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# State and fault estimation of a class of nonlinear systems with slow internal dynamics: Asymptotic Decoupling approach

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**Abstract:** This paper presents an Unknown Input Observer (UIO) for a class of Lipschitz nonlinear systems affected by disturbances/faults and having slow internal dynamics after the decoupling of the Unknown Input (UI). Thanks to the Asymptotic Decoupling notion, the constraint related to the internal dynamics is bypassed. The nonlinear UIO presents time varying dynamics that allows to obtain the Asymptotic Decoupling notion. It will be shown that the proposed approach allows to enhance the convergence rate of the UIO. The convergence enhancement of the state estimation error is demonstrated via a Lyapunov analysis and the established conditions are expressed in terms of Linear Matrix Inequalities. Finally, simulations will be given in order to illustrate the proposed approach with some comparisons to existing techniques.

*Keywords:* State estimation, Unknown Input Observers (UIO), Asymptotic Decoupling concept, Internal Dynamics, Nonlinear Lipschitz Systems, LMIs

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## 1. INTRODUCTION

Fault diagnosis and state estimation are important tasks in modern automatic control design. It allows to estimate precisely the state and detect a fault earlier and exploit these informations for control, supervision and maintenance, and consequently, enhance the performances of the system and the security of human operator and the system.

Model-based approaches have reached a certain maturity these last years and several diagnosis problems have been solved thanks to such approaches Ding (2008); Gertler (1998); Patton et al. (1998). The main commonly used approach is the one exploiting the observers as residual generators through mild modifications. For example, the design purpose differs from the observer design in the sense that the residual generators are designed in such a way to maximize the influence of faults on the residual and to minimize the effect of eventual disturbances. This concept can be found in many approaches using the  $H_\infty$  and  $H_-$  techniques Mazars et al. (2008, 2006). In Jaimoukha et al. (2006), a factorization approach is provided to solve the  $H_-/H_\infty$  fault detection problem. The same problem is explored in Liu et al. (2005), Wang et al. (2007) by using the sensitivity analysis. In Zhong et al. (2003), a LMI approach is provided for fault detection by introducing a reference model in order to shape the residual signal. This approach is developed for uncertain LTI systems affected by faults on both the state and the output equations. The problem of actuator fault diagnosis is generally more complex to study compared to sensor ones due the fact that the actuator faults do not affect directly the outputs. These techniques can be enhanced by a simple analysis of the system instead of applying the  $H_-$  approach directly. In another hand, more interesting approaches based on Proportional Integral Observers can be exploited in order to estimate both the state of the system and the fault affecting it. This technique is suitable when a knowledge on the dynamic of the

system is available. This is the case for incipient or abrupt fault which can be approximated by second order polynomials of the form  $f(t) = a_1t + a_0$  which satisfies the property  $\ddot{f}(t) = 0$ . Of course, it is possible to derive similar results for more general models of the faults (see Gao et al. (2016); Ichalal et al. (2009) for more details). There exists also other techniques based on Unknown Input Observers which are suitable for system where no information is known on the faults. In such a situation, the decoupling approach is more suitable instead of simultaneous estimation of the state and the faults by PI observers. However, the use of UIO requires two main conditions Darouach et al. (1994): The decoupling condition, and the stability of the internal dynamics. Since the internal dynamics cannot be moved or controlled in order to enhance the performances of the observer the dynamics of the UIO is constrained by such dynamics. Recently, in Ichalal and Mammar (2020), a solution is proposed in order to handle such constraint. It consists on the use of the notion "Asymptotic Decoupling" instead of the classical "Algebraic Decoupling". This notion allows to enhance the convergence rate of the UIO in the transient phase.

In this paper, the Asymptotic Decoupling approach is used to construct an UIO in order to estimate both the state and the faults, while guaranteeing a good convergence rate even in the presence of stable but slow internal dynamics. It will be shown that this fact allows to estimate correctly the UI compared to the classical approaches.

