
INVERTING AND REGULARIZING THE SADOWSKY CONSTITUTIVE RELATION FOR ELASTIC RIBBONS

SUPPLEMENTARY MATERIAL

Sebastien Neukirch*
d'Alembert Institute for Mechanics
CNRS & Sorbonne Université
UMR 7190, Paris, France

Basile Audoly
Laboratoire de mécanique des solides
CNRS, Institut Polytechnique de Paris,
UMR 7649, Palaiseau, France

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ABSTRACT

In this supplementary file, we show more figures. Please note that a Mathematica notebook is also given in the supplementary contents.

*corresponding author

1 The twist $u_3(s)$ for the Moebius configuration

$$\lambda^*(m_2, m_3) = 1 + 2a^2 - 2\sqrt{a^4 + a^2} \quad \text{with } a = m_2/m_3 \quad (1a)$$

$$u_2^*(m_2, m_3) = \frac{m_2}{(1 - \rho(\lambda^* - \epsilon))(1 + \rho\lambda^*)} \quad (1b)$$

$$u_3^*(m_2, m_3) = \frac{m_3}{2(1 + \rho\lambda^*)} \quad (1c)$$

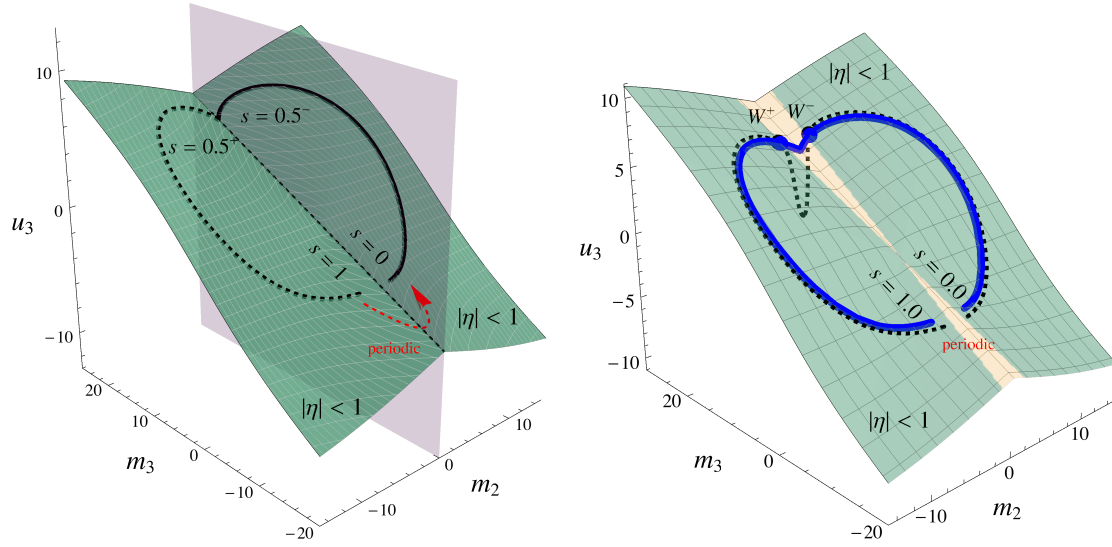


Figure 1: The twist strain u_3 as a function of m_2 and m_3 for Moebius solution. (Left) Sadowsky inverted law, Equation (1b) with $\rho = 1$ and $\epsilon = 0$, exhibiting the jump at $s = 1/2$ and the periodic closing between $s = 0$ and $s = 1$. The plane $m_2 = 0$ is also displayed. (Right) Regularized Sadowsky model with $\rho = 1$ and $\epsilon = 0.01$ (blue). Wunderlich model (black, dotted). The surface is color coded with $|\eta| > 1$ for the yellow parts and $|\eta| < 1$ for the green parts.

2 Three shapes

Figure 2 shows three Moebius solutions, computed with (1) Kirchhoff model ($\rho = 0$ and $\epsilon = 0$), (2) Sadowsky's model ($\rho = 1$ and $\epsilon = 0$), and (3) Sadowsky's regularized model ($\rho = 1$ and $\epsilon = 0.01$). The three shapes are approximately the same, with the last two virtually indistinguishable.

