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Abstract

The aim of this paper is to analyze the dynamic evolution of six liquidity proxies on time, and to find their causality with the French CAC 40 stock market index returns, over the period from January 2007 to December 2018. For that, we use a vector autoregressive approach and the impulse response function, to do causality test between the CAC 40 index returns and six differents liquidity proxies. Empirical results suggest a significant short-term relationship between the returns and the liquidity. As for Granger's causality test, the results reveal that there is unidirectional causality running from equity returns to liquidity.

Keywords: Liquidity risk, Market risk, VAR model, Granger causality, Impulse response function, CAC 40 Market.
JEL Code: C58, C22, G10, G15.

1 Introduction

In asset pricing theory, expected returns on equities are assumed to be sensitive in some way to some factors such as consumption and investment opportunities. The liquidity of an asset may impact the short-term consumption or investments decisions of an economic agent. Illiquidity is a key risk factor, which may adversely affect expected returns and, as a result, investors would be expected to earn a liquidity risk premium. Financial institutions and regulators rely on empirical studies supporting the dynamic causality between the returns of an asset or a market index and the dynamic evolution of liquidity. For a specific market (e.g. the CAC 40 index, in our paper), empirical studies could thus help investors to better assess their expected returns, but also help financial institutions to better monitor market risk. This paper examines whether market-wide liquidity is effectively priced. In other words, we investigate whether cross-sectional differences in expected stock returns are related to the sensitivity of returns to liquidity fluctuations. For the reasons mentioned above, and depending on the markets and their some their characteristics, many researchers have been interested in the interrelation that would exist between ex-post returns and liquidity. In the present paper, we adopt the same type of analysis for the French CAC 40 stock market over the period 2007-2018.

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In recent scientific literature, liquidity risk has been a focus of interest in both academic and professional research publications. Liquidity is a broad term that can refer to the liquidity of a market, an asset, a fund or a portfolio, or even the liquidity provided by a central bank. A financial asset is considered to be liquid when it can be bought and sold in large quantities over a relatively short period of time without significantly affecting its price. As noted by Foucault et al. (2013), in a liquid market it is possible to trade a considerable quantity of shares at low cost with minimal impact on the price.

The financial crisis of 2007-2008 was essentially characterized by a contagious contraction of liquidity on the money and financial markets, and it called into question the hitherto neglected management of liquidity risk in favor of other risks such as credit risk or market risk. Some authors such as van den End and Tabbae (2012) have noted that the 2007 crisis developed in three stages: first market liquidity risk, followed by funding liquidity risk, and then counterparty risk, ultimately leading to a liquidity problem for banks. In the final stage, the tightening of the conditions on the credit market created an economic slowdown. As a result, one of the stakes of financial regulation is the prevention of systemic liquidity crises that can spread to most financial institutions and markets, and not only to banks or specific markets.
Recent changes in the structure of financial markets have also raised new questions about liquidity. In particular, the development of automated trading systems and the growing prevalence of high-frequency trading have stressed the importance of regulations in the stabilization of the market and the improvement of liquidity. On the side of financial market regulators, Harris (2003) considers that one of their objectives is to ensure long-term liquidity, since less volatile markets are able to attract a large number of buyers and sellers and display the least uncertainty about volume, transaction costs, etc. As a result, long-term liquidity is crucial to the efficient functioning of equity markets. Regarding the regulatory frameworks, the "regulation national market system" (Reg NMS) and the "markets in financial instruments directive" (MiFID) represent major changes in the structure of the markets for equities trading in the U.S. and European Union respectively. Each of these regulations aimed to provide a legal framework for securities markets and trading venues in order to increase competition between market participants and thus reduce transaction costs. This has significantly improved liquidity in the markets. However, these regulations have also led to significant changes in market structure, specifically market fragmentation, the growth of automated trading, the proliferation of dark pools and the rise of high-frequency traders. These new issues have raised many concerns about market stability and the efficiency of the price formation process. In the banking sector, some regulations were imposed to negate liquidity risks of banks that played a prominent role in financial crises. The Basel Committee on Banking Supervision (BCBS) published in January 2013 the Basel III, which sets out one of the committee's key reforms to strengthen global liquidity regulations.

Most standard models in financial theory ignore the liquidity component as they assume that the financial market is frictionless and traded assets are perfectly liquid. However, practically speaking, the investor incurs an additional cost if he wants to liquidate his position immediately. Moreover, several studies confirm the existence of a liquidity premium in the markets. For example, Amihud et al. (2006) demonstrate that liquidity has large-scale effects on financial markets. Lesmond (2005) uses the five most commonly used measures of liquidity to test 31 emerging markets. He notes the presence of a liquidity premium in several markets. Bekaert et al. (2007) conclude that liquidity is an important factor in asset pricing on international markets. Some papers extend the standard capital asset pricing model (CAPM) from the perspective of asset pricing; see for example Acharya and Pedersen (2005), Pástor and Stambaugh (2003) and Wang and Chen (2012). In general, liquidity helps to better explain asset

pricing. The literature indicates that less liquid assets are allocated to investors with longer investment horizons and that the expected return on assets is an increasing function of illiquidity (Constantinides (1986)). However, counter-arguments are also present in the literature such as in Datar et al. (1998), and Brennan et al. (1998). Therefore, given the divergent findings on the relationship between liquidity and returns, this relationship needs to be examined for the CAC 40 market.
In this paper, we focus on the French CAC 40 stock market index, which is considered one of the major national indices of the Euronext N.V (the European stock exchange). Using econometric modelling, our objectives are to :

- Study the time-series properties of CAC 40 liquidity measures and returns between 2007 and 2018.
- Identify the dynamic short- and long-term relationship between liquidity and French CAC 40 index returns.
- Recognize the causal relationship between liquidity and returns on CAC 40 Stock Market.

Our paper provides a contribution to the literature on liquidity along four broad lines. First, not much research has been done on the short-term and long-term relationship and the causal link between liquidity and returns, and even less so for the CAC 40 index. Second, knowing the direction of the causal relationship between liquidity and returns (if it exists at all) is of crucial interest. For example, liquidity may have an impact on returns through a premium which corresponds to higher transaction costs (Amihud and Mendelson, 1986). However, returns may also affect how markets behave in the future, which may also affect liquidity. Third, as long as liquidity is a notion that combines (i) cost concerns, (ii) volume considerations, and (iii) time constraints, we use in this study six various measures of liquidity to analyze the dependence between liquidity and returns on the French CAC 40 market index. In fact, most empirical studies use only one measure of liquidity. This has limitations, however, as pointed out by Hasbrouck (2005), and Goyenko et al. (2009). Finally, this study covers the period 2007-2018 which includes three liquidity crunches : the 2008 Subprime crisis, the 2011 debt crisis, and more recently the 2016 recession. This would make it possible to see how liquidity moves in calm periods and how liquidity evolves in the context of severe market shocks. More importantly, the period under consideration coincides with the introduction of the MiFID 1 regime in 2008, which was a major change in the financial ecosystem in Europe. As a result, market liquidity should be re-examined.

