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# Experiments and simulations of the structure Harmony-Gamma subjected to broadband random vibrations

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## ABSTRACT

The structure Harmony-Gamma is a metallic assembly representative of an industrial structure for which the vibratory response is influenced by the apparition of nonlinear phenomena within two specific types of joints, the first corresponding to friction joints and the second to elastomer joints.

The present study extends the previous work based on experiments and numerical simulations of the structure Harmony-Gamma subjected to harmonic vibrations [1]. More specifically, the nonlinear vibrational behaviour of the assembly subjected to random broadband excitations is studied. Broadband excitations are performed experimentally, in order to provide a first understanding of the nonlinear effect of both the friction and elastomer joints. Additionally, a global numerical methodology based on finite-element modelling and the use of the Harmonic Balance Method for the prediction of the nonlinear response of the Harmony-Gamma structure subjected to stochastic excitation is proposed.

It is demonstrated that the use of a numerical model that has been validated against experimental tests can furthermore be used to achieve a refined understanding of the nonlinear phenomena and their origin.

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## 1. Introduction

Knowing the vibrational response of mechanical systems has been a systematic engineering concern for decades. The modelling and computing limitations of yesterday imposed simplified strategies to solve the equation of motion, and today's computing tools are aimed at overcoming these limitations.

Among the simplifying assumptions generally made in an engineering study, the two strongest limitations are linearisation of the motion equation, instead of considering the complex nonlinear dynamic behaviour of the mechanical system of interest, and the simplification of the vibrational input. The first advances were made in the input signal, going from simple mono-frequency signals to more realistic stochastic excitations, since most real excitations are random in nature. Today, there is an extremely good understanding of random signals [2] and it is thus possible for engineers to design mechanical systems taking into account this complex input. However, very few studies nowadays propose to predict the dynamic responses of complex mechanical systems by considering the random signals. Thus, one of the contributions of the present

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study will be to demonstrate the feasibility of predicting the vibration response of a complex industrial structure subjected to random excitations.

On the other hand, including nonlinearities in equations is not always applied in the industry. Indeed, faced with the difficulty of correctly modelling non-linear parts of mechanical systems and the need to implement specific numerical techniques to solve non-linear equations of motion, performing linear simulations is often preferred. However, experimental tests on industrial mechanical systems often exhibit a strong dependency of the vibration response on the excitation level due to the presence of nonlinear elements. It is therefore essential today to be able to model industrial structures taking into account non-linear elements, in order to be able to predict the non-linear dynamic behaviour of these complex systems more accurately. In order to respond to the growing industrial needs, over the last few years tools to deal with non-linear problems in complex mechanical structures [3,4] have emerged. However, it must be admitted today that these studies generally focus only on cases of sinusoidal-type excitations, since this first generation of software is more focused on the ability to perform nonlinear system identification, as well as to address the prediction and detection of complex nonlinear behaviour in mechanical structures. In addition, if we consider more specifically the case of nonlinear systems subjected to random excitation, simplified analytical strategies of resolution are preferred [5,6] due to the difficulty in developing a numerical tool capable of addressing the prediction of complex nonlinear dynamics for real-world structures subjected to random excitations. Faced with these observations, one of the original and additional contributions of the present study is to illustrate the capacity of a previously developed tool to address the nonlinear problem of a mechanical system with random inputs for a real industrial case. This numerical strategy is based on an extension of the Harmonic Balance Method for nonlinear systems subjected to random broadband excitation. It was only validated for two academic systems (i.e., a nonlinear beam [7] and a rubber isolator [8] subjected to random excitations).

This paper proposes to continue previous work by Roncen et al. [1] by focusing on the nonlinear response of the industrial structure Harmony-Gamma to a random broadband excitation. The first contribution of this proposed study is to analyse the nonlinear behaviour of the system through experimental tests. The second contribution concerns the demonstration of the implementation and effectiveness of a global numerical strategy for the prediction of the nonlinear response of an industrial assembly with various different types of nonlinear joints. Finally, one of the strong originalities and major contributions of the present work is to use modelling and numerical simulations to provide a complete and better understanding of nonlinear phenomena observed experimentally, such as the softening effect or modal interactions.

The paper is organised as follows. First, a brief description of the Harmony-Gamma structure and the analysis of various experimental tests are presented. Then, the description of the finite-element model with friction and elastomer joints, as well as nonlinear simulation based on the extension of the Harmonic Balance Method for mechanical systems subjected to broadband random vibrations are briefly discussed. This numerical strategy combines the previous work of Roncen et al. [1] (to develop and update the modeling of the Harmony-Gamma structure) and [8,9] (to simply go from harmonic vibrations to random vibrations in nonlinear dynamics). Finally, efficiency and accuracy of the proposed numerical methodology is evaluated and commented. The simulation results are compared to the experimental results in order to achieve a refined understanding of the nonlinear phenomena. More specifically, it will be demonstrated that it is possible to perform additional numerical simulations for a better understanding of complex nonlinear responses and their origin due to the existence of nonlinearities of distinct natures.

## 2. Harmony-Gamma structure and experiments

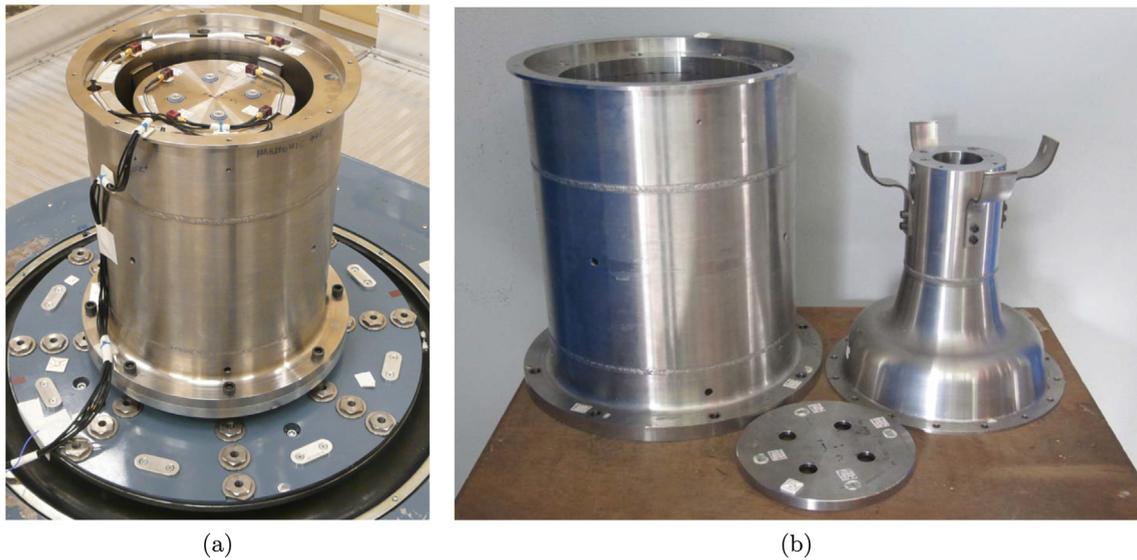
### 2.1. Presentation of the Harmony-Gamma structure

