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# Acoustic tones in the near-nozzle region of jets: characteristics and variations between Mach numbers 0.5 and 2

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(Received \*\*\*)

The characteristics of acoustic tones near the nozzle of jets are investigated for Mach numbers between  $M = 0.50$  and 2 using large-eddy simulations. Peaks are observed in all cases. They are tonal for  $M \geq 0.75$  and broaden for lower Mach numbers. They are associated with the azimuthal modes  $n_\theta = 0$  to  $n_\theta^{max}$ , with  $n_\theta^{max} = 8$  for  $M = 0.75$  and 1 for  $M = 2$ , for example. Their frequencies do not appreciably vary with the nozzle-exit boundary-layer thickness and turbulence and fall in the frequency bands predicted for the upstream-propagating guided jet waves using a vortex-sheet model. For all azimuthal modes, the peak intensities are highest for the first radial guided jet mode. They increase roughly as  $M^8$  for  $M \leq 1$  and as  $M^3$  for  $M \geq 1$ , following the scaling laws of jet noise, suggesting that they mainly result from a band-pass filtering of the upstream-travelling sound waves by the guided jet modes. In support of this, the Mach number variations of the peak width and sharpness are explained by the eigenfunctions of the guided waves. Moreover, it appears that for high subsonic Mach numbers, among the waves possibly resonating in the potential core, only those close to the cut-off frequencies of the guided jet modes can contribute to the near-nozzle peaks. Finally, the peaks are detectable in the far field for large radiation angles. For  $M = 0.90$ , for instance, they emerge for angles higher than 135 degrees.

## 1. Introduction

Considerable advancements have been made in the last few years in the field of jet aeroacoustics, as highlighted in the reviews of Brès & Lele (2019) and Lyrantzis & Coderoni (2020) on jet noise prediction and modelling using large-eddy simulations, and that of Edgington-Mitchell (2019) on aeroacoustic resonance and self-excitation in supersonic jets, for instance. They have provided new insights into jet flow and noise generation mechanisms. In particular, a number of studies have emphasized the important role played by the upstream-propagating neutral subsonic instability waves of the jets in the establishment of feedback phenomena and the radiation of acoustic tones.

These waves were first clearly identified and described by Tam & Hu (1989). They are different from the free-stream sound waves classically considered to model feedback loops in jets, as for example in Powell (1953), Ho & Nosseir (1981), Raman (1998) and Weightman *et al.* (2019). They are characterized by specific dispersion relations and can be classified into modes depending on their radial and azimuthal structures. In addition, they are essentially confined inside the jet flow. For that reason, these waves, sometimes called neutral acoustic waves in the literature, will be referred to as guided jet waves in what follows, as in the recent paper of Edgington-Mitchell *et al.* (2021).

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42 They were shown in Tam & Ahuja (1990), Tam & Norum (1992), Gojon *et al.* (2016),  
 43 Bogey & Gojon (2017) and Jaunet *et al.* (2019) to close the feedback loops encountered in  
 44 subsonic and supersonic ideally-expanded jets impinging on a flat plate, whose direct part  
 45 consists of growing aerodynamic disturbances convected downstream by the flow. Indeed,  
 46 the frequencies and axisymmetric or helical natures of the tones observed in such flow  
 47 configurations can be explained by the properties of the guided jet waves. Similar findings  
 48 were reported in Jordan *et al.* (2018) for jet-flap interaction tones for Mach numbers  
 49 between 0.6 and 1, in Tam & Chandramouli (2020) for jet-plate interaction tones based  
 50 on the experimental data of Zaman *et al.* (2015) for Mach numbers ranging from 0.5 to  
 51 1.06, as well as in Shen & Tam (2002), Gojon *et al.* (2018), Edgington-Mitchell *et al.*  
 52 (2018), Mancinelli *et al.* (2019) and Li *et al.* (2020) for some of the screech tones emitted  
 53 by non-ideally expanded supersonic jets. For round screeching jets, more precisely, the  
 54 feedback loops of the axisymmetric screech modes A1 and A2 and of the helical screech  
 55 mode C appear to be completed by waves belonging to upstream-propagating guided jet  
 56 modes, namely to the second radial axisymmetric mode and the first radial helical mode,  
 57 respectively, according to the results in Gojon *et al.* (2018). On the contrary, waves of  
 58 other kinds, e.g. free-stream acoustic waves, may be involved for the flapping screech  
 59 modes B and D.

60 The upstream-propagating guided jet waves have also been found to be responsible for  
 61 the generation of acoustic tones in the near fields of high-speed free jets. Such tones were  
 62 documented for the first time in the paper of Suzuki & Colonius (2006). These authors  
 63 noted that the tones are particularly strong near the nozzle of a jet at a Mach number  
 64 of 0.9, are weaker or undetectable at lower Mach numbers, and do not scale with the  
 65 Strouhal number in term of frequency. They stated the need for further investigation to  
 66 fully understand this phenomenon. The origin of the tones was studied a decade later  
 67 in the companion papers of Towne *et al.* (2017), Schmidt *et al.* (2017) and Brès *et al.*  
 68 (2018). Using a vortex-sheet model, Towne *et al.* (2017) showed that downstream- and  
 69 upstream-propagating guided jet waves can both exist in the potential core of subsonic  
 70 jets for Mach numbers between 0.82 and 1. They demonstrated that, combined with the  
 71 end conditions imposed by the nozzle and the contraction of the potential core with the  
 72 axial distance, this can lead to acoustic resonance and the presence of trapped waves in  
 73 the jet core within limited frequency bands, and observed that these bands are consistent  
 74 with the frequencies of the tonal peaks obtained just outside the flow in both experiments  
 75 and numerical simulations (Brès *et al.* 2018).

76 Several questions remain, however, about the acoustic tones measured in the near pres-  
 77 sure fields of free jets, as pointed out by Brès & Lele (2019). This is the case, for example,  
 78 concerning their azimuthal structures, their precise relationship with the trapped waves in  
 79 the jet core, their possible propagation to the acoustic far field and the variations of their  
 80 properties with the nozzle-exit flow conditions and with the jet Mach number. On the last  
 81 point, Towne *et al.* (2017) speculated that the tones due to the presence of trapped waves  
 82 in the jet core should gradually appear as the Mach number approaches 0.82, and reach  
 83 their strongest prominence before being damped away into a broadband spectrum for  
 84 Mach numbers higher than 1. The latter behaviour seems corroborated by the indication  
 85 of the authors that no near-nozzle tones have been detected for a jet computed at a Mach  
 86 number of 1.5. Complementary analyses and results were given in Towne *et al.* (2019)  
 87 based on large-eddy simulation data for jets at Mach numbers between 0.4 and 1.5. The  
 88 results included frequency-wavenumber spectra in the jet potential core, which enabled  
 89 to isolate the signature of the waves trapped in this flow region. The progressive emer-  
 90 gence of tones near the nozzle lips of free jets at low Mach numbers was illustrated by  
 91 the measurements of Jaunet *et al.* (2016) and Zaman & Fagan (2019) for Mach numbers

92 ranging approximately from 0.6 to 1. No discontinuity of the tone properties is seen to  
93 occur around a Mach number of 0.82, below which the guided jet waves cannot propa-  
94 gate in the downstream direction according to their dispersion relations obtained using a  
95 vortex-sheet model. This rather unexpected trend was underlined by Jordan *et al.* (2018)  
96 in their study on jet flap-interaction tones. It led them to assume that the upstream-  
97 propagating guided jet waves couple with Kelvin-Helmholtz instability waves for jets at  
98 low Mach numbers. The variations of the near-nozzle acoustic tones at high Mach num-  
99 bers were revealed in the experiments of Zaman & Fagan (2019) for free jets at Mach  
100 numbers increasing nearly up to 1.5. The tones display continuous characteristics around  
101 Mach number 1, but visibly turn into the screech tones of the axisymmetric modes A1  
102 and A2 and of the modes B and D as the jets are supersonic and not ideally expanded  
103 at the nozzle exit. This result further shows that the upstream-propagating guided jet  
104 waves are an effective means of closing the feedback loops in screeching jets. In Zaman &  
105 Fagan (2019), four axisymmetric nozzles of different diameters and geometries, providing  
106 fully turbulent or nominally laminar boundary layers at the exit, were also used. The  
107 near-nozzle acoustic tones appear poorly affected by the nozzle-exit boundary-layer state  
108 and thickness. This seems to be also the case for the two initially laminar and turbulent  
109 jets computed by Brès *et al.* (2018). Finally, regarding the propagation of the near-nozzle  
110 tones to the acoustic far field, Jaunet *et al.* (2016) reported significant coherence levels  
111 between these waves and the sound waves at 30 nozzle diameters from the jet exit at  
112 high polar angles for Mach numbers around 0.82. Zaman & Fagan (2019) observed undu-  
113 lations in the spectra measured at 25 diameters and an angle of  $60^\circ$  resembling those in  
114 the spectra close to the jet exit for a Mach number of 1.013. Therefore, the near-nozzle  
115 tones leave their footprints in the far field in both experiments. The radiation mechanism  
116 is however unclear and may involve diffraction by the nozzle lip (Jaunet *et al.* 2016) or  
117 unwanted reflections by some uncovered surfaces (Zaman & Fagan 2019).

118 In the present work, the emergence of acoustic tones in the near-nozzle spectra of  
119 isothermal round free jets is investigated using large-eddy simulation. The jets have a  
120 diameter-based Reynolds number of  $10^5$  and Mach numbers ranging from 0.5 up to 2.  
121 Their upstream boundary layers have different thicknesses, and they are tripped or not,  
122 leading to highly disturbed or fully laminar nozzle-exit flow conditions. In this way, the  
123 sensitivity of the tones to the jet initial conditions will be examined. In particular, the  
124 presence of larger velocity fluctuations early on in the mixing layers may lead to weaker  
125 tones in broader spectra. The characteristics of the tones, in terms of frequency, intensity,  
126 prominence and width, will be detailed over the jet Mach number range. Their links with  
127 the trapped waves observed in the jet potential core will be discussed, based notably  
128 on frequency-wavenumber spectra calculated inside and just outside of the jets. Their  
129 propagation to the far pressure field, computed using the Linearized Euler equations from  
130 the LES near field, will also be highlighted. The azimuthal structures of the tones will  
131 be described. For that purpose, the contributions of the first two azimuthal modes for all  
132 jets, but also of higher modes for the jets with tripped boundary layers, will be evaluated.  
133 The near-nozzle tone frequencies will be compared with the frequencies allowed for the  
134 upstream-propagating guided jet waves according to a vortex-sheet model, in order to  
135 assess the role of these wave in the tone generation. This role will also be clarified by  
136 considering the eigenfunctions of the guided waves predicted by the model, and their  
137 variations along the dispersion curves of the waves. Furthermore, the scaling of the tone  
138 intensities with the Mach number will be addressed. Specific attention will be paid at  
139 both ends of the Mach number range. For subsonic Mach numbers, for instance, the  
140 continuity of the tone properties will be scrutinized in the vicinity of the Mach number  
141 thresholds below which downstream-propagating guided jet waves cannot exist, making

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tripping	M	Re <sub>D</sub>	δ <sub>BL</sub>	δ <sub>θ</sub> (z = 0)	u' <sub>e</sub> /u <sub>j</sub>
yes	0.6 – 2	10 <sup>5</sup>	0.15r <sub>0</sub>	~ 0.018r <sub>0</sub>	~ 9%
no	0.5 – 2	10 <sup>5</sup>	0.025r <sub>0</sub> – 0.4r <sub>0</sub>	0.004r <sub>0</sub> – 0.047r <sub>0</sub>	~ 0.2%

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TABLE 1. Jet parameters: boundary-layer tripping, Mach and Reynolds numbers M and Re<sub>D</sub>, thickness δ<sub>BL</sub> of the Blasius profiles at the pipe-nozzle inlet, momentum thickness δ<sub>θ</sub>(z = 0) and peak turbulence intensity u'<sub>e</sub>/u<sub>j</sub> at the exit.

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142 their coupling with the upstream-propagating waves impossible. For supersonic Mach  
 143 numbers, the appearance of near-nozzle tones for Mach numbers greater than or equal to  
 144 1.5 is not obvious given the results mentioned above. If such tones are observed for the  
 145 present ideally expanded jets, it will be interesting to look at whether they only extend  
 146 the tones obtained for subsonic Mach numbers, or also share similarities with the tones  
 147 of screeching jets, exhibiting mode jumps as the Mach number varies, for example.

148 The paper is organized as follows. In section 2, the jet initial conditions are defined,  
 149 and the large-eddy simulation methods and parameters are documented. In section 3, the  
 150 properties of the guided jet modes obtained using a vortex-sheet model for isothermal  
 151 round jets at varying Mach numbers are presented. The simulation results are displayed  
 152 in section 5. Vorticity and pressure snapshots and the main flow features of the jets with  
 153 tripped boundary layers are briefly shown. More importantly, the peaks found in the  
 154 pressure spectra computed in the jet potential core, very near the nozzle and in the far  
 155 field are quantified and analyzed, first for the jets at a Mach number of 0.9, then over  
 156 the whole Mach number range considered. Concluding remarks are given in section 4.  
 157 Finally, results obtained for untripped jets at Mach numbers between 0.75 and 0.85 and  
 158 for untripped jets at a Mach number of 0.50 are provided in two appendices. The aim in  
 159 the second case is to explore the origin of tones appearing in the near-nozzle spectra at  
 160 the vortex-pairing frequency.

## 161 2. Parameters

### 162 2.1. Jet definition

163 Isothermal round free jets at a Reynolds number Re<sub>D</sub> = u<sub>j</sub>D/ν = 10<sup>5</sup> have been com-  
 164 puted by large-eddy simulations for various Mach numbers M = u<sub>j</sub>/c<sub>0</sub>, where u<sub>j</sub>, D, c<sub>0</sub>  
 165 and ν are the jet velocity and diameter, the speed of sound in the ambient medium and  
 166 the kinematic molecular viscosity. The jets originate at z = 0 from a straight pipe nozzle  
 167 of radius r<sub>0</sub> = D/2 and length 2r<sub>0</sub>, whose lip is 0.053r<sub>0</sub> thick, into a medium at rest at a  
 168 temperature T<sub>0</sub> = 293 K and a pressure p<sub>0</sub> = 10<sup>5</sup> Pa. At the pipe inlet, at z = -2r<sub>0</sub>, Bla-  
 169 sius laminar boundary-layer profiles of thickness δ<sub>BL</sub> are imposed for the axial velocity,  
 170 radial and azimuthal velocities are set to zero, pressure is equal to p<sub>0</sub> and temperature is  
 171 determined by a Crocco-Busemann relation. In the pipe, the boundary layers are tripped  
 172 or not, leading to highly disturbed or fully laminar flow conditions at the exit. The main  
 173 parameters of the jets are collected in table 1 and represented in figure 1. Forty-four jets,  
 174 including six tripped and thirty-eight untripped cases, are simulated.

175 The six jets with tripped boundary layers have Mach numbers equal to 0.60, 0.75,  
 176 0.90, 1.10, 1.30 and 2, and boundary layers of thickness δ<sub>BL</sub> = 0.15r<sub>0</sub> at the pipe inlet.  
 177 The boundary layers are forced by adding random low-level vortical disturbances un-  
 178 correlated in the azimuthal direction in the pipe using a procedure developed in former  
 179 simulations (Bogey *et al.* 2011b, 2012; Bogey & Marsden 2016; Bogey & Sabatini 2019),

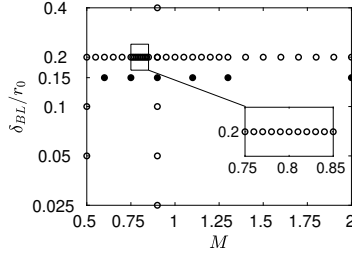


FIGURE 1. Jets with ● tripped and ○ untripped boundary layers: Mach number  $M$  and thickness  $\delta_{BL}$  of the Blasius profiles at the pipe-nozzle inlet.

180 in order to generate turbulent structures typical of those encountered in wall-bounded  
 181 flows (Bogey *et al.* 2011c). The forcing is applied at the axial position  $z = -0.95r_0$  and  
 182 the radial position  $r = r_0 - \delta_{BL}/2$  with a magnitude adjusted to obtain the desired  
 183 level of peak turbulence intensity at the pipe exit. The mean and rms velocity profiles  
 184 calculated at the nozzle-exit sections of the jets are plotted in figures 2(a,b). As intended,  
 185 they are very close to each other. The mean velocity profiles in figure 2(a) are similar to  
 186 a laminar boundary-layer profile of momentum thickness  $\delta_\theta = 0.018r_0$ , while the turbu-  
 187 lence intensities in figure 2(c) reach peak values  $u'_e/u_j \simeq 9\%$ . That was also the case in  
 188 the experiments of Zaman (1985) for a tripped jet at  $Re_D = 10^5$  with highly disturbed,  
 189 nominally laminar exit boundary layers.

