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RESEARCH ARTICLE

Initiation of deep convection through deepening of a well-mixed boundary layer

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Toulouse Cedex, France.
Email: jiy.gfder@gmail.com**Abstract**

The present study examines mechanisms by which deep moist convection may be initiated through deepening of a well-mixed convective boundary layer under large-scale low-level convergence. The process is examined using a standard formulation for the well-mixed boundary layer, in which the depth increases exponentially with time under a constant convergence with height. Consideration is also given to a case with a more realistic mean vertical velocity profile which has a maximum in the middle troposphere and attenuates to zero at the tropopause. In the latter case, the unstable well-mixed layer under convergence grows to a troposphere-deep mixed layer. Importantly, under these scenarios, the so-called convective inhibition (CIN) does not inhibit deepening of a well-mixed convective boundary layer, nor does deep moist convection need to overcome CIN to initiate.

KEYWORDS

convective inhibition, convective initiation, mixed-layer top entrainment, well-mixed convective boundary layer

1 | INTRODUCTION

It is phenomenologically well established that low-level convergence tends to induce deep moist convection (e.g., Byers and Braham, 1949; Weaver, 1979; Wilson and Schreiber, 1986). A very simple explanation for this tendency is to interpret it as a mechanism to gather moisture to the lower atmosphere, which is necessary to initiate convection. The idea has naturally led to the use of moisture convergence as a condition for closing a convection parametrization, as originally proposed by Kuo (1965, 1974). This idea has been popular during earlier developments of convection parametrization (e.g., Anthes, 1977; Bougeault, 1985; Tiedtke, 1989). However, the idea

has been gradually discarded based on a criticism that moisture convergence is better considered to be feedback of large-scale circulations to convection in response to growing convection, rather than as a prerequisite for initiating convection (cf., Emanuel *et al.*, 1994). However, the attempt to statistically predict thunderstorms by moisture convergence has hardly died out even today (e.g., van Zomeren and van Delden, 2007).

Processes leading to the initiation of deep moist convection are somehow yet not well understood. Our lack of understanding further leads to technical difficulties in properly timing initiation of deep convection in numerical modelling at large scales both for weather forecasts and global change projections. In the context of convection

parametrization, this is a ‘trigger’ problem: a certain trigger condition must be satisfied for turning on a convection parametrization (cf. review at Yano *et al.*, 2013).

A standard approach for addressing convective initiation in terms of parcel lifting (e.g., Roff and Yano, 2002; Manzato and Morgan, 2003) may be summarized as follows. A small quantity of air mass is lifted from the boundary layer (or surface) in isolation, maintaining its pressure as the same as the environment. Under a standard sounding, a lower layer of the atmosphere is stably stratified, thus a parcel lifted from the surface first loses its buoyancy with ascent until it reaches a lifting condensation level (LCL). At that level, the parcel is saturated. Beyond that level, latent heating by condensation of water vapour curbs the decrease of buoyancy with height. If the parcel is moist enough, the increase of buoyancy by latent heat overcompensates the decrease of buoyancy by adiabatic cooling, and finally, the parcel become positively buoyant, and convection may be initiated as a result of a buoyancy-induced instability. The level at which the parcel buoyancy turns from negative to positive is called the level of free convection (LFC) for this reason.

Under this parcel-based description of convective initiation, an air parcel must somehow be lifted from a surface layer to LFC for moist convection to be initiated. A certain external force is required to lift a parcel to this level, because this layer is negatively buoyant, and thus a parcel cannot be lifted spontaneously. A vertical integral of the buoyancy over this layer is called the convective inhibition (CIN), thus convection must be initiated by overcoming this barrier of CIN, according to the parcel-based description. Variations to this problem with alternative approaches are found in reviews by, for example, Nugent and Smith (2014). Nevertheless, all the existing approaches take a similar line of argument in which a barrier of CIN must somehow be overcome to initiate convection. Furthermore, this very intuitive parcel-based argument provides a basis for some convective initiation conditions in parametrizations (e.g., Mapes, 2000; Bretherton *et al.*, 2004).

The present article will present a different perspective based on the dynamics of a convectively well-mixed boundary layer. More precisely, the article presents initiation of deep moist convection as a natural consequence of continuous deepening of a well-mixed boundary layer under a presence of large-scale convergence. From a point of view of the dynamics of the well-mixed boundary layer, the inversion at the top, as a major source of CIN, does not hinder a growth of the well-mixed boundary layer, but rather it is a consequence of a growth of the well-mixed boundary layer; the convective boundary-layer air is continuously penetrating into a stable layer so that an inversion is created. Thus, if nothing else happens, the

well-mixed layer would simply keep growing even in the presence of CIN. Intuitively, a deeper well-mixed layer provides a more favourable condition for initiation of deep moist convection. Although the present study does not address the question of the initiation explicitly, it naturally follows that initiation of moist deep convection should not be hindered by the presence of CIN either, because the well-mixed layer is already growing at its expense.

The originality of the present article is to focus on the dynamics of the well-mixed boundary layer in the presence of large-scale convergence. Studies of the well-mixed boundary layer are usually focused on an environment with large-scale divergence, because it is an ideal environment for study in isolation. Once large-scale convergence kicks in, the well-mixed boundary layer is destroyed and replaced by deep moist convection, which would be a less interesting situation from a point of view of boundary-layer meteorology. In turn, the deep-convection studies are focused on a state with low-level convergence, but with fewer considerations on the processes within a pre-existing well-mixed boundary layer. The present article points out an important link of the well-mixed boundary-layer dynamics to the deep moist-convective dynamics.

