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Small numerical variations in a set of similar problems from Nippur on the area of the square

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Abstract

This study is based on a set of six tablets dated to the Old-Babylonian period in Mesopotamia, written in cuneiform script, and relating to problems on the areas of squares. Their content is very similar, except for the length measurement of the side of the square. The issue of hypothetical pedagogical issues involved in the choice of numerical values in the statements is addressed by detailing the tasks they imply in the process of problem solving.

Keywords

Nippur, Old-Babylonian, area of a square, series of problems, tasks, metrology

Classification codes

01A17, 97A30, 97G30, 97F70, 51M25

Abbreviations

- BCE: Before the Common Era
- CE: Common Era
- ca. Approximate date, from Latin “circa”.

Conventions

- Half-brackets indicate a broken but identifiable sign
- Brackets indicate a broken or erased sign
- When the notation of numbers in sexagesimal place value notation (SPVN) found in some transcriptions is written as “6.40” it is transformed as “6:40” to ensure coherence of the whole.
- Indexes: Indexes such as “3” for the measurement unit “kuš3” are indications that allow the translator to know which sign having a “kuš sound” is registered in the tablet (it is therefore a question of numbering homophone signs). In general, the criteria adopted are the following: for transliterations: kuš3 (straight characters, numerical index preserved); for translation: *kuš* (italic, no index). This notation has been used here.

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1 Introduction

1.1 Motivations

A large amount of research has been carried out internationally to promote the use of the history of science in science teaching. However, reciprocally, the tools mobilized in educational research are rarely used to analyze situations of knowledge transmission in the history of science. The strictly didactic aspects of Old Babylonian school exercises had not been studied before. To do this analysis, one has to look very closely at all the clues given by the texts, in particular the layouts, the typology of the tablets, the notations, the numerical data, the network of texts to which the exercises belong. Here, the emphasis is on numerical data and how the choice of those data affects how the calculation is carried out. This article presents an analysis of the variations in the steps of solving area calculation exercises. The choice of numbers in the statements involves different methods of resolution, which could have been aimed at by the teacher. These hypothetical intentions of having students work on different methods of solving the exercises do not appear obvious since the statements of the problems are very similar. My contribution lies in the methodical approach to the steps necessary for the resolution of the square area computation exercises, implied by the choice of the length measurement for the side of a square in the statement of these exercises. This allows us to study potential didactic intentions among teacher scribes. These variations could indicate educational intentions. The calculation tools are thus highlighted: the use of numerical and metrological tables. Can the tasks that are required in order to obtain the area of a square, and thus solve the problem, be reconstructed? These tasks are implied by the choice of the length measurement for the side of a square in the statement. Are there differences between the tablets, in terms of resolution steps? If so, how can they be explained?

1.2 Materials & Methods

This historical analysis is based on the square area computation found on cuneiform tablets from the beginning of the second millennium BC. In a corpus of exercises from Nippur on the calculation of the area of a given square, I study the different choices of the length measurement for the side of the square. The effects of the initial values on the tasks performed by the scribes are analyzed. I discuss here the interpretation of these choices in terms of "pedagogical choices", that is to say, the possible effects of the teacher's choice of length measurement on the mathematical tasks the student would have to bring into play in order to resolve the exercise.

I work on a set of six tablets bearing similar square surface problems evaluated by Proust (2007, p.190-197). These are tablets CBS 11318, UM 29 15-192, Ni 18, UM 55 21-076, IM 57846 and

IM 57828, all from Nippur. The interest of this corpus is the similarity of the content, the attested scholarly character, the period (which is the same for all the tablets - the Old-Babylonian period, beginning of the second millennium BCE), and the same provenance. I rely on Proust's historical interpretation of the numerical systems in use. I also use Proust's interpretation of the articulation of two numerical systems, one being used to compute with numbers in sexagesimal place value notation (SPVN) and the other being used for measurement. Finally, I also consider Proust's translations and reconstructions of damaged parts of the texts.

Tablet	Number
CBS 11318	1
UM 29-15-192	2
Ni 18	3
UM 55-21-076	4
IM 57846	5
IM 57828	6

Table 1: the set of six tablets from Nippur

Tablets 1 to 6 are type IV¹ and of square shape. Tablet 3 is published by Proust (2007, p.193). The other five were published for the first time by Neugebauer and Sachs (1984, pp. 246-251). Firstly, details on the numerical and metrological systems in use will be given, as well as the school curriculum in Nippur during the Old-Babylonian period. Then Tablet 6 will be used to present the area computation algorithm. Finally, the tasks needed to answer the problem will be detailed for each tablet of the corpus.

1.3 Historical context

Sexagesimal place value notation (SPVN)

The SPVN dates from 2200-2000 BCE. The date of its appearance is discussed (see for example: Friberg (2005), Ouyang and Proust (*forthcoming*)). In SPVN the decimal number “63” would appear as “1:3”, which shows the sexagesimal character of the system. But according to Proust (2013, p.24-25), the order of magnitude is not represented in the cuneiform writing of the SPVN, the system is “floating”. For example, nothing indicates whether the digit 3 effectively represents a quantity such as 3, or 3×60 , or 3×60^2 , 3×60^3 , etc., or even 3×60^{-1} . Although, the relative position is of importance: in “1:3”, the cuneiform sign for “1” represents a value sixty times greater than that of each of the signs for “3”. Houzel (in Proust 2007, p.12) explains this as being interesting for the flexibility of calculation.² In summary, SPVN is a positional and sexagesimal system: the position of a symbol “one rank to the left” indicates a value 60 times greater than the symbol positioned “one rank to the right”. However, the writing of numbers below 60 is additive (for example, 12 is written $10 + 2$, 3 is written $1 + 1 + 1$). SPVN is a “floating system”, in contrast to other numerical systems which are used for measurement and counting objects, and which keep the information of orders of magnitude. For example, “system S” is an additive numerical system. The number “661” is written “600 + 60 + 1” in this system with a different sign for each term (for an introduction to system S, see Proust (2007, p.70) or (Proust, n.d.)). In this system, the shape of the signs indicates the orders of magnitude.

School curriculum in Nippur

Proust (2007) proposed a reconstruction of the school mathematical curriculum in Nippur, based on specific clues found on mathematical tablets found in Nippur. In scribal schools, lists of syllables,

¹ There are several types of tablets depending on the number of columns, the relationship between the text of the front and the back, the school level. Type IV tablets are of square or round shape (6 to 8 cm side or diameter approximately), plano-convex profile, the reverse being in general an epigraph (Proust, 2007, p.20-21).

² “This circumstance gives great flexibility to Babylonian calculation methods, analogous to the floating-point calculations of our modern computers.” (Houzel, in Proust 2007, p.12; translation by the author).

vocabulary, acrographic classification,³ and sentences⁴ were taught to students learning Sumerian, (Veldhuis, 1997; Robson, 2002; Proust, 2007, p.57-58; Proust, 2013, p.28). In parallel, in mathematics, numerical tables and metrological lists and tables had to be memorized (Proust, 2007, p.92).

Numerical tables are tables of reciprocals, of multiplications, of squares, of square roots and of cube roots. The metrological lists are enumerations of measurement units (capacities, weights, surfaces, and lengths), which make it possible to learn metrological systems (the relations between measurement units). The metrological tables, which will be important for our purposes, make it possible to transform the various measurement values expressed in a non-floating system, into numbers expressed in SPVN.

At the advanced level, scribes had to learn to use the multiplication, division, and inversion of numbers (regarding multiplication) to solve problems, and to calculate surfaces and volumes⁵ (Proust, 2016, p.27). These sources only inform us about the written part of teaching.⁶

Metrological lists and tables

The curriculum mentioned above includes “metrological lists” and “metrological tables”. As far as the metrological lists are concerned, they are lists of measurement values, in ascending order, on a dedicated theme (capacities, weights, surfaces, length), written in columns (Proust, 2007, p.98).

Ni 2782*
Face
1/3 sila ₃ še
1/2 sila ₃ še
5/6 sila ₃ še
1 sila ₃ še
Revers
1 1/3 sila ₃ še
1 1/2 sila ₃ še
1 5/6 sila ₃ še
2 sila ₃ še
3 sila ₃ še

Table 2: metrological list (Proust, 2007, p.115)

³ Lists of cuneiform signs according to their form.

⁴ Such as contracts and proverbs.

⁵ The exercises deal with linear and quadratic problems. Some of the most common geometry problems included trapezoidal sharing problems, probably inspired by surveying and inheritance practices. Hymns, myths and compositions were also part of the curriculum (Robson, 2001, p.55), (Proust, 2007, p. 60-61).

⁶ According to Proust (2007, p.25), it is possible, for example, that music and theater have also been part of the training of scribes. Moreover, knowing the order of the lists does not say what their use was. They might have been designed to solve exercises, or used as a teaching medium, presented orally or memorized, etc.

The metrological tables differ somewhat from metrological lists. Opposite each measurement value (on the left) is a number (on the right). The interpretation of these metrological tables will be discussed later.

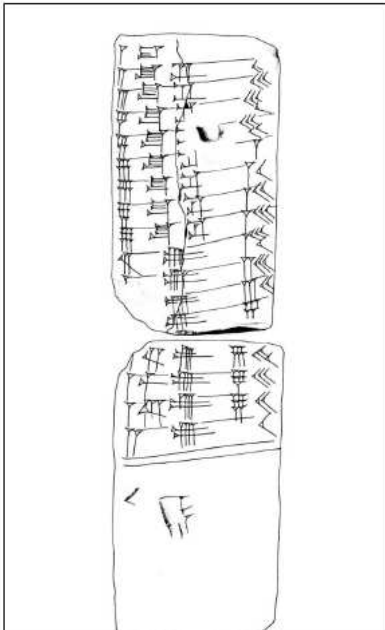
	1 šu-si	10
	2 šu-si	20
	3 šu-si	30
	4 šu-si	40
	5 šu-si	50
	6 šu-si	1
	7 šu-si	1:10
	8 šu-si	1:20
	9 šu-si	1:30
	1/3 kuš	1:40
	1/2 kuš	2:30
	2/3 kuš	3:20
	5/6 kuš	4:10
	1 kuš	5
	1 1/3 kuš	6:40
	1 1/2 kuš	7:30
	1 2/3 kuš	8:20
	2 kuš	10

Table 3: metrological table of lengths HS 241 (Proust, 2016, p.8)⁷

Using numerous tablets, historians have reconstituted lists of measurement values for each period (listing as many existing entries as possible). The list of all measurement values attested in a given set of tables is called the “composite text” of these tables.⁸ Along with other sources such as administrative texts, they make it possible for us to know, for example, the metrological systems and the factors between different measurement values of a given quantity (lengths for example). Proust (2007, p.310-314) established a composite metrological table for Nippur in the Old-Babylonian period, based on the collections held in Istanbul, Philadelphia and Jena. Below is an extract from her composite table of the metrological tables of lengths. Given the stability of the corpus of tablets containing metrological tables of surfaces, it seems reasonable to take this composite table into account in this study.

Numerical tables

As mentioned above, other tables were learned in the school curriculum: tables of reciprocals, multiplication, squares, and roots (square and cube). Some details are given here, since these tables make it possible to apprehend how calculation was carried out in SPVN.

Reciprocal tables, “regular” numbers

Reciprocal tables provide a given number in SPVN’s reciprocal in base 60. That is to say the number which, multiplied by the given number, gives a power of 60 (“1” in SPVN); like today, 3 times 20 minutes gives 1 hour. The reciprocal of 3 is 20. This is noted “igi 3” and then “20” in the table. The reciprocal table was the first numerical table learned in the curriculum (Proust, 2007, p.118). Some numbers, like 7, do not have a reciprocal that can be written with a finite sexagesimal development. These number are not “regular” in base 60, and do not appear in reciprocal tables from Nippur. The

⁷ See also the CDLI website: <http://www.cdli.ucla.edu/P388160>, first publication: Hilprecht (1906, n ° 42, pl.27).

⁸ Composite text: artificial text made from several sources. Composite texts are sometimes edited using transliteration but translation may also be used (as is the case here). It consists of the elements (signs and items) present at least once in these sources, not the most frequently statistically significant elements: it is a *maximal* version and not an *average* version.

division of two numbers in SPVN is generally carried out by multiplying the first number by the reciprocal of the second. Problems where the divisor is not regular are dealt with by some texts, but this is another type of problem, which was not addressed in the curriculum used in Nippur.

