known constants.
 404 From our numerical solutions, we find $z_m = 25 \text{ km}$.

405 Fig. 6 shows the buoyancy flux predicted by (14) as a function of the
 406 (unknown) excess plume temperature ΔT_m , for three values of W_w in the
 407 range 2000-2600 km. The buoyancy flux ranges from $1.54 \pm 0.30 \text{ Mg s}^{-1}$ for
 408 $\Delta T_m = 120 \text{ K}$ to $1.84 \pm 0.36 \text{ Mg s}^{-1}$ for $\Delta T_m = 260 \text{ K}$.

409 The foregoing estimates of B were predicted by a scaling law for a steady
 410 ridge-centered plume, and do not take into account the progressive migration
 411 of the mid-Atlantic ridge relative to the Iceland plume during the past several
 412 tens of Ma. As we noted earlier, the importance of ridge migration can be
 413 characterized by a dimensionless 'migration number' $\Pi_m = U_m/U$, where U_m

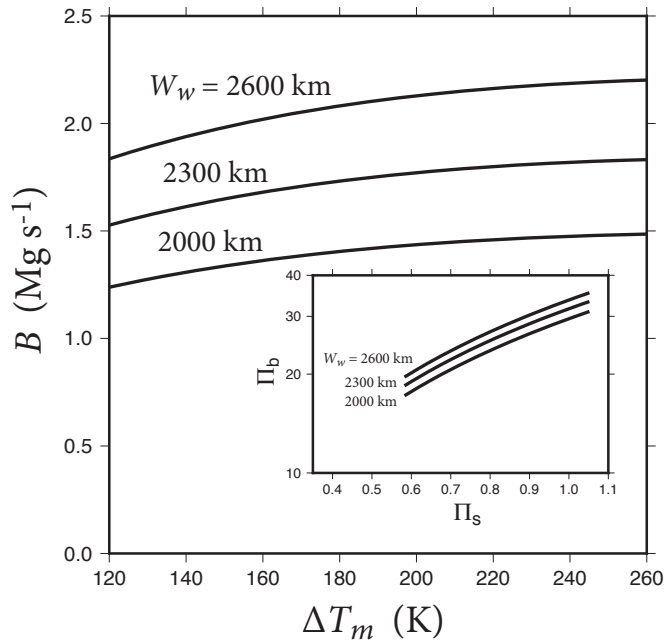


Figure 6: Predicted buoyancy flux B of the Iceland plume as a function of the plume's excess temperature ΔT_m , for three values of the waist width W_w . B is calculated from (14) as described in the text. Inset: values of Π_s and Π_b corresponding to the curves in the main figure. ΔT_m increases from 120 K to 260 K from lower left to upper right along each curve.

414 is the ridge migration speed. In Appendix B we use the HS3-NUVEL1A
 415 plate rotation vectors to estimate $\Pi_m = 1.23$ for the Iceland plume/ridge
 416 system. As we explained at the end of § 3.2, the effect of ridge migration is
 417 to reduce the waist width (by 17% for the case considered in § 3.2) relative
 418 to the case without ridge migration. This means that in the presence of ridge
 419 migration a greater buoyancy flux is required to explain a given value of the
 420 waist width.

421 The results of fig. 4 allow us to determine the factor by which the buoy-
 422 ancy flux B must be increased to compensate for the effect of ridge migration.
 423 Let W_w^{est} be some estimated value of the waist width. Then in view of the
 424 definition (6) of the length scale W_0 , the scaling law $W \propto W_0$ can be written
 425 in two ways as

$$W_w^{\text{est}} = 1.464\beta Q_{\text{no migr}}^{2/3} = 1.209\beta Q_{\text{migr}}^{2/3} \quad (16)$$

426 where $\beta = (\sigma/U^5)^{1/6}$ and the subscripts ‘migr’ and ‘no migr’ indicate values
 427 with and without ridge migration, respectively. The constants 1.464 and
 428 1.209 that appear in (16) are the values of the waist width in units of W_0
 429 from Figs. 4a and 4b, respectively. Now $B \propto Q$, whence (16) implies

$$\frac{B^{\text{migr}}}{B^{\text{no migr}}} = \left(\frac{1.464}{1.209}\right)^{3/2} = 1.33. \quad (17)$$

430 This result means that the estimated values of B in Fig. 6 must be increased
 431 by 33% to account for the effects of ridge migration that were neglected in
 432 constructing that figure. Taking into account this enhancement factor and
 433 the uncertainty of the temperature excess ΔT_m , we obtain our final estimate
 434 $B = 2.3 \pm 0.6 \text{ Mg s}^{-1}$ for the buoyancy flux of the Iceland plume.

