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Abstract

This paper investigates the design of asynchronous switched observers for continuous-time nonlinear switched systems, where Takagi-Sugeno fuzzy models represents the nonlinear modes, with mismatching switching laws and bounded output disturbances (e.g. representing measurement bias, noise or fault). The proposed switched T-S observers allow state estimations when the systems' premise variables are not necessarily measurable and when the observer's active switched-mode doesn't necessarily match the one of the switched system. To derive the design conditions, a multiple Lyapunov function candidate is considered to cope with the asynchronous switching modes, together with a H_∞ criterion to minimize the transfer between the output disturbance and the state estimation errors. Moreover, Lipschitz constraints are considered to cope with the unmeasurable premise variables. The proposed conditions are declined into four theorems as Linear Matrix Inequalities. Despite their increasing computational cost, these theorems bring successive conservatism improvements, allowing the users to select the appropriate conditions regarding to the complexity of their applications. Finally, an academic example and a practical one (dealing with a tunnel diode circuit) are given to compare the conservatism and to illustrate the effectiveness of the proposed asynchronous switched T-S observer design methodologies.

Keywords: Switched Takagi-Sugeno systems, Switched asynchronous observers, Unmeasurable premise variables, Mismatching switching laws, LMIs.

1. Introduction

Switched systems is a particular class of hybrid systems that became widespread and increasingly in demand to represent many physical systems from biological or chemical processes to robotics, embedded systems and communication networks. In the real world, many systems cannot be described by a single model due to their complexity and physical nature. Still, they can be seen as a combination of several behaviors or subsystems such as continuous dynamics and discrete events, all connected by a switching phenomenon, namely the switching rule. Hence, several works dealing with the stability analysis, the stabilization and state estimation of switched systems have been proposed, see e.g. (Daafouz et al., 2002; Cheng et al., 2005; Jabri et al., 2012; Belkhiat et al., 2011; Niu and Zhou, 2017; Zare et al., 2020; Belkhiat et al., 2019).

This work is particularly focused on the state estimation of nonlinear switched systems. Indeed, since the landmark work of Luenberger (1971), observers design caught the attention of many researchers in control engineering, and tremendous works have been done. However, with regards to switched systems, the continuous state estimation is not necessarily the only concern but the discrete state estimation too. In other words, the discrete state, i.e. the set of active modes, often needs to be estimated online. As a consequence, the switched observer could be synchronous or asynchronous with the switched system. Indeed, we call

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them synchronous when the system and the observer switches at the same instants, meaning that the active mode of the system must be known at each time instant, which is rarely realistic in practice. Therefore, a more realistic case occurs when the observer and the system exhibit asynchronous switching instants. Such asynchronicity can be due, for instance, to the time spent to detect the active mode in the case of a switched system with several modes. In that regards, observers design conditions for bi-modal piecewise affine systems were presented in (Juloski et al., 2002) for the continuous case, extended to discontinuous systems with bounded estimation error. Based on the latter work, Pettersson (2005) proposed relaxed switched observer design conditions, by considering multiple quadratic Lyapunov function candidates (Alessandri and Coletta, 2001). In the similar context, a generalized switched observer scheme was designed in (Belkhiat et al., 2015) for a class of switched linear systems in the presence of unknown bounded disturbances and sensor faults. More recently the design of synchronous and asynchronous switched observer, based on average dwell time consideration, has been proposed in (Zhao et al., 2015). In (Regaieg et al., 2019), using the average dwell-time approach, a robust dissipative synchronous switched observer has been designed for the control purpose of discrete-time linear switched systems. Also, the design of asynchronous switched observers for switched linear systems subject to external disturbances has been investigated in (Han et al., 2020), considering persistent dwell-time constraints.

Note that the above mentioned studies were only dealing with the state estimation of switched linear systems. However, it is well-known that many real systems exhibit nonlinear dynamics. Hence, many research efforts are also made in the nonlinear framework. For instance, a synchronous switched observer design methodology for switched nonlinear systems, where the nonlinear terms are handle by using Lipschitz conditions, has been proposed in (Xiang and Xiang, 2008). In the latter study, the considered class of system switches according to linear hyper-planes in their state space. Moreover, the asymptotic convergence of the designed switched observer was obtained via multiple Lyapunov function, leading to Linear Matrix Inequality (LMIs) conditions. More recently, with similar considerations of Lipschitz conditions to cope with the nonlinear terms, the design of synchronous switched adaptive observers for a class of uncertain switched nonlinear systems with average dwell time was studied in (Liu and Zhao, 2020). Nevertheless, the main disadvantage of working with such Lipschitz approximations of the nonlinear terms is that they fail to represent the global nonlinear system accurately, which may lead to a loss of information regarding to the considered systems, as so to poor observer design. An interesting framework to match nonlinear systems without loss of information rests on Takagi-Sugeno (T-S) fuzzy models (Takagi and Sugeno, 1985), when obtained by the sector nonlinearity approach (Tanaka and Wang, 2001). These are widely used to describe nonlinear systems by weighted combinations of linear systems, where many tools dedicated to linear systems can be used easily. Extending the T-S framework to switched nonlinear systems, numerous works have been proposed for their stability analysis, switched observer design and/or the stabilization, see e.g. (Benzaouia et al., 2011; Lendek et al., 2012; Zhao et al., 2016; Jabri et al., 2011; Yang et al., 2013; Lendek et al., 2014b,a; Wang et al., 2016; Jabri et al., 2018; Su et al., 2020; Garbouj et al., 2020; Shi et al., 2020). In the matter of state estimation for switched T-S systems, the design of synchronous switched T-S observers has been proposed in (Lendek et al., 2012) for a class of periodic discrete-time T-S systems, where the switching sequences and the switching instants were assumed priory known. Following this work, relaxed LMI conditions have been proposed in (Lendek et al., 2014b), where the switching sequences were no more necessarily priory known, but where an admissible set of switches was required. In (Yang et al., 2013), the synchronous observer-based robust control problem for switched T-S systems with time-delay subject to uncertainties and external disturbances has been investigated. Another work dealing with synchronous switched observers for stabilization purpose has been proposed in (Yang and Tong, 2016), where the premise variables are supposed unmeasured. Synchronous mixed H_∞ and passive Luenberger-like switched T-S filter for switched T-S systems has been investigated in (Zheng et al., 2018a). The asynchronous H_∞ filtering problem for switched T-S systems has been considered in (Hong et al., 2018), using a mode-dependent average dwell-time techniques, meaning that the asynchronous switching remains in this case as a lag between the system and the filter switches. Synchronous switched robust interval observers for switched T-S systems have been proposed in (Ifqir et al., 2017), assuming that the premise variables are measurable, and in (Garbouj et al., 2019), in the case of unmeasured premise variables.

From the above review of the literature, we can assert that the following two major issues should be

taken into consideration to design switched observers for switched T-S systems:

1) How to deal with the Unmeasured Premise Variables (UPVs)? Indeed, the representation of a nonlinear system with T-S model often leads to consider state dependent premise variables that are not always online measurable.
2) How to deal with the asynchronicity between the switching modes of the system and the observer? Indeed, as mention above, the synchronous case is unrealistic in most real-world problems due to the unavailability of the active mode from measurements.

Outside the switched nonlinear systems context, the issue of observer design with UPVs for T-S systems has been the subject of several studies in the past two decades. The first results were proposed in (Bergsten and Palm, 2000) based on Thau-Luenberger observer and Lipschitz condition, then relaxed and extended to deal with disturbances rejection in (Ichalal et al., 2010). In (Ichalal et al., 2011), the result is based on the differential mean value theorem to deal with the additive term which occurs in the dynamic of the state estimation error. Considering Lipschitz conditions and quasi input-to-state stability, relaxed results are proposed in (Ichalal et al., 2012) to ensure bounded state estimation errors. Another Lipschitz condition-based approach has been proposed in (Moodi and Farrokhi, 2014; Moodi and Bustan, 2018) to avoid UPVs by introducing the nonlinear terms in the consequent parts of the T-S models. In (López-Estrada et al., 2017), the design of a H_∞ observer for T-S systems with UPVs is considered by considering the error between the measured and unmeasured premise variables as model uncertainties. More recently, an interesting method to avoid this issue has been introduced in (Ichalal et al., 2018), where auxiliary dynamics and immersion techniques were used to transform and augment the T-S model with UPVs to a new T-S model with weighting functions depending only on measured variables, where the input-output map is preserved. However, this approach lack of generalization and may fail to provide required transformations for the initial model due to the infinite number of iterations and the nature of the nonlinear entries of the system. Furthermore, extensions to observer-based control of T-S systems with UPVs has been investigated, see e.g. (Xie et al., 2019, 2020).

The issue of asynchronous switched observer design has been first investigated in the switched linear systems framework. For instance, in (Xiang et al., 2012), the asynchronous switching was handle as a delay by dividing the active mode into two segments (asynchronous and synchronous intervals), with average dwell time consideration and Lipschitz constraints. More recently, to cope with delayed transmission or estimation of the active mode, LMI-based impulsive observer design have been proposed for switched linear systems (Etienne et al., 2020). In the switched T-S model-based framework, only few papers deals with switched anachronism but not specifically focused on observer design. For instance, in (Mao et al., 2014), the authors consider the asynchronous stabilization and exponential stability of switched T-S systems using the minimum dwell-time method. The asynchronous switched output feedback controller design was considered in (Wang and Tong, 2017), but with a synchronous switched observer. In (Zhai et al., 2018), an asynchronous switched observer is considered for adaptive tracking control of switched T-S systems with mode-dependent dwell time, without considering UPVs. In a different context, a filtering problem has been considered in (Wu et al., 2019), where an asynchronous reliable filter is proposed for nonlinear Markovian jump systems. Acknowledging the lack of suitable LMI-based conditions for the design of asynchronous switched T-S observers with mismatching switching laws and subject to external disturbances, we recently investigated this issue in (Belkhiat et al., 2019). However, in the latter study, only the case of measurable premise variables was considered.

To sum up, from the above proposed review of the literature, it appears that asynchronous observer design for switched T-S systems with unmeasured premise variables has not yet been investigated. Therefore, to cope with this challenging problem, the main contributions of this paper can be summarized by the following points:

- The first contribution consists in extending the preliminary results proposed in Belkhiat et al. (2019) to the design of robust asynchronous switched observers for a large class of switched T-S systems with UPVs by using the Lipschitz assumption. In addition, H_∞ performance specifications are considered

to cope with bounded output disturbances, which may represent measurement bias, sensors noise or faults.

- The second contribution lies to the relaxation of the proposed LMI-based design conditions. In this context, to ensure the asymptotic convergence of the state estimation error under mismatching switching sets and UPV, multiple switched Lyapunov functions will be considered. Moreover, applying a useful relaxation Lemma (Peaucelle et al., 2000), the proposed design conditions will be declined into four theorems with successive conservatism improvements. Despite their increasing computational cost, these theorems allows the users to select the appropriate conditions according to the complexity of their applications.

- The last contribution inherits from the ability to cope with arbitrary mismatching switching sets and/or sequences between the considered switched systems and observers. Indeed, despite all the above mentioned previous studies, where it is assumed that the considered switched systems and switched observers are initialized in the same switching modes, an important feature brought by this proposal is that they may now start from different initial modes (asynchronous initialization of the switched modes).

The remaining of this paper is structured as follows. Section 2 presents, as preliminaries, the considered class of switched nonlinear systems and switched T-S observers, the problem statement and some useful lemmas. The main results are presented in section 3, declined into four theorems with successive conservatism improvements, dedicated to the LMI-based design of the considered asynchronous switched T-S observers with UPVs. In section 4, a first numerical example is presented to compare the conservatism and the effectiveness of the proposed LMI-based conditions with previous related results from the literature. Then, a simulation example with physical meaning (a tunnel diode circuit) is proposed to illustrate the efficiency and practical applicability of the present proposal. Finally, section 5 presents the conclusion.