Multiplication tables and area calculation

There are only thirty-eight multiplication tables (not fifty-nine, as expected) (Proust, 2016, p.27).⁹ To multiply numbers whose product is not given in the tables, it is necessary to use a multiplicative algorithm (multiplication by steps). It is possible to imagine the use of an abacus to multiply in base 60 (Proust, 2016, p.27). For simple cases it is possible that the scribes would mentally combine the results from other tables they had memorized.

The area calculation exercises presented in the following correspond, in the reconstruction of the Nippur curriculum, to an intermediate level (between the elementary level and the advanced level). The tablets on which the exercises of this intermediate level are written are of square or lenticular shape. These intermediate exercises concern the calculation of surfaces and volumes using tables (which had been memorized at the previous level) as well as inversion algorithms using factorization.

Other tables, such as tables of squares and roots, and cube root tables were also learned. More advanced problems such as volume problems (or calculate a side, knowing a volume), geometrical series, problems of ratios of proportionality between dimensions of prisms of the same form, or transformed by affinity, also appear at the advanced level and were found in Nippur. (Proust, 2007, p.243-244).

Memorization and other methods used to multiply

The question of memorization has been studied by several authors. Veldhuis (1997, p.132) was interested in lexical texts and Delnero (2012) in literary texts. They observed the variants and errors in duplicates (tablets with the same content). According to these authors, school texts were memorized and reproduced from memory. For other authors, these texts were dictated or copied from a model. Veldhuis and Delnero draw on neuroscience to link cognitive aspects to the reproduction of memorized texts.

I will not use here the memorized character of the multiplications for my conclusions on the calculation of area exercises. I will only refer to the multiplication tables of the curriculum when they exist, and specify whether the numbers chosen for the area calculation use values that are present in these tables. The fact that they are memorized, or left materially within the pupil's reach, does not change *a priori* the need to resort to the other mathematical tasks performed by the student as part of the area calculation. On the other hand, the use of multiplications that do not appear in these tables could call for a multiplicative algorithm or an abacus. I will simply point out if the multiplication was part of this set of multiplication tables (I will use the term "elementary products" to describe this fact) or if it probably required the use of a multiplicative algorithm (which could have been performed on an abacus, a laid down calculation, dust calculation, etc.).

Proust (2007, p.165) also presents multiplication tablets that only report some of the possible multiplications: only the multiplier, the multiplicand and the result can be seen on those tablets. The algorithmic calculation does not appear explicitly. The format follows a uniform rule (Proust, 2007, p.170).

⁹ Tables of 50, 45, 44:26:40, 40, 36, 30, 25, 24, 22:30, 20, 18, 16:40, 16, 15, 12: 30, 12, 10, 9, 8:20, 8, 7:30, 7:12, 7, 6:40, 6, 5, 4:30, 4, 3:45, 3:20, 3, 2:30, 2:24, 2 1:40, 1:30, 1:20, 1:15. Outside these tables, it is possible to imagine that mental calculation or an abacus was used to multiply in base 60. To perform all the multiplications in base 60, one would in theory, memorize all the elementary products (i.e.: 59 tables of 50 products). But the existence of the intermediate base in the writing of the sexagesimal digits makes it possible to reduce the number of the products to be memorized: it suffices that only the units and the tens appear in these tables (principal numbers and multipliers 1 to 10, then 20, 30, 40, 50), i.e.: 14 tables of 14 products. For example, the multiplication of two numbers to a sexagesimal place such as "25 times 32" can be broken down into four partial products 2×20 , 2×5 , 30×20 , 30×5 (Proust, 2007, p.130-131).

2 Analysis of the six tablets dealing with the area of a square

In the six tablets of our corpus from Nippur, the text is divided into two parts. In the first part, at the left top, are numbers written in SPVN. In the second part, bottom right, is the statement of the problem, with the length measurement of the side of a square. Underneath, the result giving the area measurement of the square is also given. I will begin by analyzing the tablets in detail and deducing the possible tasks required to calculate the area of a given square, using the example from Tablet 6. Then I will give details for each tablet, and summarize all the tablets in a table that will serve my conclusions (see: 3. Results).

The example of Tablet 6

I explain below the computation of the area of the square, based on Tablet 6. I give its translation (Proust, 2007, p.194). Depending on the length measurement of the side of the square chosen in the other tablets of the corpus, some variations on the steps of the computation will be introduced afterwards.

Translation of Tablet 6 (Proust, 2007, p.194)¹⁰

1:45

1:45

<3:3:45>

1/3 *kuš* 1/2 *šu-si* on each
side (of the square).

What is its surface?

Its surface is*

9 *še* 1/5 + 3 *še*

First broad view of the steps in the calculation

Four parts of the text can be distinguished, I label them A, B, C, D.

A. The first line of the problem noted right bottom, provides the statement and the length measurement of the side of a square (non-floating system) expressed with measurement units “*kuš*” and “*šu-si*”¹¹

“1/3 *kuš* 1/2 *šu-si* on each side (of the square)”

B. The second line of the problem is the question:

“What is its surface?”

C. Considering Proust’s hypothesis (2007, p.251) on the procedure needed to solve these problems, scribes would have followed the steps that I label: “1, 2, 3”:

1. Providing a correspondence between the length measurement and the number in SPVN using the metrological table of length is necessary to be able to compute the area of the square (this step will be given more detail later with a description of the sub-steps)

1/3 *kuš* 1/2 *šu-si* → 1:45

¹⁰ According to Neugebauer and Sachs (1984, p. 246-248) and a photograph (Steele 1951, p. 25), Proust (2007, p. 194) restores the missing line (a-ša₃-bi) in Neugebauer and Sachs (1984) and confirms the end of the text (*še-kam* being indicated as uncertain in their edition).

¹¹ See also Proust (2007, p.251).

The order of magnitude for the side's length is to be kept in mind, or to be evaluated at the end.

2. Multiplication is then performed on the number in SPVN. This corresponds to the top left corner of the tablet.

$$1:45 \times 1:45 = 3:3:45$$

3. Providing a correspondence between the number in SPVN with the area measurement using the metrological table of surfaces is then carried out¹² (this step will also be given more detail later in this paper, with the sub-steps)

$$3:3:45 \rightarrow 9 \text{ še } 1/5 + 3 \text{ še}$$

D. Answer: the answer is given on the last line of the right corner of the tablet.

$$\text{Its surface is* } 9 \text{ še } 1/5 + 3 \text{ še}$$

Summary of some prior knowledge needed for the computation

The metrological table of lengths is used to provide a correspondence for the length measurement with a number in SPVN. The multiplication table (memorized or made available to the student) is used for the calculation (the multiplication of the number in SPVN by itself). Then, the metrological table of surfaces is used to provide a correspondence for the number in SPVN obtained by multiplication, with the area measurement.

Detailed reconstruction of the tasks for each tablet

While giving details for each tablet, I introduce the vocabulary related to the different tasks that may be necessary to calculate the area of the square. Where possible, I will distinguish “main tasks”, the tasks that are common to all the exercises, to compute the area of any square, and “sub-tasks” that are not common to all the exercises.

¹² The fractions of *sar* are expressed with the unit *gin* (1 *gin* is 1/60 of *sar*). In the same way, the fractions of *mana*, a unit of weight, are expressed in *gin* (1 *gin* is 1/60 *mana*). However, the sequence of *gin* appears in the metrological table of weights, but not in the metrological table of surfaces. So, it is the metrological table of weights that is used for measurements of areas below the *sar*.

Tablet 1

Translation of Tablet 1 (Proust, 2007, p.191)¹³

5
5
25

1 *kuš* the side (of the square).
What is its surface?
Its surface is
1/3 *gin* 15 *še*

The length measurement of the side of the square is given by means of a measurement value using a non-floating numerical system (the numerical value is not expressed in SPVN, see 3.2.1). The measurement is given with only one measurement value: I call it a “simple” measurement. Here, the length measurement “1 *kuš*” is “**simple**” (a length measurement such as: 1/3 *kuš* 1/2 *šu-si* would be called “composed”).

The order of magnitude for 1 *kuš* is that of the length of a large brick.

Providing a correspondence for the length measurement

1. The first task consists of providing (directly) a correspondence for the length measurement 1 *kuš* with a number in SPVN. In the metrological table of lengths, this entry exists. This is what I call providing a correspondence “directly” (in one step, the entry already existing on the table).

It gives the number 5 in SPVN. Providing a correspondence is one of the tasks I have recorded as main tasks.

Multiplication

2. The next task consists of carrying out a multiplication of 5 by itself. The multiplication table by 5 is one of the tables in the curriculum. It is not necessary to use a multiplicative algorithm. Note that this tablet is the only one in the corpus that produces a product bearing only one single sexagesimal position.

Providing a correspondence for the number in SPVN

3. Providing an area measurement by using metrological tables of surfaces is not obvious, as the SPVN numbers appear cyclically in the tables. In the example of Tablet 1, it could be carried out at least in two ways, depending on the metrological table used by the scribe.

Hypothesis 1: the scribe could provide a correspondence (directly) for 25. This would be carried out in only one step. Indeed, Old-Babylonian tablet MS 2186, which is of uncertain provenance, contains a metrological table of surfaces. The entry for 25 exists directly in the table (See table 4). Now the composite metrological table on which I draw was made up exclusively from the tablets from Nippur, preserved in Istanbul, Philadelphia and Jena. In the composite table, there is no entry for 25. Nippur tablets possibly held in other museums or lost, could provide more entries, as indicated by MS 2186 (which is not taken into account in the composite table, because its provenance is not Nippur).

¹³ According to Proust (2007, p.191) the calculation appearing in the upper left corner must have been a multiplication of 5 by 5; the 10 that appears on the copy of Neugebauer and Sachs (1984) is probably due to an error by the scribe. I do not include it here in the translation.

Hypothesis 2: the scribe could provide a correspondence *by interpolation*: this is what I now call a “sub-task” since in this case it would probably imply the combination of two main tasks: providing a correspondence (directly) for 20, then providing a correspondence (directly) for 5.

Selecting the relevant cycle to provide a correspondence for a number in SPVN to an area measurement

In the metrological tables, numbers in SPVN that correspond to measurement values, like 25 in the previous example, reappear cyclically. This is implied by the fact that in SPVN, numbers are floating (in base 60); the quantity is not fixed by the notation. I consider a cycle as a set of entries for which the numbers in SPVN range from 1 to 59.¹⁴ According to my definition of “cycle”, numbers in SPVN go from 1 to 59, then the cycle begins again.¹⁵ The previous task of providing an area measurement to a number in SPVN implies selecting the relevant cycle: this is one of the main tasks. I represent a cycle below by the entries between two horizontal lines. The extract from MS 2186’s metrological table is presented here with two columns side by side.¹⁶ The relevant cycle from the metrological table of surfaces has to be selected, in order to find the correspondence for the number in SPVN obtained by the multiplication. For the case where the number has only one sexagesimal position, such as the number 25 in SPVN, it is necessary to choose the correct cycle:¹⁷ there are several (because 25 returns cyclically).

1/2 še of silver	10	24 še	8
1 še	20	25 še	8:20
1 1/2 še	30	26 še	8:40
2 še	40	27 še	9
2 1/2 še	50	28 še	9:20
3 še	1	29 še	9:40
4 še	1:20	1/6 gin ¹⁸	10
5 še	1:40	1/6 gin 10 še ¹⁹	13:20
6 še	2	1/4 gin ²⁰	15
7 še	2:20	1/4 gin 5 še ²¹	16:40
8 še	2:40	1/3 gin	20
9 še	3	1/3 gin 15 še	25
10 še	3:20	1/2 gin	30
11 še	3:40	1/2 gin 10 še	33:20
12 še	4	1/2 gin 15 še	35
13 še	4:20	2/3 gin	40
14 še	4:40	2/3 gin 15 še	45
15 še	5	2/3 gin 25 še	48:20
16 še	5:20	5/6 gin	50
17 še	5:40	5/6 gin 15 še	55
18 še	6	1 gin	1
19 še	6:20		
20 še	6:40		

¹⁴ Arbitrarily, especially since entry 59 is not actually present on the tablets.

¹⁵ This definition is an arbitrary decision of mine, it could be the number in SPVN “2” until reaching the next “2”, or other number until it appears again in the table.

¹⁶ For MS 2186, see https://cdli.ucla.edu/search/archival_view.php?ObjectID=P250902 first publication: Friberg, (2007, MSCT 1, 119). The mention indicates that it is silver, this is a common precision at the beginning of a metrological table.

¹⁷ See for example Tablet 1.