435 5 Discussion

436 In § 2.1 we noted that our plume-ridge interaction lengthscale $(\sigma Q^4/U^5)^{1/6} \equiv$
437 W_0 corrects an erroneous length scale $(Q/U)^{1/2} \equiv S_0$ proposed by Ribe et al.
438 (1995). However, that does not mean that previous studies using the length
439 scale S_0 (Ribe et al., 1995; Ribe, 1996; Ribe and Delattre, 1998) are incorrect.
440 The reason is that S_0 and W_0 are simply related by $S_0 = W_0 \Pi_b^{-1/2}$, where Π_b
441 is the buoyancy number (7). Thus one can use S_0 as the basic length scale
442 as long as one takes the buoyancy number systematically into account. The
443 difference is essentially one of economy: scaling laws expressed in terms of
444 the correct lengthscale W_0 will be simpler and cleaner than laws expressed
445 in terms of S_0 , which will involve an extra dependence on Π_b .

446 Our estimate $B = 2.3 \pm 0.6 \text{ Mg s}^{-1}$ for the buoyancy flux of the Iceland
447 plume differs from most previous estimates of this quantity, being interme-
448 diate between lower values (Sleep, 1990; Schilling, 1991; Ribe and Delattre,
449 1998; King and Adam, 2014) and much higher ones (Parnell-Turner et al.,
450 2014). Most of the lower values are within the narrow range 1.4-1.6 Mg s^{-1} .
451 These estimates are almost certainly too low because the underlying models
452 assume $W_w = 800\text{-}920 \text{ km}$. Such widths are appropriate for the geochemical
453 anomalies along the mid-Atlantic Ridge around Iceland, but do not reflect
454 the much larger width of the elevation anomaly evident in fig. 5.

455 We now examine the much larger estimates of B obtained by Parnell-
456 Turner et al. (2014). We focus on the second and third of their three estimates
457 because equations for these cases are either given by the authors or can easily

458 be inferred. Their second estimate is

$$B_2 = \pi d^2 U \rho_0 \alpha \Delta T_m \quad (18)$$

459 where $d = W_w/2$ is the radius of a plume pool assumed to be circular. Using
460 $U = 1.25 \text{ cm yr}^{-1}$, $d = 1200 \text{ km}$, $\rho_0 = 3200 \text{ kg m}^{-3}$, $\alpha = 3 \times 10^{-5} \text{ K}^{-1}$ and
461 $\Delta T_m = 150 \text{ K}$, we obtain $B_2 = 26 \text{ Mg s}^{-1}$, the value cited by Parnell-Turner
462 et al. (2014). However, we note the appearance in (18) of the product of
463 the half-spreading rate U and the planform area πd^2 of the plume pool. It
464 seems to us more realistic to replace the quantity $\pi d^2 U$ by $2dh(2U)$, the
465 product of the cross-sectional area of a plume pool with thickness h and the
466 full spreading rate $2U$ (to account for flux of material away from the ridge
467 in both directions). Our estimate of the buoyancy flux is therefore obtained
468 from that of Parnell-Turner et al. (2014) by multiplying it by $4h/\pi d \equiv \chi_2$.
469 To estimate h , we use the scale h_0 defined by (6) with $Q = 110 \text{ m}^3 \text{ s}^{-1}$
470 and $\sigma = 4.5 \times 10^{-17} \text{ m}^{-1} \text{ s}^{-1}$, values obtained directly from one of our 3-D
471 numerical simulations of an Iceland-sized plume. We thereby find $h = 94$
472 km. This yields $\chi_2 = 0.10$, which implies a reduced estimate $B_2 = 2.6 \text{ Mg}$
473 s^{-1} of the buoyancy flux. This new value is within the range $2.3 \pm 0.6 \text{ Mg}$
474 s^{-1} that we estimated based on our scaling law.

