



HAL
open science

Optimization of the travel time of an electric vehicle with consideration of the recharging terminals

Saratou Souley, Kristan Gillet, Guillaume Colin, Antoine Simon, Cédric
Nouillant, Yann Chamailard

► **To cite this version:**

Saratou Souley, Kristan Gillet, Guillaume Colin, Antoine Simon, Cédric Nouillant, et al.. Optimization of the travel time of an electric vehicle with consideration of the recharging terminals. 16th IFAC Symposium on Control in Transportation Systems, CTS 2021, Jun 2021, Lille (virtual), France. hal-03279281

HAL Id: hal-03279281

<https://hal.science/hal-03279281>

Submitted on 6 Jul 2021

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

Optimization of the travel time of an electric vehicle with consideration of the recharging terminals

Saratou Souley* Kristan Gillet* Guillaume Colin*
Antoine Simon** Cédric Nouillant** Yann Chamaillard*

* *Laboratoire PRISME - Univ. Orléans, 8, rue Léonard de Vinci,
45072 Orléans, France (e-mail: saratou.souley@gmail.com)*

** *Groupe PSA, Division for Research Innovations, 212 Boulevard
Pelletier, 78955 Carrières-sous-Poissy, France*

Abstract: This paper deals with the computation of the energy profile of a battery electric vehicle in order to find the optimal path in terms of travel time on a defined route in the presence of recharging terminals. The model used in this study to calculate the energy profile must respect constraints such as speed limits, desired initial and final battery energy states, and maximum terminal power. This problem can be written as an Optimal Control Problem and will be solved using the dynamic programming method in order to find the optimal speed and charging quantity at the terminal to save time. The results from a defined scenario show that recharging is done in favour of terminals with a higher maximum power and that our vehicle is constantly driving at the maximum speed allowed. The vehicle can also adapt its speed to reach the next charging station with the consideration of a waiting time at a station. The Dynamic Programming method allows us to respect the final energy constraints and shows that charging at high-powered terminals saves time.

Keywords: Dynamic Programming, Electric vehicles, State of Energy, Optimal control, Recharging terminals, Travel time

1. INTRODUCTION

The automotive sector has undergone profound economic and ecological changes for many years. This has resulted in the gradual electrification of vehicles because it allows, on the one hand, the reduction in pollutant emissions and, on the other hand, reductions in cost in view of the increase in the price of fuel. However, there are drawbacks to the electrification of vehicles. Apart from the high purchase price, users face a limited autonomy, frequent and lengthy recharging and the lack of recharging infrastructure. It takes an average of 3 minutes to fill up a car with an internal combustion engine compared to 15 minutes to 6 hours for an electric car. All this contributes to increasing the user's travel time over a given cycle. There is also the *Range anxiety* syndrome, i.e. the user's fear of having an empty battery before finishing the journey or arriving at a recharging station (Neubauer and Wood, 2014). Thus, it is important to develop tools that allow the user to cope with these constraints.

Several studies have proposed methods of optimal allocation of recharging stations on the basis of a green routing solved with the method of the algorithm of the shortest path (Baouche, 2015). With the same method, (Schoenberg and Dressler, 2019) searched for a compromise between optimizing the total travel time of an electric vehicle by selecting charging stations and saving energy. Another multi-objective approach by (Wang et al., 2018) was studied based on a genetic algorithm to optimize travel time,

energy consumption and billing costs for several electric vehicles. Additionally, (Wang, 2007) proposed an integer program allowing the allocation of recharging stations for electric scooters by taking into account the waiting time at the station. With the Dynamic Programming method, (Škugor and Deur, 2015) minimized the charging energy cost while satisfying the conditions for maintaining the battery charge of an electric vehicle charged from both Renewable Energy Sources (RES) and the electrical grid.

The main contribution of this article is to propose a Dynamic Programming based optimization focusing on the optimal speed to adopt and the amount of energy to recharge at the terminals in order to find an optimal energy path to save time. A second objective was to study the influence of the waiting time at the terminal on the results.