This paper is organized as follows : Section 2 introduces the concept of liquidity and its related measures, and establishes the theoretical framework for the analysis. Section 3 reports data and results while Section 5 concludes the paper.

2 Theoretical framework

2.1 Liquidity measures

Liquidity is an elusive concept that is not easily definable and measurable since it (unfortunately) cannot be observed directly. Its various measures attempt to focus on the different aspects of liquidity including :

- Market depth or the ability to execute large transactions without significantly influencing the prices. It is often measured by volume.
- Width or the difference between the best bid and ask prices quoted by market makers. It represents the cost of the immediate consumption of liquidity.
- Immediacy or the speed with which transactions can be executed. It depends on the existence of a counterpart to the transaction, the volume of the transaction, and the trading frequency; or the speed with which prices ability to bounce back to their equilibrium value

following a large consumption of liquidity.

- Breadth measures the number of liquid securities.

Unfortunately, we are unlikely to find a single statistic that summarizes all of these attributes. Researchers rely on one or a combination of these dimensions to estimate liquidity risk. In this study, we use two measures that capture the *Bid-Ask* spread, namely : Roll (1984) measure and Corwin and Schultz (2012) measure. We also use three measures related to Trading Volume. Finally, we use a simple High-low ratio. The issue of effectiveness of liquidity proxies is comprehensively addressed by Goyenko et al. (2009), Corwin and Schultz (2012), Fong et al. (2017) and Abdi and Ranaldo (2017).

2.1.1 Spread-based measures

One way to measure liquidity risk is to look at transaction costs. These may be reflected through the *Bid-Ask* spread. However, it is not easy to obtain this measure since it is not widely published. Nevertheless, there are several *proxies* that can be calculated from price (or returns) data, which can provide an estimation of the *Bid-Ask* spread.

First, the Roll measure relies solely on the price series, by calculating the covariance between two successive price changes :

$$Cov(\triangle p_t, \triangle p_{t-1}) = \frac{-s^2}{4} \tag{1}$$

$$\hat{s} = 2.\sqrt{-Cov(\triangle p_t, \triangle p_{t-1})} \tag{2}$$

where $\triangle$ is the differential operator, p_t is the asset price on day t. In case of positive covariance, we take values with a negative sign added (Harris, 1990; Lesmond, 2005).
Secondly, the *high-low* estimator simply represents the relative difference between the highest (*high*) and lowest (*low*) price observed during the day :

$$HL_t = \frac{H_t - L_t}{H_t} \tag{3}$$

Despite its simplicity, this measure could provide an insight into the liquidity of an asset. In fact, since daily *high* prices (resp.*low*) are almost always sell (buy) orders, the ratio of *high/low* prices for a given day reflects both the variance of the security and its *Bid-Ask* spread.
The third measure of the spread is provided by Corwin and Schultz (2012) using *high* and *low* prices with a two-day interval. The intuition behind their measure is that the variance is twice as high when the *high-Low* ratios are calculated over two days, but the *Bid-Ask* spread remains unchanged. The bid-ask spread can therefore be estimated by removing the component related to price volatility. The spread can therefore be estimated by comparing the one-day High-Low ratios and those of two days interval :

$$CS_t = (\sqrt{2} + 1).(\sqrt{\beta_t} - \sqrt{\gamma_t}) \tag{4}$$

β_t is given by the sum, over two consecutive days, of the log-squared ratio between high and low prices, and γ_t is given by the log-ratio between the high price on two consecutive days and the low price on two days. More specifically:

$$\beta_t = \sum_{j=-1}^{0} \left(ln \frac{H_{t+j}}{L_{t+j}} \right)^2 \tag{5}$$

$$\gamma_t = \left(ln \frac{max(H_t, H_{t-1})}{min(L_t, L_{t-1})} \right)^2 \tag{6}$$

2.1.2 Volume-based measures

These measures seem to reflect the "consumed liquidity". Unlike spread-based measures, no theoretical basis was found to support the use of volume-based measures, other than the fact that they are a proxy gauge of market depth.

We first use the transaction volume expressed in logarithm : $Lvolume_t = Ln(Volume_t)$. The Average Daily Volume (ADV) is also widely used by traders. It consists of averaging the volume over a period of n days, in order to capture the trend.

$$ADV_{t,n} = \sum_{t-n}^{t} \frac{Volume_t}{n} \tag{7}$$

The higher the ADV, the more liquid the security and the easier it is to execute large orders without causing too much disruption on the market.
From the perspective of investors, the definition of liquidity could be tied to the amount of money they can invest without causing too much disruption to market prices. In other words, the idea is to find out how much money it would take to create a daily unit change in the price of the security. From this question, Danyliv et al. (2014) develop an easy-to-calculate liquidity ratio that enables comparisons of liquidity between several securities or between several markets :

$$LIX_t = log_{10}\left(\frac{Volume_t.p_t}{high_t - low_t}\right) \tag{8}$$

A log base of 10 would lead to a ratio between 5 (for the least liquid assets) and about 10 (for the most liquid assets).

2.2 Vector autoregression (VAR) model

The VAR representation is a generalization of autoregressive (AR) models, it allows to study the interdependencies between several variables. The VAR has been introduced by Sims (1980) as an alternative to Keynesian-inspired macroeconometric models. In a VAR models, variables are treated symmetrically so that each variable is explained by its own past values and by the past values of other variables. These models make it possible to analyze the effects of one variable on the other through random shock simulations. The definition of a k variables and p lags VAR process (VAR(p)) is given by :

$$Y_t = A_0 + A_1 Y_{t-1} + A_2 Y_{t-2} + ... + A_p Y_{t-p} + \varepsilon_t \tag{9}$$

$$\text{with} \quad Y_t = \begin{bmatrix} y_{1,t} \\ y_{2,t} \\ ... \\ ... \\ \\ ... \\ y_{k,t} \end{bmatrix} ; \quad A_{i\neq j} = \begin{bmatrix} a^1_{1,i} & a^2_{1,i} & ... & a^k_{1,i} \\ a^1_{2,i} & a^2_{2,i} & ... & a^k_{2,i} \\ .. & .. & .. & .. \\ .. & .. & .. & .. \\ \\ .. & .. & .. & .. \\ a^1_{k,i} & a^2_{k,i} & ... & a^k_{k,i} \end{bmatrix} ; \quad A_0 = \begin{bmatrix} a^0_1 \\ a^0_2 \\ ... \\ ... \\ \\ ... \\ a^0_k \end{bmatrix} ; \quad \varepsilon_t = \begin{bmatrix} \varepsilon_{1,t} \\ \varepsilon_{2,t} \\ ... \\ ... \\ \\ ... \\ \varepsilon_{k,t} \end{bmatrix}$$