2. Preliminaries and Problem Statement

Consider the class of switched nonlinear systems represented by switched T-S models given by:

$$\begin{cases} \dot{x}(t) = \sum\limits_{j=1}^{m} \sum\limits_{i_j=1}^{r_j} \sigma_j(t) h_{i_j}(z_j(t)) \left(A_{i_j} x(t) + B_{i_j} u(t)\right) \\ y(t) = Cx(t) + Ww(t) \end{cases} \tag{1}$$

where $x(t) \in \Re^n, u(t) \in \Re^p, y(t) \in \Re^v$ and $w(t) \in \Re^d$ are respectively the state vector, the input vector, the output vector and a time-varying L_2 norm bounded external disturbance vector. The number of switched modes is denoted by m and the number of fuzzy rules in the j^{th} mode by r_j $(j = 1, \ldots, m)$, $z_j(t)$ are the vectors of unmeasured premise variables and, $\forall i = 1, \ldots, r_j$, $h_{i_j}(z_j(t)) \geq 0$ are fuzzy membership functions in each switched modes j, which satisfy the convex sum property $\sum_{i_j=1}^{r_j} h_{i_j}(z_j(t)) = 1$. $A_{i_j} \in \Re^{n\times n}, B_{i_j} \in \Re^{n\times p}, C \in \Re^{v\times n}, W \in \Re^{v\times d}$ are the associated matrices to each T-S subsystems and $\sigma_j(t)$ are switching functions (switching law) defined, when the l^{th} mode is activated, as:

$$\begin{cases} \sigma_j(t) = 1 \text{ when } j = l. \\ \sigma_j(t) = 0 \text{ when } j \neq l. \end{cases} \tag{2}$$

Note that, from a switched nonlinear system, an exact switched T-S fuzzy model (1), valid on a compact subset of the state space, can be obtained by applying the well known sector nonlinearity approach (Tanaka and Wang, 2001) in each nonlinear switched modes.

Without loss of generality, we assume that the switches occur within switching sets defined by linear hyper planes S_{j,j^+} given by:

$$S_{j,j^+} = \left\{x \in \Re^\eta | s_{j,j^+} x = 0\right\}, \quad \left(j, j^+\right) \in \mathbb{I}_s \tag{3}$$

which indicates that the system switches from a subsystem j to the subsystem j^+, where $\mathbb{I}_s$ is the set of admissible switches and s_{j,j^+} are constant scalars.

Assumption 1. *In this paper, the switched T-S system (1) is assumed to be observable and, in each switched modes j, the premise variables $z_j(t)$ are assumed to be partially or totally unmeasurable.*

It should be emphasized that, contrasting with our previous work (Belkhiat et al., 2019), this study is concerned with a class of switched T-S systems (1) where the premise variables are assumed unmeasurable. In this context, our main goal is to propose LMI-based conditions for the design of asynchronous switched observers with UPVs given by:

$$\begin{cases} \dot{\hat{x}}(t) = \sum\limits_{\hat{j}=1}^{m} \sum\limits_{i_{\hat{j}}=1}^{r_{\hat{j}}} \hat{\sigma}_{\hat{j}}(t) h_{i_{\hat{j}}}(\hat{z}_{\hat{j}}(t)) \left(A_{i_{\hat{j}}} \hat{x}(t) + B_{i_{\hat{j}}} u(t) + K_{i_{\hat{j}}} (y - \hat{y}(t)) \right) \\ \hat{y}(t) = C\hat{x}(t) \end{cases} \tag{4}$$

where $\hat{x} \in \Re^n$ is the estimated state vector, $K_{i_{\hat{j}}} \in \Re^{n \times v}$ are the observer gains to be designed, $\hat{j} = 1, \ldots, m$ denotes the observer's switching modes with $(\hat{j}, \hat{j}^+) \in \mathbb{I}_s$, $\hat{z}_j$ are vectors of the observer's UPVs, and $\hat{\sigma}_{\hat{j}}$ are the observer's switching functions, which are defined similarly to (2).

Generally speaking, a problem of synchronicity can occur and can be distinguished into two cases regarding to the evolution of the considered switched observer. Indeed, in the first case, assuming synchronous switching observer, corresponds to the case where the system and the observer evolve, at each instant, in the same discrete mode $\sigma_j(t) = \hat{\sigma}_{\hat{j}}(t)$. The second case, known as the asynchronous switching observer, corresponds to the case where the observer and the system operate in different modes $\sigma_j(t) \neq \hat{\sigma}_{\hat{j}}(t)$. The latter case (asynchronous switching observer), which is the focus of this paper, is more general and more practically realistic than the first one (synchronous switching observer) but also more challenging in terms of LMI-based design. Moreover, in this paper, the switched mechanisms of the system and the observer are considered as arbitrary switching sequences according to the switching sets defined by the linear hyper planes. In that matter, two cases can be distinguished: *i*) the observer and the system switch accordingly to different switching sets; *ii*) both the system and the observer share the same switching sets, which means they switch at the same instant. Nevertheless, note that this second case is not practical due the precision in identification of the corresponding switching sets and the mismatch between the estimated states and the system states, especially during transients and when external disturbances occur. These two cases will be considered for simulation in Section IV, where the mismatching linear hyper planes $\bar{S}_{\hat{j},\hat{j}^+}$ of the switched observer are defined as follows:

$$\bar{S}_{\hat{j},\hat{j}^+} = \left\{ \hat{x} \in \Re^{\eta} | \bar{s}_{\hat{j},\hat{j}^+} \hat{x} = 0 \right\}, \quad \left(\hat{j}, \hat{j}^+ \right) \in \mathbb{I}_s \tag{5}$$

In the sequel, to lighten the mathematical expressions, the following notations will be considered.

Notations. A star $(*)$ in a matrix denotes a transpose quantity. The time t as argument of functions will be omitted when there is no ambiguity. $M^{\dagger}$ denotes the pseudo-inverse of a matrix M. Moreover, convex combinations of matrices $X_{(.)}$ with appropriate dimensions are denoted as:

$$X_{h_\sigma} = \sum_{j=1}^{m} \sum_{i_j=1}^{r_j} \sigma_j(t) h_{i_j}(z_j(t)) X_{i_j},\ X_{h_{\hat{\sigma}}} = \sum_{\hat{j}=1}^{m} \sum_{i_{\hat{j}}=1}^{r_{\hat{j}}} \hat{\sigma}_{\hat{j}}(t) h_{i_{\hat{j}}}(z_{\hat{j}}(t)) X_{i_{\hat{j}}} \text{ and } X_{\hat{h}_{\hat{\sigma}}} = \sum_{\hat{j}=1}^{m} \sum_{i_{\hat{j}}=1}^{r_{\hat{j}}} \hat{\sigma}_{\hat{j}}(t) h_{i_{\hat{j}}}(\hat{z}_{\hat{j}}(t)) X_{i_{\hat{j}}}$$

Let us denote $e = x - \hat{x}$ the estimation error, from (1) and (4), its dynamics can be expressed by:

$$\begin{aligned} \dot{e}(t) =& A_{h_\sigma} x(t) + B_{h_\sigma} u(t) - A_{\hat{h}_{\hat{\sigma}}} \hat{x}(t) - B_{\hat{h}_{\hat{\sigma}}} u(t) - K_{\hat{h}_{\hat{\sigma}}} (y - \hat{y}(t)) \\ =& (A_{\hat{h}_{\hat{\sigma}}} - K_{\hat{h}_{\hat{\sigma}}} C) e(t) + (A_{h_\sigma} - A_{h_{\hat{\sigma}}}) x(t) + (B_{h_\sigma} - B_{h_{\hat{\sigma}}}) u(t) \\ &+ f_{h_{\hat{\sigma}}}(x, u) - f_{\hat{h}_{\hat{\sigma}}}(\hat{x}, x, u) - K_{\hat{h}_{\hat{\sigma}}} W w(t), \end{aligned} \tag{6}$$

where $f_{h_{\hat{\sigma}}}(x,u) = A_{h_{\hat{\sigma}}}x(t) + B_{h\hat{\sigma}}u(t)$ and $f_{\hat{h}_{\hat{\sigma}}}(\hat{x},x,u) = A_{\hat{h}_{\hat{\sigma}}}x(t) + B_{\hat{h}_{\hat{\sigma}}}u(t)$.

In the sequel, we assume that f is Lipschitz with respect to x, then there exist positive scalars $\mu_{\hat{\sigma}}$ such that the following inequality holds (Xiang and Xiang, 2008):

$$\|f_{h_{\hat{\sigma}}}(x,u) - f_{\hat{h}_{\hat{\sigma}}}(x,\hat{x},u)\| \leq \mu_{\hat{\sigma}}\|x(t) - \hat{x}(t)\| \tag{7}$$

Problem Statement. The design objective considered in this study is summarized by the following requirements, i.e. the design of the gain matrices K_{i_j} of the asynchronous switched observer (4) such that:

- *Convergence:* the estimation error $e(t)$ converges to the origin without external disturbances ($w(t) = 0$).
- *Robustness:* for all non zero $w(t) \in L_2[0,\infty)$, the transfer between the external disturbances $w(t)$ and the estimation error $e(t)$ is minimized, which is represented by the following H_∞ criterion:

$$\int_0^\infty e^T(t)e(t)dt \leq \gamma^2 \int_0^\infty w^T(t)w(t)dt \tag{8}$$

  where $\gamma > 0$ is the disturbance attenuation level (to be minimized).

In the next section, the following useful lemmas will be used to provided the proofs of the main results.

Lemma 1. *(Zhou and Khargonekar, 1988) For any matrices X, Y with appropriate dimensions and $\lambda > 0$, the following inequality holds*

$$X^TY + Y^TX \leq \lambda X^TX + \lambda^{-1}Y^TY \tag{9}$$

Lemma 2. *S-Procedure (Derinkuyu and Pinar, 2006). Let $L \subset L^2$ be time invariant subspace and $\psi_k : L \to R, (k = 0,\dots M)$, be continuous time invariant quadratic forms. Suppose that there exist $f_* \in L$ such that $\psi_1(f_*) > 0, \dots, \psi_M(f_*) > 0$, then the following statement are equivalent:*

i. $\psi_0(f) \leq 0$ for all $f \in L$ such that $\psi_1(f) > 0, \dots, \psi_M(f) > 0$

ii. There exist $\tau_1 \geq 0, \dots, \tau_M \geq 0$ such that $\psi_0(f) + \tau_1\psi_1(f) + \dots + \tau_M\psi_M(f) \leq 0$ for all $f \in L$

Lemma 3. *(Peaucelle et al., 2000) For any matrices N, R, L, P and Q with appropriate dimensions, the following inequalities are equivalent.*

$$N^TP + P^TN + Q < 0 \iff \exists\, R, L : \begin{bmatrix} N^TL^T + LN + Q & (*) \\ P - L^T + R^TN & -R^T - R \end{bmatrix} < 0 \tag{10}$$

Remark 1. *As mentioned above, the present study is mainly focused on dealing with switching mismatches and UPVs, which are usually occurring when implementing switched T-S observers for switched nonlinear systems. In that purpose, we can argue that the proposed approach is useful when the system's switching sets are imprecisely modeled or, more generally speaking, when the system's switching signal are not online available. Moreover, dealing with UPVs for T-S models is a key-point for their state estimation. Indeed, in T-S fuzzy modeling, premise variables are usually state dependent and so, by nature, not online measurable when the state estimation is required. All of these features provide to the switched T-S observer design methodology proposed in the sequel some robustness properties against some modeling imprecision or unavailability of the considered switched nonlinear system, especially regarding to its nonlinear entries (handled as premise variables in T-S modeling) and its switching phenomena.*

This conclude the preliminaries. The following section will be dedicated to the proposed LMI-based conditions for the design of asynchronous switched observers (4), according to the above given problem statement.

3. Main Results

In this section, sufficient LMI-based conditions for the design of switched T-S observers (4) with UPVs are provided, so that the problem statement described in the previous section is satisfied. These conditions are declined into four theorems with successive conservatism improvements. The price to pay for such improvements is an increase of the computational cost when solving the related convex optimization problems. Hence, the following theorems are provided such that the users can select the appropriate one regarding to the complexity of their considered applications. This being said, the following theorem summarize the design conditions, as a basis for the next ones.