This paper contains 4 sections in which we will firstly do an energy model of the battery in Section 2. In the Section 3, we will develop the resolution method with Dynamic Programming. Then, we will show the global results based on a given scenario and compare them to other scenarios in Section 4. Section 5 gives a conclusion about the model and adds some perspectives.

2. BATTERY ENERGY MODELING

To define the remaining energy in the vehicle battery, the SOC (State of Charge) is generally used, which indicates the battery's charge level in relation to its capacity. It is

the equivalent of a fuel gauge. Following (Li et al., 2017) and (Zhang et al., 2015), several authors have referred to another variable, the state of energy (*SOE*), which is more representative of the amount of energy remaining in the battery than the SOC. Hence, in this paper we will use the variable *SOE*.

The *SOE* is defined as the ratio between the useful energy remaining in the battery $E_{bat}(t)$ and the maximum storage energy E_{max} (Miro Padovani, 2015):

$$SOE(t) = \frac{E_{bat}(t)}{E_{max}} \quad (1)$$

We can thus express the *SOE* as a function of the power exchanged in the battery P_{ech} .

$$SOE(t) = SOE(t_0) - \frac{1}{E_{max}} \int_{t_0}^t P_{ech}(\tau) d\tau \quad (2)$$

The expression of P_{ech} varies during the charging and discharging phase. When the vehicle is in motion, P_{ech} is determined by the road power law with the coefficients f_0 , f_1 , f_2 and the speed v :

$$P_{ech}(t) = f_0 v(t) + f_1 v(t)^2 + f_2 v(t)^3 \quad (3)$$

In the charging phase at the terminal, P_{ech} corresponds to the power delivered by the terminal, $P_{terminal}$:

$$P_{ech}(t, SOE) = P_{terminal}(t, SOE) \quad (4)$$

Unlike the discharging phase, P_{ech} depends on the *SOE* remaining in the battery in our model. Indeed, a recharging penalty is added to the terminal in the model. This penalty consists of the fact that the recharging speed becomes slower and slower as the recharging goes on. Thus, with an empty battery, the recharging power is close to the maximum power of the terminal and with a full battery, the recharging power tends towards 0. Thus, the recharging time will be longer at the charging point, which will impact the travel time of the vehicle. Fig.1 shows the evolution of the maximum power that can be used at the charging station as a function of the *SOE* remaining in the battery.

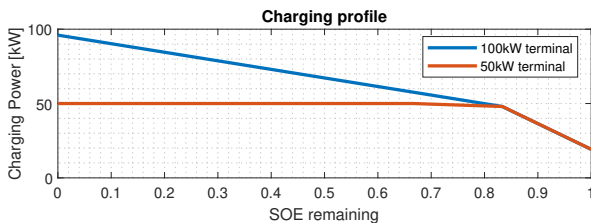


Fig. 1. Charging power profile throughout the charging process for two different maximum terminal power

In the algorithm, the amount of energy to be recharged is discretized and the power that the terminal should deliver is calculated with the *SOE* level.

3. CONSTRAINTS AND METHOD OF RESOLUTION

3.1 Assumptions and constraints

In the present study, It is assumed that:

- (i) The vehicle does not stop outside the recharging stations and that it drives at constant speed between two recharging stations.
- (ii) There are no constraints related to traffic. It is assumed that traffic is fluid and that the charging stations are not occupied.
- (iii) There is no loss model for the electric machine.

Before formulating the optimal control problem, it is necessary to define the constraints:

- (i) *Speed limit.* On a given cycle, there are maximum v_{max} and minimum v_{min} speeds not to be exceeded defined by the traffic signs. In order to respect the regulations, the maximum speed allowed is an important constraint to consider.
- (ii) *Minimum threshold for final SOE on arrival.* There is therefore a constraint on the battery final State of Energy (*SOE*).
- (iii) *Minimum and Maximum SOE.* We wish throughout the journey not to go below a threshold value of *SOE* in order to avoid premature deterioration of the battery.
- (iv) *Terminal Maximum power.* The model takes into account the maximum power delivered by the recharging station $P_{termmax}$ which varies from station to station. This a determining constraint for the calculation of the travel time.