A photograph of the Harmony-Gamma structure under study is given in Fig. 1(a). This structure is composed of three main parts: an external envelope, an upper ballast named “upper body” and a central body with four blades as illustrated in Fig. 1(b). The Harmony-Gamma structure was described in detail in [1]. As a reminder, this structure is an extension of the Harmony structure, which was previously studied in [10,11]: an upper ballast named “upper body” connected to the central body by four identical rubber isolators has been added, in order to investigate the nonlinear dynamic behavior of the whole structure, as well as the combined nonlinear effects of rubber isolators and friction between the four blades and the envelope. As previously explained in [1], the friction at each blade will be responsible for the friction dissipation at high excitation levels, while these rubber isolators will favor the softening effect for the dynamic behavior of the Harmony-Gamma structure and an increase in energy dissipation. The different properties of the assembly are detailed in Table 1. During experiments, the Harmony-Gamma structure is embedded on a base plate that couples the assembly to the shaker. Finally, the assembly is instrumented with 11 tri-axial accelerometers which are shown in Fig. 2.

### 2.2. Experimental results and discussion

Experiments were conducted to investigate the nonlinear response of the system under study and, more precisely, the dependency of the structural response on random excitations.

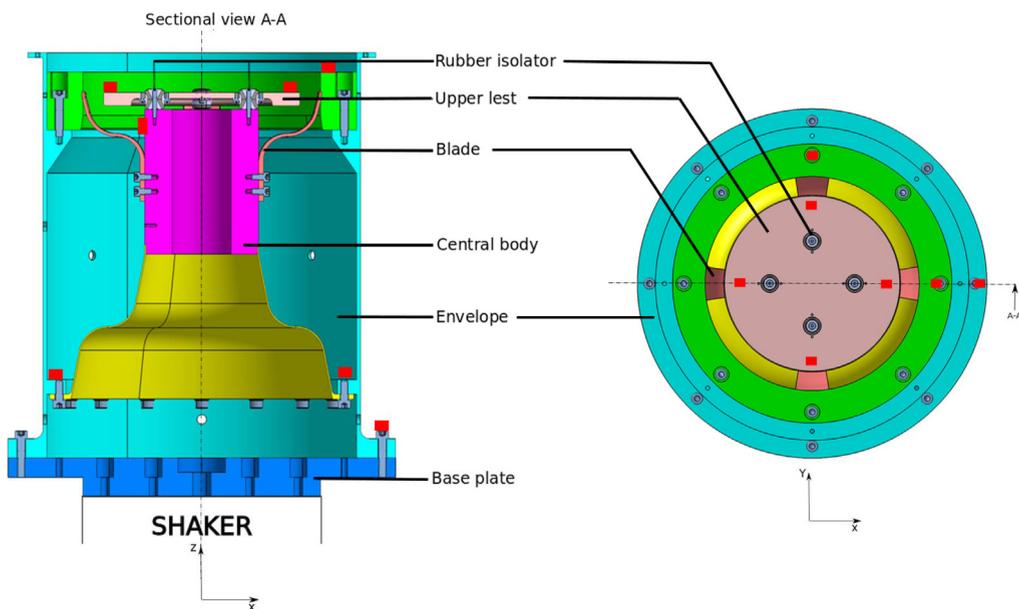
In order to achieve this goal, a set of experiments with random excitations were conducted with an increasing level of longitudinal input excitation and a constant Power Spectral Density over the frequency range [50; 1000] Hz. Fig. 3 gives



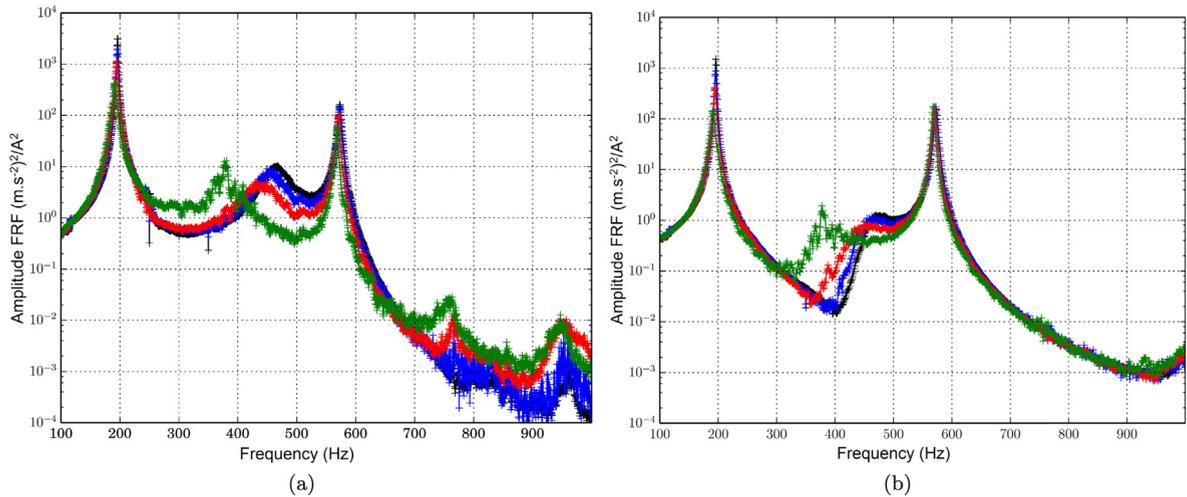
**Fig. 1.** Presentation of the mechanical structure under study with (a) the structure assembly and (b) the three main parts of the Harmony-Gamma structure.