## 2. PROBLEM STATEMENT

Let us consider the nonlinear systems of the form

$$\begin{cases} \dot{x}(t) = Ax(t) + Ed(t) + f(x(t), u(t)) \\ y(t) = Cx(t) \end{cases} \quad (1)$$

where  $x \in \mathbb{R}^n$ ,  $u \in \mathbb{R}^{n_u}$ ,  $d \in \mathbb{R}^{n_d}$  and  $y \in \mathbb{R}^{n_y}$  represent, respectively, state, known input, unknown input and output vectors

with corresponding dimensions.  $A$ ,  $E$  and  $C$  are real matrices with appropriate dimensions.  $f(x, u)$  is a Lipschitz nonlinear vector function depending on  $x$  and  $u$  which satisfies the condition

$$\|f(x, u) - f(y, u)\| \leq \nu \|x - y\|, \forall x \in D \subseteq \mathbb{R}^n \quad (2)$$

The UI  $d(t)$  may model disturbances and / or faults affecting the state equation. In order to derive the proposed UIO, the following assumptions are also assumed

- Assumption 1.* (1) The internal dynamics are stable  
(2) The unknown input  $d(t)$  is bounded  $\|d(t)\|_\infty \leq \sigma$ ,  $\sigma > 0$   
(3) The system without UI is observable  
(4)  $\text{rank}(CE) = \text{rank}(E)$

The last hypothesis is commonly used in LTI UIO design in order to ensure that there exists a constant matrix that is able to decouple the Unknown Inputs (UI) (See for example Darouach et al. (1994)). This paper focuses on this class of systems but it is possible to generalize the approach to systems that do not satisfy this rank condition (see for example Ichalal and Mammam (2015), Ichalal et al. (2015), Barbot et al. (2007), Floquet et al. (2007) for linear, LPV or nonlinear systems). In the rest of the paper it is assumed that the effect of the UI on  $x$  is taken into account in the subset  $D$  ensuring (2).

### 3. UNKNOWN INPUT OBSERVER DESIGN

#### 3.1 Structure of the UIO

The proposed UIO takes the following form

$$\begin{cases} \dot{z}(t) = N_\beta z(t) + G_\beta f(\hat{x}(t), u(t)) + L_\beta y(t) \\ \hat{x}(t) = z(t) - M_\beta y(t) \\ M_\beta = (1 - \beta(t))H \\ H = -E(CE)^- \\ \beta(t) = \kappa \exp(-\alpha t) \\ \eta(t) = \nu(1 + \beta(t)) \end{cases} \quad (3)$$

where  $(\cdot)^-$  denotes the pseudo-inverse of a matrix and  $\kappa$ ,  $\alpha$  and  $\nu$  are positive parameters to design and  $\eta(t)$  is a positive bounded function satisfying the following definitions Ichalal and Mammam (2020). The notation  $X_\beta$  means that the matrix  $X$  depend on the time-varying parameter  $\beta(t)$ . The structure of the matrices will be defined later.

The following definition presents a class of functions which will be used a a time-varying decay rate Ichalal and Mammam (2020).

*Definition 1.* (Time-Varying decay rate) A bounded time-varying function  $\eta(t) : \mathbb{R}_+ \rightarrow \mathbb{R}_+$  is a Time-Varying Decay rate if it satisfies the following conditions

- (1)  $\eta(t)$  is strictly monotonically decreasing
- (2)  $\eta_1 \leq \eta(t) \leq \eta_2$ , with  $0 < \eta_1 < \eta_2$

Notice that  $\lim_{t \rightarrow +\infty} \int_0^t \eta(\tau) d\tau = +\infty$

The objective is to design the matrices  $N_\beta$ ,  $G_\beta$ ,  $L_\beta$  and  $H$  in order to ensure asymptotic state estimation and to enhance the convergence rate even in the presence of stable but slow internal dynamics.

The state estimation error is given by

$$e(t) = x(t) - \hat{x}(t) = (I + M_\beta C)x(t) - z(t) \quad (4)$$

By defining the matrix  $P_\beta = I + M_\beta C$  and computing the time derivative of  $e(t)$ , it follows

$$\begin{aligned} \dot{e}(t) = & (P_\beta A + \dot{P}_\beta - L_\beta C - N_\beta P_\beta)x(t) + P_\beta E d(t) \\ & + N_\beta e(t) + P_\beta f(x(t), u(t)) - G_\beta f(\hat{x}(t), u(t)) \end{aligned} \quad (5)$$

Under the conditions

$$P_\beta A + \dot{P}_\beta - L_\beta C - N_\beta P_\beta = 0 \quad (6)$$

$$P_\beta = G_\beta \quad (7)$$

The state estimation error dynamics becomes

$$\dot{e}(t) = N_\beta e(t) + P_\beta \tilde{f}(x(t), \hat{x}, u(t)) + S_\beta d(t) \quad (8)$$

where

$$\tilde{f}(x(t), \hat{x}, u(t)) = f(x(t), u(t)) - f(\hat{x}, u(t))$$

and  $S_\beta = P_\beta E$ .