190 On the contrary, and unlike most high-speed jets in experiments, the jets with un-  
 191 tripped boundary layers are initially fully laminar. The computational cost for such a  
 192 jet is lower than that for a tripped jet, because it is not necessary to discretize turbu-  
 193 lent boundary-layer structures. Thus, the simulations of the untripped jets in this work  
 194 allows us to cover and describe with accuracy wide ranges of boundary-layer thicknesses  
 195 and Mach numbers at an affordable cost. Five jets have a Mach number  $M = 0.90$ ,  
 196 and pipe-inlet boundary-layer thicknesses  $\delta_{BL} = 0.025r_0, 0.05r_0, 0.1r_0, 0.2r_0$  and  $0.4r_0$ .  
 197 Past or partial simulations of the first four jets were presented in Bogey & Bailly (2010)  
 198 and Bogey (2018). The nozzle-exit mean and rms velocity profiles obtained for the five  
 199 jets are shown in figures 2(c,d). The mean profiles in figure 2(c) resemble the Blasius  
 200 profiles imposed at the inlet. They are characterized by momentum thicknesses varying  
 201 from  $0.004r_0$  up to  $0.047r_0$ , as reported in table 1. For the comparison, Zaman (1985)  
 202 measured  $\delta_\theta = 0.0062r_0$  in an untripped, initially fully laminar jet at  $Re_D = 10^5$ . There-  
 203 fore, with respect to the experiments, the boundary layer is thinner in the jet with  
 204  $\delta_{BL} = 0.025r_0$ , similar for  $\delta_{BL} = 0.05r_0$  and thicker for  $\delta_{BL} \geq 0.1r_0$ . Regarding the rms  
 205 values of velocity fluctuations in figure 2(d), they are not zero but do not exceed 0.2 per  
 206 cent of the jet velocity. In addition to the five jets at  $M = 0.90$ , thirty-two jets have the  
 207 same pipe-inlet boundary-layer thickness  $\delta_{BL} = 0.2r_0$ , yielding exit momentum thick-  
 208 nesses  $\delta_\theta \simeq 0.024r_0$ , but different Mach numbers. These ones increase from  $M = 0.50$  to  
 209  $M = 2$ , in increments of  $\Delta M = 0.05$  for  $M \leq 0.75$ ,  $\Delta M = 0.01$  for  $0.75 \leq M \leq 0.85$ ,  
 210  $\Delta M = 0.05$  for  $0.85 \leq M \leq 1.30$  and  $\Delta M = 0.10$  for  $M \geq 1.30$ . The Mach number  
 211 range  $0.75 \leq M \leq 0.85$  is particularly well discretized to carefully examine the changes  
 212 in the near-nozzle tone properties around the Mach numbers below which downstream-  
 213 propagating guided jet waves cannot exist according to the vortex-sheet model. Finally,  
 214 two jets at  $M = 0.50$  with  $\delta_{BL} = 0.05r_0$  and  $0.1r_0$  are considered in order to discuss the  
 215 emergence of acoustic tones at the vortex-pairing frequency in initially laminar jets at  
 216 low Mach numbers.

217 It can be noted that for the jets with untripped boundary layers, pressure fluctuations

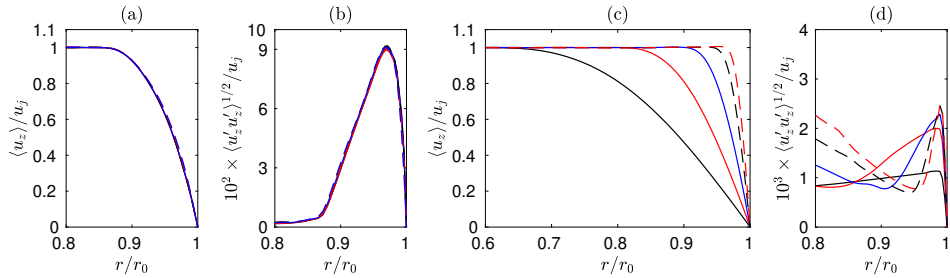


FIGURE 2. Nozzle-exit profiles for the jets (a,b) with tripped boundary layers at  $M =$  ——— 0.60, ——— 0.75, ——— 0.90, - - - 1.10, - - - 1.30 and - - - 2, and (c,d) with untripped boundary layers at  $M = 0.9$  with  $\delta_{BL} =$  ———  $0.4r_0$ , ———  $0.2r_0$ , ———  $0.1r_0$ , - - -  $0.05r_0$  and - - -  $0.025r_0$ : (a,c) mean and (b,d) rms values of axial velocity.

218 of maximum amplitude 200 Pa random in both space and time are arbitrarily introduced  
 219 from the start of the simulations between  $z = 0.25r_0$  and  $z = 4r_0$  in the shear layers, in  
 220 order to speed up the flow transitory period. At the non-dimensional time  $t = 12.5r_0/u_j$ ,  
 221 this acoustic excitation is turned off. Therefore, afterwards, the jet flow turbulent develop-  
 222 ment sustains by itself, without any external help. The acoustic waves travelling in the  
 223 upstream direction may be involved in this process, which will be investigated in future  
 224 studies.

225

## 2.2. Numerical methods

226 The numerical methods in the large-eddy simulations (LES) are identical to those used  
 227 in previous jet simulations (Bogey & Bailly 2010; Bogey *et al.* 2012, 2011*b*; Bogey 2018;  
 228 Bogey & Sabatini 2019). The LES have been carried out using an in-house solver of  
 229 the three-dimensional filtered compressible Navier-Stokes equations in cylindrical coordi-  
 230 nates  $(r, \theta, z)$  based on low-dissipation and low-dispersion explicit schemes. The axis  
 231 singularity is taken into account by the method of Mohseni & Colonius (2000). In order  
 232 to alleviate the time-step restriction near the cylindrical origin, the derivatives in the  
 233 azimuthal direction around the axis are calculated at coarser resolutions than permitted  
 234 by the grid (Bogey *et al.* 2011*a*). For the points closest to the axis, they are evaluated  
 235 using 16 points, yielding an effective resolution of  $2\pi/16$ . Fourth-order eleven-point centered  
 236 finite differences are used for spatial discretization, and a second-order six-stage  
 237 Runge-Kutta algorithm is implemented for time integration (Bogey & Bailly 2004). A  
 238 sixth-order eleven-point centered filter (Bogey *et al.* 2009) is applied explicitly to the  
 239 flow variables every time step. Non-centered finite differences and filters are also used  
 240 near the pipe walls and the grid boundaries (Berland *et al.* 2007). The explicit filtering  
 241 is employed to remove grid-to-grid oscillations, but also as a subgrid-scale high-order  
 242 dissipation model in order to relax turbulent energy at wave numbers close to the grid  
 243 cut-off wave number while leaving larger scales mostly unaffected. The performance of  
 244 this LES approach has been studied for subsonic jets (Bogey & Bailly 2006), Taylor-Green  
 245 vortices (Fauconnier *et al.* 2013) and turbulent channel flows (Kremer & Bogey 2015)  
 246 over the past years. For the jets with untripped boundary layers at  $M \geq 1.30$ , containing  
 247 weak shock cells in their potential cores as will be shown in section 4.1, a shock-capturing  
 248 filtering is applied in order to avoid Gibbs oscillations near the shocks. It consists in ap-  
 249 plying a conservative second-order filter at a magnitude determined each time step using  
 250 a shock sensor (Bogey *et al.* 2009). At the boundaries, the radiation conditions of Tam  
 251 & Dong (1996) are applied, with the addition of a sponge zone combining grid stretching  
 252 and Laplacian filtering at the outflow. At the inflow and radial boundaries, density and

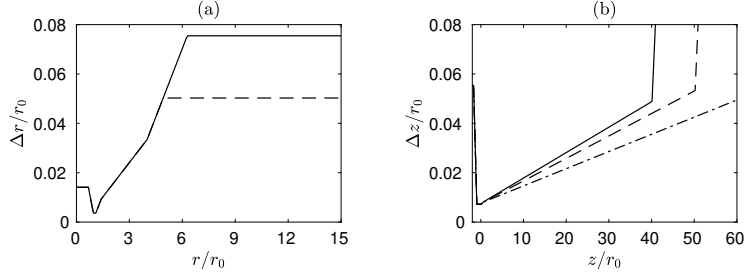


FIGURE 3. Variations (a) of radial mesh spacing  $\Delta r$  for — all jets except for - - - the jets with tripped boundary layers at  $M \geq 1.30$  and (b) of axial mesh spacing  $\Delta z$  for — all jets except for the jets with tripped boundary layers at - - -  $M = 1.30$  and - · - ·  $M = 2$ .

253 pressure are also brought back close to  $p_0$  and  $\rho_0$ , in order to keep the mean values of  
 254 density and pressure around their ambient values without generating significant acoustic  
 255 reflections. No co-flow is imposed.

### 256 2.3. Simulation parameters

257 In this study, except for the jets with tripped boundary layers at  $M = 1.30$  and  $M = 2$ ,  
 258 all the jets are simulated using the same grid in the  $(r, z)$  plane, detailed and referred  
 259 to as gridz40B in Bogey (2018). It contains  $N_r = 504$  points in the radial direction and  
 260  $N_z = 2048$  points in the axial direction, and extends radially out to  $r = L_r = 15r_0$   
 261 and axially, excluding the 100-point outflow sponge zone, down to  $z = L_z = 40r_0$ . The  
 262 variations of the mesh spacings in gridz40B are represented in figures 3(a,b). In the radial  
 263 direction, there are 96 points between  $r = 0$  and  $r = r_0$ . The mesh spacing  $\Delta r$  is minimum  
 264 and equal to  $\Delta r_{min} = 0.0036r_0$  at  $r = r_0$ . It is equal to  $0.014r_0$  at  $r = 0$  on the jet axis  
 265 and to  $0.075r_0$  between  $r = 6.25r_0$  and  $r = L_r$  in the jet near pressure field. For an  
 266 acoustic wave discretized by five points per wavelength, the mesh spacing  $\Delta r = 0.075r_0$   
 267 provides diameter-based Strouhal numbers  $St_D = fD/u_j = 10.7$  for  $M = 0.50$ ,  $St_D = 5.9$   
 268 for  $M = 0.90$ ,  $St_D = 4.1$  for  $M = 1.30$  and  $St_D = 2.7$  for  $M = 2$ , where  $f$  is the frequency.  
 269 In the axial direction, there are 169 points between  $z = -2r_0$  and  $z = 0$  along the  
 270 pipe nozzle. The mesh spacing  $\Delta z$  is minimum and equal to  $0.0072r_0$  between  $z = -r_0$   
 271 and  $z = 0$ . Farther downstream, it increases at the constant stretching rate of 0.103%  
 272 and reaches  $\Delta z = 0.049r_0$  at  $z = L_z$ . Finally, the number of points in the azimuthal  
 273 direction depends on the state and thickness of the nozzle-exit boundary layer. It was  
 274 set at  $N_\theta = 1024$  for the jets with tripped boundary layers, at  $N_\theta = 512$  for the jets at  
 275  $M = 0.90$  with untripped boundary layers of thicknesses  $\delta_{BL} \leq 0.1r_0$  and at  $N_\theta = 256$   
 276 in all other cases. This leads to a total number of points of one billion, 528 millions and  
 277 262 millions in the 3-D grids, respectively.

278 For the jets with tripped boundary layers at  $M = 1.30$  and at  $M = 2$ , the grids are  
 279 larger and contain  $N_r \times N_\theta \times N_\theta = 572 \times 1024 \times 2412 = 1.4$  billion points for  $M = 1.30$  and  
 280  $572 \times 1024 \times 2947 = 1.7$  billion points for  $M = 2$ . Compared to gridz40B, as illustrated in  
 281 figures 3(a,b), they extend farther in the axial direction in order to take into account the  
 282 lengthening of the jet potential core with the Mach number (Lau *et al.* 1979). In addition,  
 283 they are finer in the jet near pressure field to deal with the presence of sharp pressure  
 284 gradients in the acoustic field of supersonic jets (Ffowcs Williams *et al.* 1975; Laufer  
 285 *et al.* 1976). In the radial direction, the grids for  $M = 1.30$  and  $M = 2$  are the same. The  
 286 mesh spacing  $\Delta r$  is identical to that in gridz40B for  $r \leq 4r_0$ , but is constant and equal to  
 287  $0.05r_0$  between  $r = 4r_0$  and  $r = L_r = 15r_0$ , yielding  $St_D = 6.2$  for  $M = 1.30$  and  $St_D = 4$   
 288 for  $M = 2$  for an acoustic wave with 5 points per wavelength. In the axial direction,



289 the grids coincide with gridz40B for  $z \leq 0$ . From the nozzle exit, they are stretched at  
 290 the rates of 0.091% for  $M = 1.30$  and 0.070% for  $M = 2$  to obtain  $\Delta z = 0.053r_0$  at  
 291  $z = L_z = 50r_0$  and  $\Delta z = 0.050r_0$  at  $z = L_z = 60r_0$ , respectively.

292 The quality of the grids for the present jet LES has been assessed in several previ-  
 293 ous papers. In particular, studies of the sensitivity of the results to the grid resolution  
 294 in the axial and radial directions and to the number of points in the azimuthal direc-  
 295 tion were conducted in Bogey & Bailly (2010) and Bogey *et al.* (2011*b*) for some of  
 296 the jets with untripped and tripped boundary layers. The magnitude of the relaxation  
 297 filtering dissipation was also estimated and compared with that of viscous dissipation  
 298 in the wavenumber space. More recently, the grid dependence of the flow and acoustic  
 299 fields of the two jets with untripped boundary layers of thicknesses  $\delta_{BL} = 0.2r_0$  and  
 300  $0.025r_0$  and of the tripped jet at  $M = 0.90$  was discussed at length in Bogey (2018).  
 301 Moreover, for the tripped jets, the near-wall mesh spacing in the radial direction at the  
 302 nozzle exit is approximately equal to 2.4, in wall units, which is most likely sufficient  
 303 to provide accurate results according to former simulations of jets with highly disturbed  
 304 laminar boundary-layer profiles performed using the same numerical methods as in this  
 305 work (Bogey & Marsden 2016; Bogey & Sabatini 2019).

306 In the LES, with two exceptions, the time step is identical for all jets to apply the re-  
 307 laxation filtering at the same frequency, and hence not to change its magnitude compared  
 308 to that of viscous dissipation (Bogey *et al.* 2011*b*). Based on the minimum mesh spacing  
 309 and the speed of sound in the ambient medium, it is given by  $\Delta t = 0.7 \times \Delta r_{min}/c_0$ ,  
 310 ensuring numerical stability up to  $M = 2$ . The two exceptions are for the jets with  
 311 tripped boundary layers at  $M = 0.60$  and  $M = 0.75$ , for which  $\Delta t = 1.1 \times \Delta r_{min}/c_0$   
 312 and  $\Delta t = 0.9 \times \Delta r_{min}/c_0$ , respectively, in order to compensate for the increase of the  
 313 computational cost due to the lower jet velocities in these two LES performed using one  
 314 billion points. After a transient period varying from  $275r_0/u_j$  up to  $400r_0/u_j$  depending  
 315 on the jet initial conditions and on the grid extent in the axial direction, the simula-  
 316 tions have been carried out during a time period  $T$  of  $500r_0/u_j$ . The LES of the jets  
 317 with tripped boundary layers have been continued from this time onwards, leading to  
 318  $T = 3,000r_0/u_j$  at  $M = 0.90$ ,  $T = 1,250r_0/u_j$  at  $M = 0.60$  and  $0.75$  and  $T = 1,000r_0/u_j$   
 319 otherwise. This allows us to obtain a better statistical convergence for the results of the  
 320 jets with highly disturbed initial conditions, which are the main jets of interest and for  
 321 which, in addition, broadband noise components can be expected to be strong due to the  
 322 presence of fine-scale turbulence all along the mixing layers. The simulation times of the  
 323 untripped jets at  $M = 0.9$  have also been raised to  $T = 2,000r_0/u_j$  for  $\delta_{BL} = 0.2r_0$  and  
 324  $T = 1,600r_0/u_j$  for  $\delta_{BL} = 0.025r_0$ .

325 In all simulations, the signals of density, velocities and pressure have been recorded  
 326 at several locations during time  $T$ , creating a data base of the order of 150 TB, refer  
 327 for instance to Bogey (2018) and Bogey & Sabatini (2019) for a description of the data  
 328 available for the tripped jets. The data of interest in this work include those on the jet  
 329 axis at  $r = 0$ , the cylindrical surfaces at  $r = r_0$  and  $r = L_r$  and the cross sections at  
 330  $z = -1.5r_0$ ,  $z = 0$  and  $z = L_z$ . These data have been stored at a sampling frequency  
 331 corresponding to  $St_D = 12.8$ , with 256 points retained in the azimuthal direction. The  
 332 signals have also been acquired in the azimuthal planes at  $\theta = 0, \pi/4, \pi/2$  and  $3\pi/4$  for  
 333 all jets, as well as at  $\theta = \pi/8, 3\pi/8, 5\pi/8$  and  $7\pi/8$  for the tripped jet at  $M = 0.90$ , at a  
 334 sampling frequency of  $St_D = 6.4$ . The Fourier coefficients estimated over the section  $(r, z)$   
 335 for density, the velocity components and pressure have been saved in the same way for the  
 336 azimuthal modes  $n_\theta = 0$  to 8 for the six tripped jets and the untripped jets at  $M = 0.90$ ,  
 337 and for the modes  $n_\theta = 0$  and 1 for the other untripped jets. The flow and acoustic near  
 338 field statistics presented in what follows are calculated from these recordings. They are

339 averaged in the azimuthal direction, when possible. The time spectra are evaluated from  
 340 overlapping samples of duration  $90r_0/u_j$ .