3.2 Optimal control problem formulation

The problem can be written as an Optimal Control Problem (OCP) aiming to find the minimum of a cost function which in the present case is the vehicle total travel time T . Here, the control inputs are defined as the speed v and a variable p_{charge} , which is a ratio of E_{max} representing the quantity recharged at the terminal. Thus, the OCP is formulated as follows:

$$\{v^*, p_{charge}^*\} = \arg \min_{v \in V, p_{charge} \in P} T \quad (5)$$

subject to the constraints:

$$v_{min} \leq v \leq v_{max}$$

$$0 \leq p_{charge} \leq 1$$

$$P_{terminal} \leq P_{termmax}$$

And the boundary conditions are defined as follows:

$$d(0) = 0, d(t_f) = df \quad (6)$$

$$SOE(0) = SOE_{initial} \quad (7)$$

In a classical OCP, the state of the system (2) should satisfy the following equation (Trélat, 2007):

$$\frac{dx(t)}{dt} = \dot{x}(t) = f(t, x(t), u(t)) \quad (8)$$

with $u(t) = [v, p_{charge}]$ the control variables. From (2) we can deduce the differential equation verified by the *SOE*, the state of our system.

By defining $x = SOE$, we have:

$$\dot{x}(t) = -\frac{1}{E_{max}} P_{ech}(t) = f(t, x(t), u(t)). \quad (9)$$

(Ozatay et al., 2014) showed that an optimal control problem can be solved in both the spatial and temporal domains. As the goal is to trace the energy profile along the entire travel distance, we therefore chose to work in the space domain. The transition from the time domain to the space domain is determined by the relationship:

$$\frac{dv}{dt} = -\frac{dv}{dD} \frac{dD}{dt} = v \frac{dv}{dD}, \quad (10)$$

where D is the distance traveled.

In order to constrain the final condition $x_f = SOE_{final}$ on the state of Energy $x = SOE$, an external penalty function is added in (5) :

$$p(x) = \begin{cases} K_0(x_f - x)^2, & \text{if } x(d_f) < x_f \\ 0, & \text{if } x(d_f) \geq x_f \end{cases} \quad (11)$$

Fig. 2 shows the shape of this external penalty for $x_f = SOE_{final} = 0.075$ and $K_0 = 25000$

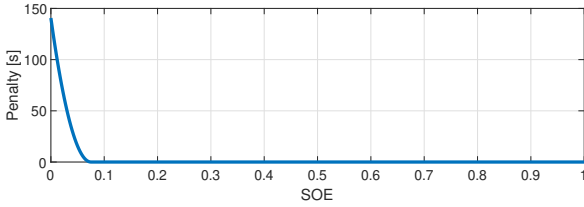


Fig. 2. External penalty for the desired final state of charge

3.3 Discretization and Dynamic Programming

In this study, the control problem is solved using Dynamic Programming (Bellman and Kalaba, 1957). Dynamic Programming is an algorithm for solving optimization problems that requires decision making (modeled by the control) with respect to a criterion to be optimized. Its principle is based on the search for an optimum of a global problem by decomposing it into sub-problems that are simpler to solve. The optimal trajectory that minimizes the cost passes through optimal sub trajectories.

Here, the OCP is discretized in terms of distance where the horizon N represents the number of sections on the cycle and a section represents the distance between two boundaries. Thus, the distance between terminals $k - 1$ and k is denoted D_k . An example of a cycle with four terminals is represented in Fig. 3.

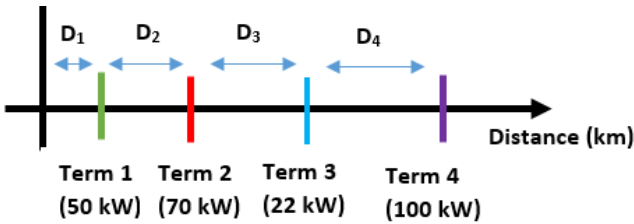


Fig. 3. Example of a cycle with four terminals

We define the steps of distance :

$$\Delta d_k = D_k - D_{k-1}$$

with $k \in \{1, \dots, N\}$

The criterion to be minimized can be written as :