In one hand, as discussed in Zemouche et al. (2008); Phomchoeng et al. (2011); Ichalal and Guerra (2019), instead of using the classical Lipschitz condition, it is much more interesting to use the polytopic form of the Lipschitz condition with the aid of the Differential Mean Value Theorem. Therefore, the term  $\tilde{f}(x(t), \hat{x}(t), u(t))$  can be expressed in the following polytopic form

$$\tilde{f}(x(t), \hat{x}(t), u(t)) = \sum_{i=1}^r h_i(z(t)) \bar{A}_i e(t) = \Phi_h e(t) \quad (9)$$

with  $z \in ]\min(x, \hat{x}), \max(x, \hat{x})[$ . From this equivalent expression, the state estimation error dynamics is written equivalently as follows

$$\dot{e}(t) = \mathcal{N}_{h,\beta} e(t) + S_\beta d(t) \quad (10)$$

where  $\mathcal{N}_{h,\beta} = N_\beta + P_\beta \Phi_h$ .

On the other hand, due to the Asymptotic decoupling concept, the matrix  $S_\beta$  converges to zero asymptotically i.e.  $\lim_{t \rightarrow \infty} S_\beta = 0$ .

Consequently, if the matrix  $\mathcal{N}_{h,\beta}$  is stable, the state estimation error  $e(t)$  converges asymptotically to zero.

The matrix  $S_\beta$  can be expanded as follows

$$S_\beta = P_\beta E = E + (1 - \beta(t))HCE$$

And by construction, the matrix  $H = -E(CE)^-$  satisfies  $(I + HC)E = 0$ , it comes

$$S_\beta = -\beta(t)HCE$$

And under the assumptions that  $\|E\|_2 \leq \lambda$  and  $\beta(t)$  is positive, one obtains the following norm bound

$$\|S_\beta\|_2 \leq \lambda \beta(t) \quad (11)$$

Consequently, due to the definition of the function  $\beta(t)$ , the norm  $\|S_\beta\|_2$  converges to zero when  $t \rightarrow \infty$ . Then, if the state estimation error dynamics is stable, it will be asymptotically converging to zero since  $S_\beta$  goes to zero too.

#### 3.2 Stability analysis

Let us consider the Lyapunov function candidate

$$V(e(t)) = e^T(t)X e(t), \quad X = X^T > 0 \quad (12)$$

Its time derivative along the trajectory of (8) is expressed by

$$\begin{aligned} \dot{V}(e(t)) = & e^T(t) \left( \mathcal{N}_{h,\beta}^T X + X \mathcal{N}_{h,\beta} \right) e(t) \\ & + e^T(t) X S_\beta d(t) + d^T(t) S_\beta^T X e(t) \end{aligned} \quad (13)$$

By adding and subtracting the term

$$-\eta(t)e^T(t)Xe(t) + \gamma d^T(t)S_\beta^T S_\beta d(t)$$

where  $\eta(t)$  is a positive bounded and monotonic decreasing scalar function ( $0 < \eta_1 \leq \eta(t) \leq \eta_2$ ) and  $\gamma$  is a positive scalar (for more details of the function  $\eta(t)$  please refer to Ichalal and Mammam (2020)). Therefore, the inequality (13) is equivalent to

$$\dot{V}(e(t)) = \zeta^T(t)\Xi_{h,\beta}\zeta(t) - \eta(t)V(e(t)) + \gamma d^T(t)S_\beta^T S_\beta d(t) \quad (14)$$

where

$$\Xi_{h,\beta} = \begin{bmatrix} \mathcal{N}_{h,\beta}^T X + X\mathcal{N}_{h,\beta} + \eta(t)X & XS_\beta \\ S_\beta^T X & -\gamma S_\beta^T S_\beta \end{bmatrix}, \zeta(t) = \begin{bmatrix} e(t) \\ d(t) \end{bmatrix} \quad (15)$$