¹⁸ It is written: “igi-6- gal₂ gin₂” (transliteration), so I write it *igi-6-gal*

¹⁹ It is written: “igi-6-gal gin 10 še”

²⁰ It is written: “igi-4-gal gin”

²¹ It is written: “igi-6-gal gin 5 še”

21 še	7
22 še	7:20
23 še	7:40

Table 4: extract from metrological table of weights used for surfaces
MS 2186 (Friberg, 2007, p.110-112)

To choose the correct one, a verification of the order of magnitude is carried out. Here, the order of magnitude is that of a large brick.²² Thus, the chosen cycle will be that of the fractions of *gin*. Once the correct cycle has been selected, it is necessary to find the area measurement corresponding to this “25” by providing a direct correspondence, because the entry exists in the table. If the entry is not in the table, other tasks need to be carried out.

Circulating within a cycle:

Following hypothesis 2 requires *circulating within a cycle*. The order of magnitude for the measurement value corresponding to the number 5 in SPVN must be smaller than that of 20. It is thus necessary to circulate within the same cycle.²³

1/2 še of silver ²⁴	10	28 še	9:20
1 še	20	29 še	9:40
1 1/2 še	30	1/6 gin ²⁵	10
2 še	40	1/6 gin 10 še	13:20
2 1/2 še	50	1/4 gin	15
3 še	1	1/4 gin 5 še	16:40
4 še	1:20	1/3 gin	20
5 še	1:40	1/2 gin	30
6 še	2	1/2 gin 10 še	33:20
7 še	2:20	1/2 gin 15 še	35
8 še	2:40	1/2 gin 25 še	38:20
9 še	3	2/3 gin	40
10 še	3:20	2/3 gin 10 še	43:20
11 še	3:40	2/3 gin 15 še	45
12 še	4	2/3 gin 25 še	48:20
13 še	4:20	5/6 gin	50
14 še	4:40	5/6 gin 10 še	53:20
15 še	5	5/6 gin 15 še	55
16 še	5:20	5/6 gin 25 še	58:20
17 še	5:40	1 gin	1
18 še	6	1 1/3 gin	1:20
19 še	6:20	1 1/2 gin	1:30
20 še	6:40	1 2/3 gin	1:40
21 še	7	1 5/6 gin	1:50
22 še	7:20	2 gin	2
23 še	7:40	3 gin	3

²² See table of order of magnitudes in Appendix.

²³ It is not possible to propose another equivalent for the number 5 in SPVN, such as for example 5 gin₂. Indeed, the order of magnitude is then that of a half-yard (this is the next cycle in the table, which also corresponds to the number in SPVN). Another smaller possibility would correspond to the 5 number in SPVN (i.e. 1/4 še). It also does not work, since the order of magnitude is much smaller (that of a small tablet). The order of magnitude for the expected measurement value is that of a small brick.

²⁴ It is written “ku.babbar” (transliteration).

²⁵ It is written “igi-6-gal₂” (transliteration).

24 še	8	4 gin	4
25 še	8:20	5 gin	5
26 še	8:40	6 gin	6
27 še	9		

Table 5: extract from composite metrological table of surfaces (translated, based on Proust, 2007, p.311)

Positioning: simply paying attention to the sexagesimal *positions* of the number 25 in SPVN, it is possible to deduce that the same cycle should be chosen for a correspondence for 5 as it was for 20, since the digits 20 and 5 occupy the same sexagesimal position.

Providing a direct correspondence for the number 20 in SPVN gives $1/3$ gin, this is an existing entry in the metrological table of surfaces.

Providing a direct correspondence for the number 5 in SPVN gives 15 še, this is also an existing entry in the metrological table of surfaces

The resulting area measurement is composed: it is “ $1/3$ gin 15 še”

With this example, I can now summarize the distinction between the main tasks and sub-tasks that have been encountered. I have called “main tasks”, the tasks that are common to all the exercises, to compute the area of any square. I underline the “main tasks”. I note what I have called “sub-tasks” in *italics*; (the tasks that are not common to all the exercises).

Here are the tasks I have called “main tasks”:

- providing a correspondence for a length measurement with a number in SPVN
- multiplication: when the multiplication draws on the tables of the curriculum (elementary products), it is “elementary multiplication”
- selecting the relevant cycle (depending on the expected order of magnitude)
- providing a correspondence for a number in SPVN to an area measurement

I have designated the other tasks encountered as “sub-tasks”. These sub-tasks require a mastery of the “main tasks”. They are not encountered in all tablets. Here are the sub-tasks hypothetically encountered in this tablet, following “hypothesis 2” (which is based on the composite metrological table based on Nippur sources):

-*providing a correspondence by interpolation* consists of providing a correspondence for a number in SPVN –here 25– with an area measurement, or vice versa, in more than one step, but using existing entries and thus a combination of main tasks)

And consequently, this sub-task implies other sub-tasks:

-first it implies *circulating within a cycle* (in this case, to look for the entry corresponding to the number 20 in SPVN, and then for the entry corresponding to the number 5, by additive linearity). Hypothetically this could imply what I designate as:

-*positioning*, which consists of relying on sexagesimal positions to navigate within the current or the previous cycle

Highlights for Tablet 1

Interestingly, Tablet 1 requires no, or almost no sub-tasks. Here all the entries that are used already exist in the metrological tables. The multiplication is elementary. Providing a correspondence for the length measurement 1 kuš in the metrological table of the lengths is direct (it is carried out in one step, and uses an existing entry).

A hypothesis linked to pedagogical choices could be that this tablet allowed the scribes to work on the task of *providing a correspondence by interpolation* in the metrological table of surfaces. This can be done, without the previous steps being too complex (providing a correspondence for the length measurement is direct, as the entry exists in the metrological table of lengths, multiplication). Moreover, it should be noted that, if hypothesis 1 is correct, then the choice of the length measurement in the statement implies there are *no* sub-tasks to be carried out to solve the problem from Tablet 1.

Tablet 2

Translation of Tablet 2 (Proust, 2007, p.193)²⁶

[2]0
20
6:40

2 *šu-si* the side (of the square).
What is its surface?
Its surface is
 $\frac{1}{3}$ *še*

Tablet 2 highlights a sub-task which differs from Tablet 1. This tablet offers only one sub-task, providing a correspondence for the number 6:40 in SPVN, which is done by *extrapolation*.

The order of magnitude is that of the length of a finger. Providing a correspondence for the length measurement 2 *šu-si* is performed in one step and directly; the entry exists in the table. It gives the number 20 in SPVN. The multiplication of 20 by 20 exists in the curriculum's multiplication tables. The number obtained, 6:40, bears two sexagesimal positions.

To select the relevant cycle, the order of magnitude sought to provide a correspondence to the number 6:40 in SPVN is smaller than an object such as a tablet. Thus, the chosen cycle will correspond to that of a tiny fraction of the *še* (the result is $\frac{1}{3}$ *še*) but this entry is not present in the metrological tables. Indeed, the initial entry in the metrological table of surfaces is the correspondence $\frac{1}{2}$ *še* → 10, whereas the area measurement sought is of a lower order of magnitude.

Extrapolation

If the measurement values are lower than the initial entry in the table, there is a need to extrapolate (for example, here regarding small fractions of *še*). The smallest value (initial entry) attested in the metrological tables was $\frac{1}{2}$ *še*. There is a great stability in the tablets that have reached us presenting metrological tables of surfaces (which bear this portion of the metrological table); they usually start at $\frac{1}{2}$ *še* or 1 *še* (Proust, 2007, p.157).

Several hypotheses are possible: either these parts of the metrological tables have not reached us, or we must use a calculation, or rounding, or even truncating this part of the measurement value. Tablets 3, 4 and 6 will show that the smallest parts of the measurement values are not always neglected, and that for some fractions of *še* at least, the exact result is actually given by the scribe.

Highlights for Tablet 2

One hypothesis would be that this tablet makes it possible to work on *extrapolation* in the metrological table of surfaces. This is possible, without the above steps being too complex (providing a direct correspondence to get the number in SPVN, carrying out a multiplication). Providing a correspondence for the number 6:40 in SPVN gives $\frac{1}{3}$ *še*, by *extrapolation*.

Tablet 3

Translation of Tablet 3 (Proust, 2007, p.193)²⁷

²⁶ UM 29-15-192. From a transcription by Neugebauer and Sachs (1894, p.251)

²⁷ Ni 18. According to Proust (2007, p.193), there is an error for the area measurement ($\frac{1}{4}$ *še* instead of $\frac{1}{3}$ *še*) and the scribe also made an error in the multiplication, which is reflected on the area computation. She points out that this detail shows that this exercise is a real calculation, and not linked to learning by heart. "sic" means that the sign identified is incorrect; it is left uncorrected in the translation.

2:10
 2:10
 4:26^{sic}:40

1/3 *kuš* 3 *šu-si* the side (of the square).
 What is its surface?
 Its surface is
 13 *še* 1/4^{sic} *še*.

Tablet 3 shows the implementation of several sub-tasks, brought about by the choice of length measurement for the side of the square. The order of magnitude is that of a large brick.

1. The first task to be carried out is to provide a correspondence by *interpolation* (in two steps) for the composed length measurement 1 *kuš* 3 *šu-si* in the metrological table of lengths, using existing entries.

3 <i>šu-si</i>	30
4 <i>šu-si</i>	40
5 <i>šu-si</i>	50
6 <i>šu-si</i>	1
7 <i>šu-si</i>	1:10
8 <i>šu-si</i>	1:20
9 <i>šu-si</i>	1:30
1/3 <i>kuš</i>	1:40

Table 6: extract from metrological table of length HS 241 (Proust, 2016, p.8)²⁸

Accordingly, adding the intermediate number in SPVN implies paying special attention to *positioning*. It is a logical consequence of the use of the correspondence for the length measurement 1/3 *kuš* 3 *šu-si*; as 3 *šu-si* corresponds to the cycle preceding that of 1/3 *kuš* in the system.

1 4 0
 3 0

The result is 2:10. This identification of the relative positions results directly from the organization of the metrological tables by cycles.

2. The multiplication task then needs to be performed. As 2:10 is not regular there is no multiplication table for it. In that case an *algorithm* would be needed, perhaps using an abacus, and the elementary products of 10 and 2. There is even an error by the scribe who finds 4:26:40 instead of 4:41:40.

Some of the multiplications are provided by the multiplication tables in the curriculum. When the multiplications are not provided in these tables (assuming that there are no other tables than those which have come to our knowledge) it is necessary to use a multiplicative algorithm (multiplication by steps).²⁹ I will specify if the multiplication table is part of those in the curriculum or not, according to the sources that have come down to us (based on Proust, 2007, p.243).

²⁸ See also CDLI: <http://www.cdli.ucla.edu/P388160>, first publication: Hilprecht (1906, n°42, pl.27)

²⁹ For example, to multiply the number in SPVN: “58:20” by itself, there could be a successive use of the multiplication tables by 50, by 8 and by 20; or by 50 and then by 8:20 (these tables belonging to the curriculum). In the example from Tablet 6, the multiplication to be performed is “1:45” multiplied by itself. The multiplication table by “45” is part of the curriculum tables.

3. On the basis of the supposed “correct calculation”, providing a correspondence for the number 4:41:40 in SPVN in the metrological table of surfaces can be done by *interpolation* in two steps: probably the providing (directly) of a correspondence for 4:40 and the providing (directly) of a correspondence for 1:40.

Selecting the relevant cycle: The expected order of magnitude for the measurement value corresponding to 4:40 is that of a brick. Thus, the scribe has to choose the cycle corresponding to the *še*.³⁰

Circulation from one cycle to another (see Appendix): the order of magnitude for the measurement value corresponding to 1:40 must be smaller than that of a brick. Based on the *position* of the sexagesimal positions of the number 4:41:40 in SPVN, it will be necessary to choose the cycle preceding that of 4:40 in the metrological table.

4 4 0
 1 4 0

I point out that there is a possibility of being wrong if a cycle is not skipped.³¹ The entry just above provides a correspondence for the number 1:40 in SPVN and the area measurement “5 *še*”.³²

2 1/2 <i>še</i>	50
3 <i>še</i>	1
4 <i>še</i>	1:20
5 <i>še</i>	1:40
6 <i>še</i>	2
7 <i>še</i>	2:20
8 <i>še</i>	2:40
9 <i>še</i>	3
10 <i>še</i>	3:20
11 <i>še</i>	3:40
12 <i>še</i>	4
13 <i>še</i>	4:20
14 <i>še</i>	4:40
15 <i>še</i>	5
16 <i>še</i>	5:20
17 <i>še</i>	5:40
18 <i>še</i>	6
19 <i>še</i>	6:20

Table 7: extract from composite metrological table of surfaces (translated, based on Proust, 2007, p.311)³³

The correspondence for 4:40 (which gives 14 *še*) is an existing entry in the metrological table of surfaces, but the number 1:40 in SPVN (which provides the correspondence 1/12 *še*) must be *extrapolated*.