$$J(u_k, D_k, x) = T = \sum_{k=1}^{N-1} (t_k + dt_k) + dt_N + p(x), \quad (12)$$

where t_k represents the charging time at terminal k defined by :

$$t_k = \frac{p_{charge}(k) E_{max}}{P_{terminal}(k, SOE)}. \quad (13)$$

and where dt_k represents the travel time between terminal $k - 1$ and k :

$$dt_k = \frac{\Delta d_k}{v_k}. \quad (14)$$

The state of charge SOE is also discretized. According to (2), we have the following discretized SOE expression, in discharging phase (3):

$$SOE(k+1) = SOE(k) - \frac{P_{ech}(k) dt(k)}{E_{max}} \quad (15)$$

and in charging phase (4):

$$SOE(k+1) = SOE(k) + p_{charge}(k) \quad (16)$$

The Dynamic Programming algorithm has two parts: backward and forward. The backward part calculates all possible paths from the end to the beginning and stores the optimal ones. The forward part traces the optimal path from the results of the backward. Algorithm 1 and Algorithm 2 show the main steps of the backward and forward algorithms implemented in the study.

Algorithm 1 Backward part of the DP algorithm

Define state and control vectors s_{tab} , p_{charge} , v_{tab}
 Form SOE , P_{charge} , P_{ech} and V , cubic matrices
 of size $\text{length}(p_{charge}) \times \text{length}(v_{tab}) \times \text{length}(s_{tab})$
 i_x : s_{tab} index
for $k = N - 1$ to 1 **do**
 $Temp_{SOE} = SOE - \frac{P_{ech} * dt}{E_{max}}$
 $SOE_{k+1} = Temp_{SOE} + P_{charge}$
 $G = dt + t_k$
 $J_k = G + J^*(SOE_{k+1}, k + 1)$
 $J(i_x, k) = \min J_k$
 $J^*(SOE_k, k) = J(i_x, k)$
 $v^*(i_x, k), p_{charge}^*(i_x, k) = \text{argmin} J_k$
end for

Algorithm 2 Forward part of the DP algorithm

Define $SOE_{initial}$
 $time = 0, a = dp; t_{charge} = 0;$
for $k = 1$ to N **do**
 $v(k, SOE) = v^*(k, SOE)$
 Calculate $dt(k), P_{ech}(v), SOE(k+1)$
 $p_{charge}(k) = p_{charge}^*(k, SOE(k))$
 $time = time + dt(k)$
 while $SOE(k+1) \leq SOE(k+1) + p_{charge}(k)$ **do**
 Find $P_{terminal}(k, SOE(k))$
 $SOE(k+1) = SOE(k+1) + a$
 $t_{charge} = t_{charge} + \frac{a E_{max}}{P_{terminal}(k, SOE(k+1))}$
 end while
 $time = time + t_{charge}$
end for

4. NUMERICAL RESULTS

4.1 Scenario used

The Dynamic Programming method was implemented on a defined scenario whose parameters are listed below:

Parameters	Values	units
Initial <i>SOE</i>	0.075	-
Desired final <i>SOE</i>	0.075	-
Minimum <i>SOE</i> allowed on the route	0.01	-
Distance of the trip	600	km
E_{max}	40000	Wh
f_0	180	N
f_1	1.1	N/m/s
f_2	0.45	N/m ² /s ²
Distance between terminals	40	km
Distance from the starting point to the first terminal	10	km
Distance from the last terminal to the end point	30	km
Maximum power of terminals	50 or 100	kW

Table 1. Parameters of the scenario used

In Dynamic Programming, a mesh on the state of the system (*SOE*) and control (v , p_{charge}) is necessary to perform the calculations. In particular *SOE* is meshed every 0.025, speed every 0.1111[m/s] and recharges every 0.012.

The terminals are placed on the cycle so that one terminal out of two has a maximum power of 100 kW from the first terminal encountered. The other terminals have a maximum power of 50 kW.

A speed limit scenario was also defined according to the sections considered, ranging from 50 km/h to 130 km/h. Fig.4 shows the limits defined for the simulation.