Finally, if  $\Xi_{h,\beta} \leq 0$  and taking into account (11) the time derivative of the Lyapunov function can be bounded as follows

$$\dot{V}(e(t)) \leq -\eta(t)V(e(t)) + \gamma\lambda^2\beta^2(t)\|d(t)\|_2^2 \quad (16)$$

The solution of the inequality (16) is given by

$$V(e(t)) \leq (V(e(0)) + \gamma\lambda^2\sigma^2\varpi) e^{-\int_0^t \eta(\tau)d\tau} \quad (17)$$

where  $\varpi$  is a positive scalar that bounds the term

$$\int_0^t e^{\int_0^\tau \eta(s)ds} \beta^2(\tau) d\tau < \varpi, \forall t$$

This proves that  $V(e(t))$  converges asymptotically to zero with the time varying rate  $\eta(t)$ .

### 3.3 LMI design procedure

In the previous section, the stability analysis is given and it is established under the condition that  $\Xi_{h,\beta} \leq 0$ . Straightforwardly, this condition can be expressed in a polytopic form. Firstly, let us recall the matrices

$$\mathcal{N}_{h,\beta} = P_\beta(A + \Phi_h) + \dot{P}_\beta - K_\beta C \quad (18)$$

which comes from the equation (6) where  $K_\beta = L_\beta + N_\beta M_\beta$  and  $P_\beta = I + M_\beta C$ .

The matrix  $\dot{P}_\beta$  is expressed as follows

$$\dot{P}_\beta = I + M_\beta C = I + (1 - \kappa \exp(-\alpha t))HC$$

Consequently, its time derivative is given by

$$\dot{P}_\beta = \kappa\alpha \exp(-\alpha t)HC$$

Then, the matrix  $\mathcal{N}_{h,\beta}$  can be expressed as follows

$$\begin{aligned} \mathcal{N}_{h,\beta} &= (A + \Phi_h) + (1 - \kappa \exp(-\alpha t))HC(A + \Phi_h) \\ &\quad + \kappa\alpha \exp(-\alpha t)HC - K_\beta C \end{aligned} \quad (19)$$

or in compact form as follows

$$\mathcal{N}_{h,\beta} = \mathcal{A}_{h,\beta} - K_\beta C$$

where

$$\mathcal{A}_{h,\beta} = (I + (1 - \beta(t))HC)(A + \Phi_h) + \alpha\beta(t)HC$$

Knowing that the function  $0 < \beta(t) \leq \kappa$ , a first polytopic form is obtained as follows

$$\begin{cases} \mathcal{A}_{h,\beta} = \sum_{j=1}^2 \mu_j(\beta(t))\bar{\mathcal{A}}_{h,j} \\ S_\beta = \sum_{j=1}^2 \mu_j(\beta(t))\mathcal{S}_j \end{cases} \quad (20)$$

where

$$\bar{\mathcal{A}}_{h,1} = (I + (1 - \kappa)HC)(A + \Phi_h) + \alpha\kappa HC \quad (21)$$

$$\bar{\mathcal{A}}_{h,2} = (I + HC)(A + \Phi_h) \quad (22)$$

$$\mathcal{S}_1 = -\kappa HCE \quad (23)$$

$$\mathcal{S}_2 = 0 \quad (24)$$

From this polytopic form, we construct the gain matrix  $K_\beta$  and the positive function  $\eta(t)$  in the same polytopic form as follows

$$K_\beta = \sum_{j=1}^2 \mu_j(\beta(t))\mathcal{K}_j, \quad \eta(t) = \sum_{j=1}^2 \mu_j(\beta(t))\eta_j$$

which allows to get the matrix  $\mathcal{N}_{h,\beta}$  in the form

$$\mathcal{N}_{h,\beta} = \sum_{j=1}^2 \mu_j(\beta(t))(\bar{\mathcal{A}}_{h,j} - \mathcal{K}_j C) = \sum_{j=1}^2 \mu_j(\beta(t))\bar{\mathcal{N}}_{h,j}$$