Theorem 1. *Consider the switched T-S system* (1) *and the asynchronous observer with UPVs* (4). *For all combinations of* $k_j \in \{1,\dots,r_j\}$, $(i_{\hat{j}}, q_{\hat{j}}) \in \{1,\dots,r_{\hat{j}}\}^2$, $(j,\hat{j}) \in \{1,\dots,m\}^2$ *and* $(\hat{j},\hat{j}^+) \in \mathbb{I}_s$, *if there exist the scalars* $\xi > 0$, $\tau > 0$, $\beta_{\hat{j}} > 0$, $\lambda_{\hat{j}} > 0$ *and the matrices* $Y_{i_{\hat{j}}} \in \Re^{n\times v}$, $G_{\hat{j}} \in \Re^{n\times v}$, $0 < P_{\hat{j}} = P_{\hat{j}}^T \in \Re^{n\times n}$ *such that the positive scalar* γ^2 *is minimized and satisfies:*

$$P_{\hat{j}^+} = P_{\hat{j}} + G_{\hat{j}}^T C + C^T G_{\hat{j}} \tag{11}$$

and the inequalities:

$$\begin{bmatrix} A_{i_{\hat{j}}}^T P_{\hat{j}} + P_{\hat{j}} A_{i_{\hat{j}}} - C^T Y_{i_{\hat{j}}}^T - Y_{i_{\hat{j}}} C + (\beta_{\hat{j}} + 1 + \tau) I & (*) & (*) & (*) & (*) \\ A_{k_{\hat{j}}}^T P_{\hat{j}} - A_{q_{\hat{j}}}^T P_{\hat{j}} & -\xi & 0 & 0 & 0 \\ B_{k_{\hat{j}}}^T P_{\hat{j}} - B_{q_{\hat{j}}}^T P_{\hat{j}} & 0 & -\xi & 0 & 0 \\ -Y_{i_{\hat{j}}} W & 0 & 0 & -\gamma^2 I & 0 \\ P_{\hat{j}} & 0 & 0 & 0 & -\lambda_{\hat{j}} \end{bmatrix} < 0 \tag{12}$$

then, with the gains $K_{i_{\hat{j}}} = P_{\hat{j}}^{-1} Y_{i_{\hat{j}}} \in \Re^{n\times v}$, *the switched observer is asymptotically convergent (without external disturbances) and satisfies the* H_∞ *criterion* (8) *with the disturbance attenuation level* γ, *providing that, at the switching instants, the updated switched observer states are computed as:*

$$\hat{x}^+ = \left(I - Q_{\hat{j}}^{-1}\left(CQ_{\hat{j}}^{-1}\right)^\dagger C\right)\hat{x} + Q_{\hat{j}}^{-1}\left(CQ_{\hat{j}}^{-1}\right)^\dagger y,\ \forall \hat{x} \in \bar{S}_{j,j^+} \tag{13}$$

which ensures the decreasing of the multiple Lyapunov function (14) *with* $Q_{\hat{j}} = V_{\hat{j}}\sqrt{\Lambda}_{\hat{j}} V_{\hat{j}}^T \in \Re^{n\times n}$, *such that* $V_{\hat{j}} \in \Re^{n\times n}$ *is a matrix composed of the orthonormal eigenvectors of* $P_{\hat{j}}$ *and* $\Lambda_{\hat{j}} \in \Re^{n\times n}$ *is the spectral matrix for* $P_{\hat{j}}$, *i.e. a diagonal matrix composed with the eigenvalues of* $P_{\hat{j}}$, *and* $\sqrt{\Lambda}_{\hat{j}}$ *a diagonal matrix composed with the square root of these eigenvalues.*

Proof 1. *Let us consider the multiple Lyapunov function candidate:*

$$V(t) = e^T(t) P_{\hat{\sigma}} e(t) \tag{14}$$

where $P_{\hat{\sigma}} = \sum_{\hat{j}=1}^{m} \sigma_{\hat{j}}(t) P_{\hat{j}}$, $P_{\hat{j}} = P_{\hat{j}}^T > 0$.
From (6), *the time derivative of* (14) *can be written as:*

$$\begin{aligned} \dot{V}(t) =& 2e^T(t) P_{\hat{\sigma}} \dot{e}(t) \\ =& 2e^T(t) P_{\hat{\sigma}} (A_{\hat{h}_{\hat{\sigma}}} - K_{\hat{h}_{\hat{\sigma}}} C) e(t) + 2e^T(t) P_{\hat{\sigma}} (A_{h_\sigma} - A_{h_{\hat{\sigma}}}) x(t) + 2e^T(t) P_{\hat{\sigma}} (B_{h_\sigma} - B_{h_{\hat{\sigma}}}) u(t) \\ &+ 2e^T(t) P_{\hat{\sigma}} (f_{h_{\hat{\sigma}}}(x,u) - f_{\hat{h}_{\hat{\sigma}}}(x,\hat{x},u)) - 2e^T(t) P_{\hat{\sigma}} K_{\hat{h}_{\hat{\sigma}}} W w(t) \end{aligned} \tag{15}$$

Moreover, the H_∞ *criterion* (8) *is satisfied if:*

$$\begin{aligned} &\dot{V}(t) + e^T(t) e(t) - \gamma^2 w^T(t) w(t) \\ &= 2e^T(t) P_{\hat{\sigma}} (A_{\hat{h}_{\hat{\sigma}}} - K_{\hat{h}_{\hat{\sigma}}} C) e(t) + 2e^T(t) P_{\hat{\sigma}} (A_{h_\sigma} - A_{h_{\hat{\sigma}}}) x(t) \\ &\quad + 2e^T(t) P_{\hat{\sigma}} (B_{h_\sigma} - B_{h_{\hat{\sigma}}}) u(t) + 2e^T(t) P_{\hat{\sigma}} (f_{h_{\hat{\sigma}}}(x,u) - f_{\hat{h}_{\hat{\sigma}}}(x,\hat{x},u)) \\ &\quad - 2e^T(t) P_{\hat{\sigma}} K_{\hat{h}_{\hat{\sigma}}} W w(t) + e^T(t) e(t) - \gamma^2 w^T(t) w(t) < 0 \end{aligned} \tag{16}$$

From Lemma 1, for any positive scalars $\lambda_{\hat{\sigma}}$, we have:

$$\begin{aligned} &2e^T(t)P_{\hat{\sigma}}(f_{h_{\hat{\sigma}}}(x,u)-f_{\hat{h}_{\hat{\sigma}}}(x,\hat{x},u)) \\ &\leq \lambda_{\hat{\sigma}}(f_{h_{\hat{\sigma}}}(x,u)-f_{\hat{h}_{\hat{\sigma}}}(x,\hat{x},u))^T(f_{h_{\hat{\sigma}}}(x,u)-f_{\hat{h}_{\hat{\sigma}}}(x,\hat{x},u))+\lambda_{\hat{\sigma}}^{-1}e^T(t)P_{\hat{\sigma}}P_{\hat{\sigma}}e(t) \end{aligned} \tag{17}$$

Moreover, assuming the Lipschitz condition (7)*, see e.g. (Bergsten and Palm, 2000; Xiang and Xiang, 2008; Ichalal et al., 2012), it follows:*

$$(f_{h_{\hat{\sigma}}}(x,u)-f_{\hat{h}_{\hat{\sigma}}}(x,\hat{x},u))^T(f_{h_{\hat{\sigma}}}(x,u)-f_{\hat{h}_{\hat{\sigma}}}(x,\hat{x},u)) \leq \mu_{\hat{\sigma}}^2 e^T(t)e(t) \tag{18}$$

Therefore, from (17) *and* (18)*, the inequality* (16) *is satisfied if:*

$$\begin{aligned} &2e^T(t)P_{\hat{\sigma}}(A_{\hat{h}_{\hat{\sigma}}}-K_{\hat{h}_{\hat{\sigma}}}C)e(t)+2e^T(t)P_{\hat{\sigma}}(A_{h_\sigma}-A_{h_{\hat{\sigma}}})x(t)+2e^T(t)P_{\hat{\sigma}}(B_{h_\sigma}-B_{h_{\hat{\sigma}}})u(t) \\ &-2e^T(t)P_{\hat{\sigma}}K_{\hat{h}_{\hat{\sigma}}}Ww(t)+\lambda_{\hat{\sigma}}\mu_{\hat{\sigma}}^2e^T(t)e(t)+\lambda_{\hat{\sigma}}^{-1}e^T(t)P_{\hat{\sigma}}P_{\hat{\sigma}}e(t)+e^T(t)e(t)-\gamma^2w^T(t)w(t)<0 \end{aligned} \tag{19}$$

Or equivalently with the augmented vector $\psi(t)=[e(t)\ x(t)\ u(t)\ w(t)]^T$ if:

$$\psi(t)^T\bar{\Omega}(t)\psi(t)<0 \tag{20}$$

where $\bar{\Omega}(t)=\begin{bmatrix} A_{\hat{h}_{\hat{\sigma}}}^TP_{\hat{\sigma}}+P_{\hat{\sigma}}A_{\hat{h}_{\hat{\sigma}}}-C^TY_{\hat{h}_{\hat{\sigma}}}^T-Y_{\hat{h}_{\hat{\sigma}}}C+(\lambda_{\hat{\sigma}}\mu_{\hat{\sigma}}^2+1)I+\lambda_{\hat{\sigma}}^{-1}P_{\hat{\sigma}}P_{\hat{\sigma}} & (*) & (*) & (*) \\ A_{h_\sigma}^TP_{\hat{\sigma}}-A_{h_{\hat{\sigma}}}^TP_{\hat{\sigma}} & 0 & 0 & 0 \\ B_{h_\sigma}^TP_{\hat{\sigma}}-B_{h_{\hat{\sigma}}}^TP_{\hat{\sigma}} & 0 & 0 & 0 \\ -Y_{\hat{h}_{\hat{\sigma}}}W & 0 & 0 & -\gamma^2I \end{bmatrix}$.

To apply the S-Procedure on (20)*, let us consider a scalar $\epsilon>0$ such that:*

$$\|e\|_2^2-\epsilon\big(\|x\|_2^2+\|u\|_2^2\big)>0 \tag{21}$$

Hence, by applying the Lemma 2, the inequality (20) *is satisfied if there exists a scalar $\tau>0$ such that:*

$$\begin{bmatrix} A_{\hat{h}_{\hat{\sigma}}}^TP_{\hat{\sigma}}+P_{\hat{\sigma}}A_{\hat{h}_{\hat{\sigma}}}-C^TY_{\hat{h}_{\hat{\sigma}}}^T-Y_{\hat{h}_{\hat{\sigma}}}C+(\lambda_{\hat{\sigma}}\mu_{\hat{\sigma}}^2+1+\tau)I+\lambda_{\hat{\sigma}}^{-1}P_{\hat{\sigma}}P_{\hat{\sigma}} & (*) & (*) & (*) \\ A_{h_\sigma}^TP_{\hat{\sigma}}-A_{h_{\hat{\sigma}}}^TP_{\hat{\sigma}} & -\tau\epsilon & 0 & 0 \\ B_{h_\sigma}^TP_{\hat{\sigma}}-B_{h_{\hat{\sigma}}}^TP_{\hat{\sigma}} & 0 & -\tau\epsilon & 0 \\ -Y_{\hat{h}_{\hat{\sigma}}}W & 0 & 0 & -\gamma^2I \end{bmatrix}<0 \tag{22}$$