341 Finally, the simulations have been carried out using an OpenMP-based in-house solver  
 342 on single nodes with 16 to 40 cores. These nodes, provided by the French regional and  
 343 national HPC centers listed in the acknowledgment section, consisted, for instance, of  
 344 four Intel Sandy Bridge E5-4650 8-core processors at a clock speed of 2.7 GHz or of two  
 345 Intel Xeon Gold 6130 16-core processors at 2.1 GHz. The LES needed between 50 GB of  
 346 memory for the jets with untripped boundary layers computed using gridz40B and 256  
 347 points in the azimuthal direction and 340 GB for the tripped jet at  $M = 2$  simulated  
 348 using the largest mesh grid. The number of iterations performed varies between 170,000  
 349 for the untripped jet at  $M = 2$  and 1.2 million for the tripped jet at  $M = 0.90$ . For  
 350 the last jet, the time per iteration is equal to 120 and 70 seconds using the two 32-core  
 351 nodes mentioned above, respectively, leading to the consumption of slightly more than  
 352 one million CPU hours in total. For the five other tripped jets, approximately three  
 353 million CPU hours have been required. For the thirty-eight untripped jets, most of which  
 354 have been simulated using four times smaller grids and over shorter time periods than  
 355 the tripped jets, between six and ten million CPU hours have been necessary. The LES  
 356 of these jets have run on a wide variety of nodes with different cores, making it difficult  
 357 to give a more accurate estimation. Thus, the cost of the full study is of the order of 15  
 358 million CPU hours.

### 359 **3. Guided jet modes in isothermal round jets at varying Mach** 360 **numbers for a vortex-sheet model**

361 The Mach number variations of the properties of the guided waves in jets are investi-  
 362 gated in this section. For that, based on the pioneering work of Tam & Hu (1989), the  
 363 dispersion relations and eigenfunctions of the neutral subsonic instability waves predicted  
 364 using a vortex-sheet model for isothermal round jets are examined. They are analyzed,  
 365 taking into account previous studies on the subject, conducted by Tam & Ahuja (1990),  
 366 Morris (2010) and Towne *et al.* (2017), among others.

#### 367 *3.1. Guided jet waves for the first two azimuthal modes*

368 As in the papers mentioned above, the two azimuthal modes  $n_\theta = 0$  and 1 are first  
 369 considered. The dispersion relations of the guided jet waves determined for these modes  
 370 at  $M = 0.70, 0.90$  and  $1.10$  using the vortex-sheet model are represented in figures 4(a-c)  
 371 as a function of wavenumber  $k$  and Strouhal number  $St_D$ . These values are chosen to  
 372 illustrate the three types of results obtained, respectively, for subsonic Mach numbers  
 373 below and above  $M \simeq 0.80$  and for supersonic Mach numbers. For each azimuthal mode,  
 374 waves are allowed for specific values of  $(k, St_D)$ . They are classified into different radial  
 375 modes, with the mode number  $n_r$  given by the number of antinodes exhibited by the  
 376 eigenfunction between the jet centerline and the shear layer. The dispersion curves start  
 377 from a limit point L on the line  $k = -\omega/c_0$ , where  $\omega = 2\pi f$ , at a Strouhal number  
 378 increasing with the mode number. The waves propagate in the upstream direction when  
 379 their group velocities  $v_g = d\omega/dk$  are negative and in the downstream direction when  
 380  $v_g > 0$ . In what follows, they will be denoted as  $v_g^-$  waves in the first case and  $v_g^+$   
 381 waves in the second one. The points on the curves where  $v_g = 0$  and  $dv_g/dk = 0$  are  
 382 marked in order to distinguish between different portions and locate waves with specific  
 383 characteristics on the dispersion curves. They will be referred to as the stationary and  
 384 inflection points S and I, respectively. The points S also correspond to the saddle points  
 385 in the complex wavenumber plane whose importance in the emergence of acoustic tones

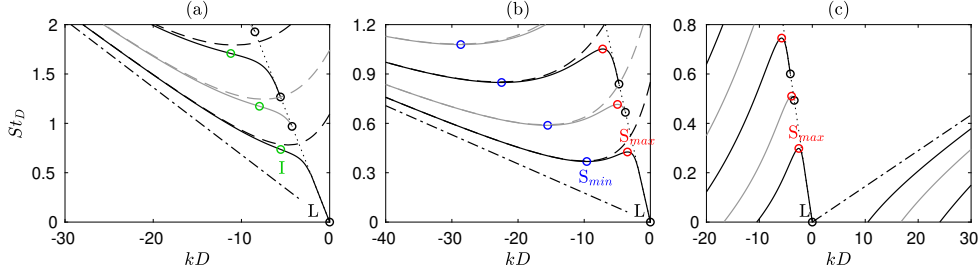


FIGURE 4. Dispersion relations obtained using the vortex-sheet model for the guided jet waves at (a)  $M = 0.70$ , (b)  $M = 0.90$  and (c)  $M = 1.10$  for  $\text{---}$   $n_\theta = 0$  and  $\text{---}$   $n_\theta = 1$  as a function of  $k$  and  $St_D$ ; points  $\circ$  L,  $\circ$   $S_{max}$ ,  $\circ$   $S_{min}$  and  $\circ$  I; dispersion relations of the acoustic waves in a duct for  $\text{---}$   $n_\theta = 0$  and  $\text{---}$   $n_\theta = 1$ ;  $\cdots$   $k = \omega/(u_j - c_0)$ ,  $\cdots$   $k = -\omega/c_0$ .

386 in the potential core of high subsonic jets was highlighted in Towne *et al.* (2017). At  
 387 these points, the waves have zero group velocity, do not propagate and are stationary  
 388 by nature. At points I, the waves have zero group-velocity dispersion. They are the  
 389 least dispersive (Whitham 1974) and most coherent waves, and travel without frequency  
 390 change. This led, for instance, Tam & Ahuja (1990) to assume that they are the most  
 391 likely to establish stable feedback loops in subsonic jets impinging on a flat plate, which  
 392 is supported by experimental data for Mach numbers between 0.7 and 0.95 in their paper.

393 For the subsonic Mach numbers  $M = 0.70$  and  $0.90$ , in figures 4(a,b), the dispersion  
 394 curves fully stand in the region with negative wavenumbers, between the two straight  
 395 lines  $k = -\omega/c_0$  and  $k = \omega/(u_j - c_0)$  indicating waves with phase and group velocities  
 396 equal to  $-c_0$  and  $u_j - c_0$ . The curves are close to the first line near the limit points L of  
 397 the modes and converge towards the second one as the wavenumber tends to  $-\infty$  and the  
 398 Strouhal number increases. To further characterize the guided waves, following Towne  
 399 *et al.* (2017), the dispersion curves obtained for the acoustic modes in a cylindrical soft  
 400 duct for  $n_\theta = 0$  and 1 are also displayed. They coincide with the dispersion curves of  
 401 the guided jet modes for high wavenumbers, in absolute value, but progressively deviate  
 402 from them as one approaches the line  $k = -\omega/c_0$ . Thus, Towne *et al.* (2017) proposed to  
 403 separate the modes into two categories, namely the duct-like modes and the free-stream  
 404 modes. For  $M = 0.90$ , in figure 4(b), they suggested that the waves belong to free-stream  
 405 modes between points L and  $S_{max}$ , named S2 in their work, and to duct-like modes  
 406 anywhere else on the dispersion curves. For  $M = 0.70$ , in figure 4(a), it appears similarly  
 407 that the waves can be considered as free-stream waves between points L and I and as  
 408 duct-like waves to the left of points I.

409 Regarding the group velocities of the waves, they are always negative for  $M = 0.70$  in  
 410 figure 4(a), implying that the waves all propagate in the upstream direction. Given that  
 411  $St_D = 0$  at the limit point L of the first axisymmetric mode,  $v_g^-$  waves can be found for  
 412 all frequencies. For  $M = 0.90$ , in figure 4(b), the group velocities of the waves are negative  
 413 between points L and  $S_{max}$ , positive between  $S_{max}$  and  $S_{min}$  and negative again to the  
 414 left of  $S_{min}$ , where  $S_{min}$  and  $S_{max}$  are the stationary points associated, respectively, with  
 415 the local minimum and the local maximum on the dispersion curves, corresponding to the  
 416 saddle points S1 and S2 in Towne *et al.* (2017). Therefore, as for  $M = 0.70$ ,  $v_g^-$  waves are  
 417 possible for all frequencies. However,  $v_g^+$  waves propagating in the downstream direction  
 418 can also exist, over limited frequency bands ranging between the Strouhal numbers at  
 419 points  $S_{min}$  and  $S_{max}$ . These waves vanish below threshold Mach numbers depending on  
 420 the azimuthal and radial modes. The threshold Mach number is equal to  $M = 0.82$  for

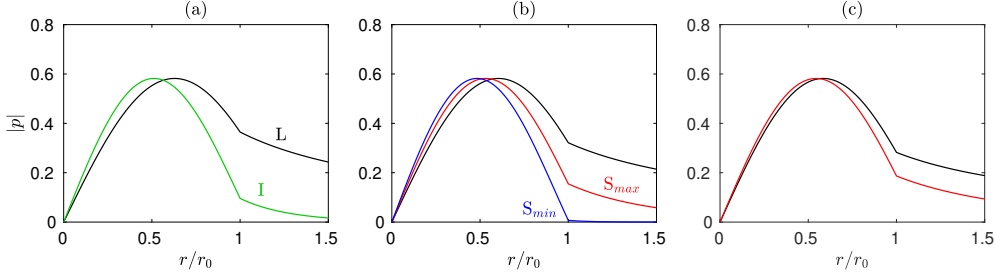


FIGURE 5. Pressure eigenfunctions obtained using the vortex-sheet model for the guided jet waves at (a)  $M = 0.70$ , (b)  $M = 0.90$  and (c)  $M = 1.10$  at points  $\text{---}$  L,  $\text{---}$   $S_{max}$ ,  $\text{---}$   $S_{min}$  and  $\text{---}$  I on the dispersion curves of the mode ( $n_\theta = 1$ ,  $n_r = 1$ ).

421 the first axisymmetric mode and to  $M = 0.80$  for the first helical mode, for example, and  
 422 decreases for higher radial modes.

423 For the supersonic Mach number  $M = 1.10$ , in figure 4(c), the dispersion curves first  
 424 extend in the region with negative wavenumbers, to the left of the limit points L on  
 425 the line  $k = -\omega/c_0$  down to  $St_D = 0$ , and then continue in the region with positive  
 426 wavenumbers, tending towards the line  $k = \omega/(u_j - c_0)$ , as illustrated in Morris (2010)  
 427 for instance. For all modes, the group velocities of the waves are negative from points  
 428 L to  $S_{max}$  and positive everywhere else. As a result, as for  $M = 0.90$ , the waves can  
 429 propagate both in the upstream and the downstream directions. Nevertheless, contrary  
 430 to the previous case, the  $v_g^-$  waves are now restricted to very narrow frequency bands  
 431 ranging between the Strouhal numbers at points L and  $S_{max}$ , whereas the  $v_g^+$  waves are  
 432 allowed for all frequencies.

433 Pressure eigenfunctions obtained using the vortex-sheet model for the guided jet mode  
 434 ( $n_\theta = 1$ ,  $n_r = 1$ ) at  $M = 0.70$ ,  $0.90$  and  $1.10$  are shown in figures 5(a-c) between  $r = 0$  and  
 435  $r = 1.5r_0$ . They are determined at the points L, I,  $S_{max}$  and  $S_{min}$ , when available. The  
 436 first helical mode is considered, but similar trends can be seen for the other azimuthal  
 437 modes. As reported in previous studies, the waves are essentially confined inside the  
 438 jet flow and they decay with the radial distance at a rate depending on the point on  
 439 the dispersion curves. Outside the jet flow, in particular, the wave magnitudes are quite  
 440 significant at the limit points L, but much lower at the other points. More precisely, they  
 441 are approximately two times smaller at the stationary points  $S_{max}$  for  $M = 0.90$  and  $1.1$ ,  
 442 and 5 times smaller at the inflection point I for  $M = 0.70$ . They are even negligible at  
 443 the stationary point  $S_{min}$  for  $M = 0.90$ , resulting in almost entirely confined waves in  
 444 that case (Tam & Ahuja 1990). These trends are consistent with the classification of the  
 445 waves into free-stream waves near the line  $k = -\omega/c_0$  and duct-like waves otherwise.  
 446 However, the changeover from free-stream to duct-like waves is gradual, which makes it  
 447 difficult to claim, for some waves such as those found in the vicinity of the points I for  
 448  $M = 0.70$  and  $S_{max}$  for  $M = 0.90$  for instance, whether they are free-stream or duct-like  
 449 waves.

450 To better quantify the amplitude of the waves outside of the jet flow, the magnitudes  
 451 of the pressure eigenfunctions obtained at  $r = 1.5r_0$  for  $n_\theta = 0$  and  $1$  at the Mach  
 452 numbers of the six jets with tripped boundary layers are represented in figures 6(a-f) as  
 453 a function of  $St_D$ . The  $v_g^-$  and  $v_g^+$  waves propagating in the upstream and downstream  
 454 directions are indicated by solid and dashed lines, respectively, and the points L, I,  $S_{max}$   
 455 and  $S_{min}$  are displayed. The variations with the frequency of the magnitude of the  $v_g^-$   
 456 waves from the limit point L depend on the Mach number and on the presence of  $v_g^+$   
 457 waves on the curves. For  $M = 0.60$  and  $0.75$ , in figures 6(a,b), in the absence of  $v_g^+$

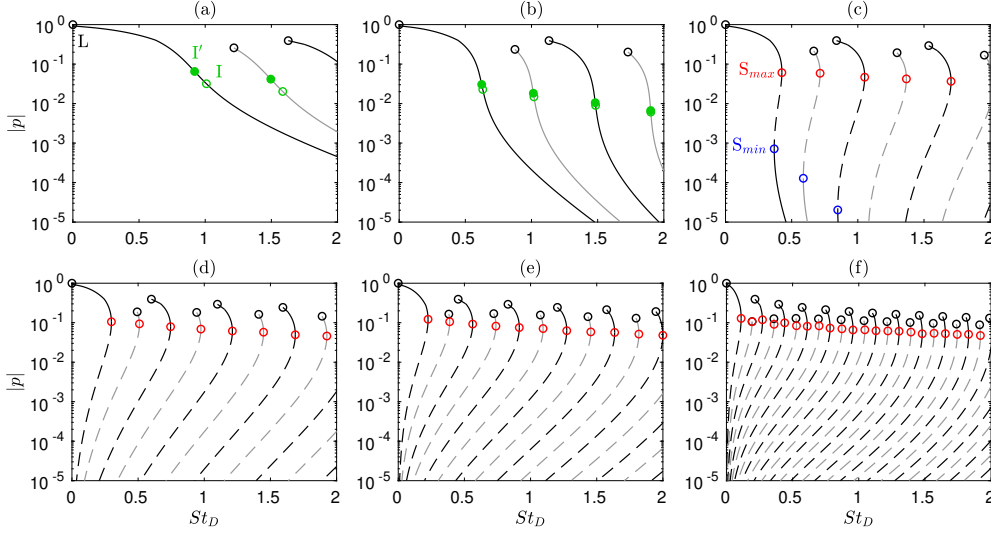


FIGURE 6. Magnitudes of the pressure eigenfunctions obtained using the vortex-sheet model for the guided jet waves at  $r = 1.5r_0$  at (a)  $M = 0.60$ , (b)  $M = 0.75$ , (c)  $M = 0.90$  (d)  $M = 1.10$ , (e)  $M = 1.30$  and (f)  $M = 2$  as a function of  $St_D$ : (solid lines) upstream- and (dashed lines) downstream-propagating waves for (black)  $n_\theta = 0$  and (grey)  $n_\theta = 1$ ; points  $\circ$  L,  $\circ$   $S_{max}$ ,  $\circ$   $S_{min}$  and  $\circ$  I on the dispersion curves;  $\bullet$  points I' of maximum rate of decrease. Only the waves with  $k \leq 0$  are considered for supersonic Mach numbers.

458 waves, the magnitude of the  $v_g^-$  waves decays continuously with the frequency, as was  
 459 noticed by Jordan *et al.* (2018) also for  $M = 0.60$ . The decay is slow for  $M = 0.60$  but  
 460 much faster for  $M = 0.75$ . It is maximum at points I', which are close to the inflection  
 461 points I for  $M = 0.60$  and nearly coinciding with them for  $M = 0.75$ . For  $M = 0.90$ , in  
 462 figure 6(c),  $v_g^-$  waves are first found between L and  $S_{max}$ , and again below  $S_{min}$  but with  
 463 an amplitude at least two orders of magnitude lower. Consequently, the magnitudes of  
 464 the  $v_g^-$  waves are significant between the Strouhal numbers of L and  $S_{max}$  and negligible  
 465 for higher frequencies. Finally, for  $M = 1.1, 1.3$  and  $2$ , in figures 6(d-f), as the waves  
 466 are all  $v_g^+$  waves below the stationary points  $S_{max}$ , the  $v_g^-$  waves are cut-off above the  
 467 Strouhal numbers of these points. Therefore, each guided jet mode can be regarded as  
 468 a band-pass filter of the upstream-propagating waves. The filter band-width appears to  
 469 decrease with the Mach number, and can be approximated by the frequency difference  
 470 between points L and I for  $M \leq 0.80$ , and points L and  $S_{max}$  for  $M \geq 0.80$ . Around  
 471 the frequencies of I or  $S_{max}$ , the filter cut-off is smooth in the first case with a slope  
 472 steepening with the Mach number, but it is sharp in the second case.