**Table 1**  
Properties of each component of the structure Harmony-Gamma.

Component	Material	Dimensions (mm) maximum/height	Mass (kg)
Central body	Stainless steel 304L	160 / 300	11.92
External envelope	Stainless steel 304L	204 / 420	44.88
Blade (x4)	High resistance steel Z8-cnd17-04	65 / 118	0.1086
Base plate	Aluminum 2017A	204 / 40	9.47
Upper body	Stainless steel 304L	206.5 / 13.5	2.49



**Fig. 2.** Cross-section view of the Harmony-Gamma structure and positioning of the accelerometers (red box). (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)



**Fig. 3.** Experimental longitudinal FRFs obtained for the accelerometers on (a) the upper body and (b) the central body for random input excitations of 30 N RMS (black), 119 N RMS (blue), 595 N RMS (red) and 2977 N RMS (green). (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

the experimental Frequency Response Functions (FRF) for different levels of random excitations, for two accelerometers located at the upper body and the central body. These amplitude FRF are defined as the measured PSD (i.e., output) divided by the PSD of the force excitation (i.e., input). By increasing the level of random excitations, nonlinear effects appear as an increase in damping (i.e., an increase in dissipation) and a softening of the resonance modes. The first peak (around 196 Hz) is slightly shifted towards lower frequencies with a decrease in amplitude when the excitation level increases, reflecting a nonlinear softening and an increase in dissipation. A more significant frequency shift is observed for the second mode (around 478 Hz), while a decrease in amplitude with absence of frequency shift is depicted for the third resonance mode (around 576 Hz). It can be noted that the linear first, second and third resonance frequencies at 196 Hz, 478 Hz and 576 Hz have been preliminarily identified by performing a low excitation level swept sine experiment [1]. Moreover the repeatability of experimental tests based on broadband random vibrations has been successfully verified.

Although it is not possible to dissociate the role of each nonlinearity through these experimental results only, it can be concluded that the decrease in the resonance frequency and the associated amplitude evolution when the input random excitation increases correspond to the combined effects of the two specific types of nonlinear joints, the first corresponding to friction joints and the second to elastomer joints. As previously explained in [1], it is difficult to define which nonlinearity is responsible for which shift in frequency or increase in energy dissipation, due to the presence of nonlinearities of distinct natures (local stick-slip behaviour for the contact blades, elastomer properties for rubber isolators, etc.). For the interested reader, a deep and complete investigation of this difficult task has been undertaken by the authors in [1] to provide some insight into the role of the nonlinearities in the nonlinear response under harmonic excitations and the more or less significant change shift in frequency peaks with increasing excitation levels. These questions will also be one of the original contributions and investigations of Section 3.3.

In conclusion, these experiments clearly illustrate the need to take into account nonlinear components, such as friction and elastomer joints, in the design of industrial structures. Indeed, even though the nonlinear joints are localised, they may induce non-negligible effects for both frequency shifts and the amplitude evolution on the global nonlinear response of the system.

### 3. Modeling and numerical simulations

This section is first devoted to a brief presentation of the modelling and the description of the numerical method used for the prediction of the nonlinear steady state response of the Harmony-Gamma structure subjected to random excitation. Secondly, the nonlinear numerical simulations are compared to the experimental results and a better understanding of the nonlinear behaviour of the structure Harmony-Gamma is undertaken.

#### 3.1. Modeling

The Finite-Element Model (FEM) of the structure Harmony-Gamma has been performed by using the finite-element software ABAQUS. The choices of modelling for the different parts of the assembly (FEM of linear sub-structures, linear updating and validation, friction and elastomer modelling, reduction strategy) have been discussed in [1]. The following section is only

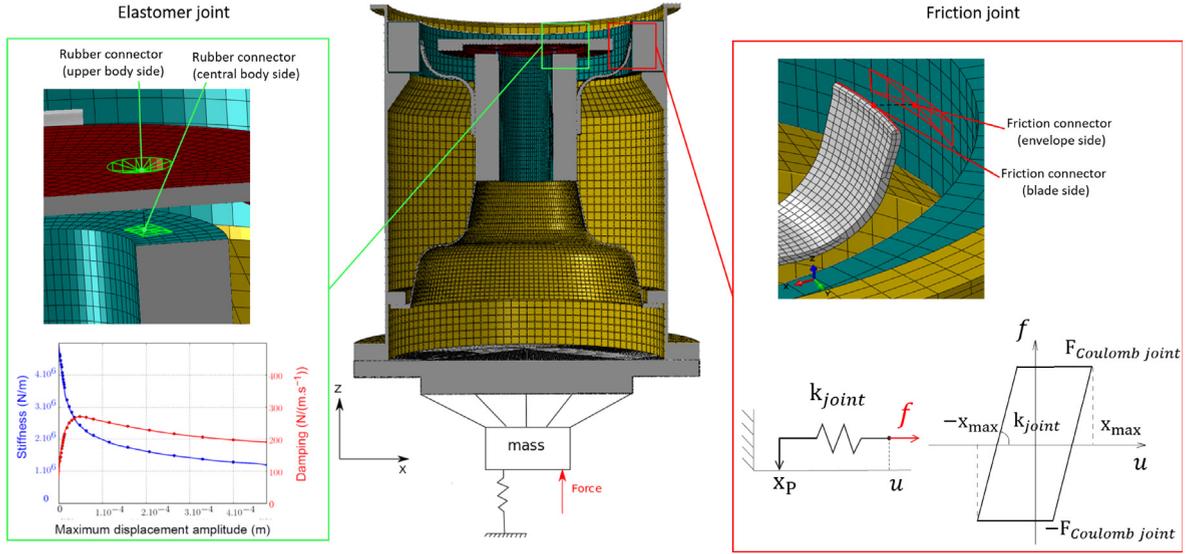


Fig. 4. Finite-element modelling of the Harmony-Gamma structure with nonlinear joints.

intended to recall the main choices for modelling the Harmony-Gamma structure. For a more complete description, the interested reader is referred to [1].

Fig. 4 presents a cross-section view of the linear FEM with specific attention to connectors for the nonlinear friction or elastomer joints. Concerning the elastomer joint, phenomenological modelling is identified by conducting swept-sine experiments with increasing excitation levels. For more details, the complete protocol and identification process are fully explained in [1]. Thereby, the nonlinear evolutions of stiffness and damping with respect to the displacement amplitude are illustrated in Fig. 4 for the longitudinal behavior of one rubber isolator. Considering the modelling of nonlinear friction joints, Jenkins modelling is implemented for the friction that occurs at the interface between the envelope and each blade. It is composed of a linear spring in series with a Coulomb friction model. Both adhesion and sliding phases at the frictional interface may occur, depending of the evolution of the force in the friction joint and the dynamic behaviour of the system (see Fig. 4). For more details, the complete modeling and the identification process are fully explained in [10]. In addition, it can also be noted that the mobile part of the shaker is modelled by a punctual inertial mass with blocked transverse and rotational degrees of freedom and the external random excitation is applied on this mass.