475 The third estimate of Parnell-Turner et al. (2014) is

$$B_3 = \pi d^2 h \rho_0 \alpha \Delta T_m / \tau \quad (19)$$

476 where $\tau = 30 \text{ Ma}$ is the time required to fill a plume pool of volume $\pi d^2 h$.
477 With the values of d , h , ρ_0 , α and ΔT_m given above, (19) gives the estimate
478 $B_3 = 17 \text{ Mg s}^{-1}$ of Parnell-Turner et al. (2014) if one assumes $h = 250 \text{ km}$.
479 However, using our more realistic value $h = 94 \text{ km}$ we find $B_3 = 6.5 \text{ Mg s}^{-1}$.

480 This estimate is 60% lower than that of Parnell-Turner et al. (2014), but still
481 more than twice the value $B = 2.9 \text{ Mg s}^{-1}$ at the upper end of our range.

482 The most recent estimate $B = 4.0 \pm 1.0 \text{ Mg s}^{-1}$ of the buoyancy flux of the
483 Iceland plume is that of Hoggard et al. (2020), based on a model in which the
484 plume pool comprises a small number of discrete fingers of thickness z_a that
485 spread radially to a maximum distance R_{max} . Upon neglecting two smaller
486 terms in eqn. (3) of Hoggard et al. (2020), that expression for the volume flux
487 becomes $Q = 3\pi z_a R_{max}^2 / 8\Delta t$, where $\Delta t = 15 \text{ Ma}$ is the time (estimated from
488 the geometry of the V-shaped ridges south of Iceland) required for pulses of
489 hot material to spread down the length of a finger. The most striking aspect
490 of the foregoing formula is that it is independent of the (half-) spreading rate
491 U . This makes the model of Hoggard et al. (2020) fundamentally different
492 than the fluid-mechanical models investigated in the present study, for which
493 $Q \propto U^{5/4}$ (eqn. (14)).

494 Finally, it is of interest to compare our estimated buoyancy flux for the
495 Iceland plume with corresponding estimates for the Hawaiian plume. The
496 classic estimates $B_{\text{Hawaii}} = 6.3 \text{ Mg s}^{-1}$ (Davies, 1988) and 8.7 Mg s^{-1} (Sleep,
497 1990) are based on equating the vertical flux of buoyancy in the plume stem
498 to the horizontal flux of buoyancy associated with the topography anomaly
499 of the Hawaiian swell moving ‘downstream’ to the NW with a speed equal
500 to the plate speed. Vidal and Bonneville (2004) used a similar flux balance
501 method to estimate B_{Hawaii} as a function of time; for the past 15 Ma, they
502 found values in the ranges $2.5\text{-}4.4 \text{ Mg s}^{-1}$ and $3.9\text{-}6.4 \text{ Mg s}^{-1}$ for two differ-
503 ent lithospheric subsidence models. As mentioned earlier, King and Adam
504 (2014) used three versions of the flux balance method to estimate the buoy-

505 ancy fluxes of 54 terrestrial hotspots, and found values $B_{\text{Hawaii}} = 4.66 \text{ Mg}$
506 s^{-1} , 7.1 Mg s^{-1} , and 4.9 Mg s^{-1} . However, the classical flux balance approach
507 does not take into account the fact that plume material moves downstream
508 at an average speed that is lower than the plate speed, due to the presence
509 of shear in the asthenosphere (Ribe and Christensen, 1994). Nor does it
510 allow for the additional buoyancy due to the depleted residuum of melting,
511 which compensates a significant portion of the swell topography (Ribe and
512 Christensen, 1999). Ribe and Christensen (1999) presented a 3-D thermome-
513 chanical model that includes both these effects, and estimated the buoyancy
514 flux of the Hawaiian plume by fitting the present-day melt production rate
515 and the width and amplitude of the Hawaiian swell. They thereby found
516 $B_{\text{Hawaii}} = 3.0 \pm 0.8 \text{ Mg s}^{-1}$. We prefer this estimate to the others cited
517 above because the underlying dynamical model is more physically realistic.
518 Provisionally accepting this estimate, we conclude that the buoyancy fluxes
519 of the Iceland and Hawaiian plumes are comparable given the uncertainties
520 involved.