The vector of innovations ε_t is i.i.d and therefore satisfies the following properties :

$$E(\varepsilon_t) = 0 \quad \text{and} \quad : E(\varepsilon_t \varepsilon_{t-1}) = \begin{cases} \Omega & j = 0 \\ 0 & j \neq 0 \end{cases}$$

In this study, two variables are used for the VAR(p) representation : the returns on the one hand, and the variable measuring liquidity on the other. More precisely, we use the following VAR(p) model :

$$\begin{cases} y_{1,t} = & a_1^0 + \sum_{i=1}^{p} a_{1i}^1 y_{1,t-i} + \sum_{i=1}^{p} a_{1i}^2 y_{2,t-i}^j + \varepsilon_{1t} \\ y_{2,t} = & a_2^0 + \sum_{i=1}^{p} a_{2i}^1 y_{1,t-i} + \sum_{i=1}^{p} a_{2i}^2 y_{2,t-i}^j + \varepsilon_{2t} \end{cases} \tag{10}$$

Where $y_1 = Returns$ denotes the returns variable, and y_2^j refers to the liquidity variable and is defined for $j = \{HL, Roll, CS, Lvolume, ADV, LIX\}$. a_k^0 represents the intercept, a_{ki}^1 is the slope coefficient with respect to the returns and a_{ki}^2 is the slope coefficient of the liquidity measure.

To set the optimal number of lags in a VAR(p), a common procedure is based on the estimation of all VAR models for orders p ranging from 0 to a given order h set arbitrarily. For each of these models, the selection method consists of checking a maximum of the following criteria : Akaike information criterion (AIC), Schwarz bayesian information criterion (SC), Hannan-Quinn information criterion (HQ) and Final prediction error criterion (FPE).
The VAR representation can be supplemented by an analysis of the impulse response functions which represents the effect of a shock (or innovation) on the current and future values of endogenous variables. A shock to any one variable may directly affect that variable, but it is also transmitted to all other variables through the dynamic structure of the VAR. The overall idea is to summarize information about the evolution of a component Y_t^j in response to a shock on Y_t^i, at date t=0, assuming that all other variables are constant for $t \leq 0$.

2.3 Granger causality

Identifying causal relationships between economic variables provides insights for a better understanding of economic phenomena. Knowing the direction of causality is as important as demonstrating a relationship between economic variables. In the sense of Granger (1969), if one variable "causes" another one, then past values of the first one should contain information that helps predict the second one. Formally, the variable y_2 does not Granger-cause the variable y_1 if and only if :
$E(y_{1,t+h}/y_{1,t}, y_{1,t-1}, ..., y_{1,1}) = E(y_{1,t+h}/y_{1,t}, y_{1,t-1}, ..., y_{1,1}, y_{2,t}, y_{2,t-1}, ..., y_{2,1})$
A variant of Granger's test, derived directly from the autoregressive representation VAR(p), proposes to estimate by the least squares method the two following equations :

$$\begin{cases} y_{1,t} = & \Psi + \sum_{i=1}^{p} \phi_i y_{1,t-i} + \sum_{i=1}^{p} \gamma_i y_{2,t-i} + \upsilon_t \\ y_{2,t} = & \alpha + \sum_{i=1}^{p} \xi_i y_{2,t-i} + \sum_{i=1}^{p} \varphi_i y_{1,t-i} + \varepsilon_t \end{cases} \tag{11}$$

A statistical hypothesis test allows us to conclude on the direction of causality. Thus the variable y_2 Granger-causes the variable y_1 if the null hypothesis H_0 is rejected in favor of the alternative hypothesis H_1 :
$\begin{cases} H_0 : \gamma_1 = \gamma_2 = ... = \gamma_p = 0 \\ H_1 : \text{at least one } \gamma_i \neq 0 \end{cases}$
at least one symmetrically, the variable y_1 Granger-causes the variable y_2 if the null hypothesis H_0 is rejected in favor of the alternative hypothesis H_1 :
$\begin{cases} H_0 : \varphi_1 = \varphi_2 = ... = \varphi_p = 0 \\ H_1 : \text{at least one } \varphi_i \neq 0 \end{cases}$

3 Results and discussion

3.1 Data and description statistics

The CAC 40 measures the performance of the 40 largest companies listed on the Euronext market. The CAC 40 was introduced on December 31, 1987 (base value is 1000) and it is considered as an indicator of both French and European economic performance. Many companies listed on the CAC 40 are highly globalized and have significant activities in foreign markets. Our database of the CAC 40 index contains daily data of log returns ($Returns_t = ln(\frac{p_t}{p_{t-1}})$) and liquidity proxies calculated over the period January 2007 - December 2018, i.e. a total of 3131 observations.

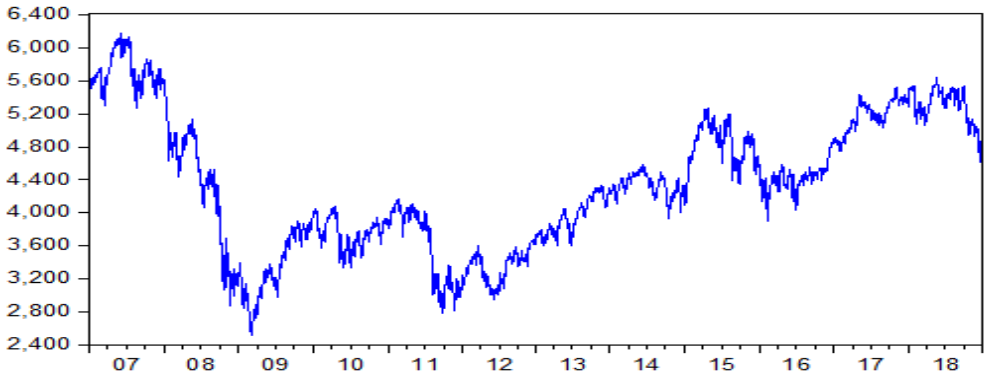

Figure 1: Time evolution of the CAC40 **spot price** between 2007 and 2018. The maximum value is reached on June 01, 2007 (6168), and the minimum value is observed on March 9, 2009 (2519).

From figure 1, the CAC 40 spot price exhibits a fairly irregular trend between 2008 and 2017, going through periods of upswings and downswings. Three recessions have occurred during this period : the December 2007 - June 2009 subprime crisis (18 months), the 2010-2011 European debt crisis, and more recently, 2016 was a high-risk year for investors, particularly for the banking sector. In February 2016, stocks of three of CAC 40 banks, Crédit Agricole, Société Générale and BNP Paribas, each lost nearly 30% of their market value. This decline was coincided with the announcement of Brexit. All three periods were characterized by market turmoil and high volatility. It is also noted that during this period, several "flash-crash" phenomena were observed (particularly in May 2010) which were characterized by a sudden disappearance of liquidity on the markets.