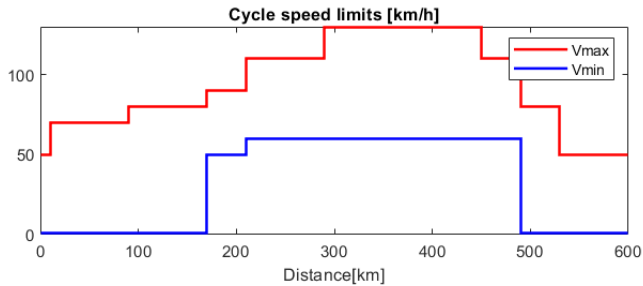


Fig. 4. Speed limits on each section of the cycle

4.2 Results with recharging penalty

With the scenario defined in Table 1, we compute the *SOE* profile for different initial *SOE*. They are plotted on a graph describing the different optimal recharging commands (Fig.5) and optimal speeds (Fig.6) thanks to the colour code.

In Fig.5, the vertical lines represent the stops at the terminals for recharging the battery. It can be seen that the vehicle stops at every two terminal corresponding to the 100 kW terminals. The algorithm therefore chooses to stop at the terminals with higher power to recharge faster and save travel time. These results are consistent with those

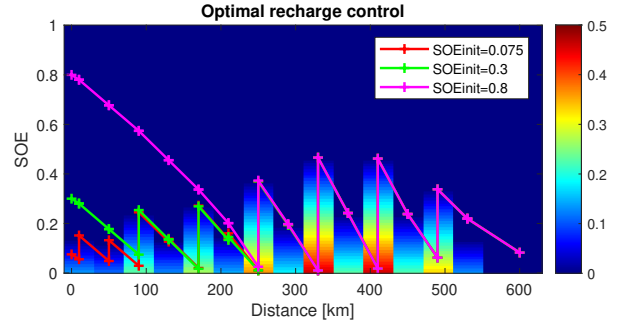


Fig. 5. Optimal energy profile and recharging commands p_{charge}^* for several initial *SOE* of 0.075, 0.3 and 0.8

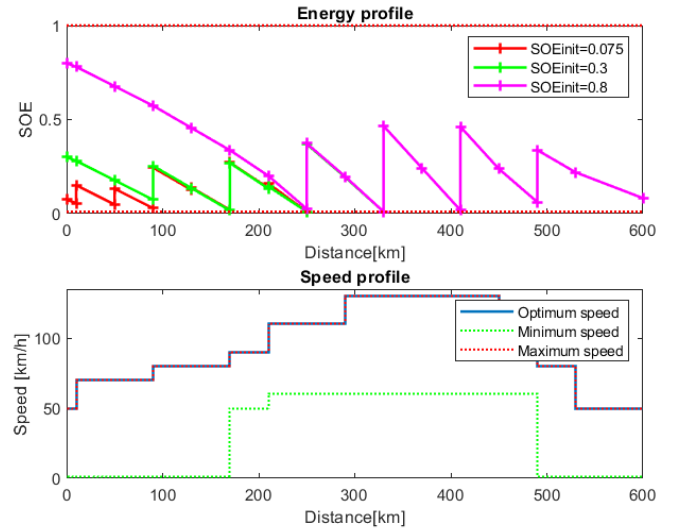


Fig. 6. Optimal energy profile and speed commands v^* [km/h] for initial *SOE* of 0.075, 0.3 and 0.8

found in the literature: (Schoenberg and Dressler, 2019) were able to show through an adaptive algorithm that travel time was optimized by charging at fast charging stations. However, due to the effects of the mesh, our algorithm has small numerical bounces that lead to charging 1 % at two 50 kW terminals if our starting *SOE* is less than 0.4. As can be seen, the higher the starting *SOE* is, the fewer stops there will be at the terminals: for an initial *SOE* of 0.8, the vehicle will stop only after 250 km. In terms of recharging, the algorithm chooses to recharge the battery to a maximum of 46 % on a given terminal in order to stay within a range of [70 kW 100kW] for the recharge power in order to save time. Note also that after a certain distance the energy profile is the same whatever the initial *SOE*. An external penalty was added on the final *SOE* constraint to come as close as possible to the minimum final *SOE* required at the finish. The final *SOE* is 0.082 (Table.2). With regard to speed Fig.6, the optimal control corresponds to the maximum speed allowed according to the section in order to have a minimum travel time between each section. In all other simulations, the optimal speed control always corresponds to the maximum allowed speed.