The expression of the area measurement is composed: 14 *še* 1/12 *še*. It should be noted that if 1/12 *še* is neglected, then this step can be avoided. Since there is a mistake in the calculation, it is not possible to know if this value would have been neglected here.³⁴

Highlights for Tablet 3

³⁰ For the orders of magnitude, see Appendix.

³¹ Between 4 from “4:40” and 1 from “1:40” there indeed has been a change in sexagesimal positions.

³² The order of magnitude is about a medium-sized tablet.

³³ The metrological table of surfaces MS 2186 (Friberg, 2007, p.110-112) is identical to composite table at this position in the table.

³⁴ Note: based on the scribe's calculation, for 4:26:40 the result should be 13 *še* 1/3 *še*. It is actually noted 13 *še* 1/4 *še*.

In summary, this tablet allows us to encounter new sub-tasks, compared to previous tablets. Providing a correspondence for 1 *kuš* 3 *šu-si* is done by *interpolation*, in two steps, and involves paying attention to the position of the number obtained in SPVN (*positioning*). The use of non-elementary multiplication involving an *algorithm* was also not encountered in tablets 1 and 2. Providing a correspondence for the number 4:41:40 in SPVN is done by a two-step *interpolation* and involves the *circulation from one cycle to the preceding one*, based on the sexagesimal positions (*positioning*). This implies a risk of error, which is new in our selection.

One hypothesis would be that this tablet involves more sub-tasks related to the presence of composed measurement values, implying subtleties in the circulation in the metrological tables, that could be asked for, by a hypothetical teacher.

Tablet 4

Translation of Tablet 4 (Proust, 2007, p.194)³⁵

[5:20
5:20
28:26:40]

1 *kuš* 2 *šu-si* on each side
(of the square).
What is its surface?
Its surface is
1/3 *gin* 25 1/3 *še*

Let us examine the sub-tasks involved as implied by the choice of the length measurement for the side of the square, in the resolution of the problem on Tablet 4. The starting order of magnitude is that of the length of a large brick.

The expression of the length measurement of the side is composed. Providing correspondence of the length measurement with the number in SPVN is therefore done by *interpolation* (two steps), but directly (entries exist in the table). The multiplication of 5:20 by 5:20 is not elementary (it is not part of the curriculum tables), so an *algorithm* is needed.

Providing a correspondence for the number 28:26:40 in SPVN in the metrological table of surfaces is done by *interpolation*, in several steps this time: probably providing the correspondence for 20, the correspondence for 8:20 and the correspondence for 6:40. The order of magnitude for the measurement value corresponding to 20 is that of a large brick. Thus, the relevant cycle chosen will be that fractions of *gin*. The order of magnitude for the measurement value corresponding to 8:20 must be smaller. It is then a matter of circulation *within the same cycle*.

2 0
8 2 0

The order of magnitude for the measurement value corresponding to 6:40 must be even smaller. The sexagesimal positions indicate that it will be necessary to *circulate towards the cycle preceding that of 8:20* (with a risk of error).

³⁵ UM 55-21-076. Based on a transcript of Neugebauer and Sachs (1984, p. 246). See also Robson (2000, p.25). Tablet says “1/3 gin2 25 2/3 še” (transliteration); the correction “1/3” is proposed by the translation (Proust, 2007, p.194). Middeke-Conlin (2020, p.168-169) notes that the difference between the expected value and that which is written is 0.33%. He concludes that it is difficult to decide if it is a typing error and / or a mapping error. The error is in any case, according to him, less clearly linked to a calculation error than for the Tablet 3 (the author identifies 5.32% of difference with the expected value, and the error is there linked to the multiplication result).

2 0
 8 2 0
 6 4 0

There is again a possibility of error if one does not change cycle. Indeed, the entry just above provides a correspondence for the measurement value 20 *še* and the number 6:40 in SPVN.³⁶ Although if sufficient attention is paid to the sexagesimal positions this should not be a problem.

1/2 <i>še</i> of silver ³⁷	10	28 <i>še</i>	9:20
1 <i>še</i>	20	29 <i>še</i>	9:40
1 1/2 <i>še</i>	30	1/6 <i>gin</i> ³⁸	10
2 <i>še</i>	40	1/6 <i>gin</i> 10 <i>še</i>	13:20
2 1/2 <i>še</i>	50	1/4 <i>gin</i>	15
3 <i>še</i>	1	1/4 <i>gin</i> 5 <i>še</i>	16:40
4 <i>še</i>	1:20	1/3 <i>gin</i>	20
5 <i>še</i>	1:40	1/2 <i>gin</i>	30
6 <i>še</i>	2	1/2 <i>gin</i> 10 <i>še</i>	33:20
7 <i>še</i>	2:20	1/2 <i>gin</i> 15 <i>še</i>	35
8 <i>še</i>	2:40	1/2 <i>gin</i> 25 <i>še</i>	38:20
9 <i>še</i>	3	2/3 <i>gin</i>	40
10 <i>še</i>	3:20	2/3 <i>gin</i> 10 <i>še</i>	43:20
11 <i>še</i>	3:40	2/3 <i>gin</i> 15 <i>še</i>	45
12 <i>še</i>	4	2/3 <i>gin</i> 25 <i>še</i>	48:20
13 <i>še</i>	4:20	5/6 <i>gin</i>	50
14 <i>še</i>	4:40	5/6 <i>gin</i> 10 <i>še</i>	53:20
15 <i>še</i>	5	5/6 <i>gin</i> 15 <i>še</i>	55
16 <i>še</i>	5:20	5/6 <i>gin</i> 25 <i>še</i>	58:20
17 <i>še</i>	5:40	1 <i>gin</i>	1
18 <i>še</i>	6	1 1/3 <i>gin</i>	1:20
19 <i>še</i>	6:20	1 1/2 <i>gin</i>	1:30
20 <i>še</i>	6:40	1 2/3 <i>gin</i>	1:40
21 <i>še</i>	7	1 5/6 <i>gin</i>	1:50
22 <i>še</i>	7:20	2 <i>gin</i>	2
23 <i>še</i>	7:40	3 <i>gin</i>	3
24 <i>še</i>	8	4 <i>gin</i>	4
25 <i>še</i>	8:20	5 <i>gin</i>	5
26 <i>še</i>	8:40	6 <i>gin</i>	6
27 <i>še</i>	9		

Table 8: abstract from metrological table of surfaces
 MS 2186 (Friberg, 2007, p.110-112)

The correspondences for 20 (which gives 1/3 *gin*) and for 8:20 (which gives 25 *še*) are entries belonging to the metrological table of surfaces.

Providing a correspondence for the number 6:40 in SPVN implies *extrapolation*. It gives 1/3 *še*. Note that Tablet 2 uses the same value. It is then very interesting to see this value used again here. The expression of the area measurement is composed: 1/3 *gin* 25 *še* 1/3 *še*.

Highlights for Tablet 4

³⁶ For MS 2186, see https://cdli.ucla.edu/search/archival_view.php?ObjectID=P250902 first publication: Friberg (2007, MSCT 1, 119). The mention, again, indicates that it is about silver, which is a common precision at the beginning of the metrological table.

³⁷ It is written “ku.babbar” (transliteration).

³⁸ It is written “igi-6-gal₂” (transliteration).

In summary, this tablet bears the same “sub-tasks” as the previous tablet; Tablet 3. But here, providing a correspondence for the number in SPVN by *interpolation* is done in three steps. It is interesting to note that the number 6:40 in SPVN has already been encountered while studying Tablet 2. This value implies *extrapolation*. It should also be noted that the area measurement value obtained by extrapolation, which gives “1/3 še”, is not neglected. It was not neglected in Tablet 2 either, but in that case, the area measurement was of simple expression, so truncating would not be relevant. Here the measurement value is composed of a measurement value of a greater order of magnitude. Thus, 1/3 še could be truncated, which is not the case and might indicate there is indeed some work on extrapolating small parts of še.

Tablet 5

Translation of Tablet 5 (Proust, 2007, p.194)³⁹

5 ? 10

2/3 kuš 9 šu-si on each
side (of the square).
What is its surface?
Its surface is
1/3 [gin 10 še]*

Let us examine the sub-tasks involved in the resolution of this exercise. The order of magnitude is that of the length of a brick. Providing a correspondence *by interpolation for* the composed measurement value, 2/3 kuš and 9 šu-si, requires two steps. These entries exist in the metrological table. The entry 2/3 kuš corresponds to the number 3:20 in SPVN. Entry 9 šu-si corresponds to the number 1:30 in SPVN.

This time, the composed measurement value for the length measurement of the side of the square implies addition of the intermediate numbers in SPVN, with particular attention to *positions*.

3	2	0
1	3	0

The result is 4:50. Note there are actually two additions.

The multiplication of 4:50 by itself requires a multiplicative *algorithm* which gives 23:21:40. Providing a correspondence for 23:21:40 also requires *interpolation* in several steps: for instance, the correspondence for 20, the correspondence for 3:20 and the correspondence for 1:40. The latter has already been detailed in the case of Tablet 3.

The selection of the relevant cycle uses the expected order of magnitude for the area measurement value corresponding to 20 which is that of a large brick and that is 1/3 gin. Thus, the cycle chosen will be that of fractions of gin (as in the previous tablet).⁴⁰

The order of magnitude for the measurement value corresponding to the number 3:20 in SPVN requires one to *select the cycle preceding that of 20*, which gives 10 še. The expected order of

³⁹ IM 57846. According to a transcription by Neugebauer and Sachs (1894, p.246, cited in Proust, 2007, p.194). There is no copy (the layout is therefore here reconstituted). CDLI number: P254368

* Note: The result written on the tablet seems to be 1/4, and the expected rounded result is 1/3 gin 10 še.

⁴⁰ For MS 2186, see https://cdli.ucla.edu/search/archival_view.php?ObjectID=P250902 first publication: Friberg (2007, MSCT 1, 119). The mention again, indicates that it is about silver.

magnitude for the measurement value corresponding to 1:40 corresponds to the *cycle preceding that of the number 3:20 in SPVN* (as for Tablet 3).

20
3 20
1 40

There is again a possibility of error if one does not change cycle. Paying attention to *positioning* makes it possible to avoid this error (there may be other ways to do so). The previous entry for the number 1:40 in SPVN, actually corresponds to the area measurement 5 še.

Interestingly, the correct entry is not in the table⁴¹ and must be *extrapolated* to find 1/12 še. This step may not exist if the smallest part of the measurement value is neglected. This is what Proust's reconstruction suggests for this tablet. It is possible to refer to Tablet 3 which uses the same value. Unfortunately, as the scribe made a mistake, it is not possible to know if the result would indeed have been neglected.

Highlights for Tablet 5

In summary, this tablet requires several sub-tasks. They are similar to those used in the two previous tablets (Tablets 3 and 4).

This tablet has the particularity of proposing, for one of the three steps for providing a correspondence, an *extrapolation* (as the correct entry does not exist in the table), with a risk of error (there is an incorrect entry present in the table) if one does not do not pay attention to *positioning*. Note this extrapolation step may not have been performed at all, if the 1/12 še area measurement was neglected.

The use of identical measurement values in tablets 1 and 4 (for 1/3 gin) and tablets 1 and 3 (for 1/12 še) should also be noted.

Tablet 6

Translation of Tablet 6 (Proust, 2007, p.194)⁴²

1:45
1:45
<3:3:45>

1/3 kuš 1/2 šu-si on each
side (of the square).
What is its surface?
Its surface is*
9 še 1/5 + 3 še

Let us focus on the tasks that the choice of the measurement values implies. The order of magnitude is again that of the length of a brick.

This time there is a peculiarity related to providing a correspondence by *interpolation* (in two steps) for the composed measurement value. It should also be noted that the length measurement value 1/3 kuš, already appears in Tablet 3.

⁴¹ In the composite table, as it has been reconstructed from many tablets.

⁴² IM 57828. "9 še igi-5 3 še-kam", after Proust (2007, p.194). Note* According to Neugebauer and Sachs (1984, p. 246-248) and a photograph (Steele 1951, p. 25), Proust (2007, p. 194) restores the line (*a-ša-bi*), missing in Neugebauer and Sachs (1984), and confirms the end of the text (the reading of *še-kam* was indicated as uncertain in their edition).