Consequently, the matrix  $\Xi_{h,\beta}$  can be expressed as follows

$$\Xi_{h,\beta} = \sum_{j=1}^2 \sum_{k=1}^2 \mu_j(\beta)\mu_k(\beta)\Theta_{h,j,k}$$

where :

$$\Theta_{h,j,k} = \begin{bmatrix} \bar{\mathcal{N}}_{h,j}^T X + X\bar{\mathcal{N}}_{h,j} + \eta_j X & X\mathcal{S}_j \\ \mathcal{S}_j^T X & -\gamma\mathcal{S}_j^T \mathcal{S}_k \end{bmatrix}$$

Now, by using the polytopic form of the matrix  $\Phi_h$ , it allows to obtain

$$\Xi_{h,\beta} = \sum_{i=1}^r \sum_{j=1}^2 \sum_{k=1}^2 h_i(z(t))\mu_j(\beta)\mu_k(\beta)\Theta_{i,j,k}$$

where

$$\Theta_{i,j,k} = \begin{bmatrix} \bar{\mathcal{N}}_{i,j}^T X + X\bar{\mathcal{N}}_{i,j} + \eta_j X & X\mathcal{S}_j \\ \mathcal{S}_j^T X & -\gamma\mathcal{S}_j^T \mathcal{S}_k \end{bmatrix} \quad (25)$$

Then, sufficient conditions for the negativity of  $\Xi_{h,\beta}$  are given by

$$\Theta_{i,j,k} \leq 0, \quad i = 1, \dots, r, \quad j, k = 1, 2$$

Finally, change of variables  $\bar{\mathcal{K}}_j = X\mathcal{K}_j$  is performed in order to obtain LMI conditions (See Ichalal and Mammam (2020) for more details).

### 3.4 Algorithm

The UIO design procedure is summarized as follows:

- (1) Define the parameters  $\kappa$  and  $\alpha$
- (2) Compute the matrix  $H = -E(CE)^-$  and compute the matrix  $M_\beta$
- (3) Compute the polytopic form of the nonlinear term (9)
- (4) Construct the polytopic matrices (20) and then the matrices (25)
- (5) Solve the LMIs

$$X = X^T > 0 \quad (26)$$

$$\begin{bmatrix} \bar{\mathcal{N}}_{i,j}^T X + X\bar{\mathcal{N}}_{i,j} + \eta_j X & X\mathcal{S}_j \\ \mathcal{S}_j^T X & -\gamma\mathcal{S}_j^T \mathcal{S}_k \end{bmatrix} \leq 0 \quad (27)$$

$i = 1, \dots, r$   
 $j, k = 1, 2$

and obtain the gain matrix  $K_\beta$

- (6) Compute the remaining matrices:

$$\begin{aligned} N_{h,\beta} &= P_\beta(A + \Phi_h) + \dot{P}_\beta - K_\beta C \\ L_\beta &= K_\beta - N_{h,\beta}H \end{aligned}$$

### 3.5 Unknown Input Estimation

After estimating the state  $x(t)$  of the system, it is often important to estimate the Unknown Inputs. Indeed, in the fault diagnosis problems or fault tolerant controllers, the availability of a precise estimation of the faults is highly required for better performances in diagnosis or fault tolerant control tasks. In order to obtain such an estimation, the following expression is used

$$\hat{d}(t) = (CE)^- (\dot{y} - CA\hat{x}(t) - Cf(\hat{x}(t), u(t)))$$

which is nothing than the inversion of the system, but since the state estimation  $\hat{x}(t)$  is precise, the estimation of the unknown input is also precise (see the simulation example). This estimation can be used for both fault diagnosis or in fault tolerant controllers to compensate the effect of the faults. Notice that due to the inherent presence of noises in the measurements  $y$ , the use of recent signal differentiation algorithms are suitable, instead of classical derivation, in order to have a precise estimation of the time derivative of  $y$ . For instance, it is possible to use the High Order Sliding Mode Differentiator Levant (2005) or ALIEN differentiator Barbot et al. (2007) or Algebraic asymptotic differentiator Ibrir (2003).

Another application of the proposed observer is in the classical bank of observers for actuator fault detection and isolation in the case where the number of inputs is greater than the number of outputs.