The present article emphasizes a simple (even trivial) fact that, under the presence of large-scale convergence, the well-mixed boundary layer simply continues to deepen with time, so long as standard mixed-layer assumptions remain valid. Especially when a standard assumption of constant convergence with height is taken, it can be shown that the well-mixed layer height deepens with time exponentially, a type of instability. I propose it as an important precondition for transformation of the well-mixed boundary layer into deep moist convection.

Considering that there is less familiarity in the deep-convection community with the problems of the well-mixed boundary layer, the article proceeds rather pedagogically; the tendency of the well-mixed boundary layer to keep deepening with low-level convergence is demonstrated in two different ways. In the next section, a simple geometrical consideration of heat (entropy) budget is employed, thus the analysis can be understood without prior knowledge of the well-mixed boundary-layer dynamics. In Section 3, the same analysis is repeated with a standard formulation for the well-mixed boundary layer. In the latter case, a solution of exponentially growing mixed-layer depth naturally arises under a condition of low-level convergence. The basic claim of the present article is better supported by presenting these two different simple boundary-layer models, with substantially different assumptions in detail, which lead essentially to the same conclusion. At the same time, the overall equivalence of these two models is established by deriving the latter from

the former in the Appendix A. The article is concluded with further discussion.

Throughout the article, a horizontally homogeneous atmosphere is considered for the sake of a lucid and simple demonstration. Thus, all the variables depend only on the height, z . To actually address the question of transformation of the well-mixed boundary layer into cumulus convection, the horizontal dependence of a growing perturbation must be considered explicitly by removing this assumption. However, as it turns out, deriving a solution becomes much more involved. For this reason, this generalization is left for a future study.

Furthermore, a dry atmosphere is assumed for the sake of simplicity throughout the article. A basic extension of the analysis to the moist case is straightforward, and essentially a matter of replacing the potential temperature by the equivalent potential temperature. Otherwise the identical formulation is still applicable, which leads to identical conclusions. However, for more careful analysis, the actual formulation becomes much more involved; Section 4.2 gives further remarks.

2 | A SIMPLE GEOMETRICAL ANALYSIS

As an initial condition, we consider an atmosphere with a constant lapse rate, $\bar{d}\theta/dz$, given in terms of the potential temperature, θ . Thus

$$\theta(z, t = 0) = \theta_0 + \left(\frac{d\bar{\theta}}{dz}\right)z. \quad (1)$$

We assume a constant heat flux, H (with units $\text{K}\cdot\text{m}\cdot\text{s}^{-1}$) from the surface at $z = 0$. As a result, a well-mixed boundary layer is formed and grows from the surface. As is phenomenologically known, we assume that the potential temperature becomes a constant value, θ_m , with height, z , over this well-mixed layer extending to a height, z_m . Above the well-mixed layer ($z > z_m$), a free atmosphere is considered (designated by a subscript + throughout), which is assumed to linearly evolve with time, t , under a constant forcing, F_+ :

$$\theta_+(z, t) = \theta(z, t = 0) + F_+t, \quad (2)$$

and we may assume

$$F_+ = -\left[\bar{w}\left(\frac{d\bar{\theta}}{dz}\right) + Q_R\right]. \quad (3)$$

Here, \bar{w} is a prescribed large-scale vertical velocity, which is assumed to be constant in this section, and Q_R is a constant radiative cooling rate.

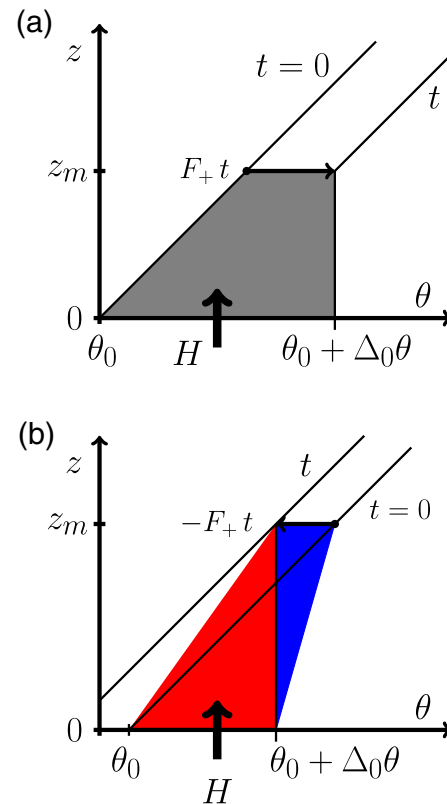


FIGURE 1 Schematics describing the well-mixed boundary layer heat budget. (a) When $F_+ > 0$, the shaded area indicates accumulated entropy, Ht , over time, t , by the surface flux, H . (b) When $F_+ < 0$, the gain of entropy (red shading) due to the surface flux, H , for the growth of mixed layer is equal to the sum of the surface-flux supply, Ht , and an excess entropy (blue shading) due to the forcing, F_+ , to the free atmosphere [Colour figure can be viewed at wileyonlinelibrary.com]

The well-mixed layer deepens in response to the surface flux, H . The supplied heat flux, Ht , over time t , must be equal to an area swept on a (θ, z) -plane from the initial condition to a state at time t as indicated by shading in Figure 1a, thus

$$Ht = \frac{\Delta_0\theta + \Delta_m\theta}{2}z_m, \quad (4)$$

where $\Delta_0\theta = \theta_m(t) - \theta_m(t = 0)$, and $\Delta_m\theta = \theta(z_m, t) - \theta_+(z_m, t = 0) = F_+t$. For now, we neglect an inversion and set $\theta_m = \theta_+(z_m, t)$. Note that when $F_+ < 0$, Equation (4) reduces to:

$$Ht = \frac{z_m\Delta_0\theta}{2} - \frac{z_m(-F_+t)}{2}.$$

Thus, the gain of entropy (red shading to the left) for the growth of the mixed layer is equal to the sum of the surface flux supply, Ht , and an excess entropy (blue shading to the right) provided

by evolution of the free atmosphere, as shown in Figure 1b.