It should be noted that the linear FE-model (i.e., the Harmony-gamma structure without the nonlinear joints) includes more than 300,000 degrees of freedom (DOFs). Thanks to a sub-structuring algorithm implemented in Abaqus [12] (i.e. a variant of the Hurty or Craig–Bampton methods [13,14]), a significant reduction in the size of the problem is achieved. The size of the reduced nonlinear FE-model is 168: 15 modes contained within the frequency bandwidth [5; 1000] Hz, 72 DOFs for the friction joints of the blades, 48 DOFs for the four rubber isolators and 33 DOFs corresponding to the closest node in the directions x, y and z for the 11 accelerometers (i.e. to allow a direct comparison between the simulation and the experimental results).

Finally the nonlinear dynamical equation of the reduced system can be written as:

$$\mathbf{M}\ddot{\mathbf{x}} + \mathbf{D}\dot{\mathbf{x}} + \mathbf{K}\mathbf{x} = \mathbf{F}_{\text{excit}}(t) + \sum_{\text{joint}} \mathbf{F}_{\text{nl,joint}}(\mathbf{V}_{\text{joint}}^T \mathbf{x}) = \mathbf{F}_{\text{excit}}(t) + \mathbf{F}_{\text{nl}}(t) \quad (1)$$

where  $\mathbf{F}_{\text{nl,joint}}$  includes all of the eight nonlinear joints (i.e., four rubber isolators plus four frictional interfaces at the blades).

### 3.2. Adaptation of the Harmonic Balance Method in the case of random excitations

The main objective of this section is to remind the development of the Harmonic Balance Method for the prediction of the nonlinear steady state response of the nonlinear system defined in Eq. 1 and subjected to random excitation. The interested reader is referred to [7,8] for more details.

The main adaptation to allow the use of the HBM concerns the modelling of the random external signal. Previous studies proposed by Shinozuka [15,16] have shown that it is possible to transpose a random excitation into an equivalent deterministic excitation with one fundamental pulsation  $\Omega$  by using trigonometric series. Thus, the random excitation can be rewritten in the form of a truncated Fourier series

$$\mathbf{F}_{\text{excit}}(t) = \sum_{k=1}^p (\mathbf{C}_{k,\text{excit}} \cos(k\Omega t) + \mathbf{S}_{k,\text{excit}} \sin(k\Omega t)) \quad (2)$$

with  $\Omega = 2\pi\Delta f$  where  $\Delta f$  defines the frequency resolution. In practice, particular attention must be paid to the choice of the two parameters  $\Delta f$  and  $p$ .

The frequency resolution is given by

$$\Delta f = f_0 \left( 1 - \sqrt{1 - 2\xi \tan\left(\frac{\pi}{16}\right)} \right) \quad (3)$$

where  $f_0$  is the frequency of the first resonance peak.  $\xi$  defines the associated damping ratio given by  $\xi = \frac{1}{2Q}$  with  $Q$  the quality factor. This definition is used in order to have 8 points in the half Nyquist circle of the first resonance mode, which gives a good resolution around the peak. In the present study the nonlinear responses will be calculated in the bandwidth of interest [50;1000] Hz. Thus, the Nyquist frequency  $f_n$  also requires a time step equal to or lower than  $\frac{1}{2f_n} = 5 \cdot 10^{-4}$  s to be used. Additionally choosing a low value of  $\Delta f$  for a given bandwidth will generate prohibitive calculation costs, as this choice directly increases the number  $p$  to be considered in Eq. 2. Considering the previous Eq. 3 it is obvious that the lowest value of  $\Delta f$  would be obtained for very high quality factors  $Q$  and very low frequency  $f_0$ . From experimental tests, the worst case scenario (i.e. the maximum value of the quality factor  $Q$  and the lowest resonance frequency) gives a quality factor of  $Q = 100$  with a resonance frequency  $f_0 = 187$  Hz. This leads to the resolution  $\Delta f = 0.186$  Hz and a minimum number of samples  $N = \frac{f_n}{\Delta f} = 10753$ . In practice,  $N$  is equal to a power of 2 that is greater than  $\frac{f_n}{\Delta f}$  to simplify the use of the FFT. In the present case, we choose  $N = 2^{14}$ . It should be noted that the order of the truncated Fourier series  $p$  has to be chosen in connection with the frequency resolution (i.e. the number of samples  $N$ ). Since the constant term will be kept on the approximate nonlinear response,  $p + 1$  is chosen to be equal to the number of sample  $N$ .

It is worth mentioning that several strategies of resolution can be used for the resolution of nonlinear problems subjected to random excitation, such as the Fokker–Planck method [5,6,17,18], the perturbation method [19], the stochastic averaging method [20,21] or the stochastic linearization technique and extensions [22,23]. In random vibrations, the use of stochastic differential equations that provided an elegant formulation for random problems has a long tradition. Analytical resolutions of these methods give good ideas of physical phenomenon, even for nonlinear dynamic problems. However, these methods can show their drawbacks when dealing with complex industrial problems. In the context of the present study, the methodology proposed allows the treatment of complex non-linear problems in the presence of random excitations. It consists in numerically solving the nonlinear equations by using a trajectory of the excitation. This strategy makes it possible to obtain pragmatic results adapted to the engineer needs even if the elegance of stochastic differential equations could be lost.

Since the Harmony-Gamma structure is subjected to an external force that corresponds to a random excitation, the frequency resolution  $\Delta f$  has to be chosen as the fundamental frequency used in the HBM process. Hence, if a nonlinear steady state solution of the system exists, the nonlinear dynamical response  $\mathbf{x}$ , as well as the vector force  $\mathbf{F}_{nl}$ , can be approximated through finite Fourier series of order  $p$ , such as

$$\mathbf{x}(t) = \mathbf{B}_0 + \sum_{k=1}^p (\mathbf{B}_k \cos(k\Omega t) + \mathbf{A}_k \sin(k\Omega t)) \quad (4)$$

$$\mathbf{F}_{nl}(t) = \mathbf{C}_0 + \sum_{k=1}^p (\mathbf{C}_k \cos(k\Omega t) + \mathbf{S}_k \sin(k\Omega t)) \quad (5)$$

As previously explained,  $p$  is chosen in connection with the frequency resolution and the number of samples. Then, the classic HBM steps can be used to solve the nonlinear problem. The nonlinear problem in time domain can be rewritten in the Fourier basis. Substituting Eqs. 2, 4 and 5 in Eq. 1 yields a set of  $(2p + 1) \times n$  equations, where  $n$  is the number of degrees of freedom of the system.