473 The dispersion relations of the guided jet waves allow us to determine the allowable  
 474 frequency bands for the  $v_g^-$  waves propagating in the upstream direction (Tam & Norum  
 475 1992). The bands predicted using the vortex-sheet model between  $M = 0.5$  and  $2$  for the  
 476 first five radial modes for  $n_\theta = 0$  and  $1$  are represented as a function of the Mach and the  
 477 Strouhal number in figures 7(a,b), along with the points L,  $S_{max}$ ,  $S_{min}$ , I and I' defined  
 478 above. The bands are highlighted in two shades of grey, depending on the presence of  
 479  $v_g^+$  waves simultaneously with the  $v_g^-$  waves. In the dark-grey regions,  $v_g^-$  and  $v_g^+$  waves  
 480 are both permitted, making acoustic resonance possible in the jet potential core (Towne  
 481 *et al.* 2017). In the light-grey ones, on the contrary, only  $v_g^-$  waves can be found. For  
 482 subsonic Mach numbers, the upstream-propagating waves can exist at all frequencies for  
 483  $n_\theta = 0$  and over a wide range of frequencies for  $n_\theta = 1$ . As the Mach number decreases,

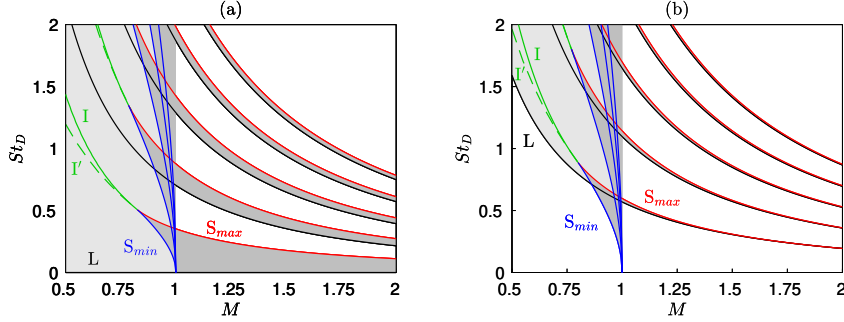


FIGURE 7. Allowable Strouhal number ranges obtained using the vortex-sheet model for the upstream-propagating guided jet waves (dark grey) with and (light grey) without downstream-propagating guided waves for (a)  $n_\theta = 0$  and (b)  $n_\theta = 1$  as a function of  $M$ ; points  $\text{---}$  L,  $\text{---}$   $S_{max}$ ,  $\text{---}$   $S_{min}$ ,  $\text{---}$  I and  $\text{---}$  I' on the dispersion curves. The first five radial modes are shown.

484 the inflection points I gradually move away from the points I' of maximum decay of the  
 485 eigenfunction magnitude outside of the jet. They remain however very close to each other  
 486 nearly down to  $M = 0.6$ . For supersonic Mach numbers, as mentioned previously, the  
 487 upstream-propagating waves are restricted to bands narrowing with the Mach number.  
 488 For a given Mach number, these bands are much smaller for  $n_\theta = 1$  than for  $n_\theta = 0$ ,  
 489 yielding extremely thin bands for  $n_\theta = 1$  at  $M = 2$  for example. As the radial mode  
 490 number increases, the bands are also smaller for  $n_\theta = 0$  but larger for  $n_\theta = 1$ .

### 3.2. Guided jet waves for the azimuthal modes $n_\theta \geq 2$

491  
 492 The properties of the guided jet waves associated with the azimuthal modes  $n_\theta \geq 2$ ,  
 493 whose contributions to the emergence of acoustic tones in the potential core and the  
 494 near field of jets have not been discussed in the literature to the best of the author's  
 495 knowledge, are briefly examined in this section. For that, the dispersion relations of the  
 496 waves have been calculated using the vortex-sheet model for  $n_\theta = 2$  to 8 over the Mach  
 497 number range considered in the present work.

498 The dispersion relations obtained at  $M = 0.70, 0.90$  and  $1.10$  for  $n_\theta = 2$  to 5 are  
 499 represented in figures 8(a-c) as a function of  $k$  and  $St_D$ , along with those for  $n_\theta = 0$   
 500 and 1. The results for  $n_\theta = 6$  to 8 are not plotted, for clarity. For each azimuthal mode,  
 501  $n_\theta$  is indicated on the curve for the first radial mode. At a higher azimuthal mode, the  
 502 dispersion curves are found for higher Strouhal numbers, but their shapes do not change  
 503 much. As a result, they are essentially aligned with each other, and are sometimes very  
 504 similar. The latter can be observed, for instance, for the solid black curve related to the  
 505 second radial axisymmetric mode and the dashed black curve of the first radial mode  
 506 for  $n_\theta = 2$ , and for the solid black curve of the third radial axisymmetric mode and  
 507 the dash-dotted grey curve of the first radial mode for  $n_\theta = 5$ . Differences between the  
 508 dispersion relations can however be noted in the vicinity of the line  $k = -\omega/c_0$ . The  
 509 portion of the curves near that line is narrower as the azimuthal mode number increases.  
 510 This leads to limit points L closer to the inflection points I for  $M = 0.70$  in figure 8(a)  
 511 and to the stationary points  $S_{max}$  for  $M = 0.90$  and  $1.10$  in figures 8(b,c). There are even  
 512 no points  $S_{max}$  at  $M = 0.90$  for  $n_\theta \geq 3$  and at  $M = 1.10$  for  $n_\theta \geq 2$ . In these cases, the  
 513 curves have a positive slope immediately to the left of the limit points L, suggesting that  
 514 free-stream upstream-propagating guided jet waves cannot exist.

515 The allowable frequency bands obtained using the vortex-sheet model between  $M = 0.5$   
 516 and 2 for the first five radial upstream-propagating wave modes for  $n_\theta = 2, 3$  and 4 are

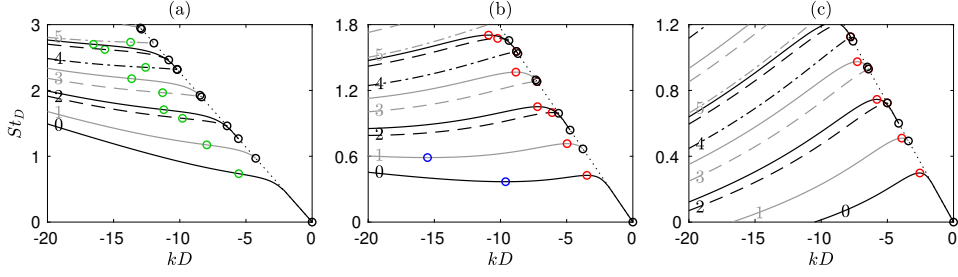


FIGURE 8. Dispersion relations obtained using the vortex-sheet model for the guided jet waves at (a)  $M = 0.70$ , (b)  $M = 0.90$  and (c)  $M = 1.10$  for  $n_\theta = \text{---} 0$ ,  $\text{---} 1$   $\text{---} 2$ ,  $\text{---} 3$   $\text{---} 4$  and  $\text{---} 5$  as a function of  $k$  and  $St_D$ ; points  $\circ$  L,  $\circ$   $S_{max}$ ,  $\circ$   $S_{min}$  and  $\circ$  I;  $\cdots$   $k = -\omega/c_0$ . For each azimuthal mode,  $n_\theta$  is indicated on the dispersion curve for the first radial mode. Only the part with  $k \leq 0$  is shown for  $M = 1.10$ .

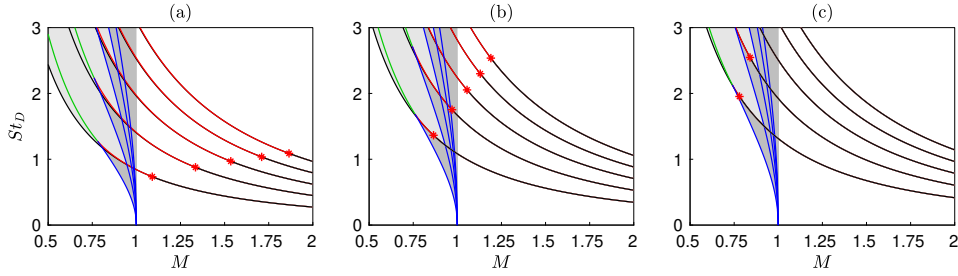


FIGURE 9. Allowable Strouhal number ranges obtained using the vortex-sheet model for the upstream-propagating guided jet waves (dark grey) with and (light grey) without downstream-propagating guided waves for (a)  $n_\theta = 2$ , (b)  $n_\theta = 3$  and (c)  $n_\theta = 4$  as a function of  $M$ ; points  $\text{---}$  L,  $\text{---}$   $S_{max}$ ,  $\text{---}$   $S_{min}$  and  $\text{---}$  I on the dispersion curves;  $*$  threshold Mach numbers for the presence of  $S_{max}$  points. The first five radial modes are shown.

517 represented as a function of the Mach and the Strouhal number in figures 9(a-c). They  
 518 resemble those obtained for  $n_\theta = 0$  and 1 in figures 7(a,b). Compared to these two modes,  
 519 however, the inflection points I and the stationary points  $S_{max}$  are nearer to the limit  
 520 points L, providing narrower bands. The bands even subsist only up to threshold Mach  
 521 numbers, indicated by red asterisks, above which there is no stationary point  $S_{max}$   
 522 on the dispersion curves. The threshold values decrease with the azimuthal mode number  
 523 and are, for example, equal to  $M = 1.09$ ,  $0.86$  and  $0.78$  for the radial modes  $n_r = 1$  for  
 524  $n_\theta = 2$ ,  $3$  and  $4$ . They also increase with the radial mode number and vary, for instance,  
 525 from  $M = 1.09$  for  $n_r = 1$  up to  $1.86$  for  $n_r = 5$  in figure 9(a) for  $n_\theta = 2$ . Again,  
 526 the vortex-sheet model predicts the non-existence of free-stream upstream-propagating  
 527 waves for the azimuthal modes  $n_\theta \geq 2$  at sufficiently high Mach numbers. The dispersion  
 528 curves of the guided jet waves being sensitive to the thickness of the mixing layer, as  
 529 shown theoretically in Tam & Ahuja (1990) for a Mach number of  $0.8$  and numerically in  
 530 Bogey & Gojon (2017) for a Mach number of  $1.5$ , one may wonder whether this will be  
 531 true for jets with mixing layers of finite thickness, such as those simulated in the present  
 532 study.

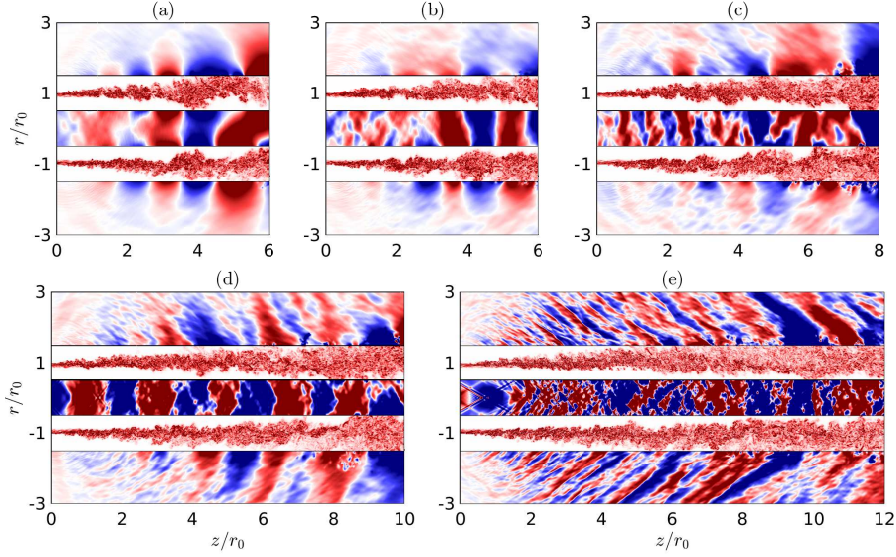


FIGURE 10. Snapshots in the  $(z, r)$  plane of vorticity norm between  $r = 0.5r_0$  and  $r = 1.5r_0$  and of pressure fluctuations otherwise for the jets with tripped boundary layers at (a)  $M = 0.60$ , (b)  $M = 0.75$ , (c)  $M = 0.90$ , (d)  $M = 1.30$  and (e)  $M = 2$ . The colour scales range between  $\pm 20u_j/r_0$  and (a)  $\pm 3.5 \times 10^{-3}p_0$ , (b)  $\pm 4.25 \times 10^{-3}p_0$ , (c)  $\pm 5.5 \times 10^{-3}p_0$ , (d)  $\pm 1.3 \times 10^{-2}p_0$  and (e)  $\pm 2.5 \times 10^{-2}p_0$ , from blue to red.

## 533 4. Simulation results

534

### 4.1. Snapshots and flow properties

535

The flow and near pressure fields of the jets with tripped boundary layers are briefly presented. For conciseness, those obtained for the jets with untripped boundary layers are not shown. In short, these jets exhibit roll-ups and pairings of vortical structures (Winant & Browand 1974) in their initially fully laminar mixing layers, which occur more rapidly and at a higher frequency as the boundary-layer thickness decreases (Bogey & Bailly 2010; Bogey 2018), and farther downstream as the Mach number increases in agreement with linear stability analyses (Michalke 1984; Morris 2010). The vortex pairings generate strong acoustic waves in the downstream direction (Colonius *et al.* 1997).

536

Snapshots of the vorticity and pressure fields obtained downstream of the nozzle-exit section for the jets with tripped boundary layers at  $M = 0.60, 0.75, 0.90, 1.30$  and  $2$  are provided in figures 10(a-e). Due to the highly disturbed exit flow conditions, turbulence is found just downstream of the nozzle, no vortex pairing can be easily detected in the shear layers, and fine-scale structures and high-frequency sound waves can be seen. In the pressure fields, fluctuations of hydrodynamic nature are visible just outside of the jets (Arndt *et al.* 1997) and waves are observed in the potential cores (Towne *et al.* 2017). For a higher jet velocity, the shear layers develop faster and Mach waves are radiated due to the convection of the flow structures at a supersonic speed. Footprints of shock cells can also be detected in the cores of the supersonic jets despite that ambient pressure is imposed at the inlet of the pipe nozzle.

537

The effects of the Mach number on the jet flow development are illustrated by the variations of the shear-layer momentum thickness, of the rms values of axial velocity at  $r = r_0$  and of the centerline mean axial velocity in figures 11(a-c). As the Mach number increases, the shear layers spread more slowly in figure 11(a) due to the lower growth rates of instability waves (Michalke 1984) and to compressibility effects (Brown & Roshko

538



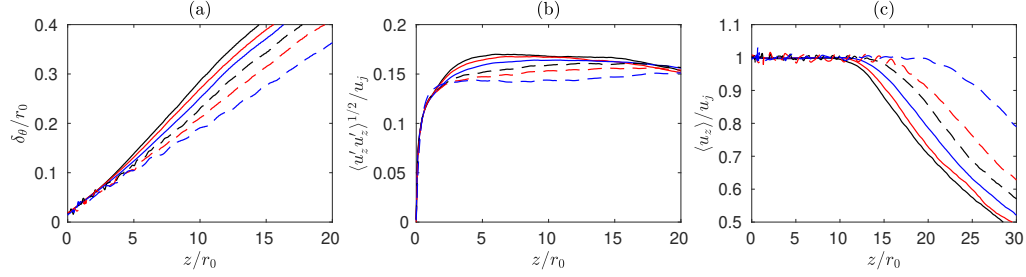


FIGURE 11. Axial variations of (a) the shear-layer momentum thickness, (b) the rms values of axial velocity at  $r = r_0$  and (c) the mean axial velocity at  $r = 0$  for the jets with tripped boundary layers at  $M =$  ——— 0.60, ——— 0.75, ——— 0.90, - - - 1.10, - - - 1.30 and - - - 2.

1974). As expected (Lau *et al.* 1979), this leads to longer potential cores in figure 11(c), ending at  $z_c = 13r_0$  for  $M = 0.60$ ,  $z_c = 14.8r_0$  for  $M = 0.90$ ,  $z_c = 17.8r_0$  for  $M = 1.30$  and  $z_c = 23.4r_0$  for  $M = 2$  for instance. In that figure, the centerline mean velocity profiles exhibit oscillations in the cores of the supersonic jets, but they are quite small, indicating that the shock-cell structures are very weak and that the jets are nearly ideally expanded at the nozzle exit. Finally, in figure 11(b), the profiles of turbulence intensities in the mixing layers are quite flat due to the high level of velocity fluctuations at the nozzle exit (Bogey *et al.* 2012), and reach peak values decreasing from 17% for  $M = 0.60$  down to 15% for  $M = 2$ .

#### 4.2. Acoustic tones for the jets at a Mach number of 0.90

The presence and properties of acoustic tones in the jet potential core, in the near-nozzle region and in the pressure far field are first examined for the six jets at  $M = 0.90$  in figure 1, with particular emphasis on the jet with tripped boundary layers.

