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# Towards a Generic Logical Encoding for Argumentation

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## Abstract

Several enrichments have been proposed for Dung abstract argumentation framework. These enrichments modify the standard setting by adding extra properties, thus changing the way arguments (and possibly other elements) are selected. The use of logical tools to encode and capture the original framework and some of its enrichments, have already been proposed. We extend these works by proposing a generic logical encoding which aims at being as modular as possible and which enables, by its genericity, not only to encompass the existing encodings, but also to define encodings for enriched frameworks that have not yet been studied through this scope.

## 1 Introduction

The formalism of abstract argumentation was introduced by [Dung, 1995]. This formalism models abstract entities which represent *arguments*, as well as an *attack* relation between them. The goal is then to select acceptable arguments in what are called *extensions*. Dung proposed several ways to select arguments such that a given extension satisfies some desirable properties; this is the notion of *semantics*.

Following Dung's paper, several works proposed to extend the initial framework. A first enrichment, developed in [Nielsen and Parsons, 2006] and pursued in [Flouris and Bikakis, 2019], is to consider coalitions of arguments: more than one argument attack another one together. [Boella et al., 2010, Nouioua and Risch, 2011, Oren and Norman, 2008] considered a second, positive, relation between the

arguments, the support relation. Relations which can target other relations directly are yet another enrichment: they are called higher-order relations [Baroni et al., 2011, Cayrol et al., 2017]. Other works have studied abstract argumentation using formal logic. Some of these works aim at giving a logical encoding of a framework and of its extensions under a given semantics (see [Besnard and Doutre, 2004, Walicki and Dyrkolbotn, 2012]). However, research in this area has mainly focused on specific frameworks, enriched or not. Other works, like Abstract Dialectical Frameworks, use logic to generalise argumentation frameworks (see [Brewka et al., 2018]).

The purpose of this paper is to propose a generic method of logical encoding for argumentation frameworks. This logical encoding should allow to retrieve the encodings which have already been defined previously. The aim is also to cover cases for which no logical encoding exists yet. By doing so, we wish to characterise in logical terms the core mechanisms common to all frameworks, no matter the number or nature of enrichments involved. In addition, such a generic encoding would provide solid foundations for the study of abstract argumentation at a general level using logical tools. The main principles of this approach are presented here by considering two enrichments: coalitions and higher-order relations. The encoding method is illustrated on frameworks which embed two, one, or none of these enrichments, using the *conflict-free* semantics. A preliminary full approach can be found in [Duchatelle, 2020]. This work is part of my PhD, whose overall goal is the study of AI systems able to provide explanations, using logic and argumentation.

## 2 Method

The definition of the generic logical encoding follows the *model checking* method of [Cayrol and Lagasquie-Schiex, 2020]. It aims at verifying that a logical model satisfies some properties. The idea here is to build a logical base, that is, a set of logical formulas, which characterises the extensions of the framework under a given semantics. Each model of the logical base encodes an extension of the framework. The point is then to define the logical base so that (1) the extensions encoded by its models are indeed extensions of the framework under the given semantics and (2) for each extension of the framework under the semantics, there exists at least one model of the base that encodes it.

To encapsulate all the frameworks studied here, we use one that possesses both enrichments considered. It is defined as such:  $\mathcal{A} = (\mathcal{A}, \mathcal{R}, s, t)$  where  $\mathcal{A} \cap \mathcal{R} = \emptyset$ ,  $s : \mathcal{R} \mapsto 2^{\mathcal{A}} \setminus \{\emptyset\}$  and  $t : \mathcal{R} \mapsto \mathcal{A} \cup \mathcal{R}$ .  $\mathcal{A}$  is the set of arguments,  $\mathcal{R}$  is the set of attacks<sup>1</sup>,  $s$  and  $t$  map each attack to its source and target respectively. The source of an attack is understood as the argument(s) from which the attack originates, and the target as the argument being attacked. The case where all sources are singletons disables the coalitions, while the case where all targets are elements of  $\mathcal{A}$  disables the higher-order relations.