Figures 2 and 3 show a substantial increase in all three spread-based liquidity measures over the three economic downturns in 2008-09, 2010-11 and 2016. In contrast, during the same periods, there was a decrease in the volume-based liquidity measures. It would thus be more appropriate to consider the volume-based measures as liquidity measures, and to view the spread-based measures as illiquidity measures. In general, market liquidity has improved following the introduction of MiFID 1 but remains fragile in times of market turmoil.
As suggested by Hendershott et al. (2011) and Laruelle and Lehalle (2018), market fragmentation has led high-frequency traders to split their orders between markets or to provide liquidity in multiple venues. Consequently, the increase in the number of transactions and the decrease in the average size of transactions is often considered as an indicator of the increasing activity of the high-frequency traders. We observe on figure 3 that after the implementation of the MiFID 1, the average volume of transactions ADV is constantly decreasing. This indicates an increasing activity of high-frequency traders on the CAC 40 market. This raises an additional concern for regulators since many authors (Baldauf and Mollner (2018), Van Kervel and Menkveld (2019) and Weller (2018)) argue that high-frequency trading can lead to a distortion of

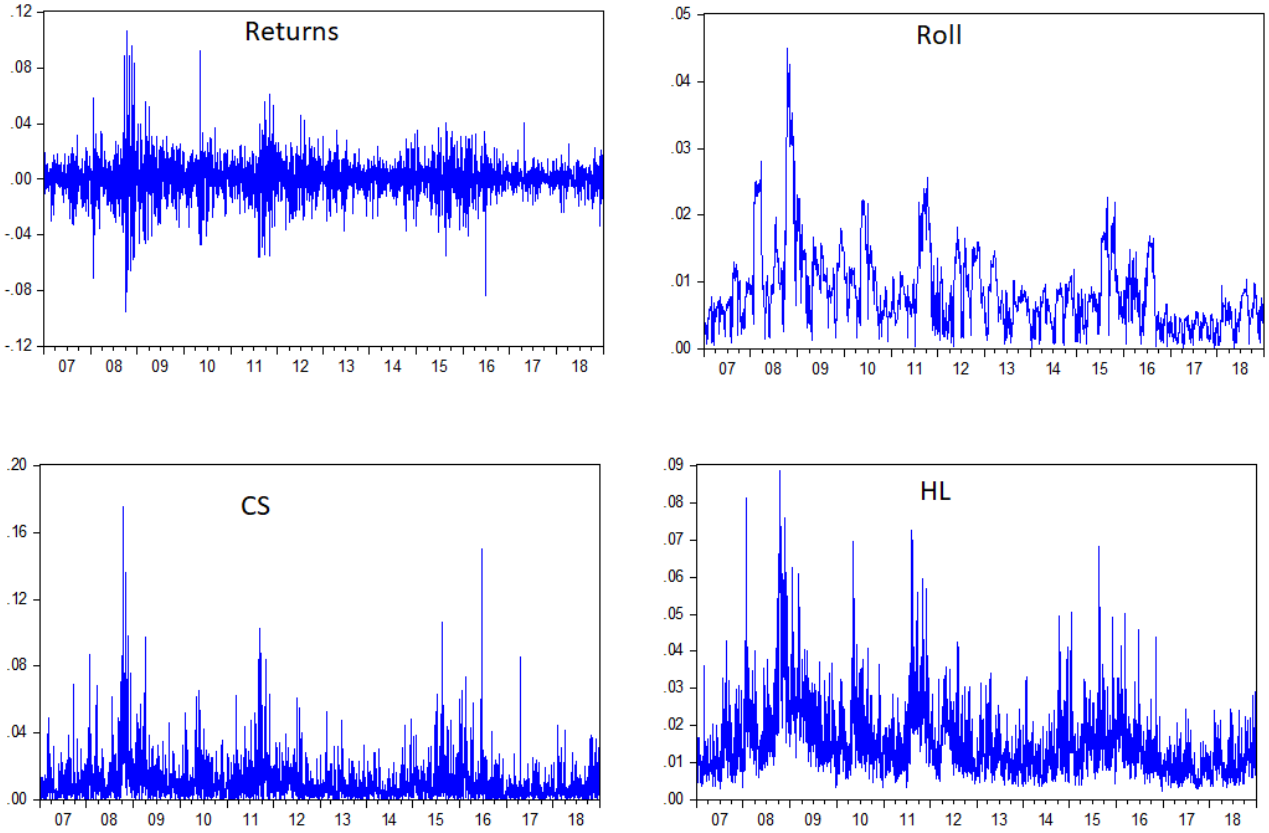

Figure 2: The temporal evolution of returns and liquidity measures reflecting the Bid-Ask spread. Returns display high volatility during 2008, 2011 and 2016. Liquidity measures show peaks during the same periods.

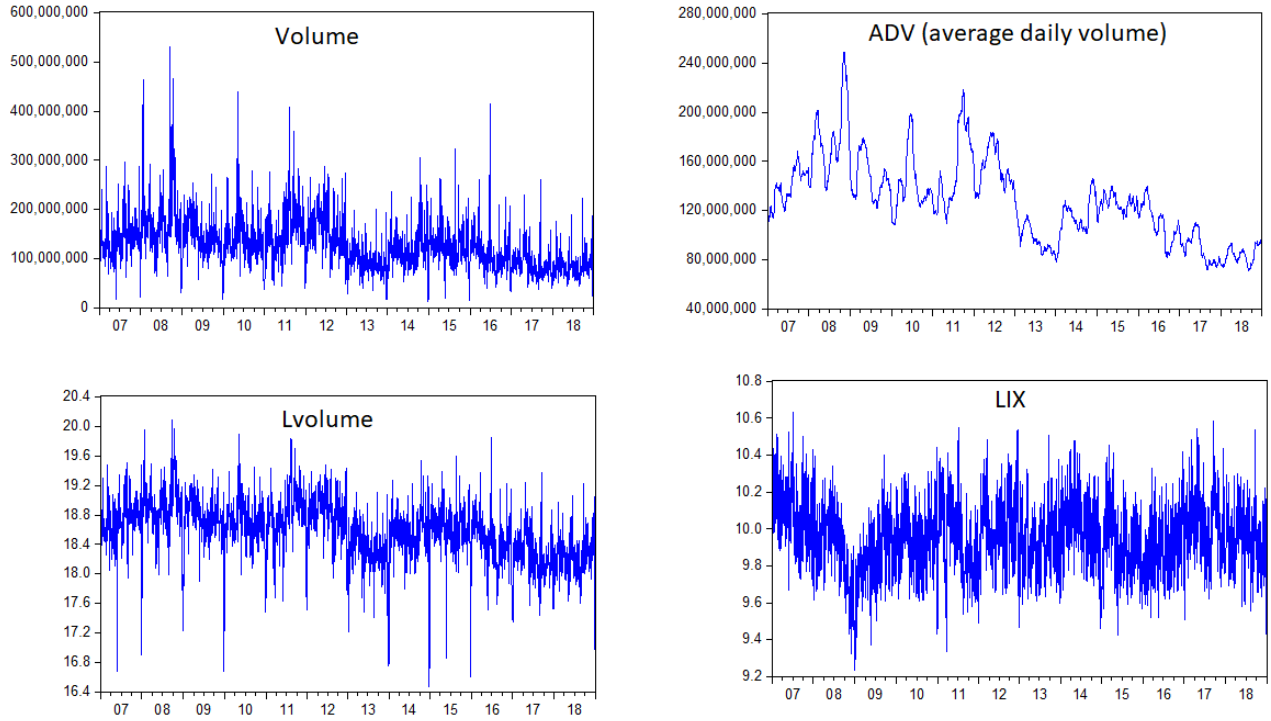

Figure 3: Time evolution of volume-related liquidity measures.

the price formation process.