By referring to Figure 1, we also find that the well-mixed layer depth, z_m , and the change, $\Delta_0\theta$, of the surface potential temperature over time t are related by

$$\Delta_0\theta - F_+ t = \left(\frac{d\bar{\theta}}{dz} \right) z_m,$$

or

$$\Delta_0\theta = \left(\frac{d\bar{\theta}}{dz} \right) z_m + F_+ t. \quad (5)$$

Substitution of Equation (5) into Equation (4) leads to:

$$H t = \frac{1}{2} \left(\frac{d\bar{\theta}}{dz} \right) z_m^2 + (F_+ t) z_m. \quad (6)$$

By solving the above for z_m , we obtain

$$z_m = \left(\frac{d\bar{\theta}}{dz} \right)^{-1} \left\{ -F_+ t + \left[(F_+ t)^2 + 2 \left(\frac{d\bar{\theta}}{dz} \right) H t \right]^{1/2} \right\} \quad (7)$$

by choosing a solution with positive z_m . When the environmental air is descending (i.e., $\bar{w} < 0$), the forcing, F_+ , is positive, and we find that the mixed-layer depth asymptotically evolves as $z_m \rightarrow H/F_+$. On the other hand, under large-scale ascent, the forcing, F_+ , may become negative, and the system never reaches equilibrium, but the mixed layer keeps deepening with time. This is the situation expected under large-scale low-level convergence.

The solution (7) can be presented in a more compact manner after non-dimensionalization by setting $z_m = z_m^* \tilde{z}_m$ and $t = t_* \tilde{t}$. Here, the non-dimensionalization scales for the mixed-layer depth and the time, respectively, are

$$z_m^* = \frac{H}{|F_+|} \sim 10^3 \text{ m}$$

$$\text{and } t_* = z_m^* \left(\frac{d\bar{\theta}}{dz} \right) \frac{1}{|F_+|} = \left(\frac{d\bar{\theta}}{dz} \right) \frac{H}{F_+^2} \sim 10^5 \text{ s} \sim 1 \text{ day}.$$

These typical scales are obtained by setting

$$|F_+| \sim 10^{-5} \text{ K} \cdot \text{s}^{-1} \sim 1 \text{ K} \cdot \text{day}^{-1},$$

$$H \sim 10^{-2} \text{ K} \cdot \text{m} \cdot \text{s}^{-1}, \quad d\bar{\theta}/dz \sim 10^{-3} \text{ K} \cdot \text{m}^{-1}.$$

The actual typical heat flux is $\rho C_p H \sim 10^2 \text{ W} \cdot \text{m}^{-2}$, and large-scale vertical velocity is $|w| \sim 10^{-2} \text{ m} \cdot \text{s}^{-1}$. After non-dimensionalization, the solution (7) reduces to:

$$\tilde{z}_m = -\text{sgn}(F_+) \tilde{t} + (\tilde{t}^2 + 2\tilde{t})^{1/2}. \quad (8)$$

The non-dimensional solution (8) is plotted in Figure 2: a well-mixed boundary layer can grow to a depth of 2 km

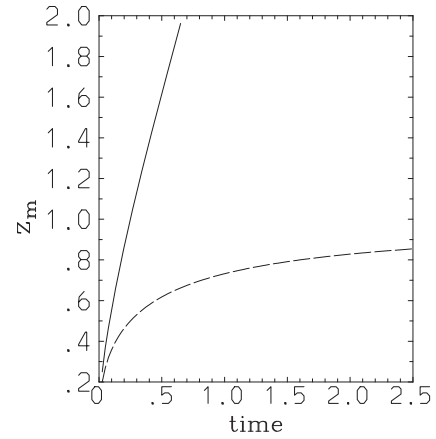


FIGURE 2 Plots of Equation (10) with $\text{sgn}(F_+) = 1$ (dashed) and $\text{sgn}(F_+) = -1$ (solid)

over half a day under a modest large-scale low-level convergence. The evolved mixed layer is deep enough to reach the level of free convection (LFC) in common situations and, under conventional wisdom, deep moist convection will be spontaneously initiated from there.

A finite inversion jump, $\Delta_i\theta = \theta_+(z_m) - \theta_m (\neq 0)$, can be included in the above analysis by simply replacing $F_+ t$ by $F_+ t - \Delta_i\theta$. Furthermore, when a finite downward entrainment flux at the top of the mixed layer (given by rH with r a constant, taken downward positive) is further considered, we just have to replace H by $(1+r)H$. It transpires that the basic conclusion of Equation (8) does not change by these generalizations.