That is to say, by applying the Schur Complement:

$$\begin{bmatrix} A_{\hat{h}_{\hat{\sigma}}}^TP_{\hat{\sigma}}+P_{\hat{\sigma}}A_{\hat{h}_{\hat{\sigma}}}-C^TY_{\hat{h}_{\hat{\sigma}}}^T-Y_{\hat{h}_{\hat{\sigma}}}C+(\beta_{\hat{\sigma}}+1+\tau)I & (*) & (*) & (*) & (*) \\ A_{h_\sigma}^TP_{\hat{\sigma}}-A_{h_{\hat{\sigma}}}^TP_{\hat{\sigma}} & -\xi & 0 & 0 & 0 \\ B_{h_\sigma}^TP_{\hat{\sigma}}-B_{h_{\hat{\sigma}}}^TP_{\hat{\sigma}} & 0 & -\xi & 0 & 0 \\ -Y_{\hat{h}_{\hat{\sigma}}}W & 0 & 0 & -\gamma^2I & 0 \\ P_{\hat{\sigma}} & 0 & 0 & 0 & -\lambda_{\hat{\sigma}} \end{bmatrix}<0 \tag{23}$$

with the bijective change of variable $\beta_{\hat{\sigma}}=\lambda_{\hat{\sigma}}\mu_{\hat{\sigma}}^2$, $\xi=\tau\epsilon$ and $Y_{\hat{h}_{\hat{\sigma}}}=P_{\hat{\sigma}}K_{\hat{h}_{\hat{\sigma}}}$.
Furthermore, when the observer switches, the decreasing of the Lyapunov Function candidate is verified if:

$$(x-\hat{x}^+)^TP_{\hat{j}^+}(x-\hat{x}^+)\leq(x-\hat{x})^TP_{\hat{j}}(x-\hat{x}) \tag{24}$$

where $\hat{j}$ and $\hat{j}^+$ denotes respectively the observer mode and its successor, $\hat{x}^+$ is the updated value of the observer's state vector.
Assuming that $\hat{x}^+$ satisfies $y=C\hat{x}^+$, we have $C(x-\hat{x}^+)=0$, and so, for any matrix T of appropriate dimension (Pettersson, 2005):

$$\left(x-\hat{x}^+\right)^T\left(T^TC+C^TT\right)\left(x-\hat{x}^+\right)=0 \tag{25}$$

Therefore, from (25)*, if there exist* $G_{\hat{j}}$ *such that the equality* (11) *is satisfied, then the inequality* (24) *yields and we can write:*

$$\left(x-\hat{x}^{+}\right)^{T} P_{\hat{j}} \left(x-\hat{x}^{+}\right) \leq (x-\hat{x})^{T} P_{\hat{j}} (x-\hat{x}) \tag{26}$$

Furthermore, the updated value of the observer's state $\hat{x}^{+}(t) \in \bar{S}_{\hat{j},\hat{j}^{+}}$ *have now to be determined such that the previous inequality is satisfied. To this end, consider the spectral decomposition* $P_{\hat{j}} = Q_{\hat{j}}^{T} Q_{\hat{j}}$*, with* $Q_{\hat{j}} = V_{\hat{j}} \sqrt{\Lambda}_{\hat{j}} V_{\hat{j}}^{T} \in \Re^{n \times n}$ *(Derinkuyu and Pinar, 2006),* (26) *is satisfied if:*

$$\|Q_{\hat{j}}(x-\hat{x}^{+})\| \leq \|Q_{\hat{j}}(x-\hat{x})\| \tag{27}$$

To find the updated value $\hat{x}^{+}$*, lying on the hyper plane* $y = C\hat{x}^{+}$*, such that the distance* $\|Q_{\hat{j}}(x-\hat{x}^{+})\|$ *is minimized, the optimization problem is defined as follows:*

$$\begin{aligned} &\min_{\hat{x}^{+}} \|Q_{\hat{j}}(\hat{x}^{+}-\hat{x})\| \\ &\textit{subject to } : C\hat{x}^{+} = y \end{aligned} \tag{28}$$

By introducing a scalar $\alpha_{\hat{j}} = Q_{\hat{j}}(\hat{x}^{+}-\hat{x})$*, we have* $Q_{\hat{j}}\hat{x}^{+} = \alpha_{\hat{j}} + Q_{\hat{j}}\hat{x}$*, then the above stated optimization problem can be reformulated as:*

$$\begin{aligned} &\min_{\hat{x}^{+}} \|\alpha_{\hat{j}}\| \\ &\textit{subject to } : CQ_{\hat{j}}^{-1}\alpha_{\hat{j}} = y - C\hat{x} \end{aligned} \tag{29}$$

which admits for solution the minimum least square length to $y - C\hat{x}$*, i.e.:*

$$\alpha_{\hat{j}} = \left(CQ_{\hat{j}}^{-1}\right)^{\dagger} (y - C\hat{x}) \tag{30}$$

and so:

$$Q_{\hat{j}}\hat{x}^{+} = Q_{\hat{j}}\hat{x} + \left(CQ_{\hat{j}}^{-1}\right)^{\dagger} (y - C\hat{x}) \tag{31}$$

Finally, left multiplying (31) *by* $Q_{\hat{j}}^{-1}$*, the updated value* $\hat{x}^{+}$ *can be computed as* (13). □

The LMI-based conditions presented in Theorem 1 being now established, let us recall that they have been obtained without any relaxation scheme and are by nature only sufficient. Therefore, there is still room for improvements. By extensions, the remaining theorems provide further relaxed LMI-based conditions, i.e. reducing the conservatism by introducing slack decision matrices from the application of Lemma 3 in various ways. The first one is summarized by the next theorem.

Theorem 2. *Consider the switched T-S system* (1) *and the asynchronous observer with UPVs* (4)*. For all combinations of* $k_j \in \{1,\ldots,r_j\}$*,* $(i_{\hat{j}}, q_{\hat{j}}) \in \{1,\ldots,r_{\hat{j}}\}^2$*,* $(j,\hat{j}) \in \{1,\ldots,m\}^2$ *and* $(\hat{j},\hat{j}^{+}) \in \mathbb{I}_s$*, if there exist the scalars* $\xi > 0$*,* $\tau > 0$*,* $\bar{\beta}_{\hat{j}} > 0$*,* $\lambda_{\hat{j}} > 0$ *and the matrices* $Y_{i_j} \in \Re^{n \times v}$*,* $L_{\hat{j}} \in \Re^{n \times n}$*,* $R_{\hat{j}} \in \Re^{n \times n}$*,* $G_{\hat{j}} \in \Re^{n \times v}$*,* $0 < P_{\hat{j}} = P_{\hat{j}}^{T} \in \Re^{n \times n}$ *such that the positive scalar* γ^2 *is minimized and satisfies the conditions expressed in Theorem 1 with, instead of* (12)*, the following inequalities:*

$$\begin{bmatrix} A_{i_j}^{T}L_{\hat{j}}^{T} + L_{\hat{j}}A_{i_j} - Y_{i_j}C - C^{T}Y_{i_j}^{T} + (\bar{\beta}_{\hat{j}} + 1 + \tau)I & (*) & (*) & (*) & (*) & (*) \\ P_{\hat{j}} - L_{\hat{j}}^{T} + R_{\hat{j}}^{T}A_{i_j} & -R_{\hat{j}}^{T} - R_{\hat{j}} & 0 & 0 & 0 & 0 \\ A_{k_j}^{T}P_{\hat{j}} - A_{q_j}^{T}P_{\hat{j}} & 0 & -\xi & 0 & 0 & 0 \\ B_{k_j}^{T}P_{\hat{j}} - B_{q_j}^{T}P_{\hat{j}} & 0 & 0 & -\xi & 0 & 0 \\ -Y_{i_j}W & 0 & 0 & 0 & -\gamma^2 I & 0 \\ P_{\hat{j}} & 0 & 0 & 0 & 0 & -\lambda_{\hat{j}} \end{bmatrix} < 0, \tag{32}$$

then, with the gains given by $K_{i_j} = P_{\hat{j}}^{-1}Y_{i_j} \in \Re^{n \times v}$*, the observer is asymptotically convergent (without external disturbances) and the* H_∞ *criterion* (8) *is satisfied with the external disturbance attenuation level* γ*.*

Proof 2. *Straightforward from the conditions expressed in Theorem 1, by the application of Lemma 3 on the first diagonal bloc of* (12). □

Theorem 2 provides the simplest proposed way to relax the conditions of Theorem 1 by introducing the slack decision variables $L_{\hat{j}}$ and $R_{\hat{j}}$ from the application of Lemma 3 on the first diagonal bloc of (12). Also, from Lemma 3, it is straightforward that Theorem 2 includes Theorem 1 as special case. Now, to further relax the proposed LMI-based conditions, let us consider the application of Lemma 3 in a more generalized way, i.e. on the whole matrix inequality (12). This result is summarized by the following theorem.

Theorem 3. *Consider the switched T-S system* (1) *and the asynchronous observer with UPVs* (4). *For all combinations of* $k_j \in \{1,\ldots,r_j\}$, $(i_{\hat{j}},\tilde{i}_{\hat{j}},q_{\hat{j}}) \in \{1,\ldots,r_{\hat{j}}\}^3$, $(j,\hat{j}) \in \{1,\ldots,m\}^2$ *and* $(\hat{j},\hat{j}^+) \in \mathbb{I}_s$, *if there exist the scalars* $\xi > 0$, $\tau > 0$, $\beta_{\hat{j}} > 0$, $\lambda_{\hat{j}} > 0$ *and the matrices* $Y_{i_{\hat{j}}} \in \Re^{n\times v}$, $G_{\hat{j}} \in \Re^{n\times v}$, $L_{\tilde{i}_{\hat{j}}} \in \Re^{(2n+p)\times(2n+p)}$, $R_{\tilde{i}_{\hat{j}}} \in \Re^{(2n+p)\times(2n+p)}$, $Z^1_{\hat{j}} \in \Re^{n\times n}$, $Z^2_{\hat{j}} \in \Re^{n\times n}$,$Z^3_{\hat{j}} \in \Re^{n\times p}$,,$Z^4_{\hat{j}} \in \Re^{p\times n}$,$Z^5_{\hat{j}} \in \Re^{p\times n}$, $Z^6_{\hat{j}} \in \Re^{p\times p}$ *and* $0 < P_{\hat{j}} = P^T_{\hat{j}} \in \Re^{n\times n}$ *such that the positive scalar* γ^2 *is minimized and satisfies the conditions expressed in Theorem 1 with, instead of* (12), *the following inequality:*

$$\left[\begin{array}{ccc:ccc:ccc} \tilde{A}^T_{i_{\hat{j}}k_jq_{\hat{j}}}L^T_{i_{\hat{j}}} + L_{\tilde{i}_{\hat{j}}}\tilde{A}_{i_{\hat{j}}k_jq_{\hat{j}}} + \tilde{H}_{i_{\hat{j}}} & & & (*) & & & (*) & & \\ \tilde{P}_{\hat{j}} - L^T_{\tilde{i}_{\hat{j}}} + R^T_{\tilde{i}_{\hat{j}}}\tilde{A}_{i_{\hat{j}}k_jq_{\hat{j}}} & & & -R^T_{\tilde{i}_{\hat{j}}} - R_{\tilde{i}_{\hat{j}}} & & & (*) & & \\ \hdashline -Y_{i_{\hat{j}}}W & 0 & 0 & 0 & 0 & 0 & -\gamma^2 I & 0 & \\ P_{\hat{j}} & 0 & 0 & 0 & 0 & 0 & 0 & -\lambda_{\hat{j}} I & \end{array}\right] < 0 \tag{33}$$

with $\tilde{A}^T_{i_{\hat{j}}k_jq_{\hat{j}}} = \begin{bmatrix} A^T_{i_{\hat{j}}} & 0 & 0 \\ A^T_{k_j} - A^T_{q_{\hat{j}}} & 0 & 0 \\ B^T_{k_j} - B^T_{q_{\hat{j}}} & 0 & 0 \end{bmatrix}$, $\tilde{P}_{\hat{j}} = \begin{bmatrix} P_{\hat{j}} & 0 & 0 \\ Z^1_{\hat{j}} & Z^2_{\hat{j}} & Z^3_{\hat{j}} \\ Z^4_{\hat{j}} & Z^5_{\hat{j}} & Z^6_{\hat{j}} \end{bmatrix}$, $\tilde{H}_{i_{\hat{j}}} = \begin{bmatrix} -C^TY^T_{i_{\hat{j}}} - Y_{i_{\hat{j}}}C + (\beta_{\hat{j}}+1+\tau)I & 0 & 0 \\ 0 & -\xi I & 0 \\ 0 & 0 & -\xi I \end{bmatrix}$,

then, with the gains given by $K_{i_{\hat{j}}} = P^{-1}_{\hat{j}}Y_{i_{\hat{j}}} \in \Re^{n\times v}$, *the observer is asymptotically convergent (without external disturbances) and the* H_∞ *criterion* (8) *is satisfied with the external disturbance attenuation level* γ.