Liquidity exhibits the property of persistence : unlike returns, the shocks that occurred in 2008, 2010 and 2016 did not fade away in the same year. This can be interpreted as the persistence of investor mistrust/confidence in the market and in illiquid/liquid securities. Amihud (2002) argues that liquidity is time-persistent, implying that if a market is illiquid in the present, it is more likely that it will not recover immediately. Empirically, Eisler and Kertész (2007), using Hurst's exponent, reported the presence of long-term memory in the frequency and size of trades.

	Returns	**HL**	**CS**	**Roll**	**Lvolume**	**ADV**	**LIX**
Observations	3131	3131	3131	3131	3131	3131	3131
Mean	-0.00005	0.01529	0.01031	0.00842	18.58727	128000000	9.95646
Median	0.00000	0.01279	0.00650	0.00722	18.61370	127000000	9.94747
Maximum	0.10595	0.08845	0.17570	0.04493	20.09074	249000000	10.63493
Minimum	-0.09472	0.00234	0.00001	0.00006	16.47700	71551395	9.23435
Std. Dev.	0.01415	0.00983	0.01257	0.00580	0.40445	33515435	0.19575
Skewness	-0.01288	2.21880	4.08862	1.81841	-0.56531	0.59883	0.11496
Kurtosis	9.62056	10.73369	31.61050	8.32304	5.34778	3.27052	3.09094
Jarque-Bera	5718.306	10371.73	115511.4	5422.015	885.8576	196.6735	7.975368
Probability	0	0	0	0	0	0	0.018543

Table 1: Description statistics of all variables

Table 1 reports the summary statistics for our variables. The maximum values for the spread-based measures are reached around 2008. These measures are expressed in percentage terms. On the other hand, the CS measure appears to be the most volatile. The values for the variable Lvolume are between 16.47 and 20.09, while for the measure LIX is between 9.23 and 10.63. The Returns and Lvolume series have a negative skewness coefficient, e.i their distributions are skewed to the left unlike the distribution of the other variables. All the series have leptokurtic distributions since their Kurtosis coefficients are greater than 3.

	Returns	**HL**	**CS**	**ROLL**	**Lvolume**	**ADV**	**LIX**
Returns	**1**	-0.13348	-0.06736	-0.03186	-0.06017	-0.00743	0.09806
HL	-0.13348	**1**	0.43541	0.52130	0.58859	0.51903	-0.64015
CS	-0.06736	0.43541	**1**	0.34347	0.38413	0.29523	-0.16041
ROLL	-0.03186	0.52130	0.34347	**1**	0.35904	0.64796	-0.28924
Lvolume	-0.06017	0.58859	0.38413	0.35904	**1**	0.55955	0.10936
ADV	-0.00743	0.51903	0.29523	0.64796	0.55955	**1**	-0.16682
LIX	0.09806	-0.64015	-0.16041	-0.28924	0.10936	-0.16682	**1**

Table 2: Correlation between our variables. Except for the LIX measure, the liquidity measures are positively correlated with each other, and weakly and negatively correlated with returns.

The Jarque Bera normality test is based on the distribution of the Skewness and Kurtosis. It evaluates the simultaneous deviations of these coefficients from the benchmark values of the normal distribution. The Jarque Bera statistic asymptotically follows a Chi-square distribution with 2 degrees of freedom. Given $\alpha = 5\%, \chi^2_{0.05}(2) = 5.99$, all series have a $JarqueBera$ statistic greater than 5.99, the hypothesis of normality is thereby rejected.
Correlation between the different liquidity indicators and returns is shown in Table 2. Except

for the LIX measure, the other measures of liquidity move in the same direction. Moreover, they are weakly and negatively correlated with returns. For the bid-ask measures, this can be explained by the fact that an increase in transaction costs leads to a decrease in returns.

	Model 1			Model 2			Model 3		
	t-Stat	1%level	5%level	t-Stat	1%level	5%level	t-Stat	1%level	5%level
Returns	-57.836	-3.961	-3.411	-57.833	-3.432	-2.862	-57.842	-2.566	-1.941
HL	-8.666	-3.961	-3.411	-7.937	-3.432	-2.862	-3.115	-2.566	-1.941
CS	-11.401	-3.961	-3.411	-10.976	-3.432	-2.862	-4.417	-2.566	-1.941
Roll	-7.372	-3.961	-3.411	-6.777	-3.432	-2.862	-3.389	-2.566	-1.941
Lvolume	-9.811	-3.961	-3.411	-5.764	-3.432	-2.862	-2.299	-2.566	-1.941
ADV	-6.275	-3.961	-3.411	-4.054	-3.432	-2.862	-1.084	-2.566	-1.941
LIX	-10.605	-3.961	-3.411	-10.606	-3.432	-2.862	-2.345	-2.566	-1.941

Table 3: T-statistic and critical value at 5% and 1% of the augmented Dickey-Fuller test, for the three models described in equations 12. The variable ADV is non-stationary (DS process, Differency stationary) while the other variables are stationary.

When constructing a VAR model, it is necessary to verify that all the variables that we incorporate into our VAR model are stationary. For each series, we test for stationarity using the augmented Dickey Fuller test following the three models :

$$\begin{cases} \Delta Y_t &= \alpha + \beta t + \rho Y_{t-1} - \sum_{i=2}^{p} \phi_j \Delta Y_{t-j+1} + \varepsilon_t \\ \Delta Y_t &= \alpha + \rho Y_{t-1} - \sum_{i=2}^{p} \phi_j \Delta Y_{t-j+1} + \varepsilon_t \\ \Delta Y_t &= \rho Y_{t-1} - \sum_{i=2}^{p} \phi_j \Delta Y_{t-j+1} + \varepsilon_t \end{cases} \tag{12}$$

With ε_t independent and identically distributed variables of standard normal distribution. The number of lags p used in this test is determined by minimizing the AIK, SC, HQ and FPE criteria and maximizing the LR statistic. Table 3 shows that, apart from the variable ADV, the null hypothesis of the existence of a unit root is rejected regardless of the model used and for the 5% and 10% confidence levels. This leads us to affirm that these series are stationary. For the variable ADV, the test procedure indicates that the ADV series is generated by a deterministic type of non-stationarity process I(1) or DS (Differency stationary). In order to make it stationary, we compute the differences between consecutive observations. We will then use in the VAR model the variable $D\_ADV_t = ADV_t - ADV_{t-1}$.