3 | ANALYSIS BASED ON A STANDARD MIXED-LAYER FORMULATION

In this section, an analysis equivalent to the last section is performed based on a standard formulation for the well-mixed boundary layer (cf. Stevens, 2006), with slightly different mixed-layer-top assumptions. As seen immediately below, in this case, an equation for the depth, z_m , of the well-mixed layer can be considered in a stand-alone manner, thus no other variable needs to be considered. This formulation can also be directly extended to the moist atmosphere without modifications, including a possibility of clouds at the top of a mixed layer. Evaluating the associated evolution of the potential temperature (and other thermodynamic variables) is straightforward, but without providing any further insight to the given problem. Section 4.2 gives further discussion.

We assume an inversion, $\Delta_i\theta = \theta_+(z_m) - \theta_m$, of the potential temperature at the top of the well-mixed

boundary layer. The mixed layer deepens relative to a background vertical motion, \bar{w} , at a rate that excess entropy is transported downwards by vertical eddy flux, $-\overline{w'\theta'}|_{z=z_m}$. Thus,

$$\left(\frac{dz_m}{dt} - \bar{w}\right) \Delta_i \theta = -\overline{w'\theta'}|_{z=z_m}. \quad (9)$$

Here, z_{m-} refers to a level immediately below the mixed-layer height, and a possible contribution of radiation is neglected. Alternatively, this contribution can be included as a part of vertical eddy flux. Furthermore, the right-hand side of Equation (9) may be expressed by

$$-\overline{w'\theta'}|_{z=z_m} = w_e \Delta_i \theta \quad (10)$$

in terms of the entrainment rate, w_e . We re-write the vertical eddy flux in this manner so that the inversion strength, $\Delta_i \theta$ no longer appears in the equation explicitly:

$$\frac{dz_m}{dt} = \bar{w} + w_e. \quad (11)$$

In the following, the entrainment rate, w_e , is considered to be a constant for the ease of analysis. It is further shown in Appendix A that Equation (9) can be derived from the geometrical formulation of the last section. As also suggested therein, Equation (11) is valid also in the limit of $\Delta_i \theta \rightarrow 0$.

Furthermore, as in many studies of the well-mixed layer (e.g., Schubert *et al.*, 1979), a background divergence, D , is assumed to be constant for now, and

$$\bar{w} = -Dz. \quad (12)$$

By substituting Equation (12), Equation (11) reduces to

$$\left(\frac{d}{dt} + D\right) z_m = w_e. \quad (13)$$

An equilibrium solution is identified from Equation (13) as

$$\bar{z}_m = w_e/D. \quad (14)$$

Note that, when the background large scale is convergent, $D < 0$, then the equilibrium depth, \bar{z}_m , becomes negative, thus unphysical. In other words, a well-mixed layer may reach an equilibrium state only under a divergent large-scale background state. Nevertheless, even an unphysical equilibrium solution provides a good reference point for performing a perturbation analysis. In the present case, the issue is inconsequential, because the

perturbation analysis can be automatically extended globally, because Equation (13) is linear.

The perturbation problem around the equilibrium solution is given by

$$\left(\frac{d}{dt} + D\right) z'_m = 0$$

with a prime designating a perturbation. The solution is immediately obtained as

$$z' = z'_0 e^{-Dt}, \quad (15)$$

with z'_0 an initial perturbation. Thus, when the system is divergent, associated with a mean descent, and $D > 0$, the equilibrium state is stable. On the other hand, when the system is convergent, associated with a mean ascent, and $D < 0$, the unphysical equilibrium state becomes unstable: a large-scale low-level convergence makes the well-mixed convective boundary layer grow exponentially with time. Note that the characteristic growth time-scale, $D^{-1} \sim 10^5 \text{ s} \sim 1 \text{ day}$ is comparable to that obtained in the last section.

Here, the result is presented in terms of a perturbation analysis. However, recall that the original full system itself (13) is linear, thus $z_m = \bar{z}_m + z'_m$ constitutes a full solution of Equation (13). In other words, we can set an initial perturbation, z'_0 , large enough so that we can make $z_m(t = 0) = \bar{z}_m + z'_0$ positive, although \bar{z}_m is negative under convergence. Thus, the exponential growth of the well-mixed layer depth also becomes physically meaningful in this respect.

However, another unphysical aspect of this solution is that the mixed-layer depth keeps growing exponentially without reaching any equilibrium. We should realize that this behaviour is due to a rather artificial assumption of a constant divergence, D , with height. Thus, when $D < 0$, the large-scale vertical velocity, given by $\bar{w} = -Dz$, simply increases linearly with height without limit. Clearly, this is unrealistic.

To amend this problem, we modify the large-scale vertical velocity to

$$\bar{w}(z) = -Dz \left(1 - \frac{z}{h}\right), \quad (16)$$

thus \bar{w} no longer grows linearly with height, but it turns back to zero at a height $z = h$. This height may be taken at the level of the tropopause, thus the existence of the top height, h , does not affect an initial exponential growth of z_m , but ultimately influences it, when z_m becomes deep enough. Here, D is a constant as before, corresponding to the divergence at $z = 0$.

As a result, Equation (11) for the well-mixed layer depth turns into:

$$\frac{dz_m}{dt} = -Dz_m \left(1 - \frac{z_m}{h}\right) + w_e. \quad (17)$$

Equilibrium solutions are, in this case, given by

$$\bar{z}_m = \frac{h}{2} \left[1 \pm \left(1 - \frac{4w_e}{hD}\right)^{1/2} \right]. \quad (18)$$

These two equilibria are approximately given by $\bar{z}_m \simeq w_e/D$ and h , when the top zero-velocity level is high enough and $w_e/hD \ll 1$. Thus we obtain two equilibrium mixed-layer depths: the first corresponds to a usual boundary-layer depth, and the second extends to a full troposphere.