The entry $1/3$ *kuš* exists in the metrological table. It corresponds to the number 1:40 in SPVN.

On the other hand, the entry $1/2$ *šu-si* does not exist in the metrological table. *Extrapolation* must then be carried out. One hypothesis is that the correspondence that associates 1 *šu-si* with the number 10 in SPVN makes it possible to provide a correspondence for $1/2$ *šu-si* to the number 5 in SPVN.

1 <i>šu-si</i>	10
2 <i>šu-si</i>	20
3 <i>šu-si</i>	30
4 <i>šu-si</i>	40
5 <i>šu-si</i>	50
6 <i>šu-si</i>	1
7 <i>šu-si</i>	1:10
8 <i>šu-si</i>	1:20
9 <i>šu-si</i>	1:30
$1/3$ <i>kuš</i>	1:40

Table 9: extract from metrological table of length
HS 241 (Proust, 2016, p.8)

Multiplying 1:45 by itself requires a *multiplicative algorithm* to produce the result: 3:3:45.

Providing a correspondence for the number 3:3:45 in SPVN is done by *interpolation*, in several steps. The order of magnitude for the measurement value corresponding to 3 is that of a brick. Thus, the selected cycle will be that of the *še*. Scribes could have provided a correspondence for 3 (which gives 9 *še*, it is an entry in the metrological table of surfaces), for 3:20 (which gives $1/6$ *še*) and for 25 (which gives $1/48$ *še*).

$1/2$ <i>še</i> of silver	10
1 <i>še</i>	20
1 $1/2$ <i>še</i>	30
2 <i>še</i>	40
2 $1/2$ <i>še</i>	50
3 <i>še</i>	1
4 <i>še</i>	1:20
5 <i>še</i>	1:40
6 <i>še</i>	2
7 <i>še</i>	2:20
8 <i>še</i>	2:40
9 <i>še</i>	3
10 <i>še</i>	3:20

Table 10: extract from metrological table of surfaces
MS 2186 (Friberg, 2007, p.110-112)

The order of magnitude for the number 3:20 in SPVN requires selecting the *cycle preceding* that of 9 *še*. This is reminiscent of the previous example, but here it is not the same order of magnitude: the entry providing a correspondence between the number 3:20 in SPVN and 10 *še* would not be suitable. Although once again, this step may not exist if the small parts of *še* are neglected. The measurement value corresponding to 25 belongs to the cycle before that of 10 *še*. This step may also be ignored if the result is neglected.⁴³

$$\begin{array}{r} 3 \\ 3 \quad 20 \\ \quad (25) \\ \hline \end{array}$$

⁴³ I put this part of the number in SPVN in parentheses because this step may not take place.

The expression of the measurement value is composed, and should be 9 še 1/6 še and 1/48 še.⁴⁴ It should be noted that the actual result expressed on the tablet is difficult to understand. Proust translates it this way: 9 še 1/5th + 3 še. Middeke-Conlin (2020, p.168-169) explains that the result could theoretically be expressed in this form: 9 še a 6th of še + an 8th of a 6th of še.⁴⁵ It would then be the same form of expression, but with appropriate values: 9 še a 5th of še of (one) third of še. Middeke-Conlin calculates a gap percentage of 1.32% with the expected value, which he attributes to rounding.⁴⁶ In any case today, the inscription remains unclear, and there is no copy or access to the original for this tablet.

Highlights for Tablet 6

In summary, this tablet bears several interesting features. The sub-tasks are similar to those for the previous three tablets, but providing a correspondence by *interpolation* for 1/3 kuš 1/2 šu-si requires *extrapolation*. There is thus extrapolation of the two last correspondences, or maybe these measurements of area are neglected which would imply that here the last extrapolation tasks do not occur.

3 Results: Summary of tasks and sub-tasks

This six-tablet corpus from Nippur allowed us to identify main tasks and sub-tasks. The main tasks are required for all the tablets, whereas the sub-task are not. It is important to note that the use of sub-tasks depends on the choice of the length measurement of the side of the square given in the statement. I represent the sub-tasks with italics.

- 1. The first main task is providing a correspondence for a length measurement with a number in SPVN.
 - This sometimes implies a sub-task if the measurement of length is between two entries existing in the table. It can be *interpolation* of a composed measurement value with a number in SPVN. This is done in a maximum of two steps in the present corpus.
 - This sub-task involves *positioning*: paying attention to sexagesimal positions of the numbers when adding intermediate numbers in SPVN.

Then, addition(s) of two numbers are carried out. There can be one or several additions.

- Another possibility when the entry is not present in the table, is *extrapolation* when the measurement of length is smaller than the smallest entry of the table.
- 2. The second main task is multiplication of a number in SPVN by itself.
 - This sometimes implies a sub-task: using a *multiplication algorithm* to multiply a number in SPVN by itself when the number is not present in the curriculum's elementary multiplication tables.
- 3. The third main task is providing a correspondence⁴⁷ for a number in SPVN with an area measurement.

⁴⁴ Or, 9 še 3/16 še.

⁴⁵ The text: 9(diš) še igi 5(diš) 3(diš) še-kam, is translated by Middeke-Conlin as “9 še one-5th (of) a 3rd is of še”

⁴⁶ The author uses statistics on the percentage deviations in the text to support his hypotheses.

⁴⁷ Another possible skill is conceivable: knowing that the beginning of the metrological table of surfaces corresponds to the beginning of the table of weights for še and gin. In fact, I have proposed in the Appendix a metrological table of surfaces, but the first part is common to that of weights. In the sources at our disposal, this part of the metrological table of surfaces appears only on the metrological table of weights. The metrological

This implies the main task of selecting the correct cycle with the expected order of magnitude of the result.

- A possible sub-task is *extrapolating* one or two measurement values inferior to the first entry in the table (small parts of *še*); or *truncating* or *rounding* them.
- Another possible sub-task is *providing a correspondence*, using *interpolation*, for a number in SPVN and with an area measurement (this takes 1 to 6 steps in this corpus).
 - This sub-task involves *circulating* within the metrological table: within a cycle or from one cycle to the preceding one.
 - This might be linked to *positioning*: using sexagesimal positions to select the correct cycle (and sometimes there is a risk of error).

3.1 Summary Table: steps of problem resolution and number of sub-tasks

For the six Nippur tablets I will detail the following points.

1. The order of magnitude for the length measurement of the side of the square.
2. The type of number in SPVN obtained using the metrological table of lengths. I also indicate whether providing a correspondence for the number in SPVN and the area measurement is *direct* (using an entry belonging to the table) and/or in one step, by the terms: "*direct*", and "*one step*" or if it was necessary to combine several entries from the table (I use the term: "*interpolation*").
3. The type of multiplication. I indicate if it is *elementary* (if it is part of the tables in the curriculum) or if it is a multiplicative algorithm (by the term: "*non elementary*").
4. With regard to the area measurement obtained using the metrological table of surfaces, I indicate whether the expression of the measurement value is *simple* (e.g. $1/3$ *kuš* for instance) or *composed* (e.g. $1/3$ *kuš* 3 *šu-si*). I also indicate if the area measurement is given directly in one step or indirectly: by *interpolation*, and / or by "*extrapolation*" (in the case of an entry not present in the table).
5. The order of magnitude for the result.
6. In the first column I summarize the number of sub-tasks implied by the choice of the length measurement given in the statement of the problem

table of surfaces begins with higher values (in written sources in any case). It is difficult to say whether, for the scribe, the beginning of the metrological table of surfaces was learned individually or if, when he learned it, the table was presented to him as that of weights.

	Order of magnitude for the side	Type of number in SPVN	Type of multiplication	Providing a correspondence for the area measurement: direct, interpolation, extrapolation	Order of magnitude for the result and cycle circulation
Tablet 1 1 or 0 sub-tasks	1 <i>kuš</i> -simple -length of a large brick ⁴⁸	5 -main task -direct -1 sexagesimal position -regular number	-main task -elementary table of 5 -result: 1 sexagesimal position	25 → 1/3 <i>gin</i> 15 <i>še</i> -direct -interpolation, -2 steps: 20 → 1/3 <i>gin</i> and 5 → 15 <i>še</i> or: -direct, -1 step, depending on the metrological table chosen (main and maybe sub-tasks) -no cycle change	1/3 <i>gin</i> 15 <i>še</i> -selection of the correct cycle ⁴⁹ (corresp. for 20 ⁵⁰) -then circulation above within the same cycle (corresp. for 5)
Tablet 2 1 extrapolation	2 <i>šu-si</i> -simple -length of a finger (smaller than a tablet length) ⁵¹	20 -direct -regular number	-main task -elementary table of 20 -result: 1 sexagesimal position	-indirect, -1 step: extrapolation 6:40 → 1/3 <i>še</i> -main task	1/3 <i>še</i> -1 step -selection of the correct cycle: -entry not present in the metrological table

⁴⁸ See the table in Appendix and Proust (2007, p.162). From 12 *šu-si* to 1 *kuš* (20 to 50 cm). See also metrological tables.

⁴⁹ Order of magnitude of a large brick, about 250 dm²

⁵⁰ From 12 1/2 *še* to 1/3 *gin* (4 to 250 dm²)

⁵¹ From 2 *šu-si* to 1/2 *kuš* (3,4 to 25 cm)

<p>Tablet 3 corrected</p> <p>main and sub-tasks:</p> <p>interpolation (2 steps)</p> <p>1 extrapolation</p> <p>1 cycle change with risk of error</p>	<p>1/3 <i>kuš</i> 3 <i>šu-si</i></p> <p>-composed measurement value</p> <p>-length of a brick</p>	<p>2:10</p> <p>-direct interpolation, 2 steps:</p> <p>1/3<i>kuš</i> → 1:40 and 3 <i>šu-si</i> → 30</p> <p>-non regular</p>	<p>-algorithm</p> <p>-result: 3 sexagesimal positions</p>	<p>corrected: 4:41:40</p> <p>-interpolation, -2 steps</p> <p>4:40 → 14 <i>še</i></p> <p>direct</p> <p>1:40 → 1/12 <i>še</i></p> <p>extrapolation or rounding</p>	<p>corrected:</p> <p>14 <i>še</i> 1/12 <i>še</i></p> <p>-selection of the correct cycle then above (risk of error)</p>
<p>Tablet 3 Scribe's (erroneous) work</p> <p>1 interpolation (2 steps)</p> <p>1 extrapolation</p> <p>1 cycle change</p>	<p>Idem</p>	<p>Idem</p>	<p>Idem with mistake</p>	<p>Scribe found 4:26:40</p> <p>-interpolation, 2 steps</p> <p>4:20 → 13 <i>še</i></p> <p>direct</p> <p>6:40 → 1/4 <i>še</i></p> <p>extrapolation</p>	<p>Scribe found 13 <i>še</i> 1/4 <i>še</i></p> <p>-selection of the correct cycle</p> <p>-then above, in the cycle</p>
<p>Tablet 4</p> <p>main and sub-tasks</p> <p>interpolation (3 steps)</p> <p>1 extrapolation</p> <p>2 cycle changes with 1 risk of error</p>	<p>1 <i>kuš</i> 2 <i>šu-si</i></p> <p>-composed measurement value</p> <p>-length of a large brick (see values from tables 1 and 2)</p>	<p>[5 :20]</p> <p>-interpolation, -2 direct steps</p> <p>1 <i>kuš</i> → 5</p> <p>2 <i>šu-si</i> → 20</p> <p>-regular</p>	<p>-algorithm</p> <p>-result: 3 positions</p>	<p>28:26:40</p> <p>-interpolation, -3 steps</p> <p>20 → 1/3 <i>gin</i> direct</p> <p>8:20 → 25 <i>še</i> direct</p> <p>6:40 → 1/3 <i>še</i></p> <p>extrapolation (Scribe wrote 2/3 and not 1/3)</p>	<p>1/3 <i>gin</i> 25 <i>še</i></p> <p>1/3 <i>še</i></p> <p>-selection of the correct cycle C1, -then above cycle C2, -then above cycle C3, (with risk of error)</p>