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527 **A Numerical implementation**

528 The code StagYY (Tackley, 2008) employs a finite-volume discretization and
 529 a multigrid solver to determine inertia-free flow in a fluid with variable vis-
 530 cosity. It can handle both Newtonian (diffusion creep) and non-Newtonian
 531 (dislocation creep) rheologies. While most of our numerical solutions used
 532 Newtonian rheology, a few used a composite diffusion creep/dislocation creep
 533 rheology. In this more general case, the viscosity is

$$\eta = \left(\frac{1}{\eta_1} + \frac{1}{\eta_{3.5}} \right)^{-1}, \quad (20)$$

534 where η_1 is the Newtonian viscosity (rheological power-law index $n = 1$) and
 535 $\eta_{3.5}$ is the non-Newtonian viscosity ($n = 3.5$; Bai et al., 1991). The general
 536 expression for both viscosities is

$$\eta_n = \eta_0 \left(\frac{\tau}{\tau_0} \right)^{1-n} \exp \left(\frac{E_n + pV_n}{RT} - \frac{E_n}{RT_0} \right) \quad (21)$$

537 where $\eta_0 = 10^{19}$ Pa s is a reference viscosity, τ is the second invariant of
 538 the deviatoric stress, τ_0 is a reference deviatoric stress, E_n is the activation
 539 energy, p is the pressure, V_n is the activation volume, R is the universal gas
 540 constant, and $T_0 = 1600$ K is the temperature of the mantle well below the
 541 cooling lithosphere. We used the values of E_n and V_n for dry olivine given
 542 in Table 1 of Karato and Wu (1993).

543 The input parameters that specify a given numerical solution are the
 544 plume temperature anomaly ΔT , the plume radius a and the half-spreading
 545 rate U . The ranges used were $180 \text{ K} \leq \Delta T \leq 275 \text{ K}$, $35 \text{ km} \leq a \leq 66 \text{ km}$,
 546 and $4 \times 10^{-10} \text{ m s}^{-1} \leq U \leq 3 \times 10^{-9} \text{ m s}^{-1}$. The horizontal dimension of each
 547 finite volume was 12.5 km for all runs. The vertical grid was refined near the

548 top and bottom of the model domain, the average finite volume height being
 549 12.5 km.

550 Turning to the boundary conditions, we recall that the velocity boundary
 551 condition on the upper surface that drives the ridge flow and the thermal
 552 boundary condition on the bottom that drives the plume were given in § 3.1.
 553 The two vertical boundaries normal to \mathbf{y} are planes of mirror symmetry. The
 554 bottom boundary and the two vertical boundaries normal to \mathbf{x} are permeable
 555 with zero shear stress but some resistance to normal flow. The normal stress
 556 σ_{nn} on these boundaries is related to the normal velocity u_n by $\sigma_{nn} = \eta u_n / D$,
 557 where η is the viscosity and D is a virtual boundary distance (Ribe and
 558 Christensen, 1994). We used $D = 500$ km for the bottom boundary and
 559 $D = 2600$ km for the side boundaries.

560 For each numerical solution corresponding to a given set of values of ΔT ,
 561 a and U , we calculated several output parameters as follows.

562 The first output parameter is the isostatic topography, defined as

$$\zeta(x, y) = \frac{\rho_0 \alpha}{\rho_0 - \rho_w} \int_0^h \delta T(x, y, z) dz, \quad (22)$$

563 where x is the horizontal coordinate perpendicular to the ridge, y is the
 564 coordinate parallel to the ridge, and z is the depth. In (22), $\rho_0 = 3300$ kg
 565 m^{-3} is the mantle density, $\rho_w = 1000$ kg m^{-3} is the density of seawater,
 566 $\alpha = 3.5 \times 10^{-5}$ K^{-1} is the thermal expansivity, $h = 400$ km is the depth of
 567 the model box, and δT is the local temperature anomaly due to the presence
 568 of the plume. It is defined as $\delta T = T(x, y, z) - T(x, y_{\max}, z)$ where y_{\max} is
 569 the width of the model box, chosen to be sufficiently large that the plume
 570 does not influence the temperature on the vertical plane $y = y_{\max}$.