Proof 3. *From Lemma 3, if the conditions of Theorem 3 holds, then the following inequalities are satisfied:*

$$\left[\begin{array}{ccc:ccc} \tilde{A}^T_{i_{\hat{j}}k_jq_{\hat{j}}}\tilde{P}_{\hat{j}} + \tilde{P}_{\hat{j}}\tilde{A}_{i_{\hat{j}}k_jq_{\hat{j}}} + \tilde{H}_{i_{\hat{j}}} & & & (*) & & \\ -Y_{i_{\hat{j}}}W & 0 & 0 & -\gamma^2 I & 0 & \\ P_{\hat{j}} & 0 & 0 & 0 & -\lambda_{\hat{j}} I & \end{array}\right] < 0 \tag{34}$$

which are equivalent, by opening the matrices $\tilde{A}_{i_{\hat{j}}k_jq_{\hat{j}}}$, $\tilde{P}_{\hat{j}}$ *and* $\tilde{H}_{i_{\hat{j}}}$, *to the inequalities* (12) *of Theorem 1.* □

Once again, Theorem 3 being obtained from the application of Lemma 3, it obviously includes Theorem 1 as special case. Moreover, it also includes Theorem 2 as special case. Indeed, from (33), the inequality (32) can be recovered by setting the slack decision matrices as the particular case:

$$Z^1_{\hat{j}} = Z^2_{\hat{j}} = \cdots = Z^6_{\hat{j}} = 0,\ L_{\tilde{i}_{\hat{j}}} = \begin{bmatrix} L_{\hat{j}} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \text{ and } R_{\tilde{i}_{\hat{j}}} = \begin{bmatrix} R_{\hat{j}} & 0 & 0 \\ 0 & -\delta I & 0 \\ 0 & 0 & -\delta I \end{bmatrix}, \tag{35}$$

then, by applying the Schur complement to cope with the last two diagonal blocs of $R_{\tilde{i}_{\hat{j}}}$ and taking the scalar $\delta > 0$ as small as possible ($\delta \to 0$).

Now, to further relax the condition (33), let us point-out that it refers to a four sum parameterized LMI (with the index $i_{\hat{j}}$, $\tilde{i}_{\hat{j}}$, k_j and $q_{\hat{j}}$). However, the structure of the slack decision variables ($L_{\tilde{i}_{\hat{j}}}$, $R_{\tilde{i}_{\hat{j}}}$, $Z^1_{\hat{j}}$, $Z^2_{\hat{j}}$,...) can be arbitrarily extended to provide full index compensations of the whole inequalities, but ineluctably, with an increase of the computational cost. This last theoretical result is summarized by the following theorem.

Theorem 4. *Consider the switched T-S system* (1) *and the asynchronous observer with UPVs* (4). *For all combinations of* $(i_{\tilde{j}}, \tilde{i}_{\tilde{j}}, k_j, \tilde{k}_j, q_{\tilde{j}}, \tilde{q}_{\tilde{j}}) \in \{1,\ldots,r_{\tilde{j}}\}^6$, $(j,\hat{j}) \in \{1,\ldots,m\}^2$ *and* $(\hat{j},\hat{j}^+) \in \mathbb{I}_s$, *if there exist the scalars* $\xi > 0$, $\tau > 0$, $\beta_{\hat{j}} > 0$, $\lambda_{\hat{j}} > 0$ *and the matrices* $Y_{i_{\hat{j}}} \in \Re^{n\times v}$, $G_{\hat{j}} \in \Re^{n\times v}$, $L_{i_{\tilde{j}}\tilde{k}_j\tilde{q}_{\tilde{j}}} \in \Re^{(2n+p)\times(2n+p)}$, $R_{i_{\tilde{j}}\tilde{k}_j\tilde{q}_{\tilde{j}}} \in \Re^{(2n+p)\times(2n+p)}$, $Z^1_{i_{\tilde{j}}\tilde{i}_{\tilde{j}}k_j\tilde{k}_jq_{\tilde{j}}\tilde{q}_{\tilde{j}}} \in \Re^{n\times n}$, $Z^2_{i_{\tilde{j}}\tilde{i}_{\tilde{j}}k_j\tilde{k}_jq_{\tilde{j}}\tilde{q}_{\tilde{j}}} \in \Re^{n\times n}$, $Z^3_{i_{\tilde{j}}\tilde{i}_{\tilde{j}}k_j\tilde{k}_jq_{\tilde{j}}\tilde{q}_{\tilde{j}}} \in \Re^{n\times p}$, $Z^4_{i_{\tilde{j}}\tilde{i}_{\tilde{j}}k_j\tilde{k}_jq_{\tilde{j}}\tilde{q}_{\tilde{j}}} \in \Re^{p\times n}$, $Z^5_{i_{\tilde{j}}\tilde{i}_{\tilde{j}}k_j\tilde{k}_jq_{\tilde{j}}\tilde{q}_{\tilde{j}}} \in \Re^{p\times p}$, $Z^6_{i_{\tilde{j}}\tilde{i}_{\tilde{j}}k_j\tilde{k}_jq_{\tilde{j}}\tilde{q}_{\tilde{j}}} \in \Re^{p\times p}$ *and* $0 < P_{\hat{j}} = P^T_{\hat{j}} \in \Re^{n\times n}$, *such that the positive scalar* γ^2 *is minimized and satisfies the conditions expressed in Theorem 1 with, instead of* (12), *the following inequality:*

$$\left[\begin{array}{c|c|c} \tilde{A}^T_{i_{\tilde{j}}k_jq_{\tilde{j}}} L^T_{i_{\tilde{j}}\tilde{k}_j\tilde{q}_{\tilde{j}}} + L_{i_{\tilde{j}}\tilde{k}_j\tilde{q}_{\tilde{j}}}\tilde{A}_{i_{\tilde{j}}k_jq_{\tilde{j}}} + \tilde{H}_{i_{\tilde{j}}} & (*) & (*) \\ \hline \tilde{P}_{\hat{j}} - L^T_{i_{\tilde{j}}\tilde{k}_j\tilde{q}_{\tilde{j}}} + R^T_{i_{\tilde{j}}\tilde{k}_j\tilde{q}_{\tilde{j}}}\tilde{A}_{i_{\tilde{j}}k_jq_{\tilde{j}}} & -R^T_{i_{\tilde{j}}\tilde{k}_j\tilde{q}_{\tilde{j}}} - R_{i_{\tilde{j}}\tilde{k}_j\tilde{q}_{\tilde{j}}} & (*) \\ \hline -Y_{i_{\tilde{j}}}W \quad 0 \quad 0 & 0 \quad 0 \quad 0 & -\gamma^2 I \quad 0 \\ P_{\hat{j}} \quad 0 \quad 0 & 0 \quad 0 \quad 0 & 0 \quad -\lambda_{\hat{j}} I \end{array}\right] < 0 \tag{36}$$

with:

$$\tilde{A}^T_{i_{\tilde{j}}k_jq_{\tilde{j}}} = \begin{bmatrix} A^T_{i_{\tilde{j}}} & 0 & 0 \\ A^T_{k_j} - A^T_{q_{\tilde{j}}} & 0 & 0 \\ B^T_{k_j} - B^T_{q_{\tilde{j}}} & 0 & 0 \end{bmatrix},\ \tilde{H}_{i_{\tilde{j}}} = \begin{bmatrix} -C^TY^T_{i_{\tilde{j}}} - Y_{i_{\tilde{j}}}C + (\beta_{\hat{j}}+1+\tau)I & 0 & 0 \\ 0 & -\xi & 0 \\ 0 & 0 & -\xi \end{bmatrix}$$

$$\tilde{P}_{\hat{j}} = \begin{bmatrix} P_{\hat{j}} & 0 & 0 \\ Z^1_{i_{\tilde{j}}\tilde{i}_{\tilde{j}}k_j\tilde{k}_jq_{\tilde{j}}\tilde{q}_{\tilde{j}}} & Z^2_{i_{\tilde{j}}\tilde{i}_{\tilde{j}}k_j\tilde{k}_jq_{\tilde{j}}\tilde{q}_{\tilde{j}}} & Z^3_{i_{\tilde{j}}\tilde{i}_{\tilde{j}}k_j\tilde{k}_jq_{\tilde{j}}\tilde{q}_{\tilde{j}}} \\ Z^4_{i_{\tilde{j}}\tilde{i}_{\tilde{j}}k_j\tilde{k}_jq_{\tilde{j}}\tilde{q}_{\tilde{j}}} & Z^5_{i_{\tilde{j}}\tilde{i}_{\tilde{j}}k_j\tilde{k}_jq_{\tilde{j}}\tilde{q}_{\tilde{j}}} & Z^6_{i_{\tilde{j}}\tilde{i}_{\tilde{j}}k_j\tilde{k}_jq_{\tilde{j}}\tilde{q}_{\tilde{j}}} \end{bmatrix},$$

then, with the gains given by $K_{i_{\tilde{j}}} = P^{-1}_{\hat{j}}Y_{i_{\tilde{j}}} \in \Re^{n\times v}$, *the observer is asymptotically convergent (without external disturbances) and the* H_∞ *criterion* (8) *is satisfied with the external disturbance attenuation level* γ.

Proof 4. *Straightforward from the proof of Theorem 3 by choosing the structure of the slack decision variables as* $L_{i_{\tilde{j}}\tilde{k}_j\tilde{q}_{\tilde{j}}}$, $R_{i_{\tilde{j}}\tilde{k}_j\tilde{q}_{\tilde{j}}}$, $Z^1_{i_{\tilde{j}}\tilde{i}_{\tilde{j}}k_j\tilde{k}_jq_{\tilde{j}}\tilde{q}_{\tilde{j}}}$, $Z^2_{i_{\tilde{j}}\tilde{i}_{\tilde{j}}k_j\tilde{k}_jq_{\tilde{j}}\tilde{q}_{\tilde{j}}}$, *and so on.* □

From the proof of Theorem 4, it is clear that it includes Theorem 3, which in turn includes Theorem 2 and Theorem 1 as well. Moreover, as pointed-out in the following remark, if they are suitable for the design of asynchronous switched T-S observers (4), they also include the synchronous case as a particular one.

Remark 2. *Let us highlight that the synchronous observer design is a particular case of the proposed approach in Theorem 1, 2, 3 and 4. Indeed, the LMI-based conditions* (11), (12) *(or* (32), *or* (33), *or* (36)*) can be readily adapted to provide sufficient LMI-based synchronous observer design. To do so, just replace the index* $\hat{j}$ *by* j *and the index* $q_{\tilde{j}}$ *by* k_j. *This leads to eliminate the terms* $A^T_{k_j}P_{\hat{j}} - A^T_{q_{\tilde{j}}}P_{\hat{j}}$ *and* $B^T_{k_j}P_{\hat{j}} - B^T_{q_{\tilde{j}}}P_{\hat{j}}$ *from LMIs* (12), (32), (33) *and* (36). *Moreover, it is also worth mentioning that the conditions of Theorem 1 for asynchronous T-S switched observer design with UPVs include, as a special case, our preliminary results in (Belkhiat et al., 2019), where the premises were assumed to be measurable.*

Note that the above proposed theorems are suitable for the design of asynchronous switched T-S observers subject to arbitrary switching sequences. Moreover, as mention above, the price to pay for the successively provided conservatism improvements is a significant increase of the computational cost. However, when the set of admissible switches $\mathbb{I}_s$ is known, there is no need to solve the LMI constraints for all combination of the m switching modes. Therefore, in order to properly implement the equality and LMI constraints in Theorems 1 to 4, the procedure proposed in the following remark may help to reduce the computational cost.