The logical language is first order logic with equality. To improve readability, bounded quantifiers will be used as such:  $\forall x \in E \psi$  means  $\forall x(E(x) \rightarrow \psi)$  and  $\exists x \in E \psi$  means  $\exists x(E(x) \wedge \psi)$ . All formulas are relative to a given argumentation framework  $\mathcal{A}$ . Accordingly, we write  $\mathfrak{L}_{\mathcal{A}}$  to denote the language. It does not use function symbols. The following predicates are used:

- $Arg(x)$  ( $x$  is an argument)
- $Att(x)$  ( $x$  is an attack)
- $Acc(x)$  ( $x$  is selected in the extension)
- $NAcc(x)$  ( $x$  cannot be selected)
- $S(\alpha, x)$  ( $x$  is in the source of  $\alpha$ )

<sup>1</sup>As they are named, attacks are separate abstract entities.

- $T(\alpha, x)$  ( $x$  is in the target of  $\alpha$ )

Now, before we explain how a logical base is built, we make explicit how we retrieve the extensions of  $\mathcal{A}$  from a model. It is done using the predicate  $Acc$ . Let  $I$  be an interpretation over  $\mathfrak{L}_{\mathcal{A}}$ ,  $S_I$  denotes the set of accepted arguments based on  $I$ . Suppose we have a logical base  $\Sigma_{cf}(\mathcal{A})$  that is meant to express the *conflict-freeness* semantics. Then, given a set  $S \subseteq \mathcal{A}$ , we want the following property to hold:

**Property 1** (Logical conflict-freeness).  $S$  is conflict-free iff there exists a model  $I$  of  $\Sigma_{cf}(\mathcal{A})$  such that  $S = S_I$ .

Semantics in argumentation frameworks with higher-order relations are defined according to a set  $S \subseteq \mathcal{A}$  and a set  $\Gamma \subseteq \mathcal{R}^2$ . Accordingly, for an interpretation  $I$  over  $\mathfrak{L}_{\mathcal{A}}$ ,  $\Gamma_I$  denotes the set of accepted attacks based on  $I$ . We thus obtain the following property:

**Property 2** (Logical higher-order conflict-freeness).  $(S, \Gamma)$  is conflict-free iff there exists a model  $I$  of  $\Sigma_{cf}(\mathcal{A})$  such that  $S = S_I$  and  $\Gamma = \Gamma_I$ .

Properties 1 and 2 represent the results that we want to obtain. They are used to prove that the formulas used to build a logical base are correct, i.e. indeed encode the desired semantics (*conflict-freeness* in this case).

We now turn to how a logical base is built so that such properties hold. Suppose that  $\mathcal{A} = \{e_1, \dots, e_k\}$ ,  $\mathcal{R} = \{e_{k+1}, \dots, e_l\}$  where  $1 \leq k \leq l$ . The following formulas encode the framework:

$$\text{for all } a \in \mathcal{A}, \quad Arg(a) \wedge \neg Att(a) \quad (1a)$$

$$\text{for all } \alpha \in \mathcal{R}, \quad \neg Arg(\alpha) \wedge Att(\alpha) \quad (1b)$$

$$\text{for all } e_i, e_j \in \mathcal{A} \cup \mathcal{R} \text{ s.t. } i \neq j, \quad \neg(e_i = e_j) \quad (1c)$$

$$\forall z \quad (z = e_1 \vee \dots \vee z = e_l) \quad (1d)$$

$$\text{for all } \alpha \in \mathcal{R}, \quad \left( \bigwedge_{y \in s(\alpha)} S(\alpha, y) \right) \wedge \left( \bigwedge_{z \in t(\alpha)} T(\alpha, z) \right) \quad (1e)$$

(1a) and (1b) encode respectively the arguments and the attacks of the framework (what is an argument is not an attack and vice-versa). (1c) and (1d)

<sup>2</sup>A couple  $(S, \Gamma)$  is called a *structure*.

say that each element of the framework is distinct and that all elements of the interpretation's domain refer to an element of the framework. (1e) encodes the sources and targets of the attacks.