The cumulative time lost throughout the trip is plotted in Fig.7. The lost time represents the percentage of time

lost in relation to the distance covered by an ICE vehicle (without stopping at the terminal).

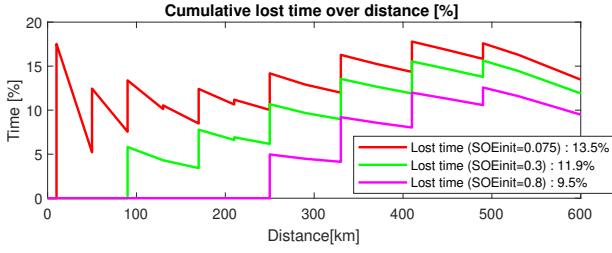


Fig. 7. Cumulated lost times for several initial SOE

In the scenario considered (Initial SOE = 0.075) the travel time corresponding to the optimal energy trajectory is 8 hours and 6 minutes. The figure above shows that even if the travel time is optimal, 13.4% of travel time is lost compared to an ICE vehicle. In the case of an electric vehicle, it is necessary to start with a full battery to save more travel time. With an Initial SOE of 0.8, there is only a 9.4% difference to an ICE vehicle.

4.3 Results with an entry ticket

In section 4.2, the optimal SOE profile is calculated along the route assuming no waiting time at the terminal. Nevertheless, this does not correspond to reality because there are other electric car drivers on the route who need to be modeled (Tang et al., 2016). The waiting time at a recharging station has to be taken into account. Waiting times have often been considered in the literature, particularly in the studies of (Liu et al., 2020) and (Luo et al., 2020) which showed respectively with a genetic algorithm and the Dijkstra algorithm that the waiting time of an electric vehicle at a station varies mainly between 0 and 10 minutes. The following simulations therefore included an entry ticket, i.e. a waiting time at the terminal in order to recharge the battery to see the impact on the optimal energy profile. The SOE profiles for 3 different waiting times, 2, 5 and 8 minutes are plotted in Fig.8 to Fig.10.

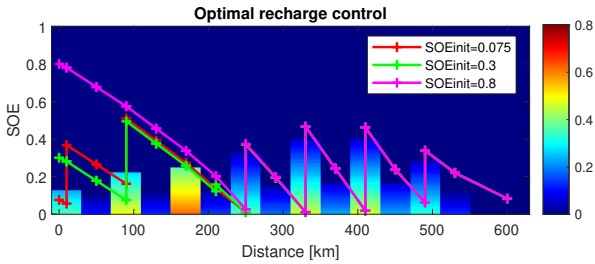


Fig. 8. Optimal energy profile and recharging commands p_{charge}^* with a 2-minute entry ticket

Adding an entry ticket means that the algorithm minimizes its number of stops. Therefore the recharging commands will be higher at a given terminal. For example in Fig.9, the battery is recharged up to 62% at the first terminal in order to skip the 100 kW terminal at 90 km. Similarly, in Figures 9 and 10, the recharging commands are constant at 81% for the terminal at 250 km. in order to skip the 100 kW terminal that follows at 330 km. Also,

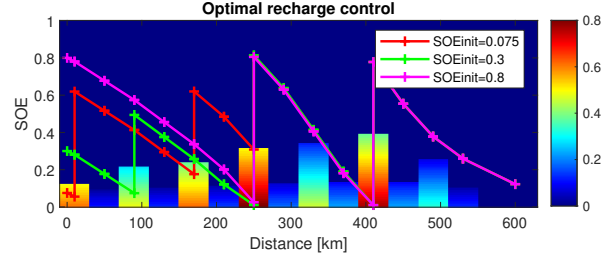


Fig. 9. Optimal energy profile and recharging commands p_{charge}^* with a 5-minute entry ticket