<p>Tablet 5</p> <p>main and sub-tasks</p> <p>interpolation 2 to 3 steps</p> <p>1 extrapolation or: rounding</p> <p>2 cycle changes</p> <p>1 risk of error</p>	<p>$2/3$ <i>kuš</i> 9 <i>šu-si</i> - composed measurement value -length of a large brick⁵² (see values from tables 1, 2, 3)</p>	<p>4:50 -interpolation, -2 direct steps $2/3$ <i>kuš</i> → 3:20 9 <i>šu-si</i> → 1:30 -non regular</p>	<p>-algorithm -result: 3 sexagesimal positions</p>	<p>23:21:40 -interpolation, -3 steps 20 → $1/3$ <i>gin</i> -direct 3:20 → 10 <i>še</i> -direct 1:40 → $1/12$ <i>še</i> (extrapolation or rounding)</p>	<p>$1/3$ [<i>gin</i> 10 <i>še</i>] -selection of the correct cycle C1, -then above cycle, C2, -then above cycle C3 (with risk of error)</p>
<p>Tablet 6</p> <p>4 sub-tasks</p> <p>+ 1 extrapolation when providing a correspondence to get number in SPVN</p>	<p>$1/3$ <i>kuš</i> $1/2$ <i>šu-si</i> -composed measurement value -length of a small brick (see value from Tablet 3 for $1/3$ <i>kuš</i>)</p>	<p>1:45 -interpolation, -2 steps $1/3$ <i>kuš</i> → 1:40 -direct $1/2$ <i>šu-si</i> → 5 -extrapolation -non regular</p>	<p>-algorithm -result: 3 sexagesimal positions</p>	<p>3:3:45 -interpolation, -1 to 3 steps 3 → 9 <i>še</i> -direct 3:20 → $1/6$ <i>še</i> -extrapolation 25 → $1/48$ <i>še</i> -extrapolation + rounding</p>	<p>9 <i>še</i> $1/5 + 3$ <i>še</i> -selection of the correct cycle C1, -then above cycle C2, -then above cycle C3</p>

Table 11: Summary of steps for problem resolution on Tablets 1 to 6

3.2 Summary of tasks for each table, depending on the variation of measurement values

Now that I have explained the tasks and sub-tasks, I present a summary of the small variations corresponding to the numerical values for the length of the side of the squares found in the Nippur corpus (tablets 1 to 6) and their implication at each stage of the resolution of the problem. This form of presentation allows me to compare the values chosen and highlight redundancies or differences. I end by summarizing in a table, the tasks and sub-tasks present for each tablet.

1. Expression of the length measurement

- The measurement units that come into play in the six tablets are *kuš* and *šu-si*. For tablets 1 and 2 the expression of the measurement value is **simple**. For tablets 3 to 6, it is **composed**.
- **Some measurement values are recurrent:** in Tablet 4 there is a use of the measurement value 1 *kuš*, that was already present in Tablet 1, and the measure value 2 *šu-si*, that was present in Tablet 2. For Tablet 4, a part of the measurement value ($1/3$ *gin*) is present in tablets 1 and 4 and another part ($1/12$ *še*) is present in Tablet 3. In Tablet 6, the measurement value $1/3$ *kuš* that was already present in Tablet 3 is used, as a part of a composed measurement value.

2. Correspondence for a length measurement and a number in SPVN

⁵² From 12 *šu-si* to 1 *kuš*: 20 to 50 cm

- In tablets 3 to 6 of the corpus, the numbers in SPVN are: “2:10”, “5:20” (according to Proust’s reconstruction), “4:50” and finally “1:45”. They are expressed with two sexagesimal positions.
- These numbers in SPVN are obtained by *interpolation*, in two steps.
- For the last number it is necessary to *extrapolate*, as providing a correspondence for 5 with $\frac{1}{2} \text{ } \check{s}u\text{-}si$ does not appear in the metrological tables at our disposal.

3. Multiplication

- The last four Nippur tablets stand out because the corresponding multiplication tables do not exist (according to the documents at our disposal), which implies using a **multiplication algorithm**.⁵³
- ### 4. Correspondence for numbers in SPVN and area measurements
- In Tablet 1, the correspondence for the area measurement is provided by *interpolation*, which requires two steps.⁵⁴ These steps are done without using extrapolation as the entries already exist in the metrological table. There is also no need to change cycle.
 - In Tablet 2, there is only one step, but it implies the sub-task that I called *extrapolation*.
 - In Tablet 3, *extrapolation* is used as well. Moreover, the values imply a two-step *interpolation*.⁵⁵ Attention also needs to be paid to *cycles*: there is a possibility of making a mistake if the previous cycle is not selected, as the current cycle also offers an entry for the number 1:40 in SPVN.
 - For the last three tablets (Tablets 4 to 6), there are a maximum of 3 steps. The second and third correspondences require **selecting the previous cycle, sometimes with a risk of error**. There is a need to *extrapolate* at least one of the steps (even two in Tablet 6).⁵⁶

3.3 Precautions

General precautions: series of problems & teaching purposes

Working with a set of similar exercises has the advantage of being able to fit into the wider debate on a *series of problems*, widely studied by historians, which leads us to express precautionary points. However, can we consider this set of tablets as a *series of problems* presenting a teaching logic? The selection of tablets comes from the same period in Nippur. Thus, it is legitimate to compare these texts given their great historical proximity. The notion of “tasks” depends on an overall logic: building a form of progression based on prior learning. It is therefore necessary to ask what the methodological implications are of looking for pedagogical intention based on a set of texts that may have been conceived by different users at different dates. From the point of view of a possible pedagogical progression, the scribal curriculum allows us to question the link that could unite these texts. The work published by Bernard and Proust (2014) leads us to beware of spontaneous interpretations of texts as automatically linked to teaching, like modern textbooks. Their study shows that nothing is known about the actual use of certain texts that may seem to have been written for teaching (or at least according to a certain pedagogical ideal).

The problems could be written in a very similar way to allow a better memorization of their structure. The logic could be that of the collection of problems, which circulated by tradition (orally or not), and thus presented differing degrees of variation; the series could also obey ideals, related to the written tradition. The educational goal is only one hypothesis among others, and the goal itself could take many forms. The variations observed do indeed have consequences on the tasks performed by the pupils in solving the problem. Did the authors of the exercises reflect on these consequences? Nothing precludes a form of chance; a tradition of learning exercises related to the transmission of a common

⁵³ For these tablets, the number in SPVN that is obtained by multiplication bears three sexagesimal positions.

⁵⁴ It could also be only one step, depending on the metrological table adopted by the scribe.

⁵⁵ Providing a correspondence for 4:40 using the metrological table, and then for 1:40, by *extrapolation*.

⁵⁶ Some measurement values that are very small parts of $\check{s}e$, might be neglected or rounded rather than extrapolated.

knowledge, or other objectives. It is also possible to consider that the authors had other projects related to teaching that depended on other lessons. I will try to point out arguments to deal with this issue.

Moreover, the use of these tablets remains unknown. While the school character is attested and they have been interpreted as exercises, it is not excluded that they served a different purpose, for example as teaching materials for the teacher, whom would have constructed his discourse on a selection of examples. The analysis of the tasks involved by the choice of the numerical values in the statement may help to provide arguments on the use that was actually made of them.

Finally, some tasks may have been obvious at this stage of schooling. It is conceivable that the tasks all bore the same level of complexity for the student at this stage of the curriculum. To what extent can these variations be interpreted as the result of pedagogical intention? This would imply a form of “complexification” based on prior learning considered acquired. The description puts all the tasks on the same plane, without ranking them from the point of view of their possible difficulty, or in order of chronological sequence. Subsequently, I will try to make assumptions about the possible order in the curriculum, relying on Proust’s reconstruction (2007, p.57-58).

Precautions regarding the tables

In the first column of Table 11, I recapitulate the subtasks by tablet. It should be noted that it has the drawback of smoothing out the differences between the tablets. Indeed, I made the choice to show only the following sub-tasks in my counting system. They seem to me to be more particularly relevant of a specific type of learning:

- interpolation for finding the number in SPVN
- interpolation for finding the area measurement
- extrapolation for finding the area measurement
- extrapolation of the number in SPVN
- multiplicative algorithm

Interpolation and extrapolation involve other sub-tasks (circulation in cycles, positioning) that I did not count. One task can also be repeated: I did not use this information in this particular summary column. I made these counting choices, to allow the highlighting of the differences on points which seem to me to reveal the presence of key learning. However, it should be noted that the presence of more steps could also bear witness of hypothetical pedagogical choices.

Extrapolation is a sub-task that depends both on the content of the chosen metrological table and on the choice of rounding/truncating or not. When tablets are broken, it is difficult to imagine if the sub-tasks do exist or if the result was rounded, which would imply not extrapolating.

3.4 Conclusion and hypotheses emerging from the results

Sub-groups of tablets

I based my analysis on a selection of six tablets from Nippur dealing with the measurement of the area of a square, made by Proust (2007, p.190-197). The study of the exercises noted on these tablets makes it possible to distinguish “main tasks” and “sub-tasks”. As mentioned in Section 2, I have called “sub-task” a task that is required as a result of the choice of measurement value for the length measurement of the side of the square in the statement of the problem, but is not common to all the tablets.

The main tasks are:

- providing a correspondence for a measurement value with a number in SPVN,
- multiplying a number by itself,
- providing a correspondence for a number in SPVN with the area measurement;
 - this involves the selection of the relevant cycle in the metrological table of surfaces.

The “sub-tasks” that are present on some tablets, are:

- the *extrapolation* of a length measurement, the *interpolation*⁵⁷ to provide a correspondence with a number in SPVN,
 - with attention paid to *positioning*;
- a non-elementary multiplication based on an *algorithm*,
- *interpolation* to provide an area measurement,⁵⁸
 - this involves the circulation from one cycle to the previous cycle or within the same cycle (with attention to the *positioning*),
- the possible *extrapolation* (implying rounding or not); and / or the choice of directly truncating some values.

Given this analysis in terms of main tasks and sub-tasks that depend on the choice of numerical values of the statement, I propose the following groupings that provide a distinction between the tablets in the use of main and sub-tasks:

1. Tablet 1 & Tablet 2:

Tablet 1 proposes only one sub-task,⁵⁹ or even none, according to the metrological table adopted. Tablet 2 proposes only one sub-task (*extrapolation*).

2.

2.1) Tablets 3, 4 and 5, offer many sub-tasks:

- interpolation* to provide the number in SPVN, use of a multiplicative *algorithm*,
- interpolation* to obtain the area measurement (implying several steps),
- extrapolation* (or choice of truncating part of the measurement value of the result),
- cycle changes*⁶⁰ with or without a risk of error, attention paid to *positioning*.

2.2) Tablet 6, which presents the same sub-tasks as the previous group of tablets but also presents a length measurement whose corresponding number in SPVN is found by *extrapolation* (since $1/2 \text{ šu-si}$ is not in the metrological tables of length that have come down to us).

It should be noted that it is conceivable that some of the sub-tasks I have distinguished are not considered difficult or new by the scribes. For example, if the multiplicative algorithm is well acquired, this sub-task may have no impact on the (hypothetical) choice of the exercise developer. However, the scribe's error confirms, for Tablet 3, that non-elementary multiplications might not all be "obvious".

Interpretation of the groups in terms of pedagogy

What is it possible to infer from this grouping of the six tablets from Nippur?

The first group (Tablets 1 and 2) is clearly distinct from the second (Tablets 3 to 6) in using sub-tasks or not, which could show an increasing level of difficulty. On this point, the hypothesis of educational choices emerges. It is conceivable that the teacher might have used sub-tasks progressively, not to introduce too many difficulties at the same time, or that some teachers may have targeted some mathematical work on sub-tasks while other teachers might not have.

Looking more precisely at sub-tasks shows the *importance of navigation within metrological tables*. The second group, is strikingly different compared to the first group, as it involves several steps

⁵⁷ Carried out in two steps maximum in this corpus.

⁵⁸ Carried out in 3 steps maximum in this corpus.

⁵⁹ This is the *interpolation* of the area measurement.

⁶⁰ It should be noted that the risk of error (which I have pointed out due to the existence of intermediate entries corresponding to the number in SPVN searched for in the metrological table of surfaces, but which should not be selected), is perhaps not one: if the cycle change is a very automatic task, these "false-friend tasks" may not be taken into account at all in the hypothetical teacher's pedagogical intention.

in providing a correspondence to obtain the area measurement and attention to cycle changes. Thanks to the information on the curriculum at our disposal, it is moreover conceivable that this navigation within and between cycles is intrinsically linked to this stage of the school curriculum, as the metrological tables are related to the learning of the area calculation.

Therefore, this makes it possible to consider navigating within metrological tables as a teaching objective, possibly progressive, linked to these sub-tasks.