Let us now define formulas that capture the semantics. As mentioned previously, we will evoke the case of *conflict-freeness*. The basic idea of *conflict-freeness* is that, for each attack, if its source is selected in the extension, then its target cannot be selected (otherwise a conflict would arise). However, its expression may vary from one argumentation framework to another depending on its potential enrichments. For example, let us consider an argumentation framework without coalitions. In this case, before defining the semantics for this framework, its encoding must be completed with (2). This axiom expresses that the source of each attack contains a single argument.

$$\begin{aligned} \forall \alpha \in \text{Att}(\forall a \in \text{Arg}(\forall b \in \text{Arg}[ \\ (S(\alpha, a) \wedge S(\alpha, b)) \rightarrow a = b])) \end{aligned} \quad (2)$$

*Conflict-freeness* is described by (3) when the framework contains no higher-order relations. The meaning of (3) is “if the source of an attack is selected to be in an extension, then its argument target is not”. Similarly, *conflict-freeness* is described by (4) when higher-order relations are present. The meaning of (4) is “if the source of an attack and the attack itself are selected, then its target is not”.

$$\begin{aligned} \forall \alpha \in \text{Att}((\forall a \in \text{Arg}[S(\alpha, a) \rightarrow \text{Acc}(a)]) \\ \rightarrow (\exists x \in \text{Arg}[T(\alpha, x) \wedge \text{NAcc}(x)])) \end{aligned} \quad (3a)$$

$$\forall x \in \text{Arg}(\text{NAcc}(x) \rightarrow \neg \text{Acc}(x)) \quad (3b)$$

$$\begin{aligned} \forall \alpha \in \text{Att}((\forall a \in \text{Arg}[S(\alpha, a) \rightarrow \text{Acc}(a)] \wedge \text{Acc}(\alpha)) \\ \rightarrow (\exists x \in \text{Arg} \cup \text{Att}[T(\alpha, x) \wedge \text{NAcc}(x)])) \end{aligned} \quad (4a)$$

$$\forall x \in \text{Arg} \cup \text{Att}(\text{NAcc}(x) \rightarrow \neg \text{Acc}(x)) \quad (4b)$$

Thus, in the case of frameworks without coalitions,  $\{(1), (2), (3)\}$  is a logical base that satisfies the logical

conflict-freeness property and  $\{(1), (2), (4)\}$  is a logical base that satisfies the logical higher-order conflict-freeness property. Let us make the following observation: while not being exactly the same, the syntactic structures of formulas (3) and (4) are still very similar. Our idea is to exploit these structural similarities in order to define one generic logical encoding.

The generic aspect of this logical encoding relies on the use of parameters. Because these parameters are not instantiated at the generic level, the encoding would encode the core mechanism of the semantics that are common to all types of argumentation frameworks. Then, to retrieve the expressions relative to each type of framework, one would only need to instantiate the parameters of the generic encoding.

In the case of (3) and (4), the difference lies on the premise of the first formula and on the boundary of the quantifiers in the conclusion of the first formula and in the second one. Let us then add two predicates, *Activable* and *Cand*, and use them to write a generic form of (3) and (4). (5) encodes the fundamental way of working of *conflict-freeness*: if an attack is “effective”, then its target cannot be selected in an extension. *Activable* can thus be understood as representing the condition for an attack to be “effective”. This condition is specific to each type of argumentation framework. For frameworks without higher-order relations, an attack is “effective” if and only if all the arguments of its source are selected. In the case of higher-order relations the attack itself must also be selected. As for *Cand*, it can be understood as representing the elements that can be part of an extension (arguments and attacks with higher-order relations, only arguments without).