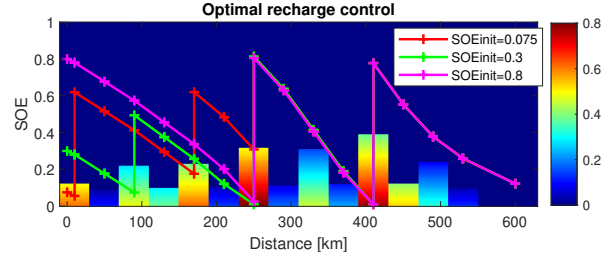


Fig. 10. Optimal energy profile and recharging commands p_{charge}^* with a 8-minute entry ticket

p_{charge} seems to increase with a higher entry ticket. By comparing Fig.9 and Fig.10, it can be seen that there are recharging commands (at 450 km) around 0.2 for 5 minutes which become recharging orders around 0.5 for 8 minutes.

The speed profile also changes by adding an entry ticket. Fig.11 plots the speed commands for a 5-minute entry ticket. The different constant speed zones have small steps of lower speeds for a well-defined SOE in order to jump on a powerful charging station and save time. Thus, for the same section, the optimum speed control varies according to the SOE in the battery. Nevertheless, these speeds are never achievable in the forward algorithm because there are enough solutions not to go through these points.

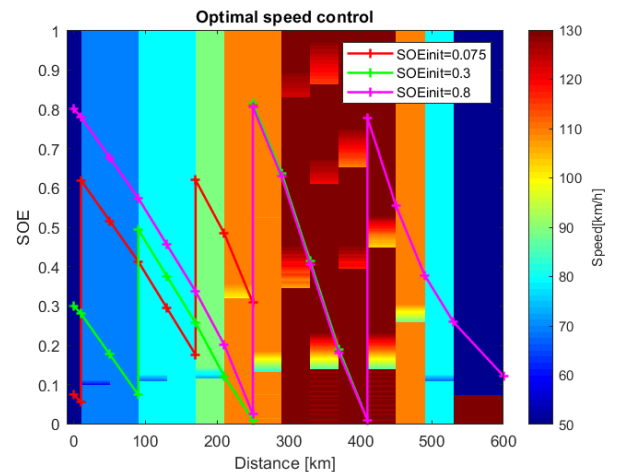


Fig. 11. Optimal energy profile and speed commands v^* [km/h] with a 5-minute entry ticket

Then, the driving and recharging times are extracted for the different situations studied. The comparison between the ICE and electric vehicles is made with respectively a

full tank and a low SOE at the beginning of the trip. The results are summarized in Table 2.

	w/o	2-min entry ticket	5-min entry ticket	8-min entry ticket
Final SOE	0.082	0.082	0.12	0.12
Total waiting time (min)	0	14	20	36
Driving time (h)	7	7	7	7
Charging time (h)	1.09	1.07	1.29	1.29
Travel time (h)	8.1	8.27	8.62	8.89
Lost time compared to ICE (%)	13.4	16	19	22

Table 2. Comparison for different waiting times or entry ticket at the charging stations

The driving time between two terminals remains the same regardless of the scenario for an initial SOE of 0.075 because the vehicle is always driven at the maximum speed allowed to optimize the time spent. What changes is the recharging time: the higher the entry ticket, the higher the overall recharging time. This is because the recharging command is much higher in order to skip as many terminals as possible. Recharging more at a given charging station saves more time than waiting 2, 5 or 8 minutes at all 100 kW charging terminals because the time gained in these three cases is respectively 36 seconds, 3 minutes and 12 minutes. So, In the scenario with an entry ticket, more time is saved by skipping terminals instead of waiting at each 100 kW terminal.