Initialization

$j \leftarrow j_0$, $\hat{j} \leftarrow \hat{j}_0$

Main Loop

Solve (12) (or (32), or (33), or (36), depending on the chosen theorem),
Compute the Lyapunov matrix P_{j^+} according to (11),
Set $j_0 \leftarrow j^+$ as the forthcoming mode of the system in $\mathbb{I}_s$,
Set $\hat{j}_0 \leftarrow \hat{j}^+$ as the forthcoming mode of the observer in $\mathbb{I}_s$,

Repeat for all the admissible switches in $\mathbb{I}_s$.

Algorithm 1: Implementation procedure for the LMI-based conditions provided in Theorems 1 to 4.

Remark 3. *Let j_0 and $\hat{j}_0$ be respectively the initial modes of the considered switched T-S system* (1) *and of the asynchronous switched T-S observer* (4) *(j and $\hat{j}$ are their actual modes in the set of admissible switches $\mathbb{I}_s$). The equality and LMI conditions of Theorems 1 to 4 can be implemented according to Algorithm 1 in order to reduce their computational cost.*

The theoretical part of our proposal being now established, this section is concluded. In the following section, simulation tests, discussion and comparisons are proposed to illustrate the effectiveness of the above proposed asynchronous switched T-S observer design methodologies.

4. Simulation Results

In this section, two simulation examples are proposed to illustrate the effectiveness of the proposed robust asynchronous observer design methodology for switched T-S systems with UPVs. The first example is an academic one, dedicated to compare the conservatism and the effectiveness of the proposed conditions with regards to several previous related studies (Garbouj et al., 2019; Zheng et al., 2018b; Hong et al., 2018; Belkhiat et al., 2019). Then, the second example shows the effectiveness of the proposed methodology for the design of a robust asynchronous observer with UPVs for a switched nonlinear system having a physical meaning, i.e., a switched tunnel diode circuit. Note that these simulation examples have been implemented in Matlab (using the ode23 solver) and the LMI conditions of the above proposed theorems have been solved using the YALMIP Toolbox (Lofberg, 2004) with the semidefinite programming solver SeDuMi (Labit et al., 2002).

4.1. Academic example for conservatism comparison

The goal of this academic example is to discuss the conservatism of the LMI-based conditions proposed in Theorems 1-4, with respect to previous results (Garbouj et al., 2019; Zheng et al., 2018b; Hong et al., 2018; Belkhiat et al., 2019). Note that, from the previous literature, we failed to find suitable LMI-based conditions for switched T-S observers that exhibit both UPVs and asynchronous switched modes. Therefore, for the conservatism comparison purpose, we consider the following recent and closely related studies:

- Theorem 1 in Garbouj et al. (2019), which considers the design of interval observers for switched T-S systems with UPVs in the synchronous case,
- Theorem 1 in Hong et al. (2018), which proposes the design of H_∞ filters for switched T-S systems with asynchronous switched modes but without UPVs,
- Theorem 1 in Zheng et al. (2018b), where an average dwell-time approach is proposed for the design of switched T-S Luenberger-like filters, assuming that the premises variables are fully measurable and with synchronous switched modes,

- Theorem 1 in Belkhiat et al. (2019), which constitutes a special case (preliminary conference paper) of the present study where the premises variables are assumed fully measurable, and without the consideration of relaxation techniques such like the use of Lemma 3.

Let us consider a switched T-S system (1), with $r_j = 2$ fuzzy rules in each $m = 4$ switched modes ($j = 1, ..., 4$), specified by the following matrices:

$$A_{1_1} = \begin{bmatrix} -3.6 & 10 \\ -2 & -1 \end{bmatrix}, A_{2_1} = \begin{bmatrix} -2 & 12 \\ -2 & -1 \end{bmatrix}, A_{1_2} = \begin{bmatrix} 4b + 2.5 + a & -2 + b \\ -6a & -0.5 \end{bmatrix}$$

$$A_{2_2} = \begin{bmatrix} b + 2.5 + a & 2a \\ -2.5 & 1.9 + 4b \end{bmatrix}, A_{1_3} = \begin{bmatrix} -1.2 & 1 \\ -1.1 & -3.2 \end{bmatrix}, A_{2_3} = \begin{bmatrix} -1.5 & 0 \\ -1 & -3.2 \end{bmatrix}$$

$$A_{1_4} = \begin{bmatrix} -2.2 & 0 \\ -1.1 & -3.2 \end{bmatrix}, A_{2_4} = \begin{bmatrix} -2.3 & 1 \\ -1 + b & -3.2 \end{bmatrix}, B_{i_1} = B_{i_3} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, B_{i_2} = B_{i_4} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$C = \begin{bmatrix} 0 & 1 \end{bmatrix}, W = 0.5,$$

where a and b are two scalar parameters dedicated to check the feasibility fields of the proposed LMI-based conditions, and with the membership functions given by:

$$\begin{aligned} &\text{Mode 1: } \begin{cases} h_{1_1}(z_1(t)) = \frac{1}{2}\big(1 - sin(z_1(t))\big) \\ h_{2_1}(z_1(t)) = 1 - h_{1_1}(z_1(t)) \end{cases} \qquad \text{Mode 2: } \begin{cases} h_{1_2}(z_2(t)) = \frac{1}{2}\big(1 - sin(z_2(t))\big) \\ h_{2_2} = 1 - h_{1_2}(z_2(t)) \end{cases} \\ &\text{Mode 3: } \begin{cases} h_{1_3}(z_3(t)) = cos^2(z_3(t)) \\ h_{2_3}(z_3(t)) = 1 - h_{1_3}(z_3(t)) \end{cases} \qquad \text{Mode 3: } \begin{cases} h_{1_4}(z_4(t)) = sin^2(z_4(t)) \\ h_{2_4}(z_4(t)) = 1 - h_{1_3}(z_4(t)) \end{cases} \end{aligned} \tag{37}$$

where $x_1(t)$ and $x_2(t)$ denote the state variables, and the state dependent premise variables $z_1(t) = z_2(t) = z_3(t) = z_4(t) = x_1(t) \in \mathbb{R}$.

In this example, we consider the switching sequence $\mathcal{G} = \{\mathcal{V}, \mathcal{E}\}$ depicted in Fig. 1, where $\mathcal{V} = \{1, 2, 3, 4\}$ denotes the sets of switched modes and $\mathcal{E} = \{(1, 2), (2, 3), (3, 4), (4, 1), (1, 4), (3, 2)\}$ the set of the admissible switches between modes (Lendek et al., 2014a).

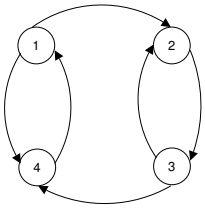

Figure 1: Graph representation of the switched system.

For several values of $a = [-15, 15]$ and $b = [-10, 25]$, with a step of 1, the feasibility of the conditions proposed in Theorems 1-4, as well as the ones proposed in (Garbouj et al., 2019; Zheng et al., 2018b; Hong et al., 2018; Belkhiat et al., 2019), has been checked with the Yalmip toolbox and the semi-definite programming solver SeDumi in Matlab (Lofberg, 2004). This results to the feasibility fields plotted in Fig. 2 and 3.

As illustrated in Fig. 2, over the 1116 points (a, b) that have been tested for each considered LMI conditions, the solutions obtained from Theorem 1 in (Garbouj et al., 2019) provide 208 feasible solutions (18.6%), which are mostly included in those obtained by solving Theorem 2 (581 feasible solutions, i.e. 52.1%), excepted for 3 points when $a = 0$. Moreover, all the feasible solutions obtained from Theorem 1 in Zheng et al. (2018b) (119 feasible solutions, 10.7%) and from Theorem 1 in Hong et al. (2018) (86 feasible solutions, 7.7%) are included in those obtained from Theorem 2.

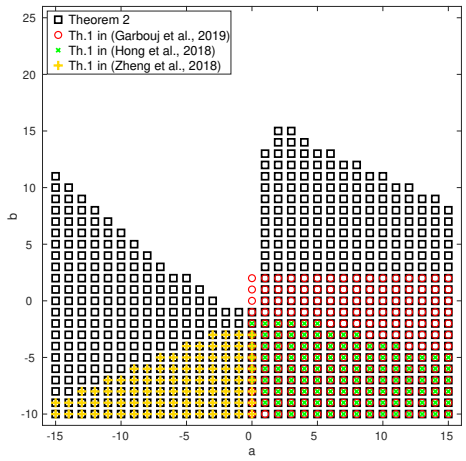

Figure 2: Feasibility fields obtained from of Theorem 2, Theorem 1 in Garbouj et al. (2019), Theorem 1 in Hong et al. (2018) and Theorem 1 in Zheng et al. (2018b).

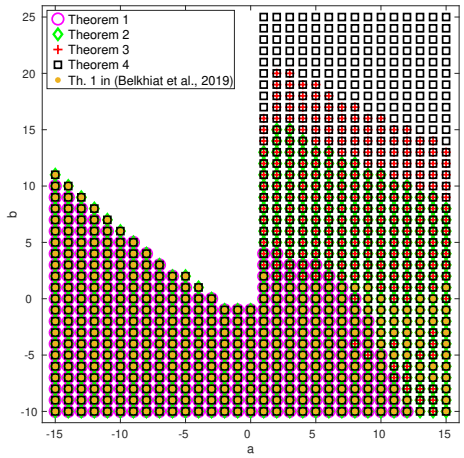

Figure 3: Feasibility enhancement of the provided theorems in this paper and Theorem 1 in Belkhiat et al. (2019) "yellow star"."

Remark 4. *Let us recall that the LMI-based conditions provided as Theorem 1 in (Garbouj et al., 2019) are only valid for the design of synchronous switched interval observers, while the conditions of Theorem 2 are valid for the more general case of asynchronous switched observers design. This explains why Theorem 1 (Garbouj et al., 2019) succeeded to find a feasible solution for the three points* $(a, b) = \{(0, 0), (0, 1), (0, 2)\}$*, whereas Theorem 2 failed. Nevertheless, excepted for these three particular points, Theorem 2 provides an overall larger feasibility fields than the one obtained from (Garbouj et al., 2019), which comfort the conservatism improvements bring by the present proposal.*

Fig. 3 shows the comparison of the feasibility fields obtained from Theorems 1-4 and Theorem 1 in (Belkhiat et al., 2019), which constitutes a special case (synchronous switched modes) of the present study. The feasibility field obtained from Theorem 4 (781 solutions, 70.0%) includes the one from Theorem 3 (655 solutions, 58.7%), which in turn includes the one of Theorem 2 (581 solutions, 52.1%), then Theorem 1 (361 solutions, 32.4%). Moreover, note that the feasibility field obtained from Theorem 1 in (Belkhiat et al., 2019) (396 solutions, 35.5%) is always included and outperformed by the ones obtained from Theorems 2-4.

These comparisons of the feasibility fields clearly indicate that, over the whole tested area, the LMI conditions proposed in this paper provide significant improvements in terms of conservatism reduction regarding to the considered previous related results (Garbouj et al., 2019; Zheng et al., 2018b; Hong et al., 2018; Belkhiat et al., 2019).

Let us now discuss the computational complexity of the proposed LMI-based conditions, compared with the complexity of the LMI-based results proposed in (Belkhiat et al., 2019; Garbouj et al., 2019; Zheng et al., 2018b; Hong et al., 2018). This comparison is detailed in Table 1, for this numerical example ($n = 2$ and $m = 4$), with regards to the conservatism achievements (Feasibility in % of the whole tested area $(a, b) \in [-15, 15] \times [-10, 25]$) for each results. Three criteria have been considered to evaluate the computational complexity of each LMI-based conditions: the number of decision variables (v) and LMI constraints (c) to be optimized, and finally, the ratio $\eta = v/c$. The latter constitutes the overall computational performance index we chose to evaluate, that is to say, the higher η is, the more is the computational complexity. Therefore, we observe that the price to pay for less conservative results is the increase of the computational complexity. Moreover, the computational complexity of Theorems 1 and 2 are somewhat comparable, but with significant conservatism improvements, to the one of the previous considered results (Belkhiat et al., 2019; Garbouj et al., 2019; Zheng et al., 2018b; Hong et al., 2018). Theorems 3, and even more Theorems 4, suffer from a significant increase of their computational complexities. However, if this can be seen as a drawback, especially for systems with large orders, let us recall that LMI computation are done offline and, as far as the computational capacity grows for daily computers, such complexity should be alleviated.