$$\begin{aligned} \forall \alpha \in \text{Att}(\text{Activable}(\alpha) \\ \rightarrow (\exists x \in \text{Cand}[T(\alpha, x) \wedge \text{NAcc}(x)])) \end{aligned} \quad (5a)$$

$$\forall x \in \text{Cand}(\text{NAcc}(x) \rightarrow \neg \text{Acc}(x)) \quad (5b)$$

To retrieve (3) and (4), we use these instantiations.

$$\forall x(\text{Activable}(x) \leftrightarrow \quad (6a)$$

$$\forall a \in \text{Arg}[S(x, a) \rightarrow \text{Acc}(a)])$$

$$\forall x(\text{Cand}(x) \leftrightarrow \text{Arg}(x)) \quad (6b)$$

$$\forall x(Activable(x) \leftrightarrow (\forall a \in Arg[S(x, a) \rightarrow Acc(a)] \wedge Acc(x))) \quad (7a)$$

$$\forall x(Cand(x) \leftrightarrow (Arg(x) \vee Att(x))) \quad (7b)$$

$\{(1), (2), (5), (6)\}$  satisfies the logical conflict-freeness property and  $\{(1), (2), (5), (7)\}$  satisfies the logical higher-order conflict-freeness property. Since it is (2) that constrains the absence of coalitions in the argumentation frameworks encoded with this method, it suffices to remove it from the logical base to consider frameworks with coalitions. Thus, in the case of frameworks with coalitions,  $\{(1), (5), (6)\}$  satisfies the logical conflict-freeness property and  $\{(1), (5), (7)\}$  satisfies the logical higher-order conflict-freeness property. This shows that frameworks with coalitions work fundamentally in the same way as frameworks without. In this respect, axiom (2) can be seen as a parameter of its own, inside the generic logical encoding.

### 3 Discussion

While only *conflict-freeness* was discussed in this paper, the method of generic logical encoding presented can be used to express the other usual semantics defined in [Dung, 1995], the *admissible*, *complete*, *preferred*, *grounded* and *stable* semantics. Moreover, the encoding is defined so that it can express these semantics for all types of argumentation frameworks obtainable with the combinations of the three enrichments presented in the introduction: coalitions, higher-order relations and the support relation.

It should however be pointed out that higher-order relations and the support relation have several interpretations. That is to say, there are several ways to modify the usual semantics once one of these enrichments is present in a framework. For now, only the interpretation of higher-order relations as RAF semantics, presented in [Cayrol et al., 2017], and the interpretation of the support relation as evidential support (see [Oren and Norman, 2008]) are taken into account. In this respect, the generic logical encoding encompasses the more specific logical encodings presented in [Cayrol and Lagasque-Schiex, 2020].

As future work, we wish to extend our generic logical encoding to include the missing interpretation of higher-order relations, namely the AFRA semantics, discussed in [Baroni et al., 2011]. Similarly, we wish to take into account the other interpretations of the support relation, namely the deductive support ([Boella et al., 2010]) and the support with necessities ([Nouioua and Risch, 2011]). While the AFRA semantics should fit into the work that has already been done, we have the intuition that the evidential support works in such a specific way that it will not be possible to express all three interpretations of the support relation using the same generic logical formulas. However, since the support with necessities and the deductive support are very closely linked, it should be possible to include both of them in the same generic logical theory. Works on Abstract Dialectical Frameworks have shown that some relations cannot be expressed using a two-valued logic in some cases. Abstract argumentation falls in these cases. Hence, there is certainly an underlying three-valued logic in our method, which should be investigated.

### 4 Conclusion

To conclude, we presented a method of generic logical encoding for abstract argumentation. The objective of this encoding is to factor as much as possible the selection mechanisms of the usual semantics that are common to a variety of argumentation frameworks, and to retrieve the cases of specific frameworks via the parameterization of these mechanisms. For now, the encoding encompasses the argumentation frameworks that can be defined using the combinations of three enrichments: coalitions, the RAF interpretation of higher-order relations and the evidential interpretation of the support relation.

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