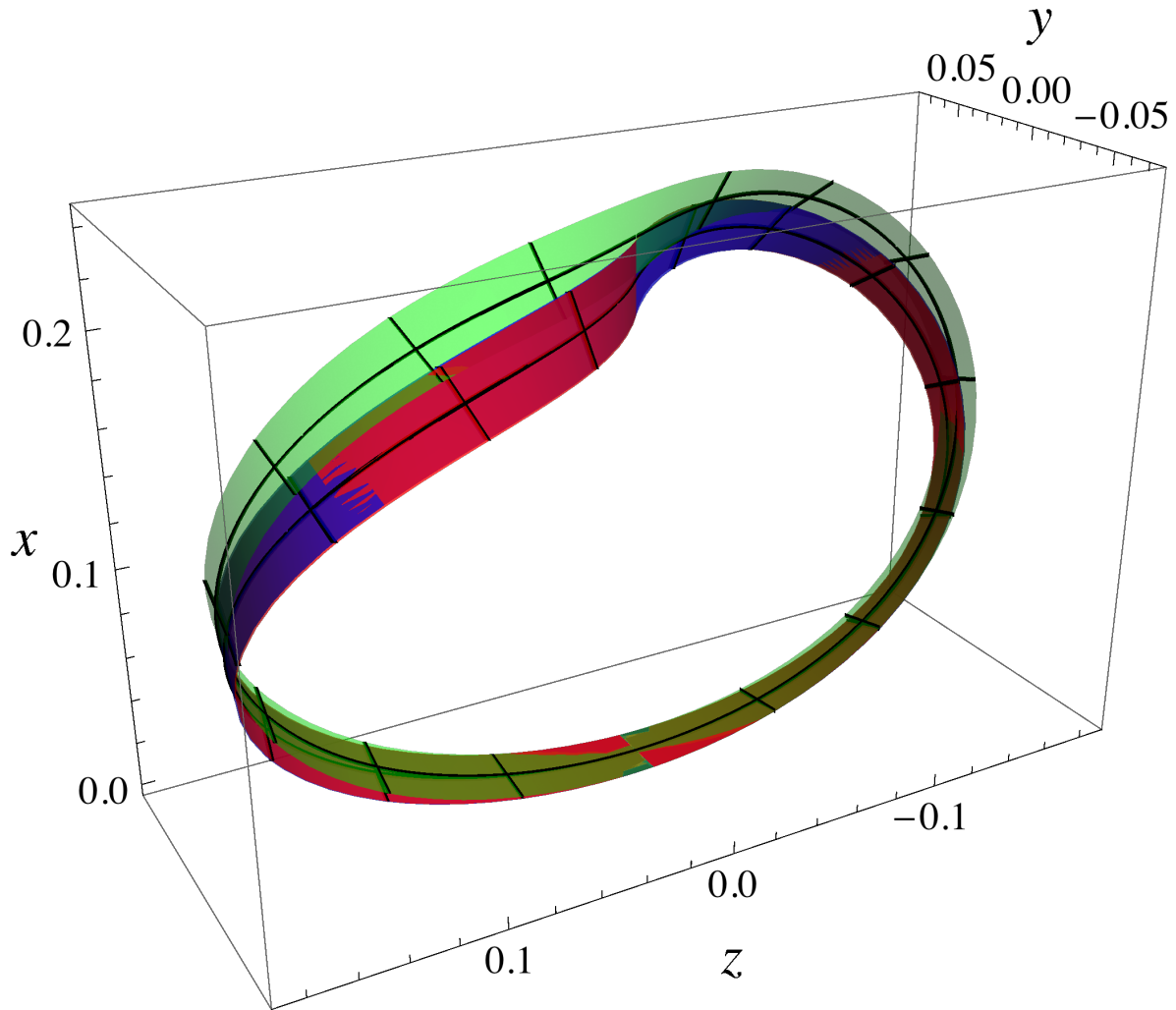


Figure 2: Moebius solution for (Green) Kirchhoff model ($\rho = 0$ and $\epsilon = 0$), (Red) Sadowsky's model ($\rho = 1$ and $\epsilon = 0$), and (Blue) Sadowsky's regularized model ($\rho = 1$ and $\epsilon = 0.01$).

3 From Kirchhoff to Sadowsky's models

Figure 3 shows the curvature $u_2(0)$, opposite to the Wunderlich point, computed with the constitutive relation (1) with $\epsilon = 0$, for the Moebius configuration. The limit $\rho = 0$ corresponds to the Kirchhoff model, while the limit $\rho = 1$ corresponds to the Sadowsky's model.

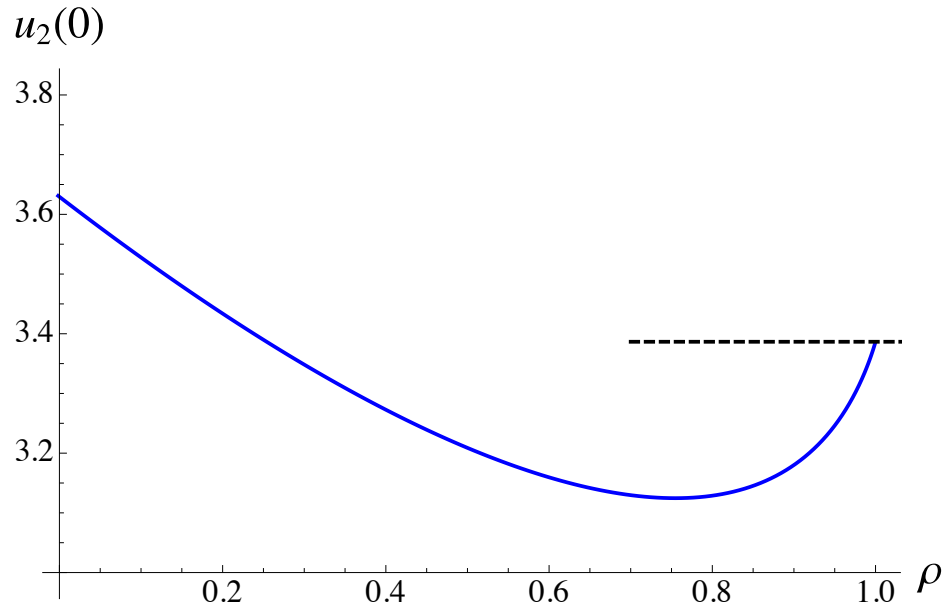


Figure 3: Curvature at $s = 0$ (opposite to the Wunderlich event) for the Moebius solution computed with (1). The black dashed line corresponds to the Sadowsky's limit, $u_2(0) = 3.3865$.