By setting the lower and the upper equilibrium solutions, respectively, to be z_1 and z_2 (thus $z_1 < z_2$), Equation (17) is rewritten as

$$\frac{dz_m}{dt} = \frac{D}{h}(z_m - z_1)(z_m - z_2). \quad (19)$$

This form is much easier to solve analytically. The solution is more conveniently presented in two equivalent but different forms, depending on the sign of D , so that exponential contributions decay with time in both cases: when a background state is diverging,

$$z_m = \frac{z_1 + \varepsilon z_2 e^{-Dt}}{1 + \varepsilon e^{-Dt}}, \quad (20)$$

and when a background state is converging,

$$z_m = \frac{z_2 + (z_1/\varepsilon)e^{Dt}}{1 + e^{Dt}/\varepsilon}, \quad (21)$$

with

$$\varepsilon = \frac{z_0 - z_1}{z_2 - z_0}, \quad (22)$$

and z_0 is an initial depth. Thus, the depth, z_m , asymptotically approaches the lower and the upper equilibria, z_1 and z_2 , respectively, when $D > 0$ and $D < 0$.

So long as the stability of the standard mixed-layer with a depth $z_m = z_1 \simeq w_e/D$ is concerned, we recover the same conclusion as previously: it is stable and unstable under divergence and convergence, respectively. However, by introducing a more realistic large-scale vertical velocity profile, we can infer the subsequent evolution of an unstable boundary layer; it no longer keeps growing, but it asymptotically approaches a state of a troposphere-deep well-mixed layer. In reality, the well-mixed layer would be transformed into deep moist convection before getting

to that stage. However, to describe this transformation process, the horizontal dependence of the perturbation growth must be explicitly taken into account by removing the assumption of horizontal homogeneity.

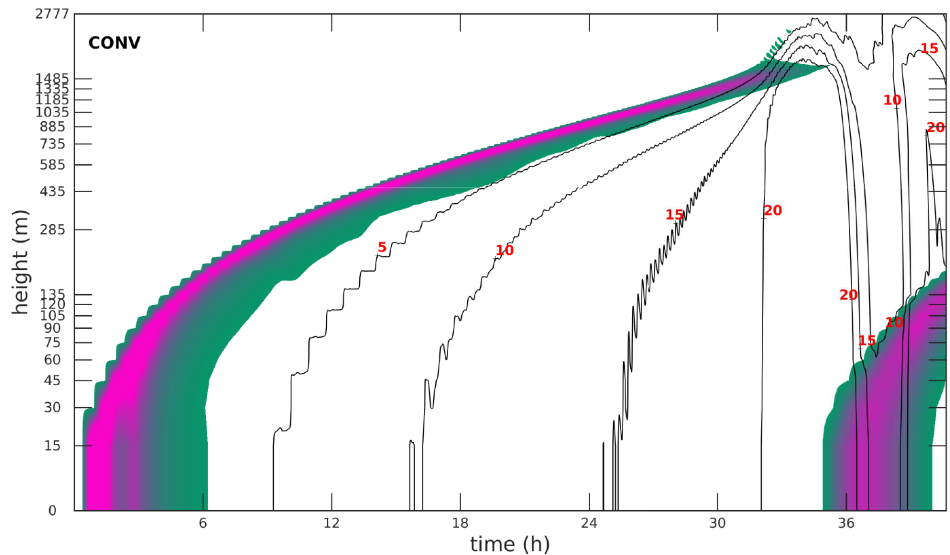
4 | DISCUSSION

4.1 | Summary and implications

The present study suggests that deep moist convection is initiated through the deepening of a well-mixed convective boundary layer under a large-scale low-level convergence. This tendency of the well-mixed boundary layer is demonstrated by two similar, simple theoretical models, but which are substantially different in detail. The first, considered in Section 2, is based on a simple geometrical argument of the heat budget of the well-mixed boundary layer, assuming a constant large-scale vertical velocity with height. In this case, the well-mixed layer depth increases in quasi-linear manner under a large-scale convergence over a half day period up to a depth of a few kilometres. The second, considered in Section 3, is based on a standard formulation for the depth of a well-mixed layer. When the large-scale divergence (or convergence) is assumed to be constant with height, as is commonly assumed in theoretical analyses, the well-mixed layer simply grows exponentially with time under convergence. A more realistic solution is obtained by assuming a profile of the large-scale vertical velocity, which reaches the maximum at the middle troposphere, and vanishes at the top of the troposphere. In this case, the unstable standard well-mixed layer grows into a troposphere-deep mixed layer. This final state could even be very crudely interpreted as a state of deep moist convection.

Various alternative interpretations of deep-convection initiation under convergence are already available in literature. A historically important one is to interpret it in terms of moisture budget. Superficially, the present interpretation may somehow appear to be very akin to this historical interpretation, but is actually fundamentally different. This historical interpretation is based on an analysis of the budget of the column-integrated moisture in the atmosphere. Convergence is simply a mechanism of providing more moisture to an atmospheric column. There is no role of the boundary layer in this interpretation. In contrast, the moisture does not play any active role in the present analysis. The argument is solely based on a tendency of the boundary layer to deepen with time under a converging flow. The top of boundary layer may ultimately reach a LCL, and further to the LFC, thus leading to conditional instability. However, these aspects are not considered explicitly, but only implied as what follows.