By considering the first  $n^{th}$  equations, the vector  $\mathbf{B}_0$  of the constant terms for the approximation of the nonlinear solution  $\mathbf{x}(t)$  is given by

$$\mathbf{K}\mathbf{B}_0 = \mathbf{C}_0 \quad (6)$$

The  $2 \times p \times n$  remaining equations defining the vectors  $\mathbf{A}_k$  and  $\mathbf{B}_k$  of the  $k$ th Fourier coefficients are given by

$$\begin{bmatrix} \mathbf{K} - (k\Omega)^2 \mathbf{I} & -k\Omega \mathbf{D} \\ k\Omega \mathbf{D} & \mathbf{K} - (k\Omega)^2 \mathbf{I} \end{bmatrix} \begin{bmatrix} \mathbf{A}_k \\ \mathbf{B}_k \end{bmatrix} = \begin{bmatrix} \mathbf{S}_k \\ \mathbf{C}_k \end{bmatrix} + \begin{bmatrix} \mathbf{S}_{k,excit} \\ \mathbf{C}_{k,excit} \end{bmatrix} \quad \forall k \in [1, p] \quad (7)$$

where  $\mathbf{I}$  is the identity matrix.

Due to the fact that the coefficients  $\mathbf{C}_0$ ,  $\mathbf{S}_k$  and  $\mathbf{C}_k$  depend on the coefficients  $\mathbf{B}_0$ ,  $\mathbf{A}_k$  and  $\mathbf{B}_k$ , an adaptation of the classical Alternate Frequency Time domain method (AFT-method) [24] is used. Two versions of the AFT are used, depending on the type of nonlinearity. Friction joints have a natural definition with respect to the relative displacement of the joint with respect to time, and the classical AFT can be used, following this implementation:

$$\mathbf{X} = [B_0, A_1, B_1 \dots A_p, B_p] \xrightarrow{FFT^{-1}} X(t) \xrightarrow{Friction} F_{nl}(t) \xrightarrow{FFT} \mathbf{B}_{NL} = [C_0, S_1, C_1, \dots, S_p, C_p] \quad (8)$$

The elastomer modeling is based on the amplitude of deformation, and this notion is not time-dependent in itself. The modification of the AFT to accept this type of nonlinearity was first proposed in [8]. The following process is proposed and implemented

$$\mathbf{X} = [B_0, A_1, B_1 \dots A_p, B_p] \xrightarrow{FFT^{-1}} X(t) \xrightarrow{HilbertFiltering} \hat{X} \xrightarrow{Elastomer} F_{nl}(t) \xrightarrow{FFT} \mathbf{B}_{NL} = [C_0, S_1, C_1, \dots, S_p, C_p] \quad (9)$$

Let's start from  $B_0$  and  $(A_k, B_k)_{k \in [1:p]}$ , the harmonic components of the response. The calculation of the amplitude of  $X(t)$  for each  $t$  is performed by using an inverse FFT procedure. In the general case, the amplitude of  $X(t)$  over time cannot be defined properly, but in many cases,  $X(t)$  is a narrow-band signal, typical of the response of a single degree-of-freedom system. In this case, the amplitude of excitation is slowly evolving over time and a filtering technique can follow the evolution of the amplitude. Hilbert analytical filtering [25] is used and gave excellent results in a previous work [8]. A Hilbert filtering for the HBM method is implemented. Then the calculation of the nonlinear force  $F_{nl}(t)$  can be performed based on the approximation of  $\hat{X}$ . Finally, using a FFT procedure, the harmonic components  $C_0$  and  $(S_k, C_k)_{k \in [1:p]}$  of the nonlinear force in the frequency-domain are estimated.

Following this iterative process, the Fourier coefficients of Eq. 4, which define the steady state nonlinear solution over the entire frequency bandwidth, are determined by solving the  $(2p + 1) \times n$  nonlinear equations of motion 6 and 7. This solution can also be used to estimate the PSD of the solution. It should be noted that compared to the classical use of the HBM method for a non-linear system subjected to harmonic excitations (i.e. for which a resolution of Eqs. 6 and 7 is performed at each frequency step), the methodology proposed in the case of random excitations requires only one calculation for the entire frequency band concerned.

In order to allow an efficient and optimal use of the proposed method, it is important to question the numerical efficiency and cost of this approach and the recommendations to be taken. A brief reminder on this fact previously discussed in [9] is now proposed. The final system to be solve consists of  $(2p + 1) \times n$  equations where  $n$  defines the number of degrees of freedom and  $p$  is the order of the truncated Fourier series (related to the number of samples  $N$  needed for the modelling of the random external signal,  $N = p + 1$ ). So considering more specifically the numerical efficiency of the proposed approach three main points have to be highlight:

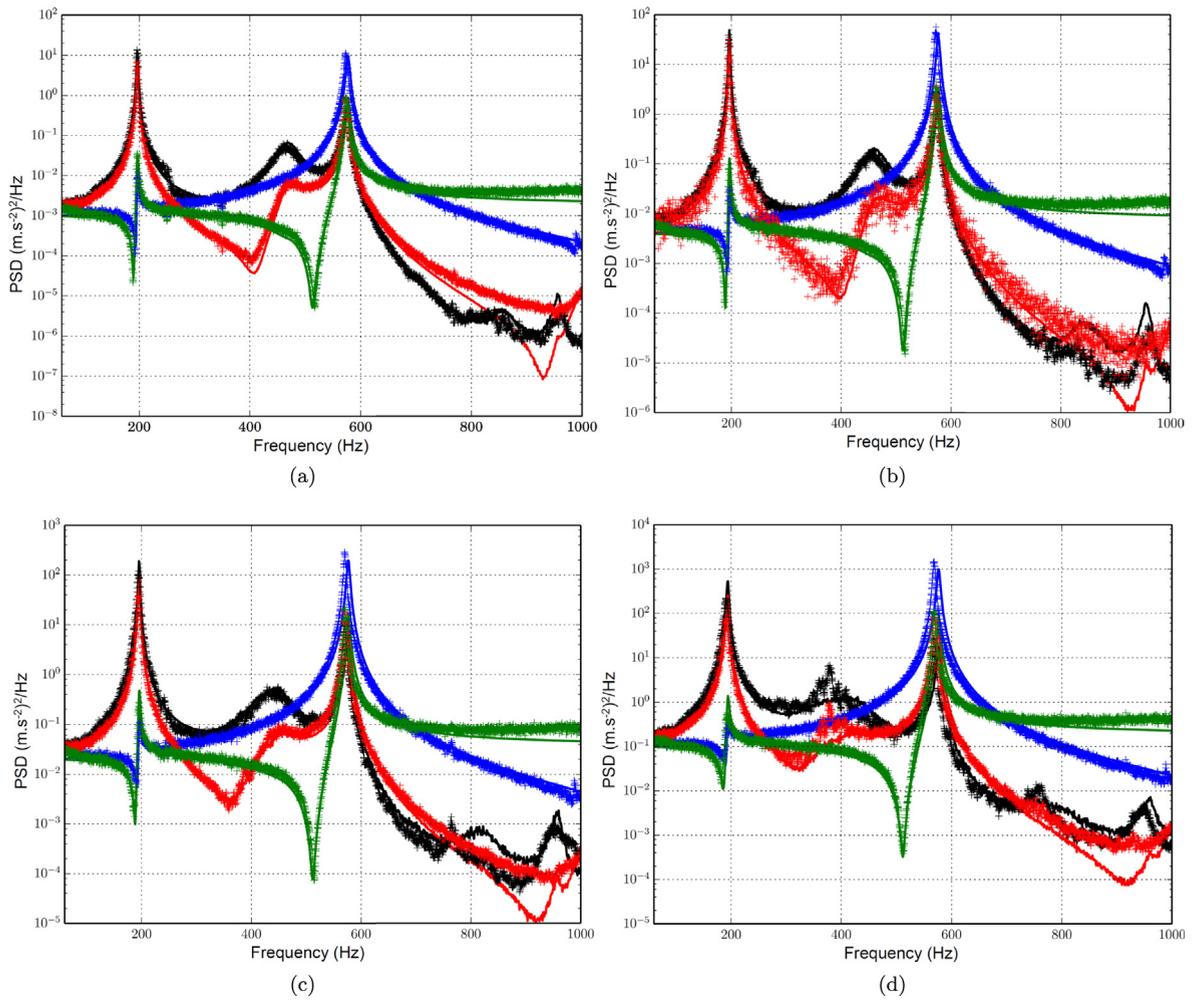
- the first part of the numerical process based on the model order reduction and its validity is crucial to drastically reduce the initial number of degrees of freedom (see Section 3.1);
- a condensation process based on the separation of linear and nonlinear degrees of freedom [26] as well as an adaptation of the AFT-method in the associated condensed Fourier space [1], are performed in order to reduce the size of the system;
- the representation of the random signals in terms of highly resolved Fourier series is potentially numerically expensive and may generate computational problems. Indeed considering  $N$  the number of samples, the size of the Jacobian of the system is  $(2N, 2N)$ . As a result, two processes are potentially numerically expensive. The first one is due to the inversion of the Jacobian of complexity class  $O(N^3)$ . The second one corresponds to the construction of the Jacobian matrix. In fact, each column needs the calculation of nonlinear forces  $F_{nl}(t)$ , and the latter is based on the use of one FFT and one inverse FFT with signals of size  $N$ , as indicated in Eqs. 8 and 9. This results in a well-known complexity of  $O(N \log(N))$ . Finally this leads to the complexity class of the algorithm of  $O(N^2 \log(N))$  for to the construction of the Jacobian matrix. One issue to facilitate numerical calculations during the HBM process is the use of a simplified Jacobian matrix as previously proposed in [9].

### 3.3. Comparison between numerical and experimental results

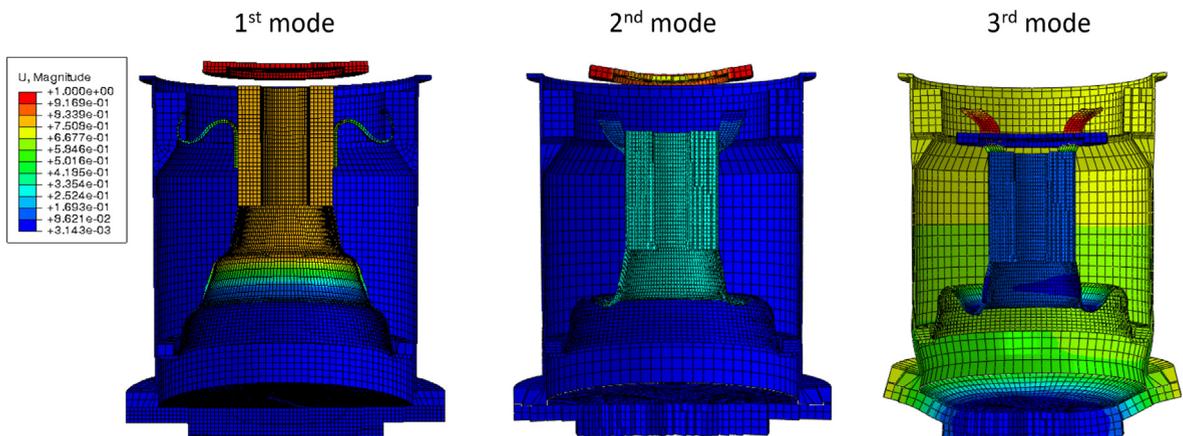
The main objective of this section is to validate the numerical nonlinear modeling by carrying out comparison between experiments and numerical results for the Harmony-Gamma structure subjected to broadband random vibrations. Fig. 5 gives a comparison between the simulation and experimental results for four random input excitations of 30 N RMS, 119 N RMS, 595 N RMS and 2977 N RMS. It is observed without any ambiguity that the numerical model very faithfully reproduces the dynamic behaviour of the structure Harmony-Gamma over the entire frequency interval of interest and for all of the different tested parts of the structure (i.e., the upper body, the central body, the upper part of the envelope and the base plate).