571 The second output parameter is the waist width W_w , which is defined as
 572 (twice) the value of y at which the isostatic topography along the ridge falls
 573 below 30 m.

574 A third output parameter is the buoyancy flux B of the plume defined as

$$B = -\rho_0\alpha \int \mathbf{u} \cdot \mathbf{e}_z (T - T_0) dx dy. \quad (23)$$

575 where \mathbf{e}_z is a downward-pointing vertical unit vector and the integral is over
 576 the bottom of the model box.

577 A fourth output parameter is the minimum viscosity η_m in the plume
 578 directly beneath the hotspot $(x, y) = (0, 0)$. Let T_m be the temperature at
 579 the point where the viscosity is η_m .

580 The aforementioned output parameters are sufficient for calculating the
 581 volume flux Q and the spreadability σ , which are in turn needed to calculate
 582 the plume width scale W_0 and the dimensionless parameters Π_b and Π_s . We
 583 calculated the volume flux as

$$Q = \frac{B}{\rho_0\alpha\Delta T_m}, \quad (24)$$

584 where $\Delta T_m = T_m - T_0$. The spreadability was calculated as

$$\sigma = \frac{g\rho_0\alpha\Delta T_m}{12\eta_m}. \quad (25)$$

585 Note that Q and σ are calculated using the temperature T_m and the viscosity
 586 η_m at the point where the viscosity is minimum, because that point corre-
 587 sponds best to the point at which the plume fluid is injected into the pool.
 588 Once Q and σ are known, the length scale W_0 and the buoyancy number Π_b
 589 were calculated using (24), (25) and the input value of U . Finally, the slope
 590 number Π_s was calculated assuming $\kappa = 7.6 \times 10^{-7} \text{ m}^2 \text{ s}^{-1}$.

591 **B Rates of spreading and ridge migration at**
 592 **Iceland**

593 In this appendix we estimate the half-spreading rate U and the ridge mi-
 594 gration speed U_m for the mid-Atlantic ridge at Iceland, where the North
 595 American plate (plate 1) and the Eurasian plate (plate 2) meet. Let \mathbf{n} be
 596 the unit normal vector to the ridge pointing from plate 1 to plate 2, and let
 597 \mathbf{U}_1 and \mathbf{U}_2 be the velocities of plates 1 and 2 in the hotspot reference frame.
 598 Then

$$U = \frac{1}{2} |(\mathbf{U}_2 - \mathbf{U}_1) \cdot \mathbf{n}|, \quad U_m = \frac{1}{2} |(\mathbf{U}_2 + \mathbf{U}_1) \cdot \mathbf{n}|. \quad (26)$$

599 To estimate \mathbf{U}_1 and \mathbf{U}_2 , we use the angular velocities of model HS3-
 600 NUVEL1A, found in Table 12 of Gripp and Gordon (2002). It is most con-
 601 venient to use a mixture of spherical and Cartesian coordinates. The radial
 602 unit vector is

$$\mathbf{r} = \cos \theta \mathbf{z} + \sin \theta (\cos \phi \mathbf{x} + \sin \phi \mathbf{y}), \quad (27)$$

603 where θ is the colatitude, ϕ is the longitude, and \mathbf{x} , \mathbf{y} and \mathbf{z} are Cartesian
 604 unit vectors in the directions indicated. Transforming the angular velocities
 605 from HS3-NUVEL1A into Cartesian coordinates, we have

$$\boldsymbol{\omega}_1 = 5.443 \cdot 10^{-17} \mathbf{x} + 1.297 \cdot 10^{-17} \mathbf{y} - 2.046 \cdot 10^{-16} \mathbf{z}, \quad (28a)$$

$$\boldsymbol{\omega}_2 = 1.517 \cdot 10^{-17} \mathbf{x} + 5.112 \cdot 10^{-17} \mathbf{y} - 9.987 \cdot 10^{-17} \mathbf{z}, \quad (28b)$$