FIGURE 3 Explosive evolution of an Arctic well-mixed boundary layer under a constant convergence, simulated by Dimitrelos *et al.* (2020). Shading denotes the domain-averaged droplet mixing ratio, ranging from 0.0125 g·kg⁻¹ (green) to 0.35 g·kg⁻¹ (red) on a logarithmic scale. Contours are the ice-crystal mixing ratio (g·kg⁻¹). Reproduced from Figure 3(h) of Dimitrelos *et al.* (2020), but using a logarithmic scale in the vertical [Colour figure can be viewed at wileyonlinelibrary.com]



Another common approach is to interpret the convective initiation process in term of a lifting parcel from the boundary layer. Under this picture, the main obstacle is convective inhibition (CIN), which is re-interpreted as an inversion strength, $\Delta\theta$, under the well-mixed layer formulation adopted herein. Importantly, the inversion does not hinder the deepening of the well-mixed layer at all under the standard description presented in Section 3. Phenomenologically, the well-mixed layer deepens by a downward vertical eddy flux, $-\overline{w'\theta'}|_{z=z_m}$, at the top, which is known to be approximately proportional to the surface flux. Although a stronger inversion may reduce the entrainment rate proportionally (cf. Equation 12), the inversion itself does not hinder the deepening of the well-mixed layer. It is rather other way round: the downward flux at the mixed-layer top generates the inversion. A stronger downward flux creates a stronger inversion, as a simple extension of a geometrical argument in Section 2 can show. In this manner, CIN does not play a role of convection inhibition in the context of a well-mixed convective boundary layer. If deep moist convection can be interpreted as a transformation of a continuously deepening well-mixed convective boundary layer, this transformation process should not be hindered by a presence of CIN either, because the well-mixed layer is already growing at its expense.

Here, growth of the convective well-mixed layer is not a sole mechanism for generation of an inversion and CIN. A notable example is the so-called elevated mixed layer, which is a consequence of advection of warm moist air originating from the Gulf of Mexico to higher latitudes (e.g., Lanicci and Warner, 1997). However, the main point remains the same: regardless of being self-generated or externally generated, the convective well-mixed layer

continues to grow by penetrating its air into an inversion layer, thus the existence of CIN does not block its growth.

Under large-scale descent, a standard environment considered for studies of a well-mixed boundary layer, it reaches an equilibrium state due to the presence of a large-scale descent which tends to suppress the growth driven by entrainment. However, once the large-scale movement turns from descent to ascent, there is no longer a mechanism to restrict the well-mixed boundary layer to an equilibrium. Thus, it keeps growing, leading to deep convection. Based on this very simple picture, the deep-convection community should not be concerned with the existence of the so-called CIN. Unfortunately, this is not the case. In fact, for many of the parametrizations in current operational models, the initiation of convection is controlled by CIN (e.g., Zhao *et al.*, 2009, 2016; Walters *et al.*, 2019)

4.2 | Limits of the present study

The present study is based on very simple theoretical analyses, and it hardly presents a full picture of the problem.

At the most technical level, analyses have been performed under rather simple treatments at the top of the well-mixed boundary layer, assuming a constant heat flux and a constant entrainment rate in Sections 2 and 3, respectively. Validity of these simplifications can be questioned. However, a large-eddy simulation (Dimitrelos *et al.*, 2020) indeed demonstrates that such an explosive growth of a well-mixed layer actually is realized. The case here assumes a large-scale divergence of $1.5 \times 10^{-6} \text{ s}^{-1}$ constant with height under Arctic conditions. An exponential tendency of the growth of the mixed-layer depth from

100 to 1,500 m over a period of $t = 6\text{--}30$ hr is convincingly shown in Figure 3 (reformatting their figure 3(h)) by making the vertical axis logarithmic. Note that in this simulation, by assuming an Arctic situation, the surface-flux values are by an order of magnitude smaller than those typical over the tropical oceans. Thus, if a similar simulation were performed in a tropical environment, the growth of the well-mixed layer would also be about ten times faster.

The most obvious limit in the physical formulation is the use of dry dynamics, and the contribution of moisture remains implicit. However, extension to the moist case is conceptually rather straightforward. Recall that the basic premise of the well-mixed layer is simply to assume that the main role of the convective dynamics is to mix the conserved variables of a system homogeneously in the vertical. Such a variable is the potential temperature in a dry well-mixed layer. When moisture is added to a system, the main modification is to replace it by the equivalent potential temperature, because the latter is conserved along with the total water when the precipitation can be neglected. Otherwise, the procedure of the analysis overall remains the same, although it becomes more involved in the detail; full discussions appear in Lilly (1968); Carson (1973); Deardorff (1976); Schubert *et al.* (1979). For the geometrical formulation in Section 2, the development of the well-mixed layer of moisture must be considered separately under this generalization, because of decreasing moisture with height rather than increasing. Appendix B shows that an identical result (Equation 8) as for the potential temperature is obtained in the end. So long as an equation for the evolution of the well-mixed layer height, z_m , is concerned, its form remains identical to Equation (13), as already emphasized in Section 3. The only necessary modification is to replace the potential temperature, θ , in Equations (11) and (12) by the virtual potential temperature, θ_v . Thus, as far as our very crude description of the evolution of well-mixed layer height is concerned, the given formulations can be extended to the moist case without any fundamental modifications.

From a more formal point of view, all those extra factors can be taken into account by including the possible time dependencies of various parameters (H , F_t , D , w_e) in both formulations. As it turns out, analytical solutions are still available in both cases even under those circumstances, and it can also be easily shown that the main conclusions do not change so long as these parameters evolve smoothly with time.