##### 4.2.1. Tones in the jet potential core

In order to shed light on the waves in the jets, a space–time Fourier transform has been applied to the pressure fluctuations inside the jet potential core for azimuthal modes  $n_\theta = 0$  and 1, as in previous work on free, impinging and screeching jets (Towne *et al.* 2017, 2019; Bogey & Gojon 2017; Gojon *et al.* 2018), but also for modes  $n_\theta = 2$  to 8. The pressure fluctuations are taken at a fixed radial position, depending on the azimuthal mode, from the nozzle exit at  $z = 0$  down to  $z = 0.7z_c$ . The latter position allows us to reduce the contributions of the pressure disturbances of aerodynamic nature, particularly significant around the end of the potential core, while permitting a substantial spatial extent in the axial direction.

The spectra obtained for the jets with tripped boundary layers for  $n_\theta = 0$  to 5 are represented in figures 12(a–f) as a function of wavenumber and Strouhal number. For positive wavenumbers, strong components lie near the line  $k = \omega/(0.75u_j)$ . They correspond to the footprints left in the jet potential core by the shear-layer turbulent structures convected by the flow. For negative wavenumbers, high levels appear along bands near the dispersion curves predicted for the guided jet waves using the vortex-sheet model. This is clearly visible for  $n_\theta = 0$  and 1, as in Towne *et al.* (2017) also for a Mach number 0.9 jet, as well as for higher azimuthal modes. Differences can be noted between the bands and the dispersion curves. They are less important in the spectra for the untripped jets with thinner boundary layers, not shown for brevity. Therefore, they can be attributed to the assumption of an infinitely thin shear layer in the vortex-sheet model. In agreement with previous studies (Tam & Ahuja 1990; Bogey & Gojon 2017), the bands are slightly

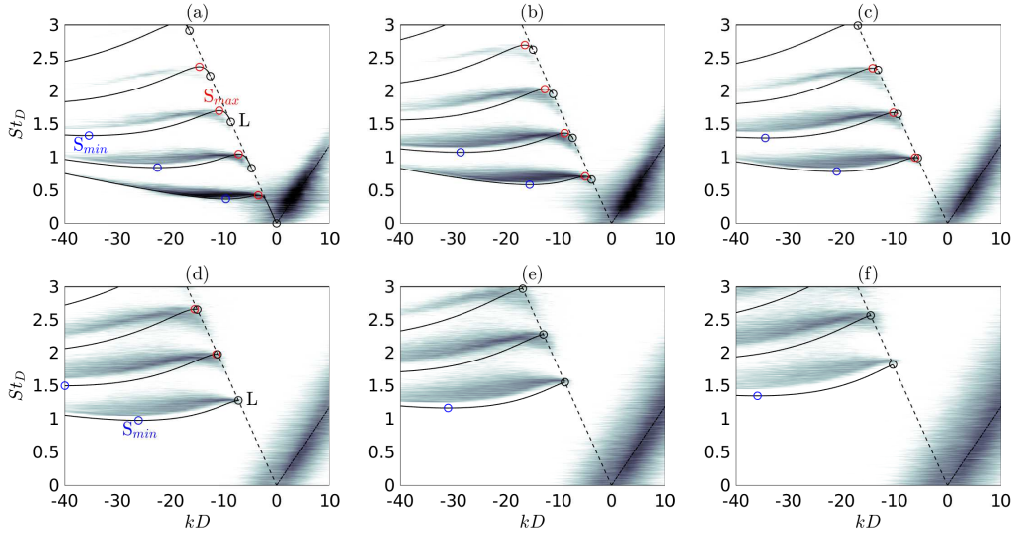


FIGURE 12. Frequency-wavenumber spectra of pressure fluctuations in the potential core of the jet at  $M = 0.90$  with tripped boundary layers at (a)  $r = 0$  for  $n_\theta = 0$ , (b)  $r = 0.2r_0$  for  $n_\theta = 1$ , (c)  $r = 0.3r_0$  for  $n_\theta = 2$ , (d)  $r = 0.3r_0$  for  $n_\theta = 3$ , (e)  $r = 0.4r_0$  for  $n_\theta = 4$  and (f)  $r = 0.4r_0$  for  $n_\theta = 5$  as a function of  $(kD, St_D)$ ; ——— dispersion curves of the guided jet waves, points  $\circ$  L,  $\circ$   $S_{max}$  and  $\circ$   $S_{min}$ ; - - -  $k = -\omega/c_0$ , - · - ·  $k = \omega/(0.75u_j)$ . The greyscale levels spread over 25 dB.

594 above the dispersion curves far from the line  $k = -\omega/c_0$ . Near the line, the opposite  
 595 trend is observed for  $n_\theta = 0$  to 2 in figures 12(a-c). In this zone, the bands have larger  
 596 portions with a negative slope than the dispersion curves, yielding waves with a negative  
 597 group velocity over wider frequency ranges. This is especially true for  $n_\theta = 3$  to 5 in  
 598 figures 12(d-f), for which upstream-propagating waves are found near  $k = -\omega/c_0$  for  
 599 all radial modes, contrary to the model prediction. In that case, they are restricted to  
 600 Strouhal numbers around that of the limit points L given by the model. Moreover, the  
 601 bands are thicker as the azimuthal mode number increases. In the bands, the energy is  
 602 rather evenly distributed and no peak can be detected, unlike the results for impinging  
 603 and screeching jets (Bogey & Gojon 2017; Gojon *et al.* 2018) for instance. However, the  
 604 energy levels are, overall, highest between the Strouhal numbers of the two stationary  
 605 points  $S_{min}$  and  $S_{max}$ , as pointed out by Towne *et al.* (2017). In most cases, the maximum  
 606 levels even appear mainly located between  $S_{min}$  and  $S_{max}$  where the waves propagate in  
 607 the downstream direction.

608 Spectra of pressure fluctuations obtained for  $n_\theta = 0$  to 5 in the potential core of the jets  
 609 with tripped boundary layers and with untripped boundary layers with  $\delta_{BL} = 0.025r_0$   
 610 are plotted in figures 13(a-f) as a function of  $St_D$ . To be consistent with the frequency-  
 611 wavenumber spectra of figure 12, they are computed at the same radial positions, by  
 612 averaging between  $z = 0$  and  $z = 0.7z_c$ . The allowable frequency ranges predicted using  
 613 the vortex-sheet model for the upstream-propagating guided jet waves are displayed using  
 614 the same colour code as in figures 7 and 9. To avoid an overlapping of the dark-grey bands,  
 615 in which upstream-propagating but also downstream-propagating waves can exist, only  
 616 the bands for the first four radial guided jet modes are represented for  $n_\theta = 0$ . In the  
 617 same way, only the bands for  $n_r = 1 - 3$  and  $n_r = 1 - 2$  are shown for  $n_\theta = 1, 2$  and  
 618  $n_\theta = 3 - 5$ , respectively. For all azimuthal modes, despite the presence of strong broadband  
 619 aerodynamic components, visible in the spectra of figure 12 for positive wavenumbers,

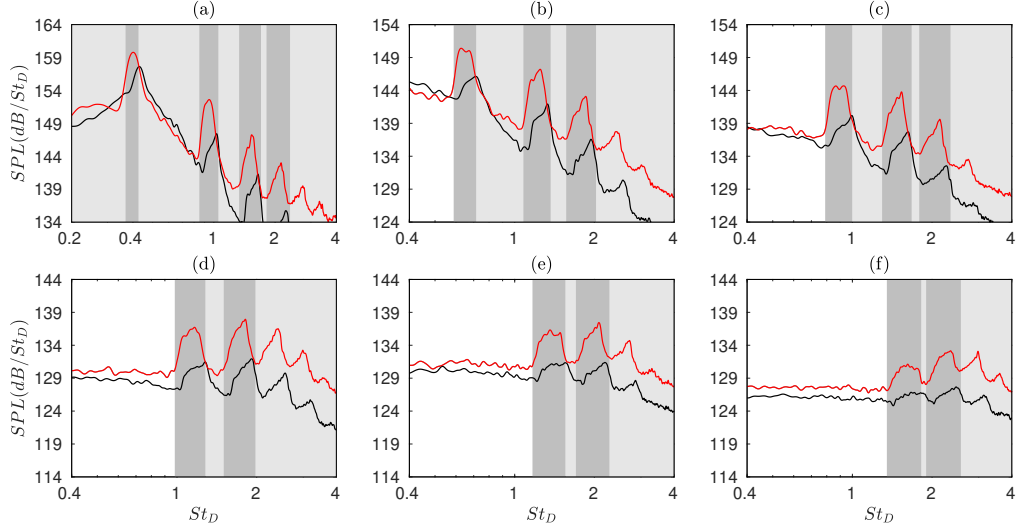


FIGURE 13. Sound pressure levels in the potential core of the jets at  $M = 0.90$  — with tripped boundary layers and — with untripped boundary layers with  $\delta_{BL} = 0.025r_0$  at (a)  $r = 0$  for  $n_\theta = 0$ , (b)  $r = 0.2r_0$  for  $n_\theta = 1$ , (c)  $r = 0.3r_0$  for  $n_\theta = 2$ , (d)  $r = 0.3r_0$  for  $n_\theta = 3$ , (e)  $r = 0.4r_0$  for  $n_\theta = 4$  and (f)  $r = 0.4r_0$  for  $n_\theta = 5$  as a function of  $St_D$ ; allowable ranges for the upstream-propagating guided jet waves (dark grey) with and (light grey) without downstream-propagating guided waves for (a)  $n_r = 1 - 4$ , (b-c)  $n_r = 1 - 3$  and (d-f)  $n_r = 1 - 2$ .

620 large acoustic peaks emerge in the spectra. They lie exclusively within the dark-grey  
 621 bands, indicating that they are closely linked to the presence of  $v_g^+$  guided jet waves.  
 622 These results are in line with the findings of Towne *et al.* (2017), who demonstrated  
 623 the possibility of resonant interactions between the former waves and  $v_g^-$  guided waves  
 624 in high subsonic jets between the frequencies of the stationary points  $S_{min}$  and  $S_{max}$   
 625 for  $n_\theta = 0$  and 1. In the present work, these interactions are found to be possible for  
 626 higher azimuthal modes. On the basis of the frequency-wavenumber spectra of figure 12,  
 627 two types of resonances can occur (Towne *et al.* 2017). The first one involves  $v_g^+$  and  $v_g^-$   
 628 duct-like waves located on both sides of points  $S_{min}$  and the second one happens between  
 629  $v_g^+$  duct-like and  $v_g^-$  free-stream waves around points  $S_{max}$ .

#### 630 4.2.2. Tones in the jet near-nozzle region

631 To identify which of the guided jet waves have a significant radial support outside of  
 632 the jets, a space-time Fourier transform has been applied to the pressure fluctuations at  
 633  $r = 1.1r_0$ , between  $z = 0$  and  $z = 0.7z_c$  as previously in the potential core, for  $n_\theta = 0$   
 634 to 8. The spectra for  $n_\theta = 0$  to 5 are represented in figures 14(a-f) as a function of  $k$   
 635 and  $St_D$  for  $k \leq 0$  only. Strong components of aerodynamic nature are observed near  
 636  $k = 0$  for low Strouhal numbers due to the proximity of the shear layer. In spite of this,  
 637 spots of high levels are found for all azimuthal modes in the vicinity of the dispersion  
 638 curves predicted for the guided jet waves using the vortex-sheet model. With respect to  
 639 the elongated bands obtained in the in-core spectra of figure 12, the spots do extend  
 640 along the curves but are restricted to very limited parts. Their levels are highest close  
 641 to the line  $k = -\omega/c_0$ , rapidly decrease farther from it and are negligible to the left of  
 642 the local maximum point. Therefore, the waves located between the limit points L and  
 643 the stationary points  $S_{max}$  of the dispersion curves, *i.e.* the so-called free-stream waves  
 644 in Towne *et al.* (2017), are detected just outside of the jets, whereas the other, duct-like,  
 645 waves are not. The frequency ranges of the free-stream waves also appear to be narrower

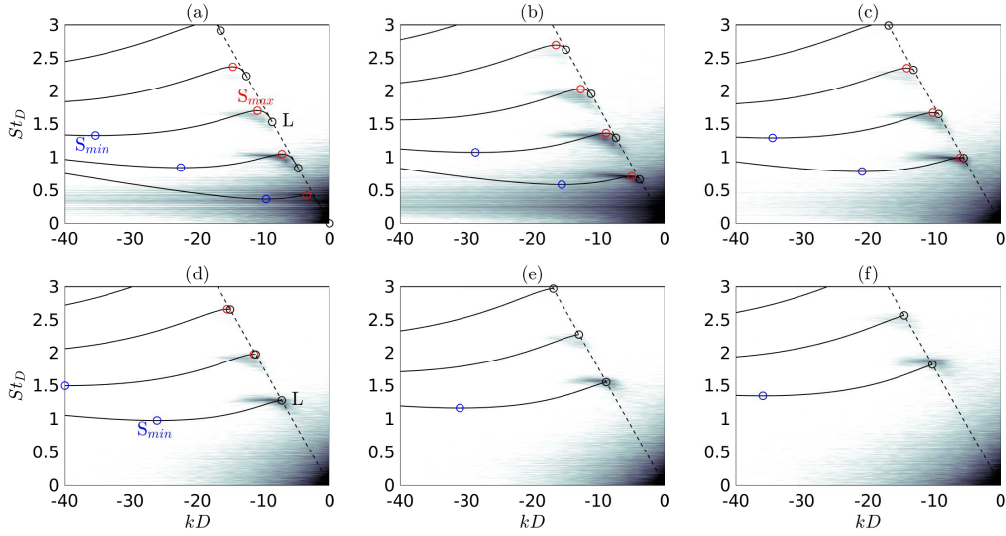


FIGURE 14. Frequency-wavenumber spectra of pressure fluctuations of the jet at  $M = 0.90$  with tripped boundary layers at  $r = 1.1r_0$  for (a)  $n_\theta = 0$ , (b)  $n_\theta = 1$ , (c)  $n_\theta = 2$ , (d)  $n_\theta = 3$ , (e)  $n_\theta = 4$  and (f)  $n_\theta = 5$  as a function of  $(kD, St_D)$ ; ——— dispersion curves of the guided jet waves, points  $\circ$  L,  $\circ$   $S_{max}$  and  $\circ$   $S_{min}$ ; - - -  $k = -\omega/c_0$ . The greyscale levels spread over 25 dB. Only  $k \leq 0$  is shown.

646 at a higher azimuthal mode. These results are in agreement with the variations of the  
 647 eigenfunction magnitude of the guided jet waves outside of the shear layer in figures 5(b)  
 648 and 6(c), and with the merging of points L and  $S_{max}$  as the azimuthal mode number  
 649 increases in figures 7 and 9 for  $M = 0.90$ . Given their negative group velocities, the  
 650 free-stream waves propagate in the upstream direction, and can be expected to mark the  
 651 pressure spectra in the near-nozzle region.

652 The spectra of pressure fluctuations computed near the nozzle exit at  $z = 0$  and  
 653  $r = 1.5r_0$  for the jets with tripped boundary layers and with untripped boundary layers  
 654 with  $\delta_{BL} = 0.025r_0$  are represented in figures 15(a,b) as a function of  $St_D$ . The contri-  
 655 butions of the first six azimuthal modes are also depicted. Tonal peaks emerge in the full  
 656 spectra as well as for the azimuthal modes. They are very similar to those found in the  
 657 near-nozzle spectra reported in Suzuki & Colonius (2006), Towne *et al.* (2017) and Brès  
 658 *et al.* (2018) for jets at the same Mach number as the present jets but at Reynolds num-  
 659 bers  $Re_D \simeq 10^6$ . They are stronger and narrower for the jet with untripped boundary  
 660 layers than for the other one. However, the peak frequencies are almost identical in the  
 661 two cases. For a given azimuthal mode, the first peak falls very near the dash-dotted line  
 662 indicating the Strouhal number of point  $S_{max}$ , or L in the absence of  $S_{max}$ , obtained on  
 663 the dispersion curves for the radial guided jet mode  $n_r = 1$  using the vortex-sheet model.  
 664 This supports that the peaks are due to the upstream-propagating waves highlighted in  
 665 figure 14. Regarding the full spectra, the first, second and third peaks at  $St_D \simeq 0.4, 0.6$   
 666 and 1 coincide with the first peaks of modes  $n_\theta = 0, 1$  and 2 in red, blue and green,  
 667 respectively. The fourth peak corresponds to the first peak of mode  $n_\theta = 3$  in magenta,  
 668 enhanced by the second peak of mode  $n_\theta = 1$  in blue. The higher peaks also consist of  
 669 combinations of peaks of different modes, for instance modes  $n_\theta = 2$  and 4 for the fifth  
 670 peak and modes  $n_\theta = 1, 3$  and 5 for the sixth peak. The complex structure of the peaks  
 671 can be explained by the great resemblance, and quasi superposition in some instances,  
 672 of the dispersion curves for different azimuthal modes, discussed in section 3.2 based on

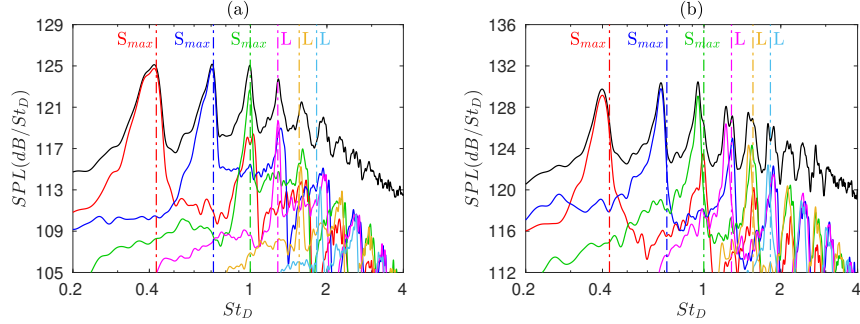


FIGURE 15. Sound pressure levels obtained at  $z = 0$  and  $r = 1.5r_0$  for the jets at  $M = 0.90$  (a) with tripped boundary layers and (b) with untripped boundary layers with  $\delta_{BL} = 0.025r_0$  as a function of  $St_D$ : — full spectra and  $n_\theta =$  — red — 0, — blue — 1, — green — 2, — magenta — 3, — orange — 4 and — cyan — 5; (dash-dotted lines) Strouhal numbers at points  $S_{max}$  or L on the dispersion curves for the guided jet modes ( $n_\theta, n_r = 1$ ) using the same colours for as for the solid lines.