607 in units of radians s^{-1} . Next, we note that the center of Iceland is located
 608 approximately at 65° N latitude and 341.5° E longitude. The radial vector
 609 from the center of the Earth to this point is

$$\mathbf{R}_0 = 2.553 \cdot 10^6 \mathbf{x} - 8.543 \cdot 10^5 \mathbf{y} + 5.774 \cdot 10^6 \mathbf{z} \quad (29)$$

610 in units of m. Noting now that $\mathbf{U}_j = \boldsymbol{\omega}_j \times \mathbf{R}_0$, we find

$$\mathbf{U}_1 = -9.992 \cdot 10^{-11} \mathbf{x} - 8.366 \cdot 10^{-10} \mathbf{y} - 7.961 \cdot 10^{-11} \mathbf{z} \quad (30a)$$

611

$$\mathbf{U}_2 = 2.098 \cdot 10^{-10} \mathbf{x} - 3.426 \cdot 10^{-10} \mathbf{y} - 1.435 \cdot 10^{-10} \mathbf{z} \quad (30b)$$

612 in units of m s^{-1} .

613 The final step is to determine the normal vector \mathbf{n} . To estimate the
 614 strike of the ridge, we use the line connecting the southernmost point of the
 615 Kolbeinsey Ridge at 66.65° N latitude and 340.5° E longitude (point \mathbf{R}_+)
 616 and the northernmost point of the Reykjanes Ridge at 64.1° N latitude and
 617 337.5° E longitude (point \mathbf{R}_-). In Cartesian coordinates,

$$\mathbf{R}_+ = 2.380 \cdot 10^6 \mathbf{x} - 8.429 \cdot 10^5 \mathbf{y} + 5.849 \cdot 10^6 \mathbf{z}, \quad (31a)$$

618

$$\mathbf{R}_- = 2.571 \cdot 10^6 \mathbf{x} - 1.065 \cdot 10^6 \mathbf{y} + 5.731 \cdot 10^6 \mathbf{z}. \quad (31b)$$

619 Now the three components of \mathbf{n} satisfy the three simultaneous equations
 620 $(\mathbf{R}_+ - \mathbf{R}_-) \cdot \mathbf{n} = (\mathbf{R}_+ + \mathbf{R}_-) \cdot \mathbf{n} = \mathbf{n} \cdot \mathbf{n} - 1 = 0$. The solution is

$$\mathbf{n} = 0.6956 \mathbf{x} + 0.6948 \mathbf{y} - 0.1829 \mathbf{z}. \quad (32)$$

621 Substituting (30) and (32) into (26), we obtain

$$U = 2.85 \cdot 10^{-10} \text{ m s}^{-1} \quad (0.90 \text{ cm yr}^{-1}) \quad (33a)$$

622

$$U_m = 3.51 \cdot 10^{-10} \text{ m s}^{-1} \quad (1.11 \text{ cm yr}^{-1}). \quad (33b)$$

623 **C Regression for the waist width of a ridge-** 624 **centered plume**

625 We represent the function $f_2(\Pi_s, \Pi_b)$ shown in fig. 3 by a bicubic polynomial
626 of the form

$$f_2 = \sum_{i=0}^3 \sum_{j=0}^{3-i} c_{ij} (\Pi_s)^i (\log_{10} \Pi_b)^j. \quad (34)$$

627 Using a standard least-squares procedure, we find that the coefficients that
628 provide the best fit to the 32 points in fig. 3 are

$$629 \quad c_{00} = 3.306, \quad c_{01} = -4.852, \quad c_{02} = 4.733, \quad c_{03} = -2.223, \quad (35a)$$

$$630 \quad c_{10} = -3.916, \quad c_{11} = -1.096, \quad c_{12} = 7.000, \quad c_{20} = 7.922, \quad (35b)$$

$$c_{21} = -16.72, \quad c_{30} = 8.685 \quad (35c)$$

631 The RMS error of the fit is 0.028. Note that (34)-(35) is only reliable for
632 points (Π_s, Π_b) that lie within the ‘cloud’ of points in fig. 3, and should not
633 be used to extrapolate to points lying outside the cloud.

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