## 4. SIMULATION RESULTS AND COMPARISONS

Let us consider the system (1) defined by the following matrices

$$A = \begin{bmatrix} 0 & 1 & 0 \\ -3 & -2 & 1 \\ -5 & -6 & -0.03 \end{bmatrix}, E = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix},$$

$$C = [1 \ 1 \ 0],$$

$$f(x, u) = [0 \ u(t) \ x_2 \sin(x_2)]$$

The input is  $u(t) = \sin(t)$  and the fault signal is defined by

$$d(t) = \begin{cases} 0 & 0 \leq t < 5 \\ 4 & 5 \leq t < 15 \\ t - 11 & 15 \leq t < 20 \\ t - 11 + 2 \sin(1.5t) \cos(0.5t) & t \geq 20 \end{cases}$$

In order to construct the UIO, the time varying function  $\beta(t) = 10 \exp(-0.1t)$  ( $\kappa = 10$  and  $\alpha = 0.1$ ) is considered and the matrix  $H$  is computed from  $H = -E(CE)^-$ . By following the procedure of constructing the matrices of the UIO, the proposed LMIs provide a solution and the obtained results are depicted in the figures 1 and 2. Notice that, for comparison, the classical UIO is constructed according to the procedure given in Chen and Saif (2006) with a Lipschitz constant  $v = 10$  in (2) computed for a subset  $D = \{x \in \mathbb{R}^3 : |x_2| \leq 10\}$ . This same subset is used also, in the proposed approach, to express the polytopic form of the nonlinear term  $f(x, u)$  as in (9). From the figure 1, it can be seen that the slow internal dynamics affect the convergence rate of the observer, especially, for the third state which is only detectable with the classical Exact Decoupling approach. However, with the proposed Asymptotic Decoupling notion, the state estimation is better. Additionally, since the state estimations are used to compute an estimation of the fault, these lasts will be considerably affected if the estimated states are obtained from a classical UIO which is not the case with the proposed UIO as illustrated in figure 2.

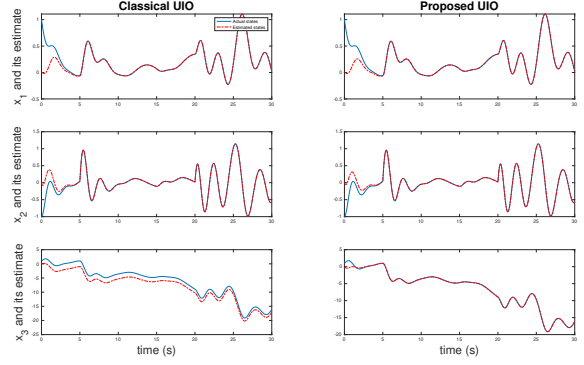


Fig. 1. Real states (blue) and estimations (red). Classical UIO (Left), Proposed UIO (Right)

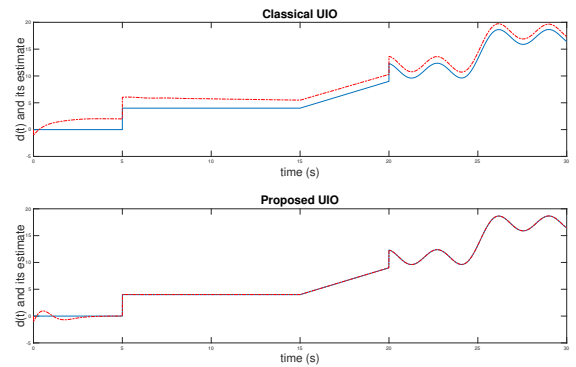


Fig. 2. Fault  $d(t)$  and its estimation with both a classical approach and the proposed one

## 5. CONCLUSION

This paper presents a novel approach to design an observer-based fault diagnosis by using a new UIO. The particularity of the proposed UIO is its ability to handle stable but slow internal dynamics of a system and then enhance the convergence rate of the observer. The proposed UIO uses a time varying function aiming to ensure the asymptotic decoupling property. This property allows to preserve the observability (lost due exact decoupling) at least in a short time interval in the transient phase. Consequently, this allows to enhance the convergence rate of the state estimation error dynamics, and then provide an acceptable estimation of the system states and faults.

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