673 figure 8. This issue was mentioned by Suzuki & Colonius (2006) who remarked that the  
 674 frequency of the first peak of mode  $n_\theta = 2$  is nearly the same as that of the second peak  
 675 of mode  $n_\theta = 0$  in their experimental spectra.

676 The spectra calculated at  $z = 0$  and  $r = 1.5r_0$  for the jets with tripped boundary  
 677 layers and untripped boundary layers with  $\delta_{BL} = 0.025r_0$  for modes  $n_\theta = 0$  to 5 are  
 678 plotted in figures 16(a-f) as a function of  $St_D$ . The allowable ranges for the upstream-  
 679 propagating guided jet waves according to the vortex-sheet model are represented in  
 680 grey as in figure 13. The frequency ranges of the free-stream waves between points L  
 681 and  $S_{max}$  on the dispersion curves are also indicated by oblique hatching, when possible.  
 682 Compared to the peaks obtained in the potential core in figure 13, the near-nozzle peaks  
 683 are narrower and exhibit a sharper decrease on the right side of the stationary points  
 684  $S_{max}$ . Moreover, instead of fully filling the dark-grey bands where  $v_g^-$  and  $v_g^+$  guided jet  
 685 waves can both exist, they appear limited to the hatched bands. This trend is clearly  
 686 observed for modes  $n_\theta \geq 1$  in figures 16(b-f), for which points L and  $S_{max}$  are very  
 687 close or superimposed on the dispersion curves, which gives rise to very tonal peaks. The  
 688 present results are in agreement with the eigenfunctions of figure 5(b) and the transfer  
 689 functions of 6(c). They show that the near-nozzle peaks are mainly related to the free-  
 690 stream guided jet waves. In particular, it appears that among the resonant interactions  
 691 possibly occurring between  $v_g^-$  and  $v_g^+$  guided waves in the jet potential core, only those  
 692 involving  $v_g^-$  free-stream waves can contribute significantly to the near-nozzle pressure  
 693 field. Given that the peak levels are higher at point  $S_{max}$  than at point L when the  
 694 two points are sufficiently distinct from each other, as for mode ( $n_\theta = 0, n_r = 2$ ) in  
 695 figure 16(a) and for mode ( $n_\theta = 1, n_r = 1$ ) in figure 16(b), this may be especially true  
 696 for the waves resonating around the stationary point  $S_{max}$ .

697 The Strouhal numbers of the peaks obtained for the six jets at  $M = 0.90$  in the spectra  
 698 at  $z = 0$  and  $r = 1.5r_0$  for  $n_\theta = 0, 1$  and 2 are represented in figures 17(a-c) as a  
 699 function of boundary-layer thickness  $\delta_{BL}$  at the nozzle-pipe inlet. The allowable bands  
 700 predicted for the upstream-propagating guided jet waves using the vortex-sheet model,  
 701 as well as the points L,  $S_{max}$  and  $S_{min}$  on the dispersion curves given by the model,  
 702 are also displayed. Over the wide range of boundary-layer thicknesses considered, the peak  
 703 Strouhal numbers do not vary appreciably despite the ratio of 16 between the largest and  
 704 the smallest values of  $\delta_{BL}$ . This is in line with the experimental results of Zaman & Fagan  
 705 (2019) for two jets with boundary-layer thicknesses differing by a factor of 3. In the same

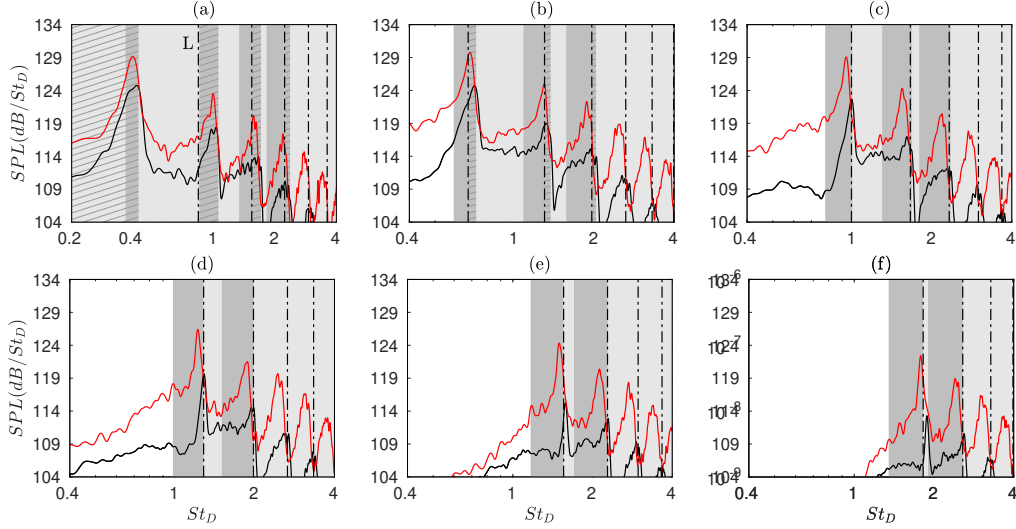


FIGURE 16. Sound pressure levels obtained at  $z = 0$  and  $r = 1.5r_0$  for the jets at  $M = 0.90$  — with tripped boundary layers and — with untripped boundary layers with  $\delta_{BL} = 0.025r_0$  for (a)  $n_\theta = 0$ , (b)  $n_\theta = 1$ , (c)  $n_\theta = 2$ , (d)  $n_\theta = 3$ , (e)  $n_\theta = 4$  and (f)  $n_\theta = 5$  as a function of  $St_D$ ; allowable ranges for the upstream-propagating guided jet waves (dark grey) with and (light grey) without downstream-propagating guided waves and (hatched) between the  $St_D$  at points L and  $S_{max}$  for (a)  $n_r = 1 - 4$ , (b-c)  $n_r = 1 - 3$  and (d-f)  $n_r = 1 - 2$ ,  $- \cdot - \cdot -$   $St_D$  at points L on the dispersion curves.

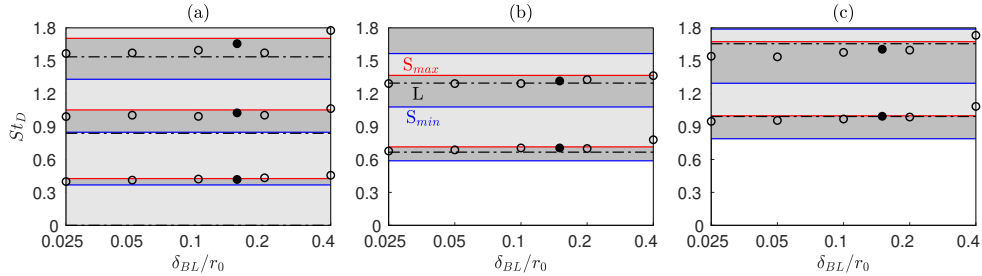


FIGURE 17. Peak Strouhal numbers in the spectra of pressure fluctuations at  $z = 0$  and  $r = 1.5r_0$  for the jets at  $M = 0.90$  with  $\bullet$  tripped and  $\circ$  untripped boundary layers for (a)  $n_\theta = 0$ , (b)  $n_\theta = 1$  and (c)  $n_\theta = 2$  as a function of  $\delta_{BL}/r_0$ ; allowable ranges for the upstream-propagating guided jet waves (dark grey) with and (light grey) without downstream-propagating guided waves,  $St_D$  at points  $- \cdot - \cdot -$  L,  $-$   $S_{max}$  and  $-$   $S_{min}$  on the dispersion curves.

706 way, the peak frequencies are very similar for tripped and untripped boundary layers,  
 707 as was the case for the two initially laminar and turbulent jets computed by Brès *et al.*  
 708 (2018). For all jets, even for the one with  $\delta_{BL} = 0.4r_0$  for which the vortex-sheet model  
 709 is a very rough approximation, the peaks are located between the points L and  $S_{max}$ ,  
 710 or near the point L when  $S_{max}$  does not exist. These results provide further evidence  
 711 about the links between the near-nozzle peaks and the free-stream guided jet waves. In  
 712 the zones between points L and  $S_{max}$ , the peaks are closer to the second point than to  
 713 the first one for the first two radial modes for  $n_\theta = 0$  in figure 17(a). This again supports  
 714 the possible contributions of waves resonating around the stationary point  $S_{max}$  to the  
 715 near-nozzle pressure fields.

716 The sensitivity of the near-nozzle acoustic peaks to the jet initial conditions is exam-  
 717 ined by comparing some properties of the near-nozzle peaks obtained for the six jets at

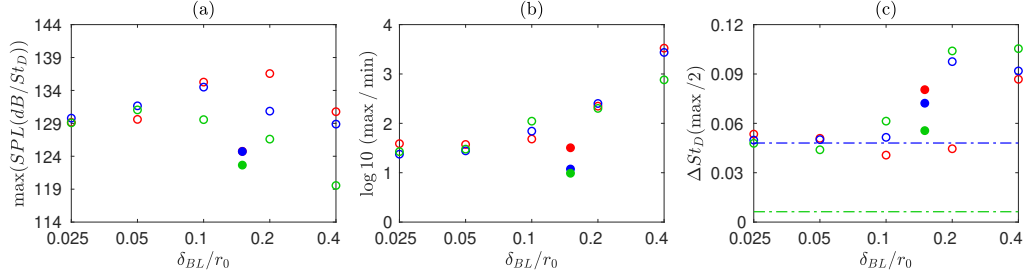


FIGURE 18. Near-nozzle peaks for the jets at  $M = 0.90$  with (bullets) tripped and (circles) untripped boundary layers for the radial modes  $n_r = 1$  of the guided jet waves for (red)  $n_\theta = 0$ , (blue)  $n_\theta = 1$  and (green)  $n_\theta = 2$ : (a) peak levels, (b) ratio between the peak levels and the minimum values for higher  $St_D$  and (c) peak widths as a function of  $\delta_{BL}/r_0$ ; (dash-dotted lines)  $\Delta St_D$  between points  $S_{max}$  and L using the above colours for  $n_\theta$ .

718  $M = 0.90$ . Only the peaks associated with the guided jet modes  $n_r = 1$  for  $n_\theta = 0, 1$  and  
 719 2 are considered. The peak intensities are depicted in figure 18(a). They are lower for the  
 720 jet with tripped boundary layers than for the jets with untripped ones, and for the latter  
 721 jets, they grow with the boundary-layer thickness except for  $\delta_{BL} \geq 0.2r_0$ . Thus, overall,  
 722 the more noise generated by the jets (Bogey & Baily 2010; Bogey 2018), the higher the  
 723 levels of the near-nozzle acoustic peaks.

724 In order to quantify the degree of emergence of the peaks, the ratios between the  
 725 peak levels and the first minimum values reached for a higher frequency are plotted  
 726 in figure 18(b). As for the peak intensities, they are minimum for the jet with tripped  
 727 boundary layers and are higher as the boundary layer is thicker for the initially fully  
 728 laminar jets. Therefore, the near-nozzle peaks are more prominent for the jets with mixing  
 729 layers containing stronger large-scale coherent structures, and inversely weaker fine-scale  
 730 turbulent structures, yielding a weaker broadband noise in the upstream direction.

731 Finally, the peak widths at half of maximum are given in figure 18(c). They increase  
 732 with the boundary-layer thickness. This trend can be attributed to the effects of the  
 733 shear-layer thickness on the dispersion curves of the guided jet waves near the line  $k =$   
 734  $-\omega/c_0$  (Tam & Ahuja 1990; Bogey & Gojon 2017). Indeed, as mentioned in section 4.2.1  
 735 and illustrated by the frequency-wavenumber spectra of figure 12, the free-stream guided  
 736 jet waves are obtained over wider frequency ranges for a thicker shear layer. In that  
 737 case, the band-pass filtering of the upstream-propagating waves by the guided jet modes  
 738 has a larger width. For comparison, the widths estimated as the frequency differences  
 739 between points L and  $S_{max}$  on the dispersion curves using the vortex-sheet model for  
 740 modes ( $n_\theta = 0, n_r = 1$ ) and ( $n_\theta = 1, n_r = 1$ ) are shown in figure 18(c). A fairly good  
 741 agreement is found with the peak widths for  $n_\theta = 1$ . For  $n_\theta = 2$ , the peak width is  
 742 underestimated by the model, which is expected due to the discrepancies between the  
 743 numerical and theoretical dispersion curves near  $k = -\omega/c_0$  in figure 12(c).

#### 744 4.2.3. Tones in the jet acoustic far field

745 The LES near-field fluctuations obtained for the jet at  $M = 0.90$  with tripped boundary  
 746 layers have been propagated to the far field using an in-house OpenMP-based solver of  
 747 the isentropic linearized Euler equations (ILEE) in cylindrical coordinates based on the  
 748 same numerical methods as the LES. Two calculations are performed as in previous  
 749 studies (Bogey & Sabatini 2019; Bogey 2021). They are carried out from the velocity  
 750 and pressure fluctuations recorded during time  $T = 3,000r_0/u_j$  at  $r = 15r_0$  and at  
 751  $z = -1.5r_0$  and  $z = L_z = 40r_0$  at a sampling frequency corresponding to a Strouhal  
 752 number of  $St_D = 12.8$ . They allow us to compute the pressure waves radiated at a

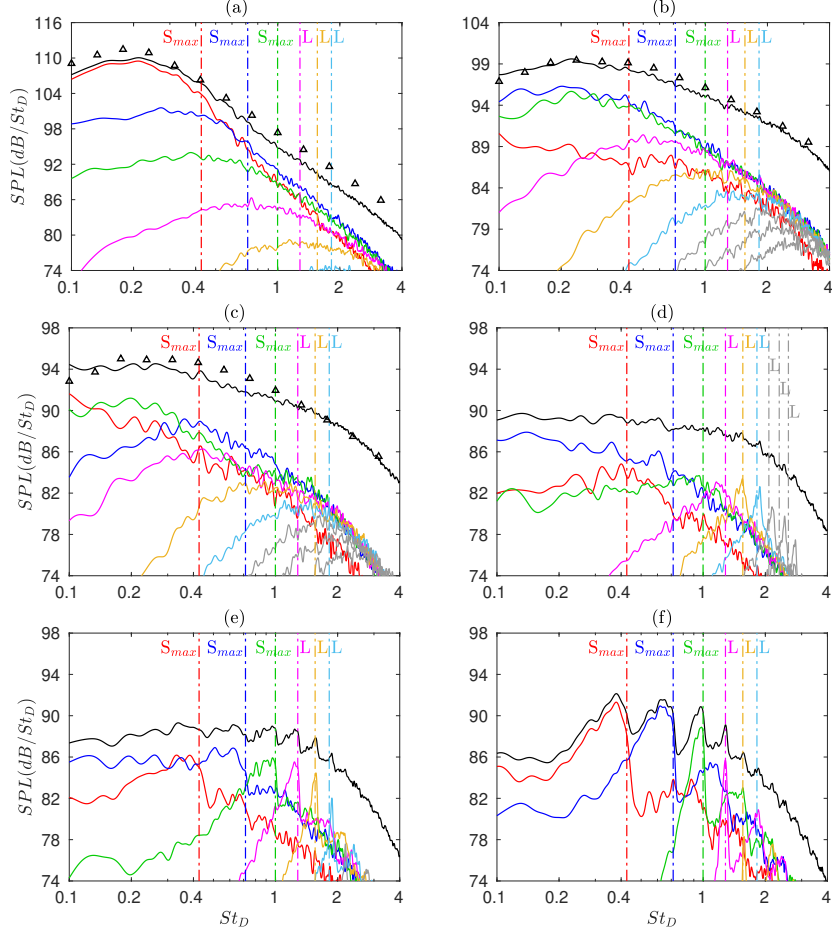


FIGURE 19. Sound pressure levels obtained for the jet at  $M = 0.90$  with tripped boundary layers at  $150r_0$  from the nozzle exit for (a)  $\phi = 30^\circ$ , (b)  $\phi = 60^\circ$ , (c)  $\phi = 90^\circ$ , (d)  $\phi = 135^\circ$ , (e)  $\phi = 150^\circ$  and (f)  $\phi = 165^\circ$  as a function of  $St_D$ : — full spectra and  $n_\theta =$  — 0, — 1, — 2, — 3, — 4, — 5, — 6, 7 and 8; (dash-dotted lines) Strouhal numbers at points  $S_{max}$  or L on the dispersion curves for the guided jet modes ( $n_\theta, n_r = 1$ ) using the same colours as for the solid lines;  $\Delta$  measurements of Bridges & Brown (2005) for an isothermal jet at  $M = 0.9$  and  $Re_D = 10^6$ .