3.2 Vector autoregression analysis

By taking two variables, we apply the VAR model specified in equation 10. With $y_1 = Returns$ the variable reflecting returns, and y_2 the variable reflecting liquidity. For the latter, we use one of the six liquidity measures: HL, Roll, CS, Lvolume, ADV, Lix. In total, we estimate six VAR models.
First, we need to determine the optimal VAR model lag. To this end, we estimate various VAR processes for lag orders p ranging from 1 to 8, and then use the previously defined information criteria (Table 4).

- For the variables Roll and Lvolume, the VAR(6) model is preferred.
- the VAR(8) model is retained for the variables HL and CS.
- For the measure LIX, the model VAR(7) is selected.
- For the variables D_ADV, the VAR(5) model is preferred.

The estimation results of the different VAR models are summarized in Table 5.

We can see from both tables that many of the coefficients associated with the lagged terms are not significantly different from 0 for the confidence levels α= 1% (i.e. 2.33), α= 5% (i.e.

VAR Lag Order Selection Criteria	LR	FPE	AIC	SC	HQ
Endogenous variables: Returns HL Exodenous variables :C	Lag 8	Lag 8	Lag 8	Lag 6	Lag 8
Endogenous variables Returns ROLL Exodenous variables: C	Lag 6	Lag 6	Lag 6	Lag 2	Lag 5
Endogenous variables Returns CS Exodenous variables: C	Lag 8	Lag 8	Lag 8	Lag 5	Lag 8
Endogenous variables Returns Lvolume Exodenous variables: C	Lag 6	Lag 6	Lag 6	Lag 5	Lag 6
Endogenous variables Returns D_ADV Exodenous variables: C	Lag 5	Lag 6	Lag 6	Lag 5	Lag 5
Endogenous variables Returns LIX Exodenous variables: C	Lag 7	Lag 7	Lag 7	Lag 5	Lag 6

Table 4: Determination of the optimal number of VAR model lags, using the information criteria : LR, AIC, SC, HQ and FPE. The optimal VAR lag is one that minimizes the AIC, SC and HQ statistics, and maximizes the LR statistic.

1.96) and α= 10% (i.e. 1.64). In general, we have a negative dependence between returns and liquidity measures (except for the LIX measure).
In most estimations, returns are negatively dependent on their lagged value, but are also negatively dependent on liquidity measures with p lags. This is consistent with the fact that a wide bid-ask spread often reflects high transaction costs that are directly translated into the daily return. This confirms previous results observed in the US market (Amihud, 2002; Goyenko et al., 2006), Australia (Marshall and Young, 2003; Vu et al., 2015; Chang et al., 2010). The results partially agree with (Copeland, 1976; Jennings et al., 1981) who argue that new information is not disseminated to all market players at the same time. Therefore, the lagged trading volume contains useful information for predicting current returns and the lagged returns contain useful information for predicting current trading volume.
There is also a positive and significant dependence between liquidity measures and their own lagged values. This suggests (as for returns) the existence of a self-correcting mechanism in the short term,as described in Chordia et al. (2000) and Hasbrouck and Seppi (2001).
Regarding liquidity measures related to the ask bid spread, there is a significant dependence between them and returns in both directions. Conversely, returns are not significantly dependent on volume-based liquidity measures. This means that the French market can absorb a large volume of transactions without significantly disrupting prices. This is consistent with the work of Gębka (2012), who finds little evidence of the impact of volume on yields. Lee and Rui (2002) even think that trading volume cannot be used as an explanatory variable for returns.
From an informational perspective, if the market liquidity observed through the bid-ask spread, for example, reflects all the available information about market liquidity, then it would be impossible to predict its future level based on the available information. The implication of such a hypothesis is that liquidity is driven by a 'random walk' process where only naive forecasts are possible. On the other hand, if liquidity has a long-term dependency structure, thus the knowledge of the past provides useful information for predicting the future level of liquidity. Our results (from Table 5 and from our discussion about the liquidity persistence) support the latter assumption. Therefore, the ability to predict future trends in liquidity would allow investors to take advantage of the reduction of transaction costs through a "best execution" strategy.
In conclusion, liquidity displays a dependence characteristic i.e. liquidity shock does not therefore fade instantaneously, and investors will continue to suffer the consequences over a varying period of time. However, the perception of this liquidity shock by investors is not always easy.

In some cases, when market liquidity decreases, the market may not immediately and fully integrate this illiquidity. It is plausible that illiquidity may hinder the price formation process. The hypothesis of under-reaction of prices (or returns) following a liquidity shock is studied using the impulse response function.

	Returns	X=Roll	Returns	X=HL	Returns	X=CS
Constant	0.00016	0.0002***	-0.0001	0.0024***	-0.0005	0.0037***
Returns(-1)	-0.0359**	-0.0052***	-0.0361**	-0.1077***	-0.0301*	-0.0755***
Returns(-2)	-0.0468**	-0.0063***	-0.0421**	-0.0774***	-0.0426***	-0.0814***
Returns(-3)	-0.0595***	-0.0046**	-0.0419**	-0.0625***	-0.0396**	-0.059***
Returns(-4)	0.0169	-0.0043**	0.036*	-0.0597***	0.029~	0.1085***
Returns(-5)	-0.0671***	-0.002	-0.0619***	-0.0084	-0.0521***	-0.0642***
Returns(-6)	0.0018	-0.0071***	-0.0012	-0.0036	0.0163	-0.0537***
Returns(-7)			0.007	0.0033	0.0256	-0.01
Returns(-8)			0.0135	0.0017	0.0187	-0.021
X(-1)	-0.3051*	0.7851***	0.0027	0.2427***	0.0587***	0.0384**
X(-2)	0.4209**	0.1206***	0.0231	0.1252***	0.0346	0.0716***
X(-3)	-0.2488	0.0067	0.056	0.0733***	0.0554***	0.0774***
X(-4)	-0.1611	0.0653***	0.0423	0.0783***	-0.054***	0.0992***
X(-5)	0.6572***	-0.0168	-0.0947***	0.1165***	0.0243	0.1316***
X(-6)	-0.3898**	0.005	-0.0722*	0.0774***	-0.0281	0.0645***
X(-7)			-0.0054	0.0543***	-0.015	0.0599***
X(-8)			0.0538	0.0769***	-0.0313	0.0982***