Table 1: Comparison of the computational complexity (Example 4.1).

Method	Feasibility (%)	Nb of dec. var. (v)	Nb of LMIs (c)	$\eta = v/c$
Theorem 1	**32.2**%	26	62	**0.42**
(Zheng et al., 2018b)	7.7%	16	32	0.5
(Belkhiat et al., 2019)	**35.5**%	16	29	**0.55**
Theorem 2	**52.1**%	34	62	**0.55**
(Hong et al., 2018)	10.7%	23	26	0.88
(Garbouj et al., 2019)	16.6%	21	21	1
Theorem 3	**58.7**%	193	110	**1.75**
Theorem 4	**70.0**%	3193	398	**8.02**

For the rest of this subsection and for simulation purpose, let us now consider this numerical example at the particular point $(a, b) = (0, -1)$. The switched T-S observer gain matrices and parameters, listed in Table 2, have been obtained from Theorem 2 and Theorem 1 in (Belkhiat et al., 2019). Moreover, we assume that the premise $x_1(t)$ is not measured and the designed observers and the switched system share the same switching sets (3), which hyper planes are defined by:

$$\begin{array}{c} S_{1,2} = -0.5x_1 + x_2,\ S_{2,3} = x_1 - 20x_2, \\ S_{3,2} = x_1 + 10x_2,\ S_{3,4} = x_1 + 6x_2, \\ S_{1,4} = -x_1 + x_2,\ S_{4,1} = 4x_1 + x_2. \end{array} \tag{38}$$

Fig. 4 shows the simulation results for both observers designed from Theorem 1 and Theorem 1 in (Belkhiat et al., 2019). The observers and the system have been respectively initialized in different modes (2^{nd} and 3^{rd}) with the initial conditions $x^T(0) = \begin{bmatrix} 2 & 1 \end{bmatrix}$ and $\hat{x}^T(0) = \begin{bmatrix} 5 & 5 \end{bmatrix}$, as shown in Fig. 4 to Fig. 6.

Table 2: Observer gains and parameters obtained from Theorem 2 and Theorem 1 in (Belkhiat et al., 2019).

Method	Observer gains	Scalar parameters
Theorem 2	$K_{1_1} = \begin{bmatrix} -15.64 \\ 50.51 \end{bmatrix}, K_{2_1} = \begin{bmatrix} -7.37 \\ 43.48 \end{bmatrix}$ $K_{1_2} = \begin{bmatrix} -224.9057 \\ 188.9234 \end{bmatrix}, K_{2_2} = \begin{bmatrix} -129.3769 \\ 114.2768 \end{bmatrix}$ $K_{1_3} = \begin{bmatrix} -6.56 \\ 7.40 \end{bmatrix}, K_{2_3} = \begin{bmatrix} -7.42 \\ 7.50 \end{bmatrix}$ $K_{1_4} = \begin{bmatrix} -2.69 \\ 4.33 \end{bmatrix}, K_{2_4} = \begin{bmatrix} -1.59 \\ 4.28 \end{bmatrix}$	$\bar{\beta}_1 = 7.5\times10^{-9}, \lambda_1 = 4.0$ $\bar{\beta}_2 = 0.28, \lambda_2 = 18.0$ $\bar{\beta}_3 = 11.0, \lambda_3 = 10.0$ $\bar{\beta}_4 = 0.61, \lambda_4 = 8.0$ $\xi = 150, \tau = 10^{-3}, \gamma = 3.94$
(Belkhiat et al., 2019)	$K_{1_1} = \begin{bmatrix} -87.97 \\ 52.37 \end{bmatrix}, K_{2_1} = \begin{bmatrix} -49.69 \\ 32.84 \end{bmatrix}$ $K_{1_2} = \begin{bmatrix} -61.69 \\ 8.30 \end{bmatrix}, K_{2_2} = \begin{bmatrix} -59.19 \\ 8.0 \end{bmatrix}$ $K_{1_3} = \begin{bmatrix} -98.12 \\ 59.26 \end{bmatrix}, K_{2_3} = \begin{bmatrix} -52.25 \\ 35.16 \end{bmatrix}$ $K_{1_4} = \begin{bmatrix} -12.97 \\ 2.59 \end{bmatrix}, K_{2_4} = \begin{bmatrix} -11.67 \\ 2.53 \end{bmatrix}$	$\lambda = 10.0$ $\epsilon = 7.3\times10^{3}$ $\tau = 10^{-3}$ $\gamma = 1.14$

Moreover, the following external disturbance and the input signal are applied to the system:

$$w(t) = \begin{cases} \sin(1.6\pi t + 0.5),\ \forall t \in [2.5, 3.5], \\ 0 \text{ otherwise.} \end{cases} \quad u(t) = \begin{cases} 1,\ \forall t \in [0, 2], \\ 10\sin(\pi t + 0.25),\ \forall t \in [2, 5], . \end{cases} \tag{39}$$

As we can see on Fig. 4 and 6, the switched T-S observer with UPVs designed from Theorem 1 is properly estimating the states, while with the observer designed from (Belkhiat et al., 2019) provides poor estimations. This is confirmed by the state errors and switched modes estimations plotted in Fig. 5, where the observer with the gains design from (Belkhiat et al., 2019) shows poor results. Of course, this was expected since (Belkhiat et al., 2019) doesn't cope with UPVs, which emphasizes the importance of the present proposal, for the design of switched T-S observers with UPVs.

To conclude this first example, let us now post-verify that the gamma-level attenuation (defined by the H_∞ criterion (8)) is achieved by the proposed design. To do so, from the the simulation shown in Fig. 4 and 6, where the final simulation time is $t_f = 5s$, we can compute an approximation of the effective disturbance attenuation level as:

$$\sqrt{\frac{\int_0^{t_f} e^T(t)e(t)dt}{\int_0^{t_f} w^T(t)w(t)dt}} = 3.75$$

which is lower than $\gamma = 3.94$ minimized from Theorem 2. This confirm the effectiveness of the proposed H_∞ design.

4.2. Case study of a Switched Tunnel Diode Circuit

This example is devoted to illustrate the effectiveness of the proposed switched observer design methodology (Theorem 2) on a system having a physical meaning. To do so, let us consider the modified tunnel diode circuit system, depicted in Fig. 7, which state space realization is given by (Shen et al., 2020):

$$\begin{cases} \dot{x}_1(t) = \frac{0.2}{C}x_1(t) + \frac{0.01}{C}x_1^3(t) + \frac{1}{C}x_2(t) \\ \dot{x}_2(t) = -\frac{1}{L}x_1(t) - \frac{R_{\sigma(t)}}{L}x_2(t) + \frac{1}{L}u(t) \end{cases} \tag{40}$$

where $x_1(t) = v_D$ and $x_2(t) = i_D$ are respectively the voltage and current of the tunnel diode (state variables); $\sigma(t) \in \{1, 2\}$ denotes the switching modes, whereas the resistances $R_{\sigma(t)}$ switches between two distinct values ($R_1 = 1\,\Omega$ and $R_1 = 2\,\Omega$); $C = 0.1\,F$ is the circuit capacitance; $L = 1\,H$ is the circuit inductance.

In the sequel, we assume that only $x_2(t)$ is measured and the output signal is affected by a disturbance $w(t)$ such that $y(t) = Cx(t) + Ww(t)$ with $C = \begin{bmatrix} 0 & 1 \end{bmatrix}$ and $W = 1$. Moreover, assuming $x_1(t) \in [-3, 3]$, the state dependent premise variables $z_1(t) = z_2(t) = x_1^2(t) \in [0, 9]$ and $x(t) = \begin{bmatrix} x_1(t) & x_2(t) \end{bmatrix}^T$, the switched

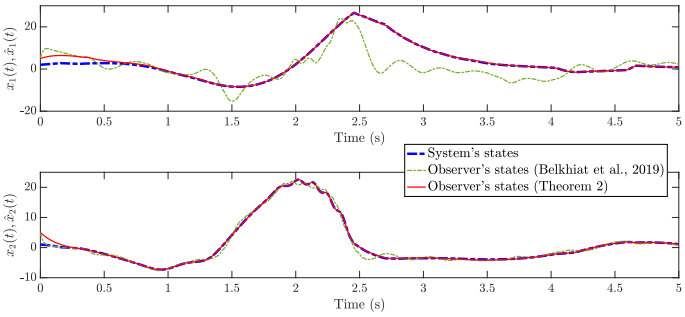

Figure 4: States and estimated states evolution.

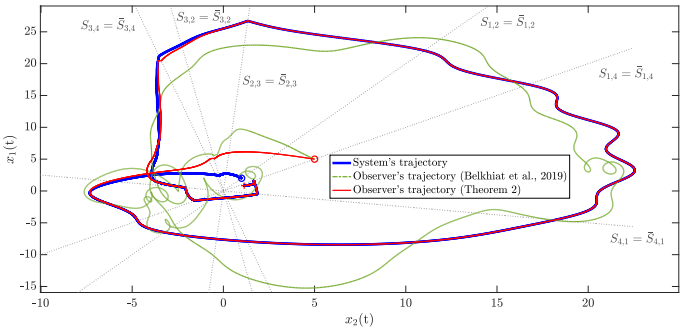

Figure 5: System's and observer's phase planes.

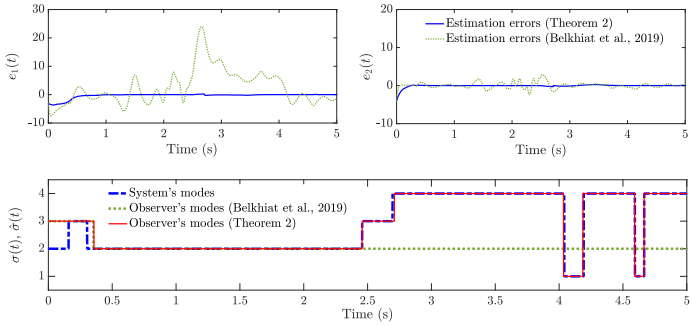

Figure 6: a) Evolution of the estimation errors, b) Evolution of system and observer modes.

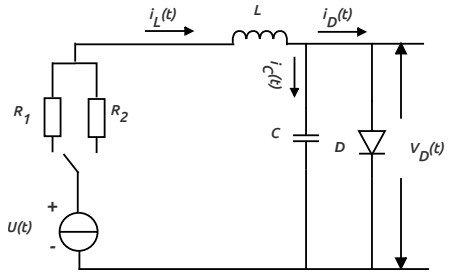

Figure 7: Switched Tunnel diode circuit

nonlinear system (40) can be exactly rewritten as a switched T-S system (1), by applying the sector nonlinearity approach (Tanaka and Wang, 2001), with $m = 2$, $r_1 = r_2 = 2$,

$$A_{1_1} = \begin{bmatrix} 2 & 10 \\ -1 & -1 \end{bmatrix}, A_{2_1} = \begin{bmatrix} 2.9 & 10 \\ -1 & -1 \end{bmatrix}, A_{1_2} = \begin{bmatrix} 2 & 10 \\ -1 & -2 \end{bmatrix}, A_{2_2} = \begin{bmatrix} 2.9 & 10 \\ -1 & -2 \end{bmatrix}, B_{1_1} = B_{1_2} = B_{2_1} = B_{2_2} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

and the membership functions:

$$h_{1_1}(z_1(t)) = h_{1_2}(z_2(t)) = 1 - \frac{z_1(t)}{9},\ h_{2_1}(z_1(t)) = h_{2_2}(z_2(t)) = 1 - h_{1,1}(z_1(t)) \tag{41}$$

The conditions of Theorem 2 have been solved using MATLAB and YALMIP (Lofberg, 2004) (with parameters $\tau = 10^{-3}$, $\xi = 10^3$). With the attenuation level $\gamma = 3.13$, we obtain the following switched T-S observer gain matrices:

$$K_{1_1} = \begin{bmatrix} -156.5504 \\ 13.0947 \end{bmatrix}, K_{2_1} = \begin{bmatrix} -142.7867 \\ 12.0289 \end{bmatrix}, K_{1_2} = \begin{bmatrix} -523.8499 \\ 34.9150 \end{bmatrix}, K_{2_2} = \begin{bmatrix} -453.8294 \\ 30.2984 \end{bmatrix},$$

the Lyapunov matrices:

$$P_1 = \begin{bmatrix} 0.23 & 1.54 \\ 1.54 & 19.66 \end{bmatrix}, P_2 = \begin{bmatrix} 0.23 & 2.52 \\ 2.52 & 38.24 \end{bmatrix},$$

and the decision variables $G_1 = \begin{bmatrix} 0.97 & 9.29 \end{bmatrix}$, $\bar{\beta}_1 = 0.18$, $\bar{\beta}_2 = 0.54$, $\lambda_1 = 307.89$ and $\lambda_2 = 193.41$.