5. CONCLUSION

In this study, an optimization method using Dynamic Programming was presented to find an optimal route in terms of travel time by selecting the charging stations. The model considers battery state of energy as a state and then the speed and the amount recharged at the stops as control variables. After implementation, the numerical results show a preference for charging at high power terminals while keeping the SOE below a particular threshold to avoid inefficient charging regions and reduce the time lost along the trip. Also, in order to optimize the driving time, the algorithm shows that the optimal speed is equal to the maximum speed allowed. However, such a model is very simplistic because it does not take into account the waiting time, which has a considerable impact on the results. By considering the waiting time, the algorithm skips as many terminals as possible. It also prioritizes the powerful charging station and is ready to adapt speed to reach the next good one even if it means recharging more at a given terminal. Nevertheless, Dynamic Programming proves to be effective ensuring coherence results of the model which is a good basis for adding other parameters to be taken into account in future work : the battery efficiency in charge and discharge phase (Škugor and Deur, 2015), traffic constraints that have been the object of several studies (Tang et al., 2016), the price of the trip and speed which is not assumed to be constant in reality because of traffic constraints and stops at the terminals. Several parameters in future work should be taken into account not only offline but in real time to achieve greater concordance in the results.

REFERENCES

- Baouche, F. (2015). *Outils pour l'optimisation de la consommation des véhicules électriques*. Ph.D. thesis.
- Bellman, R. and Kalaba, R. (1957). Dynamic programming and statistical communication theory. *Proceedings of the National Academy of Sciences of the United States of America*, 43(8), 749.
- Li, K., Wei, F., Tseng, K.J., and Soong, B.H. (2017). A practical lithium-ion battery model for state of energy and voltage responses prediction incorporating temperature and ageing effects. *IEEE Transactions on Industrial Electronics*, 65(8), 6696–6708.
- Liu, J., Peper, J., Lin, G., Zhou, Y., Awasthi, S., Li, Y., and Rehtanz, C. (2020). A planning strategy considering multiple factors for electric vehicle charging stations along german motorways. *International Journal of Electrical Power & Energy Systems*, 124, 106379.
- Luo, Y., Feng, G., Wan, S., Zhang, S., Li, V., and Kong, W. (2020). Charging scheduling strategy for different electric vehicles with optimization for convenience of drivers, performance of transport system and distribution network. *Energy*, 194, 116807.
- Miro Padovani, T.M. (2015). *Loi de gestion d'énergie embarquée pour véhicules hybrides: approche multi-objectif et modulaire*. Ph.D. thesis.
- Neubauer, J. and Wood, E. (2014). The impact of range anxiety and home, workplace, and public charging infrastructure on simulated battery electric vehicle lifetime utility. *Journal of power sources*, 257, 12–20.
- Ozatay, E., Onori, S., Wollaeger, J., Ozguner, U., Rizzoni, G., Filev, D., Michelini, J., and Di Cairano, S. (2014). Cloud-based velocity profile optimization for everyday driving: A dynamic-programming-based solution. *IEEE Transactions on Intelligent Transportation Systems*, 15(6), 2491–2505.
- Schoenberg, S. and Dressler, F. (2019). Planning ahead for ev: Total travel time optimization for electric vehicles. In *2019 IEEE Intelligent Transportation Systems Conference (ITSC)*, 3068–3075. IEEE.
- Škugor, B. and Deur, J. (2015). Dynamic programming-based optimisation of charging an electric vehicle fleet system represented by an aggregate battery model. *Energy*, 92, 456–465.
- Tang, T.Q., Xu, K.W., Yang, S.C., and Ding, C. (2016). Impacts of soc on car-following behavior and travel time in the heterogeneous traffic system. *Physica A: Statistical Mechanics and its Applications*, 441, 221–229.
- Trélat, E. (2007). *Commande optimale*. *Notes de cours*.
- Wang, Y.W. (2007). An optimal location choice model for recreation-oriented scooter recharge stations. *Transportation Research Part D: Transport and Environment*, 12(3), 231–237.
- Wang, Y., Bi, J., Guan, W., and Zhao, X. (2018). Optimising route choices for the travelling and charging of battery electric vehicles by considering multiple objectives. *Transportation Research Part D: Transport and Environment*, 64, 246–261.
- Zhang, Y.Z., He, H.W., and Xiong, R. (2015). A data-driven based state of energy estimator of lithium-ion batteries used to supply electric vehicles. *Energy Procedia*, 75, 1944–1949.