	Returns	X=Lvolume	Returns	X=D_ADV	Returns	X=LIX
Constant	0.0083	2.9108***	0.0006	-3951.49	0.0066	2.3184***
Returns(-1)	-0.0341*	-1.8918***	-0.0358**	-7217861***	-0.037**	1.431***
Returns(-2)	-0.0476***	-1.2055***	-0.0467**	-7012598***	0.0496***	0.8391***
Returns(-3)	-0.0543***	-0.99328***	-0.0533***	-6457904***	0.0564***	0.8345***
Returns(-4)	0.0212	-1.2728***	0.0224	-2270768	0.0223	0.38*
Returns(-5)	-0.0681***	0.2481	-0.0689***	-5944207***	-0.0681***	-1223
Returns(-6)	0.0021	-0.1844			0.0037	-0.1248
Returns(-7)					0.017	0.0865
Returns(-8)						
X(-1)	0.0003	0.5057***	«0.01	0.4646***	0.0018	0.2541***
X(-2)	-0.0003	0.0556***	«0.01	0.0644***	-0.0004	0.1422***
X(-3)	0.0002	0.0666***	«0.01	-0.0150	-0.0002	0.0854***
X(-4)	0.0006	0.0741***	«0.01	0.1134***	-0.0028*	0.062***
X(-5)	-0.0007	0.2337***	«0.01	0.1092***	0.0022	0.0877***
X(-6)	-0.0006	-0.0924***			-0.0003	0.0686***
X(-7)					-0.0009	0.067***
X(-8)						

Table 5: Estimation of the six VAR models using returns and variable X denoting the liquidity which is represented each time by one of the six liquidity measures: Roll, HL, CS, Lvolume, D_ADV and LIX. *** means that the parameter is significant at the 1% confidence level. ** means that the parameter is significant at the 5% confidence level. * means that the parameter is significant at the 10% confidence level.

3.3 Granger causality findings

Causal analysis helps to identify the statistically significant influences of the two variables (returns and liquidity) on each other. We will carry out the Granger causality test in order to identify these influences and also to find out the direction of any causality. The causality test is therefore conducted using the six previously estimated VAR(p) representations.
Recall that in the Granger test, a variable is the cause of another if the predictability of the

first is improved when information relating to the second is incorporated into the analysis.
We test: H_0: No causality between returns and liquidity. Against H_1: Causality between returns and liquidity.
The probability values for F statistics as well as the p-values are given on the right-hand side of the Table 6. If these probability values are less than any confidence level α level, the H_0 hypothesis would be rejected at that level.

Pairwaise Granger Causality Test		Sample:1/01/2007 12/31/2018	
Null Hypothesis	Obs	F-statistic	Prob
ROLL does not Granger cause Returns	3125	2.70228	0.0128
Returns does not Granger cause ROLL		5.38521	«0.01
Returns does not Granger cause HL	3123	32.0559	«0.01
HL does not Granger cause Returns		1.76584	0.0790
CS does not Granger cause Returns	3123	3.13717	0.0015
Returns does not Granger cause CS		18.0898	«0.01
Lvolume does not Granger cause Returns	3125	0.28284	0.9453
Returns does not Granger cause Lvolume		9.70566	«0.01
D_ADV does not Granger cause Returns	3125	1.03611	0.3945
Returns does not Granger cause D_ADV		18.8970	«0.01
LIX does not Granger cause Returns	3124	0.78237	0.6021
Returns does not Granger cause LIX		11.1209	«0.01

Table 6: Granger causality test. The null hypothesis H_0 is the absence of causality. Unidirectional causality is accepted between returns and the variables ROLL, HL and CS. However, there is unidirectional causality from the returns towards the measures Lvolume, D_ADV and LIX.

- For the spread-based measures :
  The null hypothesis of no causality is rejected in all cases, there is a bidirectional causality between liquidity and returns. This two-way causality is supported by other studies including Chen et al. (2001) and Kumar et al. (2009). Czauderna et al. (2015) also confirm a bidirectional relation between illiquidity and market returns, for the German stock market between July 2006 and June 2010.
- For trading volume measures, we can state that there is an unidirectional causality between returns and liquidity. Indeed, returns cause liquidity, but liquidity does not cause returns. At the 5% confidence level, these results are consistent with the conclusions obtained from the estimation of the VAR models. However, the results are in total contradiction with Saatcioglu and Starks (1998) who find that trading volume in the Latin American market causes returns, but stock returns do not cause volume.

3.4 Impulse response results

The impulse response function is a simple tool for the quantitative study of the dynamic propagation of shocks or innovations. It measures the effect of a shock on the current and future values of endogenous variables. All the variables are initially considered to be potentially endogenous. A shock to the *ith* variable may directly affect this *ith* variable, but it is also transmitted to all the other variables through the dynamic structure of the VAR.
Figures 4, 5 and 6 plot the responses to shocks on the errors of the two variables of the VAR model: returns and liquidity . The time horizon is set at 10, which represents the time required for the variables to return to their long-term levels.

- Returns' response to a liquidity shock : returns respond weakly and slowly to a liquidity shock. The response of returns is slightly greater for the spread-based liquidity measures than for the volume-based measures. The impact of the shock fades after six periods of time, before the returns converge oscillatorily towards their equilibrium value.
- Liquidity response to shock on returns : Liquidity, on the other hand, reacts differently to returns innovations. A returns shock has an immediate impact on liquidity. The effect of the shock is long-lasting and does not even fade after 10 days. Note, however, that the responsiveness of liquidity is almost the same regardless of the liquidity measure used.

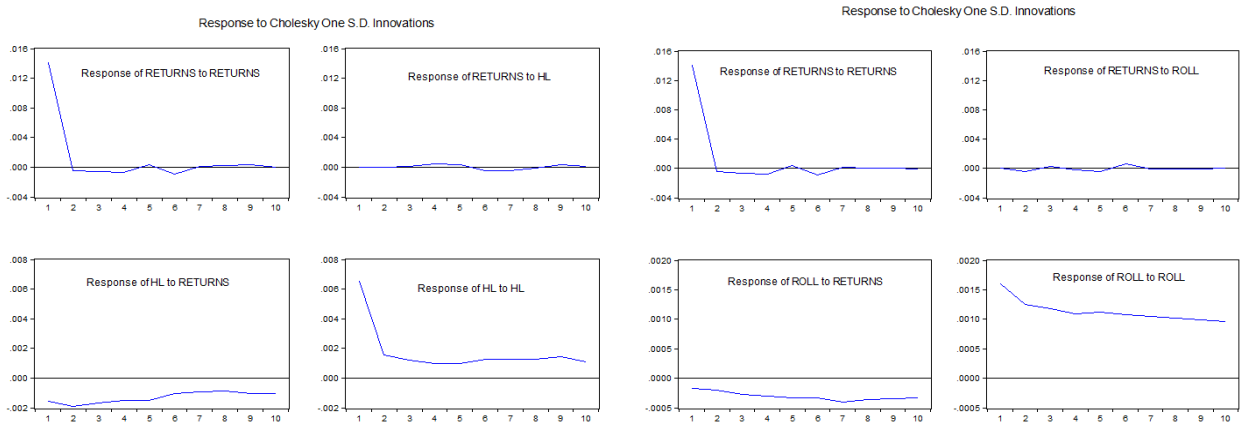

Figure 4: On the left are the impulse response functions returns/HL. On the right, the impulse response functions returns/ROLL.