To check the effectiveness of the designed switched T-S observer, the following input signal is set in simulation to the tunnel diode circuit:

$$\begin{cases} u(t) = \sin(1.6\pi t + 0.5),\ \forall t \in [1.4, 5] \\ u(t) = 0, \text{ otherwise} \end{cases}$$

Also, a noisy output disturbance signal $w(t)$ is set as:

$$\begin{cases} w(t) = 0.02\sin(1.65\pi t + 0.4) + r(t), \forall t \in [1.6s, 2.6s] \\ w(t) = r(t), \text{ otherwise} \end{cases}$$

where $r(t)$ is an additive white Gaussian noise with a signal to noise ratio equal to $20dB$.

For a first simulation (*case 1*), let us consider that the switched system and the switched observer share the same switching sets $\bar{S}_{\hat{j},\hat{j}^+} = S_{j,j^+}$, defined by the hyper planes $S_{1,2} = -0.01x_1 + 2x_2$ and $S_{2,1} = 2x_1 + 0x_2$. In this first case, we choose the initial conditions $x^T(0) = [0.3\ \ -0.06]$ and $\hat{x}^T(0) = [-0.3\ \ -0.7]$, so that the system and the observer are respectively initialized in their second and first modes. The simulation

results are shown in Fig. 8, 9 and 10. Fig. 8 shows the trajectories of both states of the system and their estimates. Fig. 9 exhibits the estimation errors and the evolution of asynchronous switched modes of the system and the observer. Fig. 10 shows the output of the system subject to noisy disturbances, the same without noise (for indication), and the estimated output from the observer. It is worth to point-out that, due to the employed H_∞ criterion, the transfer between the external disturbances to the state estimation error is successfully attenuated, so that the estimated output provides a filtered estimation of the system's one (despite the presence of noise). Moreover, even if the system and the observer share the same switching sets, their switching modes evolve asynchronously because they are not initialized in the same mode and the system output is affected by external disturbances. However, in this case, when the state estimation error converges, the observer modes provide fine estimates the systems ones (see Fig. 9). To conclude this first simulation case, from Fig. 8 and Fig. 9, we observe that the switched system's states are accurately estimated by the designed asynchronous switched T-S observer, despite the presence of output disturbances, as illustrated by Fig. 10 where the system noisy output and the observer filtered one are plotted.

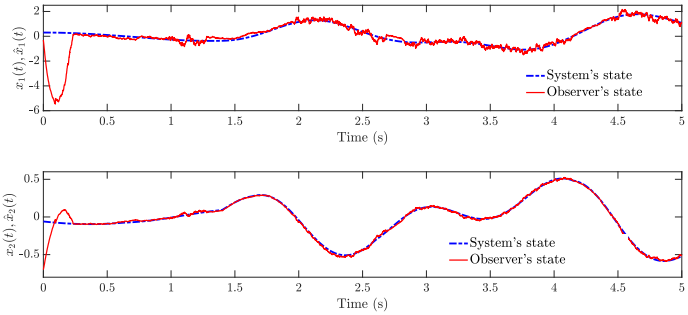

Figure 8: States and estimated states evolution in the presence of noise (example 2, *case 1*).

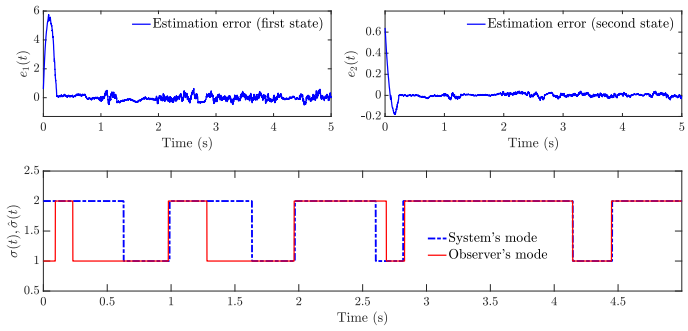

Figure 9: Evaluations of the estimation errors and asynchronous switched modes (example 2, *case 1*).

For the second simulation case (*case 2*), we assume that the switching sets of the system and the observer are mismatched That is to say $\tilde{S}_{\hat{j},\hat{j}^+} \neq S_{j,j^+}$ defined by the hyper planes $S_{1,2} = -0.01x_1 + 2x_2, S_{2,1} = 2x_1 + 0x_2$ for the switched T-S system, and for the switched T-S observer by the hyper planes

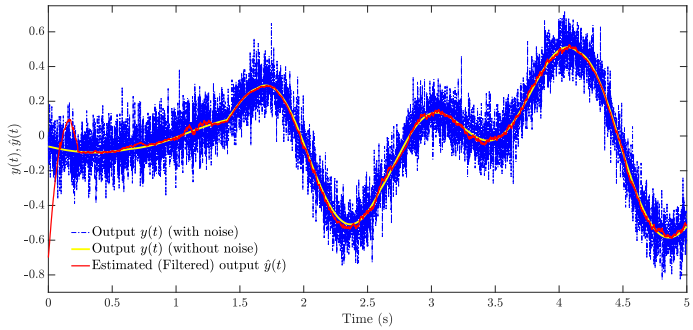

Figure 10: System's and observer's output with and without noise (example 2, *case 1*).

$\bar{S}_{1,2} = -0.1\hat{x}_1 + 3\hat{x}_2, \bar{S}_{2,1} = 3\hat{x}_1 + 1\hat{x}_2$. The system and the observer are respectively initialized in their second and first modes, with the same initial conditions as in *case 1*. The simulation results are shown in Fig. 11 to Fig. 13. The trajectories of both state and their estimates are depicted in Fig. 11. Fig. 12 shows the estimation errors as well as the evolution the asynchronous switched modes of the system and observer. In addition, Fig. 13 depicts the observer's output, which provide a filtered estimation of the system's one, affected by the external disturbance with noise (the system's output without noise is given for indication). From these figures, we can conclude that the observer successfully estimates the states of the system despite the mismatch of the switching sets, the presence of the noisy modes disturbance, and the fact that the system and the observer are initialized in different modes.

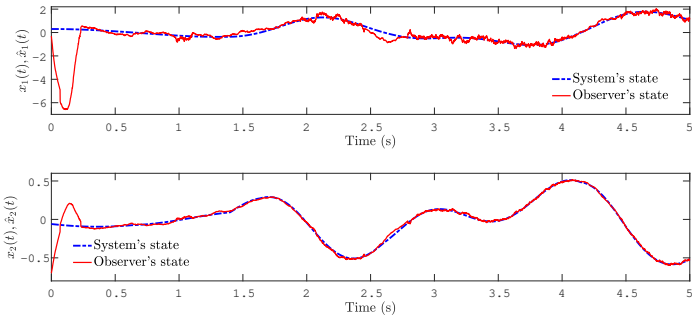

Figure 11: States and estimated states evolution (example 2, *case 2*).

Remark 5. *A systematic way to obtain a switched T-S system from a switched nonlinear one is to apply the well known sector nonliterary approach (Tanaka and Wang, 2001) on each nonlinear mode. In this case, the resulting T-S model is valid globally for global nonlinear sectors (like in the first example given in the subsection 4.1) or locally in a compact subset of the state space for local nonlinear sectors, which is the case of the second example (presented in the subsection 4.2) where we assume $x_1 \in [-3, 3]$. Of course, in this second case, if the state variables exit the validity domain of the switched T-S model, the convergence of the state estimation error cannot be guaranteed, which is one of the main limitation of T-S model-based*

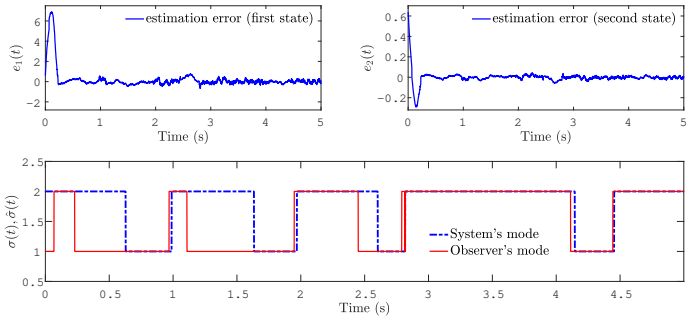

Figure 12: Evolution of the estimation errors and asynchronous switched modes (example 2, *case 2*).

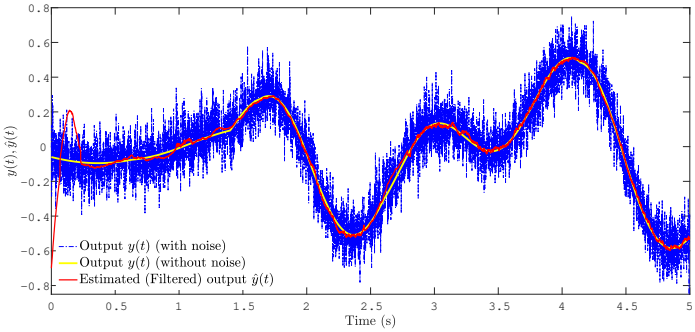

Figure 13: System's and observer's outputs with and without noise (example 2, *case 2*).

observers obtained from local sector nonlinearity approaches. In this context, the estimation of the domain of attraction of the designed state estimation error dynamics should be investigated more deeply, for instance by computing the maximum Lyapunov level set included in the validity domain of the switched T-S model, and by considering bounded input signal $u(t)$ constraints. This point is left-out from this study and will be the subject of one of our further prospects.

5. Conclusion

This paper has investigated the design of asynchronous switched Takagi-Sugeno observer with unmeasurable premise variables for a class of switched continuous-time Takagi-Sugeno systems subject to external output disturbances. To obtain the design conditions, expressed in terms of Linear Matrix Inequalities, Lipschitz conditions were assumed to cope with the unmeasurable premise variables. Also, to attenuate the output disturbances on the state estimation error, a H_∞ criterion has been employed. The design conditions have been declined into four theorems with successive conservatism improvements, naturally at the expense of the computational cost. Two simulation examples have been proposed to illustrate the effectiveness of the proposed asynchronous switched Takagi-Sugeno observer design methodologies. The first academic example

illustrates the conservatism improvement raised by this proposal regarding to previous related results from the literature. The second example have illustrated the applicability of the designed asynchronous switched Takagi-Sugeno observer on a system with physical meaning, namely a tunnel diode circuit. In perspective of this work, our future research will be focused on relying to the state estimation of a wider class of nonlinear switched systems, including network-induced phenomena, which is a key point in many daily applications. Also, recall that the present study assumed arbitrary switching sequences driven by given switching sets. If mismatches are now allowed between the switching laws of the observer and the system, there is still open issues to be dealt with or improved, such like the online system's switched modes estimation or identification. Furthermore, another interesting way to follow could be the co-design of the observer switching mechanism to further improve the transient of the estimation error dynamics, as well as to maximize the domain of attraction of the state estimation error dynamics. Finally, further works can also be made to cope with modeling uncertainties, which is an important topic for real applications, for instance by considering well-known norm-bounded uncertainty conditions of by considering extensions to the Type-2 fuzzy logic modeling framework.

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