More specifically, the nonlinear effects, such as the frequency shifts and the evolution of amplitudes, are perfectly reproduced in the vicinity of the three main resonances (around 196 Hz, 478 Hz and 576 Hz, respectively). The significant softening effect at the second resonance peak, as well as an increase in dissipation for the three peaks when the excitation level increases, are predicted by numerical simulations. Likewise, the slight shift towards lower frequencies (an absence of frequency shift, respectively) is also reproduced for the first peak (third peak, respectively), with a decrease in amplitude when the excitation level increases.

To better understand the role of each nonlinear joint (i.e., the rubber joint and the friction joint) in the nonlinear response of the Harmony-Gamma structure and, more specifically, the more or less pronounced frequency shift (i.e., softening effect and dissipation) for the first three longitudinal modes, a view of the first three longitudinal numerical modes of the



**Fig. 5.** Experimental (crosses) and numerical (plain curves) PSD obtained for the accelerometers on the upper body (black), the central body (red), the upper part of envelope (blue) and the base plate (green) for random input excitations of (a) 30 N RMS (b) 119 N RMS (c) 595 N RMS (d) 2977 N RMS. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)



**Fig. 6.** View of the the first three longitudinal numerical modes of the structure Harmony-Gamma.

Harmony-Gamma structure is plotted in Fig. 6. The first mode mainly requires both the four blades and the upper body. This leads to the simultaneous nonlinear contributions of the four rubber joints and of the friction interface between the four blades and the central body. For the second mode, the vibrational behaviour is more located on the upper body with a pumping mode of the latter. This leads to a sollicitation of the four rubber isolators between the upper body and the central body. Thus, it can be concluded that the softening effects at the second peaks (around 478 Hz) observed during experiments are mainly due to the activation of the elastomer nonlinearity. Finally, the third mode puts the bottom part of the envelope in motion. As a result, the friction nonlinearity at each blade is activated due to the longitudinal motion of the entire envelope. Moreover, it is also clearly shown that the four rubber joints are not sollicitated for this third longitudinal mode, due to the fact that no relative motion is observed between the upper body and the upper part of the envelope. In conclusion, the numerical results allow us to undertake a qualitative study of the non-linear contributions due to friction blades and rubber insulators.

To go further in the comparison between experiments and numerical results, a slight difference can be observed for high excitation levels around the second mode (see Fig. 5(d) around 380 Hz for the central body and the upper part of envelope). This is only due to the interaction between the second harmonic of the first mode and the resonance of the second mode of vibration. Indeed, based on experimental results at a high level of random excitation (see Fig. 5(d)), it is shown that the frequency of the first mode is situated at 190 Hz, while the second mode resonance peak is at 380 Hz, twice the frequency of the first mode resonance peak. The potential explanation of this slight difference between experiments and simulation is the choice of the modelling for elastomer joints, which does not allow all of the harmonics generated to be captured and taken into account. Obviously, this result defeats the hypothesis of the first harmonic preserved for the rubber modelling. However, this phenomenon appears only at the highest level of random excitation and does not affect the overall perfect correlation of the nonlinear solution for the entire frequency range of interest. Thus, the proposed modelling of rubber joints appears to be a very good compromise between complexity of modelling and quality of numerical results.

Fig. 7 presents numerical results for the nonlinear behaviour of the Harmony-gamma structure on the upper body, for six levels of broadband random excitations, the last two levels having not been tested experimentally. The objective of these results is to confirm the trends and general observations already described with regard to the role of the non-linear joints. For the first mode, the resonance peak is first dampened and flattened increasing the excitation level (see experiments and numerical results for the four lowest levels of excitation). Then, increasing the excitation level leads to less dissipation and a softening of the resonance peak (see numerical results for the two strongest levels of excitation). This nonlinear phenomenon is characteristic of the impact of friction between the four blades and the envelope, even though the contribution of the four rubber joints is still active. Then, the second resonance peak is shifted continuously towards lower frequencies with a decrease in amplitude when the excitation level increases. This softening effect is consistent with the previous description of a pumping mode for the second resonance peak with the sollicitation of the four rubber isolators between the upper body and the central body. Finally, the third resonance peak decreases in amplitude with an absence of frequency shift. Our understanding is that the third mode is purely linear, and the decrease in amplitude is simply the result of the modal interaction between the second mode and the third mode: as the excitation increases, the second mode becomes farther away from the third mode because of the elastomeric softening, leading to a classical linear decrease of amplitude for the third mode.

Finally it can be mentioned that the computational time to calculate the output results for the case under study is about one hour (based on a Dell Inc. PowerEdge C6320 with a Processor Intel Xeon E5-2680 v4, 2 sockets, 14 Cores, 2.46 GHz, CPU

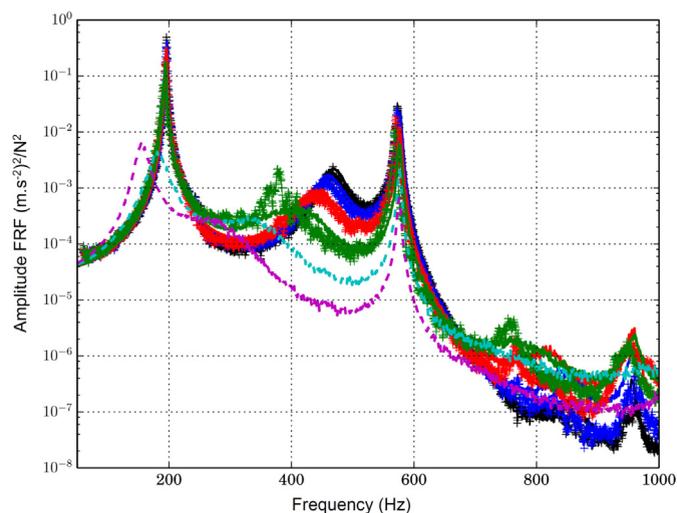
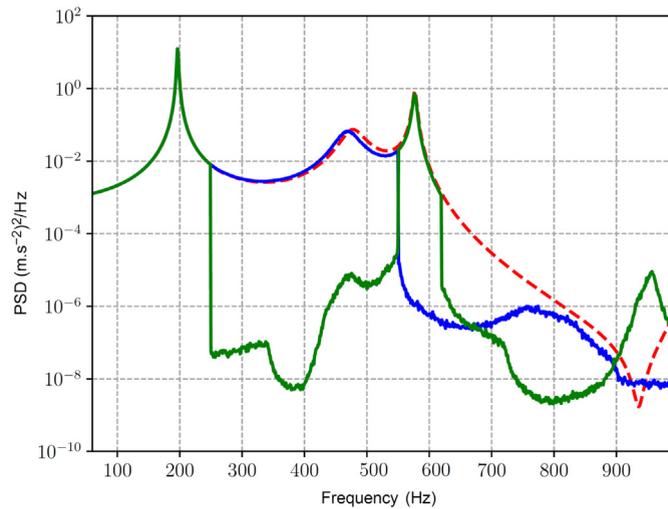