The most crucial limit of the present study is in assuming a horizontal homogeneity of the well-mixed layer. As long as we consider only a proper convective well-mixed boundary layer, this assumption would not be serious. However, this very assumption hinders us from describing

its transformation into deep moist convection, because the latter is characterized by localized updraughts surrounded by environmental descent. Instead, a well-mixed layer deepens all the way through the troposphere, as a solution from the present analysis shows. Mathematically, it is straightforward to add horizontal dependence to a standard mixed-layer formulation. As a result, the formulation must also be modified in such manner that resulting local circulations arising from horizontal inhomogeneity are also taken into account. This turns out not to be a difficulty, either. However, my own preliminary investigation reveals that the resulting analysis is rather involved and lengthy. Thus, it will be reported in a separate article.

Probably, the most serious limit of the present analysis is in its perturbation approach. It would be relatively easy to envisage a fully nonlinear analysis under a certain numerical procedure, but with the latter detail still left to be sorted out. Inclusion of precipitation and the associated microphysics would be another aspect to be incorporated into this nonlinear formulation. Under such an approach, transformation of a convectively well-mixed boundary layer into deep moist convection may be described in full, still assuming a local vertical mixing for homogenization of variables.

4.3 | Implications for the convection parametrization problem

The present study may be considered a first step for constructing a convective initiation condition in a more formal and robust manner. A main problem to note, in this respect, is the fact that the majority of current convection parametrizations are constructed without any explicit coupling with the boundary layer. The present study suggests the crucial importance of such a coupling, especially for initiating deep moist convection as a result of large-scale low-level convergence. No doubt many operational forecast centres are already working towards this goal. However, importantly, this issue should not be reduced to a narrow question of identifying, say, a coupling parameter. In this respect, it is worthwhile revisiting the original formulation of mass-flux convection parametrization by Arakawa and Schubert (1974); they introduced a specific well-mixed layer model, similar in design to the present study, for the purpose of coupling convection with the boundary layer. Likewise, any renewed efforts for coupling between deep convection and the boundary layer should also involve modifications in the treatment of both components. It is hoped that the present theoretical investigation suggests a general direction towards this goal.

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APPENDIX A. DERIVATION OF EQUATION (11) FROM THE SYSTEM OF SECTION 2

In this Appendix, Equation (11) for the evolution of the mixed-layer height, z_m , is derived from the system of Section 2. Two derivations are presented. The first is heuristic, which elucidates an implicit role of an inversion in the system of Section 2. The second is a full derivation.

A.1. Heuristic derivation

For a heuristic derivation, we begin with the simplest case with $F_+ = 0$ and $\Delta_i\theta = 0$. In this case, Equations (4) and (7) reduce to

$$Ht = \frac{\Delta_0\theta}{2}z_m, \quad (\text{A1})$$

$$\Delta_0\theta = \left(\frac{d\bar{\theta}}{dz}\right)z_m. \quad (\text{A2})$$

By substituting Equation (A2) into Equation (A1), we obtain:

$$\frac{z_m^2}{2} = \left(\frac{d\bar{\theta}}{dz}\right)^{-1} Ht.$$

By taking a time derivative of the above, we obtain:

$$\frac{dz_m}{dt} = \frac{H}{\Delta_0\theta}, \quad (\text{A3})$$

also using Equation (A2). In a final step, we need to recover a small, but non-vanishing inversion, $\Delta_i\theta (\neq 0)$. Note that the heat flux must decrease linearly with height, thus a simple geometrical consideration suggests

$$H/(z_m - \Delta_i z) = -H_{(-)}/\Delta_i z.$$

Here, $H_{(-)} = \overline{w'\theta'}|_{z=z_m-}$ is the heat flux at $z = z_m-$ just below the inversion. By taking a limit of $\Delta_i z \rightarrow 0$, we obtain

$$H/z_m \rightarrow -H_{(-)}/\Delta_i z.$$

Further recalling Equation (A2) and $\Delta_i\theta = (d\bar{\theta}/dz)\Delta_i z$, thus

$$H/\Delta_0\theta \rightarrow -H_{(-)}/\Delta_i\theta. \quad (\text{A4})$$

Finally, by substituting Equation (A4) into Equation (A3), we obtain

$$\Delta_i\theta \frac{dz_m}{dt} = -H_{(-)}.$$

In the presence of large-scale ascent, \bar{w} , the height, z_m , of the well-mixed layer is uplifted accordingly, thus

$$\Delta_i\theta \left(\frac{dz_m}{dt} - \bar{w}\right) = -H_{(-)}, \quad (\text{A5})$$

which is equivalent to Equation (11).

The derivation here suggests that the existence of a small, but finite, inversion is essential for the growth of a well-mixed layer: it grows by generating an inversion by penetrating convective plumes into a free atmosphere.

A.2. Full derivation

When a full system of Section 2 is considered, reduction to Equation (11) appears only under approximations, most likely due to minor inconsistencies in the formulation of Section 2, but not of any serious nature.