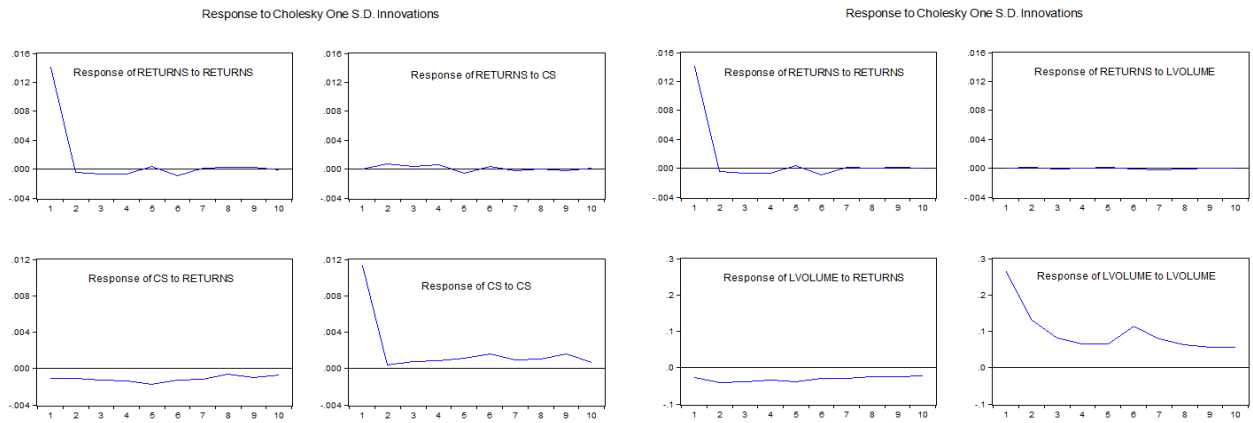

Figure 5: On the left, the impulse response functions returns/CS. On the right, the impulse response functions returns/LVolume.

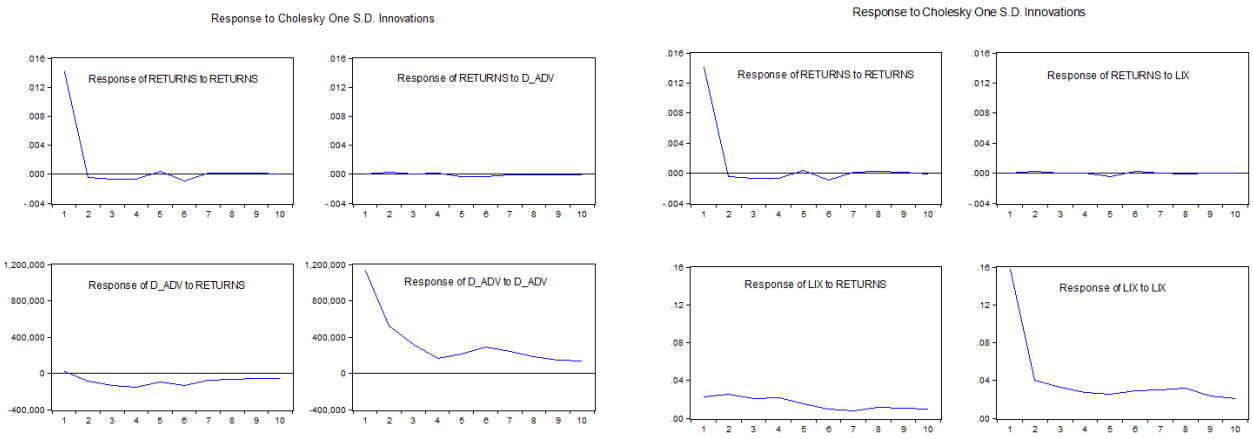

Figure 6: On the left, the impulse response functions returns/A_ADV. On the right, the impulse response functions returns/LIX.

4 Conclusion

The purpose of this study is to analyze the short-term dependence between liquidity, measured by six different proxies, and market returns. We evaluate how liquidity impacts market returns and whether past returns determine liquidity. Three of the liquidity measures used are related to the bid-ask spread, and the other three measures are related to volume. After checking the stationarity of the variables, we applied VAR modelling to the French CAC 40 index data using 3131 historical observations over the period 2007-2018, including three periods of recession: the 2008 subprime crisis, the 2011 debt crisis and, more recently, the 2016 recession. The study period was also chosen to take into account the impact of the implementation of the MiFID 1 regulation. The results emphasize the importance of taking into account the level of liquidity but also the liquidity dynamics. Our results show an improvement in liquidity following the introduction of MiFID 1 but it still appears to be vulnerable in times of market disruption.
The conclusion of this study is that there is a negative dependence between returns and liquidity measures. However, the impact of liquidity on returns is very small or even insignificant when using volume measures. We also found a positive and significant dependence between liquidity measures and their own lagged values. Causality tests in the sense of Granger led to mixed results. On the one hand, for the Bid-ask spread measures, we identified a bi-directional causality between returns and liquidity. Causality is only unidirectional from returns to liquidity when using volume measures. Finally, we found that returns respond weakly and slowly to a liquidity shock. A key implication of our results is that, since liquidity exhibits the property of persistence, investors can adapt their trading strategy to take advantage of periods of high liquidity and mitigate the impacts of periods of low liquidity. The persistence for volume-related liquidity measures (LIX and ADV) reflects the market's ability to resist large volumes without a significant impact on prices. However, we believe that the activity of high-frequency traders (seen through the decline in average volume) must be better supervised in order to improve market liquidity. On the other hand, persistence in the bid-ask spread liquidity measures indicates the persistence of transaction costs. We therefore believe that the knowledge specific information on liquidity would provide an advantage in terms of reducing transaction costs through a 'best execution' strategy. The results can be considered to be robust, as we use several liquidity measures.
In this study, we have used various proxies as indicators of liquidity in order to refine our understanding of the concept of liquidity. Since there is no consensus within the academic community on the measurement of liquidity, other dimensions of liquidity could be included (such as resilience, thickness, immediacy). We used data at daily frequencies, but other frequencies

could be considered.

Funding No external funding has been received.

Data availability statement Manuscript contains data which will be made available on reasonable request.

Compliance with ethical standards

Confict of interest There is no confict of interest.

Ethical approval No experiments have been conducted.

Informed consent This study enjoys the consent of all its authors.

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