753 distance of  $150r_0$  from the nozzle exit, where far-field acoustic conditions are expected  
 754 to apply (Ahuja *et al.* 1987; Viswanathan 2010), between the angles  $\phi = 15^\circ$  and  $165^\circ$   
 755 relative to the jet direction. Grids containing up to  $1.6 \times 10^9$  points with a uniform mesh  
 756 spacing of  $0.075r_0$  in the axial and radial directions and  $N_\theta = 256$  points in the azimuth  
 757 are used. This mesh spacing, leading to  $St_D = 5.9$  for an acoustic wave discretized by  
 758 five points per wavelength, is identical to that in the LES near pressure field. The sound  
 759 pressure spectra thus determined at six angles  $\phi$  between  $30^\circ$  and  $165^\circ$  are represented  
 760 in figures 19(a-f) as a function of  $St_D$ . The contributions of modes  $n_\theta = 0$  to 8 are also  
 761 shown. Because of the different shapes of the spectra (Mollo-Christensen *et al.* 1964; Tam  
 762 1998), the level axis ranges from 74 up to 116 dB/ $St_D$  in figure 19(a) for  $\phi = 30^\circ$ , but  
 763 only up to 104 dB/ $St_D$  in figure 19(b) for  $\phi = 60^\circ$  and to 98 dB/ $St_D$  in figures 19(c-f)  
 764 for  $\phi \geq 90^\circ$ .

765 In the downstream and sideline directions, in figures 19(a-c), the pressure spectra ex-



hibit the characteristics typically found in the far field of subsonic jets. The axisymmetric mode is dominant in the downstream direction and is soon overwhelmed by modes  $n_\theta = 1$  and 2 as the radiation angle increases (Juvé *et al.* 1979; Cavalieri *et al.* 2012; Brès *et al.* 2018). More importantly given the focus of the present work, there are peaks neither in the spectra for the full pressure signals, nor in those for the different azimuthal modes. This is obvious in figure 19(c) for the angle  $\phi = 90^\circ$ , for instance. In particular, there is no trace of the undulations noticed around  $St_D = 1$  in the spectra of Zaman & Fagan (2019) at an angle of  $60^\circ$  for jets at Mach numbers close to 1. This supports the hypothesis of the authors that these undulations are due to unwanted reflections by some uncovered surfaces in their experiments.

In the upstream direction, as suggested by the experiments of Jaunet *et al.* (2016), peaks appear in the full spectra at high radiation angles. They are barely detectable at  $\phi = 135^\circ$  in figure 19(d), clearly visible at  $\phi = 150^\circ$  in figure 19(e) and predominant at  $\phi = 165^\circ$  in figure 19(f). Their frequencies and azimuthal structures at the latter angle are very close, if not identical, to those of the near-nozzle peaks in figure 15(a). In summary, the  $i^{th}$  peak in the full spectrum corresponds to the first peak of the azimuthal mode  $n_\theta = i - 1$ . The latter peak is located near the Strouhal number of the point  $S_{max}$ , or L when  $S_{max}$  is lacking, obtained on the dispersion curve for the guided jet mode ( $n_\theta = i - 1$ ,  $n_r = 1$ ) using the vortex-sheet model. Therefore, the free-stream guided jet waves contribute significantly to the far-field noise for very large radiation angles. It can be noted that the prominence and tonal shape of the peaks vary with the angle. Thus, the most apparent peaks are the peaks for modes  $n_\theta = 4 - 8$  at  $\phi = 135^\circ$ , for  $n_\theta = 2 - 5$  at  $\phi = 150^\circ$  and for  $n_\theta = 0 - 3$  at  $\phi = 165^\circ$ . Finally, the emergence of peaks in the spectra results in stronger noise components at  $\phi = 165^\circ$  than at  $150^\circ$ . This trend is similar to that observed for supersonic jets generating screech tones in the upstream direction.

To better describe the noise variations with the radiation angle, the overall sound pressure levels computed at  $150r_0$  from the nozzle exit are plotted in figure 20 as a function of  $\phi$ . They decrease monotonically with the angle between  $\phi = 25^\circ$  and  $150^\circ$ , in agreement with the experimental data of the literature (Bridges & Brown 2005; Bogey *et al.* 2007), and then are nearly constant between  $\phi = 150^\circ$  and  $165^\circ$ . The effects of the emergence of peaks in the spectra for large radiation angles are more visible on the sound levels associated with the different azimuthal modes. For  $\phi \leq 135^\circ$ , in line with previous studies (Cavalieri *et al.* 2012; Brès *et al.* 2018), the stronger modes are the axisymmetric mode for  $\phi \leq 45^\circ$  and modes  $n_\theta = 1$  and 2 for  $\phi \geq 45^\circ$ . In addition, for all modes, the levels decrease between  $\phi = 60^\circ$  and  $135^\circ$ . For  $\phi \geq 135^\circ$ , more surprisingly, the contributions of modes  $n_\theta = 0$  and 1 sharply grow with the radiation angle. As a consequence, at  $\phi = 165^\circ$ , the first helical mode predominates, closely followed by the axisymmetric mode. For larger angles, given the tendencies obtained between  $\phi = 150^\circ$  and  $165^\circ$ , one can expect a further increase of the sound levels for modes  $n_\theta = 0$  and 1 and the predominance of mode  $n_\theta = 0$  near  $\phi = 180^\circ$ . This could be checked in future studies.

#### 4.3. Acoustic tones for the jets at Mach numbers between 0.50 and 2

The persistence and properties of acoustic tones in the jet potential core and in the near-nozzle region are now investigated for the jets at Mach numbers varying from  $M = 0.50$  to 2 in figure 1. As previously, greater attention is paid to the jets with tripped boundary layers.

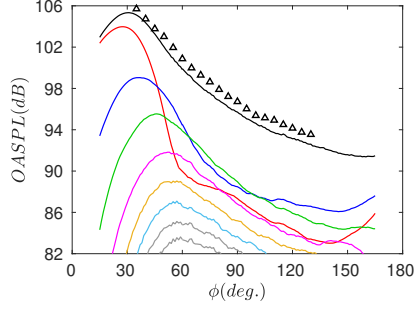


FIGURE 20. Overall sound pressure levels obtained for the jet at  $M = 0.90$  with tripped boundary layers at  $150r_0$  from the nozzle exit as a function of the angle  $\phi$ : — full spectra and  $n_\theta =$  — 0, — 1, — 2, — 3, — 4, — 5, — 6 and 7;  $\Delta$  measurements of Bridges & Brown (2005) for an isothermal jet at  $M = 0.9$  and  $Re_D = 10^6$ .

#### 812 4.3.1. Tones in the jet potential core

813 A space–time Fourier transform has been applied to the pressure fluctuations inside the  
 814 jet potential core for modes  $n_\theta = 0$  to 8. The fluctuations are located between  $z = 0$  and  
 815  $0.7z_c$  at radial positions depending on the azimuthal mode as in section 4.2.1. The spectra  
 816 for the jets with tripped boundary layers at  $M = 0.75$ , 1.10 and 2 for  $n_\theta = 0$  to 2 are  
 817 represented in figures 21(a–i) as a function of wavenumber and Strouhal number. As for  
 818 the tripped jet at  $M = 0.90$  in figure 12, strong aerodynamic components are found along  
 819 the line  $k = \omega/(0.75u_j)$ . Bands of high energy are also observed near the dispersion curves  
 820 of the guided jet modes predicted by the vortex-sheet model, for negative wavenumbers  
 821 but also for positive wavenumbers in the supersonic cases, as expected. Similar results  
 822 were obtained by Towne *et al.* (2019) for jets at  $M = 0.70$ , 0.80, 0.9 and 1.50 for  $n_\theta = 0$ .  
 823 The agreement between the bands and the dispersion curves is good in figures 21(a–c)  
 824 for  $M = 0.75$ , fair in figures 21(d–f) for  $M = 1.10$  and rather poor in figures 21(g–i)  
 825 for  $M = 2$ . This may be due to the fact that the dispersion curves are determined  
 826 from the linearized equation of motion of a compressible flow by assuming disturbances  
 827 of small amplitudes, which is not necessarily true for supersonic jet velocities. For all  
 828 Mach numbers, the energy is fairly well distributed along the bands. This is the case in  
 829 particular for  $M = 0.75$ , in figures 21(a–c), where the levels are significant from the limit  
 830 point L of the dispersion curves on  $k = -\omega/c_0$  to the inflexion point I and far beyond  
 831 that point. For  $M = 2$ , in figures 21(g–i), the levels are however much stronger for positive  
 832 wavenumbers than for negative ones. Focusing on the region near the line  $k = -\omega/c_0$ , the  
 833 bands all exhibit a portion with negative slopes between the limit point on the line and  
 834 the local maximum point. This indicates the presence of upstream-propagating guided  
 835 jet waves for all modes, including those for which such waves should not exist according  
 836 to the vortex-sheet model. This is the case, for instance, in figure 21(f) for the mode  
 837 ( $n_\theta = 2$ ,  $n_r = 1$ ) at  $M = 1.10$ . This discrepancy, also noticed for the tripped jet at  
 838  $M = 0.90$  for  $n_\theta \geq 3$  in previous section, is most likely due to the infinitely thin shear  
 839 layer in the vortex-sheet model.

840 The pressure spectra calculated in the potential core of the six jets with tripped bound-  
 841 ary layers for  $n_\theta = 0$  and 1, by averaging between  $z = 0$  and  $0.7z_c$  at  $r = 0$  and  $r = 0.2r_0$ ,  
 842 respectively, are depicted in figures 22(a,b) as a function of  $St_D$ . For the two modes and  
 843 for modes  $n_\theta \geq 2$ , not shown for brevity, the spectra are smooth for  $M = 0.60$  but  
 844 peaks can be seen for higher Mach numbers. The peaks clearly appear for  $M = 0.75$  at  
 845  $St_D \geq 1.5$ , dramatically emerge for  $M = 0.90$  and are hardly visible for supersonic Mach  
 846 numbers. This trend is in good agreement with the statement of Towne *et al.* (2017)

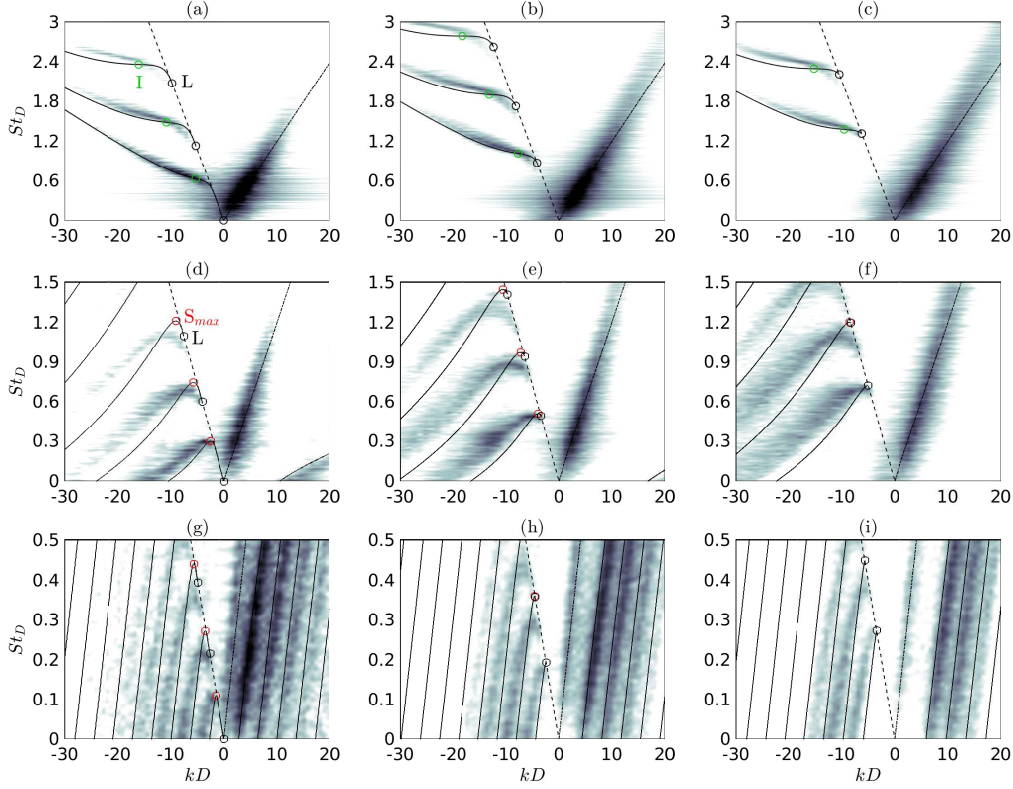


FIGURE 21. Frequency-wavenumber spectra of pressure fluctuations in the potential cores of the jets with tripped boundary layers at (a-c)  $M = 0.75$ , (d-f)  $M = 1.10$  and (g-i)  $M = 2$ , at (a,d,g)  $r = 0$  for  $n_\theta = 0$ , (b,e,h)  $r = 0.2r_0$  for  $n_\theta = 1$  and (c,f,i)  $r = 0.3r_0$  for  $n_\theta = 2$ , as a function of  $(kD, St_D)$ ; — dispersion curves of the guided jet waves, points  $\circ$  L,  $\circ$   $S_{max}$ ,  $\circ$   $S_{min}$  and  $\circ$  I; - - -  $k = -\omega/c_0$ , - · - ·  $k = \omega/(0.75u_j)$ . The greyscale levels spread over 25 dB.

847 that the acoustic tones in the jet potential core, attributed to resonating trapped waves,  
 848 should reach their strongest prominence between  $M \simeq 0.82$  and 1. For  $M = 0.90$ , as illus-  
 849 trated in figure 13, the peaks lie within the frequency bands of the  $v_g^+$  duct-like waves.  
 850 For  $M = 0.75$ , the peaks are close to the Strouhal numbers of the inflexion points I on  
 851 the dispersion curves. For supersonic Mach numbers, they are near the frequencies of  
 852 the stationary points  $S_{max}$ , around which interactions are possible between  $v_g^+$  and  $v_g^-$   
 853 guided jet waves. Finally, it can be remarked that in the frequency-wavenumber spec-  
 854 tra, the dominant components are those associated with aerodynamic fluctuations for  
 855  $M = 0.75$  in figures 21(a-c) and with the guided jet waves with positive wavenumbers for  
 856  $M = 2$  in figures 21(g-i). The strengthening of these two components, in comparison with  
 857 those related to the guided jet waves with negative wavenumbers, may be one reason for  
 858 the weakening of the peaks in the spectra as the Mach number deviates from  $M = 0.90$ .

#### 859 4.3.2. Tones in the jet near-nozzle region

860 To get information on the pressure waves just outside of the jet flow, a space-time  
 861 Fourier transform has been applied to the pressure fluctuations at  $r = 1.1r_0$  between  
 862  $z = 0$  and  $0.7z_c$  for  $n_\theta = 0$  to 8, as in section 4.2.2. The spectra for the jets with tripped  
 863 boundary layers at  $M = 0.75$ , 1.10 and 2 for  $n_\theta = 0$  to 2 are presented in figures 23(a-i)  
 864 as a function of  $k$  and  $St_D$  for  $k \leq 0$ . In the region with  $k \geq 0$ , not shown for clarity, the

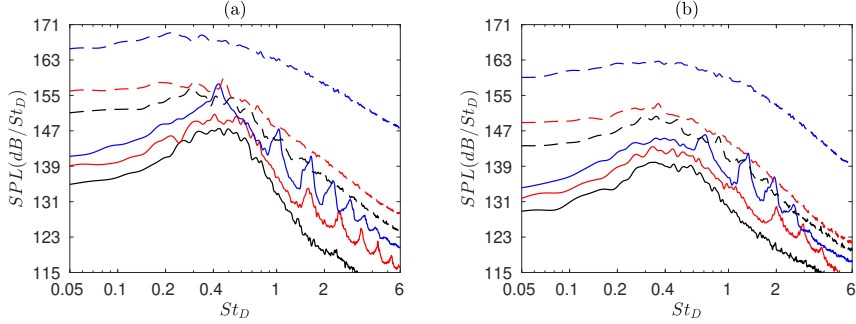


FIGURE 22. Sound pressure levels obtained at in the potential core of the jets with tripped boundary layers at (a)  $r = 0$  for  $n_\theta = 0$  and (b)  $r = 0.2r_0$  for  $n_\theta = 1$  as a function of  $St_D$  for  $M =$  ——— 0.60, ——— 0.75, ——— 0.90, - - - 1.10, - - - 1.30 and - - - 2.