Fig. 7. Experimental (crosses) and numerical (plain curves) longitudinal FRFs on the upper body 30 N RMS (black), 119 N RMS (blue), 595 N RMS (red), 2977 N RMS (green), 95258 N RMS (cyan) and 7602061 N RMS (magenta). (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)



**Fig. 8.** PSD of the non-linear response via the HBM method for the linear case (red), a excitation centered on the first and second modes (blue) and an excitation centered on first and third modes (green). (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

128Go RAM).. This proves the numerical efficiency of the proposed strategy and its eventual applicability in academia and industry. Of course, it would be possible to further reduce calculation times by parallelizing the Jacobian calculation or by applying other optimization techniques.

#### 3.4. On the use of numerical simulation for a better understanding of nonlinear phenomena observed experimentally

In this last section specific attention will be paid to better understanding the nonlinear behaviour of the Harmony-Gamma structure within the frequency bandwidth [800; 1000] Hz. Fig. 8 shows two resonance peaks within the frequency bandwidth [800; 1000] Hz: at low levels, the first peak is located at 850 Hz, and the second peak is located at 950 Hz. As the excitation increases, the first resonance peak shifts to the left (i.e., softening effect) and becomes more and more damped, while the second resonance peak is damped but remains at 950 Hz. At the highest excitation levels (not reached experimentally), numerical results indicate that both peaks have disappeared. First of all, it is worth recalling that these two peaks are present because of nonlinear phenomena, due to the fact that a classical modal analysis indicates the nonexistence of these two resonance peaks in the absence of non-linearity.

To go further in this prior analysis of this phenomenon and to better understand the role of nonlinearities in the appearance of these two additional resonance peaks, additional numerical simulation calculations were carried out, keeping only one of the two sources of non-linearity (i.e., the blade friction or the elastomer joint). These calculations (not presented here for the sake of brevity) show that these two resonance peaks located in the range [800; 1000] Hz depend only on the non-linearity of the four rubber joints, without any contribution from the friction blades. Moreover, the origin of these two additional peaks cannot be linked to the presence of harmonics. A nonlinear modal interaction phenomenon could be at the origin of the appearance of these two resonance peaks in the frequency bandwidth [800; 1000] Hz. To verify this, two additional simulations are carried out: the first calculation corresponds to a random excitation centred only around the first and second modes (with a random excitation over the frequency bandwidth [50; 550] Hz), while the second is achieved by considering a random excitation only centred around the first and third (applying a random excitation over the two frequency bandwidths [50; 250] Hz and [550; 620] Hz). Numerical results are shown in Fig. 8, which illustrates the PSD of the nonlinear response for the upper body. It is clearly observed that the first peak at 850 Hz (the second peak at 950 Hz, respectively) is present for a random excitation centred only around the first and second modes (the first and third modes, respectively), while it disappears for a random excitation only centred around the first and third modes (the first and second modes, respectively).

It can be concluded without any ambiguity that a modal interaction for the first and second modes is the origin of the appearance of the resonance peak at 850 Hz, whereas a modal interaction for the first and third modes is the origin of the appearance of the resonance peak at 950 Hz. A slight frequency shift can be observed for the first “secondary” peak between Figs. 5 (a) (850 Hz) and 8 (800 Hz). One explanation is that the first and second modes are necessary to produce the modal interaction and create the first “secondary” peak, while the third mode affects the modal interaction and increases the frequency of the peak.

In conclusion, the complete modelling methodology and the proposed numerical simulations allow greater insight into the underlying nonlinear phenomena to be achieved.

#### 4. Conclusion

A finite-element model for a nonlinear industrial structure with friction and elastomer joints is proposed to predict the nonlinear vibrational behaviour of the system subjected to broadband random vibrations.

Based on experiments, it is demonstrated that the Harmony-Gamma structure may exhibit complex nonlinear behaviour due to the fact that the two nonlinearities have a simultaneous contribution. More or less significant changes are observed with frequency shifts (i.e., softening effects) and decreases in amplitude peaks by increasing excitation levels.

Although it is not possible to dissociate the role of each nonlinearity through experiments, numerical simulations can be carried out in addition to the experiments to allow a better understanding of the complex nonlinear behaviour of the Harmony-Gamma structure. Thanks to numerical simulation, some insight into the role of the nonlinearities (i.e., the friction blades and/or the elastomer joints) in the nonlinear response under random excitations is possible. Finally, numerical results provided excellent comparisons with the experimental results, validating not only the global strategy, but also the fact that a simplified phenomenological nonlinear modelling of the friction and elastomer joints is sufficient to capture all of the essential nonlinear phenomena and to go further in the understanding of the role of each nonlinearity in the structural nonlinear response of the Harmony-Gamma structure.

In this paper, the discussion of the validity of the approach has exclusively been conducted by comparing power spectral density measures between experiments and numerical simulations. Higher order moments have not been studied numerically nor investigated experimentally. One potential perspective could be to undertake the validity of the numerical strategy in the context of higher order moments and related effects. In fact, there is no a priori incompatibility between the proposed approach and the study of higher order moments. The method is robust enough to conduct such studies since it can manage inputs and create rare events by changing the phases of inputs as needed. Moreover, the wide-spread assumption of random phases is followed in the present study. However, the random phases assumption is not necessarily the most common case for real-world random excitations, either by their origin, or also as a consequence of interaction of structure and exciter. So one interesting perspective could be to extend the study for complex mechanical systems subjected to random excitations without considering a random-phase property. This additional challenge is a priori not incompatible with the proposed methodology.

#### Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

#### CRedit authorship contribution statement

**T. Roncen:** Methodology, Software, Validation, Investigation, Resources, Writing - review & editing, Visualization. **J.-J. Sinou:** Conceptualization, Methodology, Validation, Investigation, Writing - original draft, Writing - review & editing, Visualization, Supervision. **J.-P. Lambelin:** Conceptualization, Methodology, Validation, Investigation, Resources, Supervision, Project administration, Funding acquisition.

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