Under the presence of an inversion, Equations (4) and (7) are modified into:

$$(H - H_{(-)})t = \frac{\Delta_0\theta + \Delta_m\theta - \Delta_i\theta}{2}z_m, \quad (\text{A6})$$

$$\Delta_0\theta = \left(\frac{d\bar{\theta}}{dz}\right)z_m + \Delta_m\theta - \Delta_i\theta. \quad (\text{A7})$$

From Equation (A7), we find:

$$\frac{\Delta_0\theta + \Delta_m\theta - \Delta_i\theta}{2} = \frac{1}{2} \left(\frac{d\bar{\theta}}{dz}\right)z_m + \Delta_m\theta - \Delta_i\theta.$$

Substituting this expression into the right-hand side of Equation (A6), and taking a time derivative of the result:

$$\Delta_0\theta \frac{dz_m}{dt} + z_m \frac{d}{dt}(\Delta_m\theta - \Delta_i\theta) = H - H_{(-)}. \quad (\text{A8})$$

In writing the above result, Equation (A7) has already been applied to the first term.

In deriving the final result, we need to introduce several approximations. First, the temporal change of $\Delta_i\theta$ is expected to be much smaller than that of $\Delta_m\theta$, thus

$$\frac{d}{dt}(\Delta_m\theta - \Delta_i\theta) \simeq \frac{d}{dt}\Delta_m\theta. \quad (\text{A9})$$

Furthermore, radiation is neglected in Equation (3), and it should be realized that it is always a good approximation to set:

$$\left(\frac{d\bar{\theta}}{dz}\right)z_m \simeq \Delta_0\theta.$$

Using these two approximations,

$$\frac{d}{dt}\Delta_m\theta \simeq -\bar{w} \left(\frac{d\bar{\theta}}{dz}\right) \simeq -\frac{\bar{w}\Delta_0\theta}{z_m}. \quad (\text{A10})$$

By applying Equations (A9) and (A10) into Equation (A8), we find:

$$\Delta_0\theta \left(\frac{dz_m}{dt} - \bar{w}\right) = H - H_{(-)}. \quad (\text{A11})$$

Finally, by invoking a geometrical consideration,

$$(H - H_{(-)})/\Delta_0\theta = -H_{(-)}/\Delta_t\theta,$$

Equation (A11) reduces to Equation (11)

APPENDIX B. DEVELOPMENT OF A WELL-MIXED MOISTURE LAYER

Development of a well-mixed layer with any conserved variable can be formulated as for the potential temperature, as presented in Section 2. However, when a background value decreases with height, a special consideration becomes necessary. This Appendix considers this issue by taking the moisture as an example. In extending the formulation of Section 2 to the moist case, this consideration becomes important, because the equivalent potential temperature also decreases with height in the lower troposphere.

The derivation proceeds in parallel to Section 2. For simplicity, some notations for the moisture are kept the same as for the potential temperature in Section 2. This should not cause any confusion, because none of the results in this Appendix are directly quoted in the main text. Thus, as an initial condition for the moisture mixing ratio, q , we assume

$$q(z, t = 0) = q_0 + \left(\frac{d\bar{q}}{dz}\right)z \quad (\text{B1})$$

in place of Equation (1). Here, $d\bar{q}/dz (< 0)$ is assumed constant with height. Though this rather strong assumption can be relaxed, as moisture typically decreases exponentially with height, but more involved algebra follows. As before, we assume a constant moisture flux, H , from the surface at $z = 0$. As a result, a well-mixed layer of moisture is formed and grows from the surface.

However, as a major difference from the potential temperature treatment in Section 2, the growth is inevitably associated with a development of an inversion (discontinuity), Δq , of the moisture at the top of the mixed layer. To close the problem with ease we assume, in turn, that the surface value, q_0 , is maintained constant with time, thus all the supplied moisture flux is used for deepening of the mixed layer. Thus, the inversion is defined by

$$\Delta q = q_0 - q_+(z_m, t). \quad (\text{B2})$$

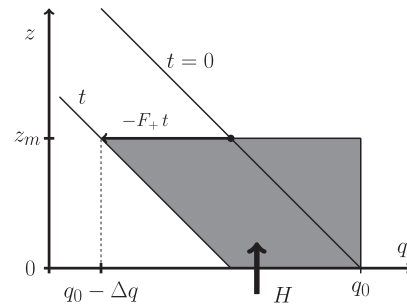


FIGURE B1 Schematic describing the well-mixed moisture layer

Evolution of moisture above the well-mixed layer ($z > z_m$) may be given by

$$q_+(z, t) = q(z, t = 0) + F_+t, \quad (\text{B3})$$

with

$$F_+ = -\bar{w} \left(\frac{d\bar{q}}{dz} \right). \quad (\text{B4})$$

Note that, in contrast to the case with potential temperature, $F_+ < 0$ applies in a descending environment, and $F_+ > 0$ in an ascending environment.

Expected temporal growth of a well-mixed moisture layer is schematically shown in Figure B1: the well-mixed moisture layer deepens in response to the surface flux, H . The supplied heat flux, Ht , over time t , must be equal to an areas swept on a (q, z) -plane from the initial condition to a state at time t as indicated by the shading, thus

$$Ht = \frac{\Delta q - F_+t}{2} z_m. \quad (\text{B5})$$

Also from a geometrical consideration suggested by Figure B1,

$$\Delta q = - \left(\frac{d\bar{q}}{dz} \right) z_m + (-F_+t). \quad (\text{B6})$$

By substituting Equation (B6) into Equation (B5), we find:

$$Ht = -\frac{1}{2} \left(\frac{d\bar{q}}{dz} \right) z_m^2 + (-F_+t)z_m. \quad (\text{B7})$$

This equation becomes equivalent to Equation (8) by replacing $-d\bar{q}/dz$ and $-F_+$ by $d\bar{\theta}/dz$ and F_+ , respectively.