This hypothesis is reinforced in terms of the choice of numerical values, as there are clearly common values in this small selection of tablets.⁶¹ It is still too early to understand these similarities with certainty, for which it would take a wider corpus. But they allow us to infer on the existence of a choice and / or a tradition.

The choice of measurement value in the statement of the problem also implies whether to *extrapolate* or not. This is a common sub-task for five out of the six tablets. It could be argued that today we are only aware of some of the metrological tables, and there might have been others, which would make the hypothesis of such type of work less plausible. This however seems unlikely, as there is great stability in the metrological tablets used for establishing the composite table. The hypothesis of work on extrapolation is therefore conceivable. Some fractions of $\frac{1}{2}$ were perhaps also neglected or rounded. Extrapolation or approximation could thus be part of a learning objective.

Conclusions

I have analyzed the choice of the initial length measurement for the side of a square in the statements of a group of six tablets from Nippur⁶² from the Old-Babylonian period, dealing with area calculation. The group was studied by Proust (2007, p.190-197). My approach was inspired by a didactic perspective on the texts, especially the notions of *didactic variables* (Brousseau, 1998). The work has benefited from precautions from historians working on interpretation of educational texts as well as *series of problems* (regarding the interpretation of similarities between exercises).

The analysis of the corpus made it possible to identify two types of tasks needed to solve a problem. There are main tasks, essential to be carried out for each tablet; and sub-tasks, present or not depending on the choice of the length measurement. There are therefore profound differences in these exercises of homogeneous appearance. The analysis led to distinguishing two groups of tablets, some bearing no, or very few, sub-tasks. This might show a progression in the difficulty of the exercises. These groups create strong distinctions between tablets of similar appearance, differences that do not appear without detailing the tasks and sub-tasks needed to solve the problems. The necessity of performing tasks to solve a problem depends strongly on the (apparently) small variations on the initial length measurement for the side of the square in the statement.

Another important conclusion of this work is that the grouping of the tablets reveals the importance of the sub-tasks related to navigating within the metrological tables and extrapolating new entries (or truncating results). The pedagogical interpretation can only be expressed in the form of hypotheses, but the position of this work in the curriculum suggests that these sub-tasks could be related to a teaching objective. The question of the place of tasks linked to using metrological tables is therefore particularly emergent from this study. It is likely that this type of mathematical work was indeed targeted, given the link between the area calculation exercise and the elementary curriculum of the scribes.

Opening questions

The educational interpretations can only be formulated as hypotheses, but they seem interesting to follow up in the future. They could be cross-referenced with other types of analysis on student

⁶¹ Tablet 4 shows measurement values used in other tablets within the composed expression. Some of these measurement values are the main simple measurement values for tablets 1 and 2. Tablet 6 uses values in common with Tablet 3.

⁶² Tablets: CBS 11318, UM 29-15-192, Ni 18, UM 55-21-076, IM 57846, IM 57828, NBC 8082 and NCBT 1913.

curricula It would also be interesting to investigate which measurement values were neglected or rounded (for example here, $1/12 \text{ še}$). The work of Middeke-Conlin (2020) seems to indicate that small fractions of še could indeed be truncated or rounded. These elements would make it possible to know if the task of extrapolation did exist systematically and thus, to know whether this sub-task could result, or not, from a choice of the measurement values (variations on the length measurement of the side of the square) by the teacher. In addition, to advance in the metrological findings, it would be interesting to work with Middeke-Conlin's data (2020) on values that have been rounded in other contexts in the same period, to allow us to advance in the hypotheses on the study on sub-tasks.

It is also striking to note that the six Nippur exercises imply the use of the same lower part of the metrological table; unlike two other tablets found in places other than Nippur that I discuss in my doctoral dissertation (de Varent, 2018). The choice of these measurement values clearly indicates, the intention or the habit of working with small area measurements (with the order of magnitude of about a brick) for some exercises. There are many hypothetical reasons: such surface sizes could have been used in a context that remains unknown today, these surfaces could have served as basic standards for calculating larger areas, etc. Although, given the position of these exercises in the curriculum, and the sub-tasks described here, the hypothesis of exercises dedicated to work on a part of the metrological table seems plausible, perhaps especially around what could be the new mathematical task of extrapolation. Behind what seemed to be a single learning objective (calculating the area of a square), many others could hide. In any case, the assumption of a choice of numerical values that do not involve working on *all* the sub-tasks (when a potentially new teaching element is brought in) might suggest pedagogical intention to complexify the work progressively. This is corroborated by the distinctions between groups 1 and 2, which could be "level" groups. How are the sub-tasks relative to working on cycles (choosing the relevant cycle and the navigating from one cycle to the preceding one) taught? When does this teaching take place in the curriculum? Is it precisely this teaching that we are dealing with here?

The question of the position in the curriculum of tasks related to cycles and extrapolation emerges particularly from this study. Knowing the position of area calculations in the curriculum in relation to that of multiplicative algorithms, cycle navigation, rounding, etc. is important in order to be able to carry out a continuation of this reflection.

Methodology, curriculum, didactics and series of problems:

From a methodological point of view, the notion of *didactic variables*, which applies in the very precise framework of Brousseau's theory of didactic situations, allowed me to wonder about the effects of the choice of numerical values in similar exercises and led me to analyze them precisely. This historical analysis has made it possible to identify what is probably a situation of progressive transmission around the metrological tables. In (de Varent, 2018) I discuss other tools from education research to interpret differences with some tablets of unknown provenance.

These tablets may not belong to the same localization or exact time-period, and the teachers and students may be distinct. However, if we consider that the common point that brings them together is that of a progression of work on the metrological tables, tablets could belong to the same historical series, in terms of a progression in the school curriculum. Of course we must be cautious, since the curriculum could have evolved during the Old-Babylonian period, and the progression that I propose by distinguishing groups 1 and 2 is still formulated as hypothesis.

4 References

- Bernard, A., Proust, C. (Eds.), 2014. Scientific Sources and Teaching Contexts Throughout History: Problems and Perspectives. Springer, Dordrecht. <https://doi.org/10.1007/978-94-007-5122-4>
- Brousseau, G., 1998. Théories des situations didactiques. La pensée sauvage. <https://doi.org/10.4000/educationdidactique.1005>
- Delnero, P., 2012. Memorization and the Transmission of Sumerian Literary Compositions. Journal of Near Eastern Studies 71 (2), 189-208. <https://doi.org/10.1086/666645>
- Friberg, J., 2005. On the Alleged Counting with Sexagesimal Place Value Numbers in Mathematical Cuneiform Texts from the Third Millennium BC. https://cdli.ucla.edu/pubs/cdlj/2005/cdlj2005_002.html
- Friberg, J., 2007. A Remarkable Collection of Babylonian Mathematical Texts: Manuscripts in the Schøyen Collection: Cuneiform Texts I. Springer, New York. <https://doi.org/10.1007/978-0-387-48977-3>
- Hilprecht, H.V., 1906. Mathematical, Metrological, and Chronological Tablets from the Temple Library of Nippur. Philadelphia: University of Pennsylvania Museum of Archaeology and Anthropology. <https://www.upenn.edu/pennpress/book/863.html>
- Middeke-Conlin, R., 2020. The Making of a Scribe: Errors, Mistakes, and Rounding Numbers in the Old Babylonian Kingdom of Larsa. Springer, Cham. <https://doi.org/10.1007/978-3-030-35951-5>
- Neugebauer, O.E., Sachs, A.J., 1984. Mathematical and Metrological Texts. Journal of Cuneiform Studies 36, 243-251. <https://doi.org/10.2307/1360058>
- Ouyang, X., Proust, C., *forthcoming*. Place value notations in the Ur III period: marginal numbers in administrative texts. In: Chemla, K., Keller, A., Proust, C., (Eds.), Cultures of computation and quantification. Springer.
- Proust, C., 2007. Tablettes mathématiques de Nippur. Istanbul: Institut Français d'Études Anatoliennes. De Boccard. <http://www.deboccard.com/fr/category/11361-Produit-9782906053922.html>
- Proust, C., 2008. Tablettes mathématiques de la collection Hilprecht. Harrassowitz, Wiesbaden. https://www.harrassowitz-verlag.de/reihe_250.ahtml

- Proust, C., 2013. Du calcul flottant en Mésopotamie. *Gazette des Mathématiciens* 138, 23-48.
<https://smf.emath.fr/node/109902>
- Proust, C., 2016. Mathématiques en Mésopotamie: étranges ou familières? In: Theis, L. (Ed.), *Proceedings from the EMF Conference 2015*. Algiers, Algeria, 2015. pp. 17-39. <http://emf.unige.ch/emf-2015/>
- Proust, C., n.d. Une numération sexagésimale de principe additif en Mésopotamie: le système S.
<https://cm2.ens.fr/print.php?nid=2111&print=yes>
- Robson, E., 2000. *Mathematical Cuneiform Tablets in Philadelphia. Part 1: problems and calculations. Sources and Commentaries in Exact Sciences* 1, 11-48. <https://www.sciamvs.org/2000.html>
- Robson, E., 2001. The tablet House: a scribal school in old Babylonian Nippur. *Revue d'assyriologie et d'archéologie orientale* 93 (1), 39-66. <http://doi.org/10.3917/assy.093.0039>
- Robson, E., 2002. More than metrology: mathematics education in an Old Babylonian scribal school. In: Steele, J.-M., Imhausen, A. (Eds.), *Under One Sky. Astronomy and Mathematics in the Ancient Near East*. Ugarit-Verlag, Münster, pp. 325-367. <https://ugarit-verlag.com/products/under-one-sky-astronomy-and-mathematics?variant=31649385939006>
- Steele, F., 1951. Writing and History: The New Tablets from Nippur. *University Museum Bulletin* 16 (2), 21-27. <https://www.penn.museum/sites/bulletin/3454/>
- Varent (de), C. (2018). Pluralité des concepts liées aux unités de mesure : liens entre histoire des sciences et didactique, le cas de l'aire du carré dans une sélection de textes anciens [Doctoral dissertation, Sorbonne Paris Cité University]. (NNT : 2018USPCC106). <https://hal.archives-ouvertes.fr/tel-02411352/>
- Veldhuis, N.C., 1997. *Elementary education at Nippur. The lists of trees and wooden objects*. University of Groningen. [https://www.rug.nl/research/portal/publications/elementary-education-at-nippur-the-lists-of-trees-and-wooden-objects\(d7539f72-9851-4b83-841f-886ef19cb1be\)/export.html](https://www.rug.nl/research/portal/publications/elementary-education-at-nippur-the-lists-of-trees-and-wooden-objects(d7539f72-9851-4b83-841f-886ef19cb1be)/export.html)

5 Appendices

Composite text of metrological tables from Nippur

Lengths

Table 1 – Lengths from 1 šu-si to 1 UŠ⁶³

The table gives the corresponding number in SPVN for a given measurement of length. I chose to represent it on two columns to save space.

1 šu-si	10	1/2 ninda 4 kuš	50
2 šu-si	20	1/2 ninda 5 kuš	55
3 šu-si	30	1 ninda	1
4 šu-si	40	1 1/2 ninda	1:30
5 šu-si	50	2 ninda	2
6 šu-si	1	2 1/2 ninda	2:30
7 šu-si	1:10	3 ninda	3
8 šu-si	1:20	3 1/2 ninda	3:30
9 šu-si	1:30	4 ninda	4
1/3 kuš	1:40	4 1/2 ninda	4:30
1/3 kuš 1 šu-si	1:50	5 ninda	5
1/3 kuš 2 šu-si	2	5 1/2 ninda	5:30
1/3 kuš 3 šu-si	2:10	6 ninda	6
1/3 kuš 4 šu-si	2:20	6 1/2 ninda	6:30
1/2 kuš	2:30	7 ninda	7
1/2 kuš 1 šu-si	2:40	7 1/2 ninda	7:30
1/2 kuš 2 šu-si	2:50	8 ninda	8
1/2 kuš 3 šu-si	3	8 1/2 ninda	8:30
1/2 kuš 4 šu-si	3:10	9 ninda	9
2/3 kuš	3:20	9 1/2 ninda	9:30
2/3 kuš 1 šu-si	3:30	10 ninda	10
2/3 kuš 2 šu-si	3:40	20 ninda	20
2/3 kuš 3 šu-si	3:50	30 ninda	30
2/3 kuš 4 šu-si	4	40 ninda	40
5/6 kuš	4:10	45 ninda	45
5/6 kuš 1 šu-si	4:20	50 ninda	50
5/6 kuš 2 šu-si	4:30	55 ninda	55
5/6 kuš 3 šu-si	4:40	1 UŠ	1
5/6 kuš 4 šu-si	4:50		
1 kuš	5		
1 1/3 kuš	6:40		
1 1/2 kuš	7:30		
1 2/3 kuš	8:20		
2 kuš	10		
3 kuš	15		
4 kuš	20		
5 kuš	25		
1/2 ninda	30		
1/2 ninda 1 kuš	35		
1/2 ninda 2 kuš	40		
1/2 ninda 3 kuš	45		

⁶³ The metrological table reconstructed by Proust (2007, p.314) continues up to 50 *danna*, *danna* being the measurement value following the UŠ.

Surfaces

Table 2 – Surfaces up to 2 (*bur*) GAN⁶⁴

The table gives, for a given measurement of area, the corresponding number in SPVN. Again, I chose to represent it on two columns to save space.

The first part of the table represented here is an extract from the composite table of weights, also used for surfaces.

1/2 <i>še</i> of silver ⁶⁵	10	1/2 <i>gin</i> 25 <i>še</i>	38:20
1 <i>še</i>	20	2/3 <i>gin</i>	40
1 1/2 <i>še</i>	30	2/3 <i>gin</i> 10 <i>še</i>	43:20
2 <i>še</i>	40	2/3 <i>gin</i> 15 <i>še</i>	45
2 1/2 <i>še</i>	50	2/3 <i>gin</i> 25 <i>še</i>	48:20
3 <i>še</i>	1	5/6 <i>gin</i>	50
4 <i>še</i>	1:20	5/6 <i>gin</i> 10 <i>še</i>	53:20
5 <i>še</i>	1:40	5/6 <i>gin</i> 15 <i>še</i>	55
6 <i>še</i>	2	5/6 <i>gin</i> 25 <i>še</i>	58:20
7 <i>še</i>	2:20	1 <i>gin</i>	1
8 <i>še</i>	2:40	1 1/3 <i>gin</i>	1:20
9 <i>še</i>	3	1 1/2 <i>gin</i>	1:30
10 <i>še</i>	3:20	1 2/3 <i>gin</i>	1:40
11 <i>še</i>	3:40	1 5/6 <i>gin</i>	1:50
12 <i>še</i>	4	2 <i>gin</i>	2
13 <i>še</i>	4:20	3 <i>gin</i>	3
14 <i>še</i>	4:40	4 <i>gin</i>	4
15 <i>še</i>	5	5 <i>gin</i>	5
16 <i>še</i>	5:20	6 <i>gin</i>	6
17 <i>še</i>	5:40	7 <i>gin</i>	7
18 <i>še</i>	6	8 <i>gin</i>	8
19 <i>še</i>	6:20	9 <i>gin</i>	9
20 <i>še</i>	6:40	10 <i>gin</i>	10
21 <i>še</i>	7	11 <i>gin</i>	11
22 <i>še</i>	7:20	12 <i>gin</i>	12
23 <i>še</i>	7:40	13 <i>gin</i>	13
24 <i>še</i>	8	14 <i>gin</i>	14
25 <i>še</i>	8:20	15 <i>gin</i>	15
26 <i>še</i>	8:40	16 <i>gin</i>	16
27 <i>še</i>	9	17 <i>gin</i>	17
28 <i>še</i>	9:20	18 <i>gin</i>	18
29 <i>še</i>	9:40	19 <i>gin</i>	19
1/6 <i>gin</i> ⁶⁶	10		
1/6 <i>gin</i> 10 <i>še</i> ⁶⁷	13:20		
1/4 <i>gin</i> ⁶⁸	15		
1/4 <i>gin</i> 5 <i>še</i> ⁶⁹	16:40		
1/3 <i>gin</i>	20		
1/2 <i>gin</i>	30		

⁶⁴ The metrological table reconstructed by Proust (2007, p.313) reaches up to 1 (*šar*₂)-gal *šu-nu-tag* GAN₂, another cycle and a half.

⁶⁵ The translation « of silver » corresponds to « *ku₃ babbar* » in the text.

⁶⁶ It is written: « *igi-6-gal₂ gin₂* »

⁶⁷ It is written: « *igi-6-gal₂ gin₂ 10 še* »

⁶⁸ It is written: « *igi-4- gal₂ gin₂* »

⁶⁹ It is written: « *igi-4- gal₂ gin₂ 5 še* »

The second part of the table represented here is an extract from the composite table of surfaces.

surface of 1/3 sar ⁷⁰	20	1(iku) GAN	1:40
1/2 sar	30	1(iku) 1(ubu) GAN	2:30
2/3 sar	40	2(iku) GAN	3:20
5/6 sar	50	2(iku) 1(ubu) GAN	4:10
1 sar	1	3(iku) GAN	5
1 1/3 sar	1:20	3(iku) 1(ubu) GAN	5:50
1 1/2 sar	1:30	4(iku) GAN	6:40
1 2/3 sar	1:40	4(iku) 1(ubu) GAN	7:30
1 5/6 sar	1:50	5(iku) GAN	8:20
2 sar	2	5(iku) 1(ubu) GAN	9:10
3 sar	3	1(eše) GAN	10
4 sar	4	1(eše) 1(iku) GAN	11:40
5 sar	5	1(eše) 2(iku) GAN	13:20
6 sar	6	1(eše) 3(iku) GAN	15
7 sar	7	1(eše) 4(iku) GAN	16:40
8 sar	8	1(eše) 5(iku) GAN	18:20
9 sar	9	2(eše) GAN	20
10 sar	10	2(eše) 1(iku) GAN	21:40
11 sar	11	2(eše) 2(iku) GAN	23:20
12 sar	12	2(eše) 3(iku) GAN	25
13 sar	13	2(eše) 4(iku) GAN	26:40
14 sar	14	2(eše) 5(iku) GAN	28:20
15 sar	15	1(bur) GAN	30
16 sar	16	1(bur) 1(eše) GAN	40
17 sar	17	1(bur) 2(eše) GAN	50
18 sar	18	2(bur) GAN	1
19 sar	19		
20 sar	20		
30 sar	30		
40 sar	40		
1(ubu) GAN	50		
1(ubu) GAN 10 sar	1		
1(ubu) GAN 20 sar	1:10		
1(ubu) GAN 30 sar	1:20		
1(ubu) GAN 40 sar	1:30		

Tablets references

Tablets are always cited in the text with reference to the corresponding CDLI number. The complete reference can be found in bibliography.

Tablet name	CDLI number	First Publication Other Publication	Number
CBS 11318	P254478	Neugebauer & Sachs (1984, p. 247, 251) Proust (2007, p.191)	1
UM 29-15-192	P254900	Neugebauer & Sachs (1984, p. 250-251) Proust (2007, p.193)	2
Ni 18	P368708	Proust (2007, p.193)	3

⁷⁰ It is written: “1/3 sar a-ša₃”

UM 55-21-076		Neugebauer & Sachs (1984, p. 246) Robson (2000, p. 25)	4
IM 57846		Neugebauer & Sachs (1984, p. 246)	5
IM 57828 Or : 2N-T 30		Steele (1951, p. 25) Neugebauer & Sachs (1984, p. 246-248)	6
NBC 8082	P254774	Neugebauer & Sachs (1945, p. 10) Nemet-Nejat ⁷¹ (1995, p. 260)	7
NCBT 1913	P254777	Neugebauer & Sachs (1945, p. 10) Nemet-Nejat (1995, p. 260)	8

Orders of magnitude

Tablet	1/2 še to 10 še (17 to 314 cm ²)
Brik	12 1/2 še to 1/3 gin (4 to 250 dm ²)
Yard	13 gin to 1/2 sar (8 m ² to 15m ²)
House	1 1/2 sar to à 3 1/2 sar (50 m ² to 120 m ²)

Table of orders of magnitude – extract from Proust (2007, p.162)

Details on circulating from one cycle to the previous one, and circulating inside a cycle

In some cases, as the measurement of area is obtained indirectly. There has to be an additive decomposition of number in SPVN, using the previous cycle, or navigating inside the cycle. Let us take the example of the number “1:46:40” in SPVN with the order of magnitude of a *sar*.⁷² Providing a corresponding measurement of area for the number 1:40 (the leftmost) gives the measurement of area 1 2/3 of *sar* (because of the expected order of magnitude). Then the number in SPVN: “6” (which follows one step to the right) must be looked for in the metrological table of surfaces, same cycle, higher in the table. This makes it possible to associate the measurement of area “6 *gin*” with the number 6 in SPVN.

⁷¹ Nemet-Nejat, K. R., 1995. Systems for learning mathematics in Mesopotamian scribal schools. JNES 54 (4), 241-260.

⁷² This example exists in tablet NBC 8082.

2/3 gin	40
2/3 gin 10 še	43:20
2/3 gin 15 še	45
2/3 gin 25 še	48:20
5/6 gin	50
5/6 gin 10 še	53:20
5/6 gin 15 še	55
5/6 gin 25 še	58:20
1 gin	1
<hr/>	
1 1/3 gin	1:20
1 1/2 gin	1:30
1 2/3 gin	1:40
1 5/6 gin	1:50
2 gin	2
3 gin	3
4 gin	4
5 gin	5
6 gin	6
7 gin	7
8 gin	8
9 gin	9
10 gin	10
11 gin	11
12 gin	12
13 gin	13
14 gin	14
15 gin	15
16 gin	16
17 gin	17
18 gin	18
19 gin	19
1/3 sar	20
1/2 sar	30
2/3 sar	40
5/6 sar	50
1 sar	1
<hr/>	
1 1/3 sar	1:20
1 1/2 sar	1:30
1 2/3 sar	1:40
1 5/6 sar	1:50

Extract from composite metrological table of surfaces (Proust, 2007, p.311)

Then, the number 40 in SPVN, which follows to the right, must still be looked for inside the metrological table of surfaces, in the cycle preceding that of number 6. This makes it possible to associate the measurement of area “2/3 of gin” with the number in SPVN 40. There is here proof of a committed competence, necessary to be able to carry out the exercise: that of knowing how to circulate a cycle to that which precedes.

To make it understandable, I materialized the cycle changes in the table with a black line. The final measurement of area is “1 2/3 sar 6 gin 2/3 gin”. To know in advance whether or not one has to change cycles, it is possible to break down the number into sexagesimal positions.

1 40
6
40

In this example, the cycle change is natural and is therefore probably made by the ancient scribes without even thinking about it. Going up higher up the table to find the match for the number 6, then for the number 40, shows no difficulty.

In some cases several values seem to match the number in SPVN up the table, and it is necessary to know the sexagesimal position of the number in SPVN to match it correctly (and for example, skip an erroneous value in SPVN).

Positioning

I therefore call “positioning” the possible special attention given by the scribe to choosing to circulate (or not) to the previous cycle, not to select a possible (but erroneous) measurement of area.

For this, the scribe can rely on sexagesimal positions of the number in SPVN that he wants to find a correspondence for.

Positioning can also mean giving a special care when providing a correspondence for a composed measurement of length into two numbers in SPVN, to be added.

1/48 <i>še</i>	25
1/24 <i>še</i>	50
1/12 <i>še</i>	1:40
1/6 <i>še</i>	3:20
1/5 <i>še</i>	4
1/4 <i>še</i>	5
1/3 <i>še</i>	6:40
1/2 <i>še</i>	10
1 <i>še</i>	20

Hypothetical technique of extrapolation

To obtain this reconstructed extrapolated table, scribes could have divided 20 by 2, to get the match for 1/2 *še*. The division of 20 by 3 gives the number 6:40 in SPVN (3 times 6:40 is 20) which makes it possible to obtain the correspondence for 1/3 *še*, etc.

Thus, some exercises might have been designed by the teachers to make the scribes work on this calculation, making extrapolation needed while choosing small lengths for the square's sides in the statements. It should be noted that if the tables do start with the entry 1/2 *še*, it is possible that this extrapolation exercise is not a practical exercise but rather an exercise designed for mathematical practice. It cannot be ruled out, however, that the missing table portion may have existed. This is unlikely (since the tablets on which the composite table is based do not bear lower entries) but this would call into question the hypothesis of an extrapolation mathematical work.⁷³

⁷³ In my vocabulary, extrapolation consists of using an entry below the smallest of the metrological table entries, (or beyond the largest entry), not between two existing entries (interpolation). It is not specific to the metrological table of surfaces: such action can occur for all tables. The extrapolation can also relate to providing a correspondence between a measurement of length and a number in SPVN. Or beyond the largest entry, but this case does not appear in the sources considered here.