865 spectra are dominated by very strong aerodynamic components as the in-core spectra  
 866 of figure 21. However, contrary to the latter, they do not reveal spots of notable energy  
 867 near the dispersion curves of the guided jet modes provided by the vortex-sheet model for  
 868 supersonic Mach numbers. For  $k \leq 0$ , despite the wide dark patch due to aerodynamic  
 869 disturbances at low Strouhal numbers, high levels are found close to the dispersion curves.  
 870 As for the jet at  $M = 0.90$  in figure 14, the levels are significant only in the vicinity of  
 871  $k = -\omega/c_0$ . For  $M = 0.75$ , in figures 23(a-c), they are negligible on the left side of the  
 872 inflexion point I of the dispersion curves. For  $M = 1.10$ , in figures 23(d-f), they quickly  
 873 decrease to the left of the local maximum point, corresponding to the stationary points  
 874  $S_{max}$  of the dispersion curves. For  $M = 2$ , in figures 23(g-i), a similar trend is observed  
 875 for the first radial modes. For modes  $n_r \geq 2$ , however, the decay on the left side of the  
 876 local maximum point is less rapid. This may be caused by the fact that for the jet at  
 877  $M = 2$ , represented in figure 10(e) between  $z = 0$  and  $12r_0$ , some of the points considered  
 878 from  $z = 0$  down to  $z = 0.7z_c = 16.4r_0$  at  $r = 1.1r_0$  to compute the spectra lie inside the  
 879 jet flow. Therefore, only the guided jet waves located approximately between the limit  
 880 points L on  $k = -\omega/c_0$  and points I in the subsonic case, and between points L and  $S_{max}$   
 881 in the supersonic cases, clearly extends out of the jets. These waves, with a negative  
 882 group velocity, are able to propagate in the upstream direction up to the near-nozzle  
 883 region. These results are consistent with the eigenfunction magnitudes obtained for the  
 884 guided jet waves at  $r = 1.5r_0$  using the vortex-sheet model, displayed as a function of  
 885 the Strouhal number in figures 6(b,d,e) for  $M = 0.75, 1.10$  and  $2$ .

886 The spectra of pressure fluctuations obtained at  $z = 0$  and  $r = 1.5r_0$  for the jets with  
 887 untripped boundary layers with  $\delta_{BL} = 0.2r_0$  are presented in figure 24(a) as a function of  
 888 the Mach number, using a logarithmic scale. They are normalized by their respective peak  
 889 values, yielding a maximum value of 1 for each jet velocity. The contributions of modes  
 890  $n_\theta = 0$  and 1 to the spectra are highlighted in figures 24(b,c). A zoom between  $M = 0.75$   
 891 and  $0.85$  is provided in appendix A, based on the results for the jets at Mach numbers  
 892 increasing in increments of  $\Delta M = 0.01$  in figure 1. Well-organized peaks are visible in  
 893 the spectrograms. For  $M \leq 0.65$ , the dominant peaks are at  $St_D \simeq 0.70$  for both  $n_\theta = 0$   
 894 and 1. As discussed in appendix B, they happen at the vortex-pairing frequency and may  
 895 result from the establishment of a feedback loop between the Kelvin-Helmholtz instability  
 896 waves and the upstream-propagating sound waves generated by the first stage of vortex  
 897 pairings in the initially laminar shear layers. For higher Mach numbers, the peaks form  
 898 continuous bands, with the first two bands associated with the axisymmetric and the  
 899 first helical modes, respectively. The bands have central Strouhal numbers decreasing

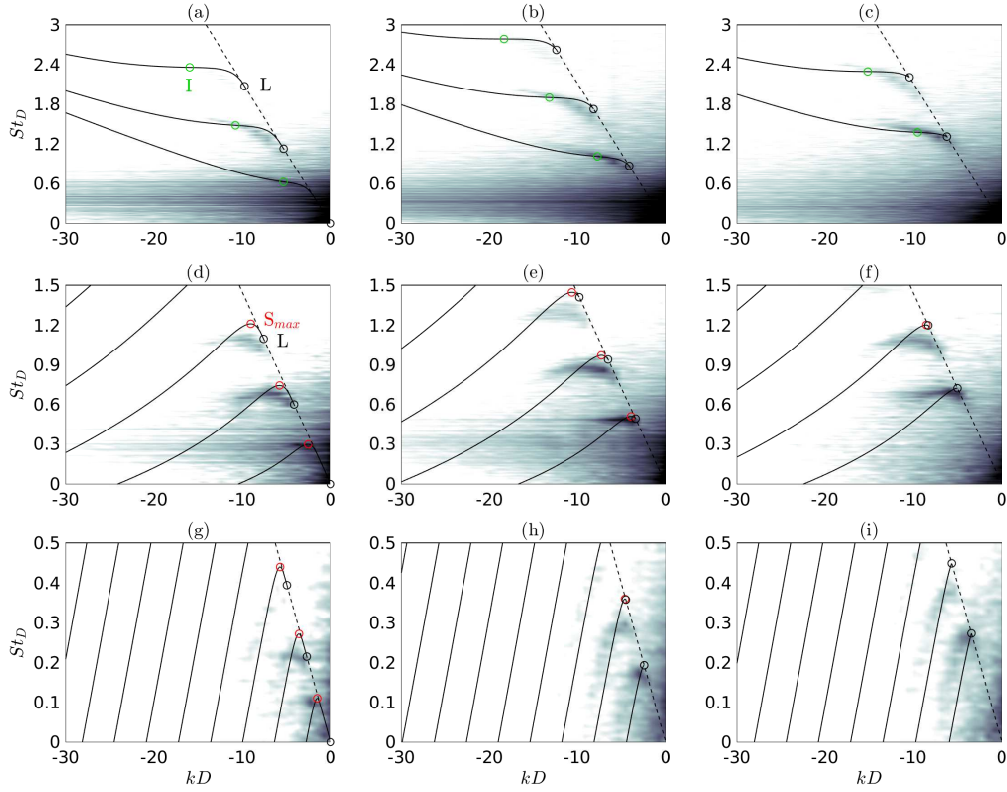


FIGURE 23. Frequency-wavenumber spectra of pressure fluctuations of the jets with tripped boundary layers at (a-c)  $M = 0.75$ , (d-f)  $M = 1.10$  and (g-i)  $M = 2$ , at  $r = 1.1r_0$  for (a,d,g)  $n_\theta = 0$ , (b,e,h)  $n_\theta = 1$  and (c,f,i)  $n_\theta = 2$ , as a function of  $(kD, St_D)$ ; — dispersion curves of the guided jet waves, points  $\circ$  L,  $\circ$   $S_{max}$ ,  $\circ$   $S_{min}$  and  $\circ$  I; - - -  $k = -\omega/c_0$ . The greyscale levels spread over 25 dB. Only  $k \leq 0$  is shown.

900 with the Mach number, and emerge hardly from the background noise below  $M = 0.75$   
 901 but quite distinctly above. They are similar to those measured near the nozzle lips of  
 902 free jets by Jaunet *et al.* (2016) and Zaman & Fagan (2019) for  $0.6 \lesssim M \leq 1$ , and  
 903 resemble the allowable frequency bands obtained for the upstream-propagating guided  
 904 jet waves in figure 7. They persist for supersonic Mach numbers up to  $M = 2$ , without any  
 905 discontinuity or energy jump from one band to another. In particular, they do not turn  
 906 into screech-tone bands, as it was the case for the non-ideally expanded jets of Zaman &  
 907 Fagan (2019).

908 The spectra of the pressure fluctuations at  $z = 0$  and  $r = 1.5r_0$  for the six jets with  
 909 tripped boundary layers at  $M = 0.60, 0.75, 0.90, 1.10, 1.30$  and  $2$  are represented in  
 910 figures 25(a-f) as a function of  $St_D$ , along with the contributions of modes  $n_\theta = 0$  to  $8$ .  
 911 The Strouhal numbers of specific points on the dispersion curves predicted by the vortex-  
 912 sheet model for the guided jet modes ( $n_\theta, n_r = 1$ ) are also indicated by dash-dotted lines.  
 913 For a given mode, the reported point is the inflexion point I when the dispersion curve  
 914 has no portion with a positive slope. Otherwise, the point is the stationary point  $S_{max}$ ,  
 915 or the limit point L on the line  $k = -\omega/c_0$  when there is no local maximum on the curve.  
 916 The first case happens for all modes for  $M = 0.60$  in figure 25(a), and the second one for  
 917 all modes for  $M \geq 0.90$  in figures 25(c-f). For  $M = 0.75$  in figure 25(b), both cases occur  
 918 and the dash-dotted lines are associated with points I for  $n_\theta = 0-3$ ,  $S_{max}$  for  $n_\theta = 4$  and

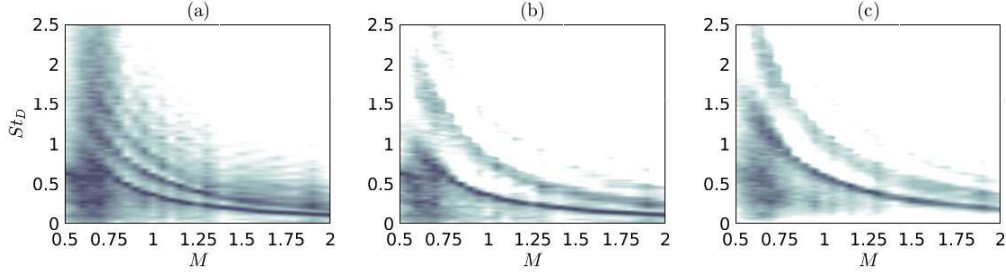


FIGURE 24. Power spectral densities of (a) pressure fluctuations at  $z = 0$  and  $r = 1.5r_0$ , normalized by their peak values, and contributions of modes (b)  $n_\theta = 0$  and (c)  $n_\theta = 1$  for the jets with untripped boundary layers with  $\delta_{BL} = 0.2r_0$  as a function of  $(M, St_D)$ . The greyscale ranges logarithmically from  $10^{-2}$  to 10.

919 L for  $n_\theta = 5 - 8$ . According to figure 6, these lines reveal the cut-off Strouhal numbers of  
 920 the band-pass filtering of the sound waves propagating in the upstream direction outside  
 921 of the jets by the guided jet modes.

922 In figure 25, peaks emerge in the spectra for the full pressure signals, strongly for  
 923  $M \geq 0.75$  but hardly for  $M = 0.60$ . This is in agreement with the experimental results of  
 924 Suzuki & Colonius (2006) and Zaman & Fagan (2019). They correspond to peaks in the  
 925 spectra for the modes  $n_\theta = 0$  to  $n_\theta^{max}$ , with  $n_\theta^{max}$  depending on the Mach number and  
 926 being equal to 8 for  $M = 0.75$  and to 1 for  $M = 2$ , for instance. For a given azimuthal  
 927 mode, the first peak falls very near the associated dash-dotted line. The second peak  
 928 is much weaker than the first one for subsonic Mach numbers in figures 25(a-c), but is  
 929 significant for supersonic ones for  $n_\theta = 0$  and 1 in figures 25(d-f). Coming back to the first  
 930 peak for each  $n_\theta$ , it is tonal and well above the background noise when the dash-dotted  
 931 line is plotted for a point  $S_{max}$  or L. For a point I, in contrast, the peak can be more or  
 932 less broadband and prominent. For  $M = 0.75$ , in figure 25(b), the peak is very narrow  
 933 for mode  $n_\theta = 3$  in magenta, and broadens for a lower azimuthal mode. The peaks are  
 934 still larger and less pronounced for  $M = 0.60$  in figure 25(a) than for  $M = 0.75$ . These  
 935 results demonstrate the close links between the near-nozzle peaks and the upstream-  
 936 propagating guided jet modes over the wide range of Mach numbers considered in this  
 937 study. Moreover, they suggest that the tonal character of the peaks is related to the  
 938 shape of the transfer function of the band-pass filtering associated with the latter modes  
 939 outside of the jet, illustrated in figure 6. Indeed, the peaks are tonal for a sharp cut-off,  
 940 but are gradually broader for a smoother one. This can explain the different shapes of  
 941 the peaks for Mach numbers above and below  $M = 0.75$ .

942 As was done in figures 16(a-c) for jets at  $M = 0.90$ , the pressure spectra obtained  
 943 at  $z = 0$  and  $r = 1.5r_0$  for  $n_\theta = 0, 1$  and 2 for the jets at  $M = 0.75$  with tripped  
 944 boundary layers and with untripped boundary layers with  $\delta_{BL} = 0.2r_0$  are represented  
 945 in figures 26(a-c). The allowable ranges for the upstream-propagating guided jet waves  
 946 according to the vortex-sheet model are displayed in grey using the same colour code  
 947 as in figures 7 and 9. Among these waves, those located between points L and I on  
 948 the dispersion curves are specifically highlighted with greenish grey. For the two jets,  
 949 and especially for the untripped one, peaks appear in the spectra, despite the fact that  
 950 downstream-propagating guided jet waves cannot exist at  $M = 0.75$  for  $n_\theta = 0, 1$  and 2.  
 951 This result, along with those reported in appendix A for the jets at Mach numbers  
 952 varying from  $M = 0.75$  up to  $M = 0.85$  in increments of  $\Delta M = 0.01$ , may cast doubt  
 953 on the importance of the acoustic resonance possibly occurring in the jet potential core  
 954 in the generation of the near-nozzle peaks. In figure 26, overall, the peaks lie in the

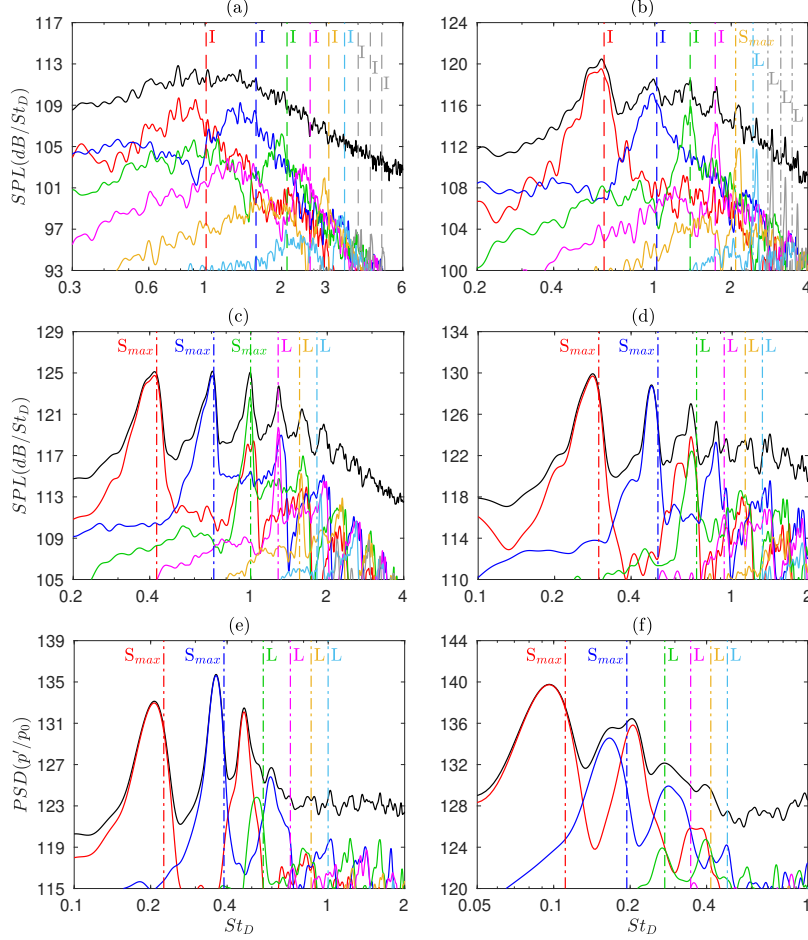


FIGURE 25. Sound pressure levels obtained at  $z = 0$  and  $r = 1.5r_0$  for the jets with tripped boundary layers at (a)  $M = 0.60$ , (b)  $M = 0.75$ , (c)  $M = 0.90$ , (d)  $M = 1.10$ , (e)  $M = 1.30$  and (f)  $M = 2$  as a function of  $St_D$ : — full spectra and  $n_\theta =$  — 0, — 1, — 2, — 3, — 4, — 5, — 6, 7 and 8;  $St_D$  at points (dashed lines) I and (dash-dotted lines)  $S_{max}$  or L on the dispersion curves for the guided jet modes ( $n_\theta, n_r = 1$ ) using the same colours as for the solid lines.

955 greenish-grey bands. They strongly decrease on the right side of the bands, all the more  
 956 rapidly that the peak is at a higher frequency. This is clearly observed, for instance,  
 957 for the untripped jet in figure 26(b) for  $n_\theta = 1$ . These trends are in agreement with the  
 958 transfer functions of figure 6(b), revealing that along the dispersion curves of the guided  
 959 jet modes, the decay of the wave magnitude outside of the jet is maximum around the  
 960 inflexion point I, and that the decay rate at that point increases with the radial mode  
 961 number. They provide additional evidence of the importance of the steepness of the band-  
 962 pass filter associated with the latter modes on the near-nozzle peak shape. Regarding  
 963 the Mach number effects for  $M \leq 0.75$ , the decrease of the spectra around point I is  
 964 slower for  $M = 0.60$  in figure 25(a) than for  $M = 0.75$  in figure 25(b). Again, this can be  
 965 explained by the lower decay rates in the transfer functions for  $M = 0.60$  in figure 6(a)  
 966 than for  $M = 0.75$  in figure 6(b).

967 To examine the near-nozzle peak properties for a supersonic Mach number, the pressure  
 968 spectra obtained at  $z = 0$  and  $r = 1.5r_0$  for  $n_\theta = 0, 1$  and 2 for the jets at  $M =$

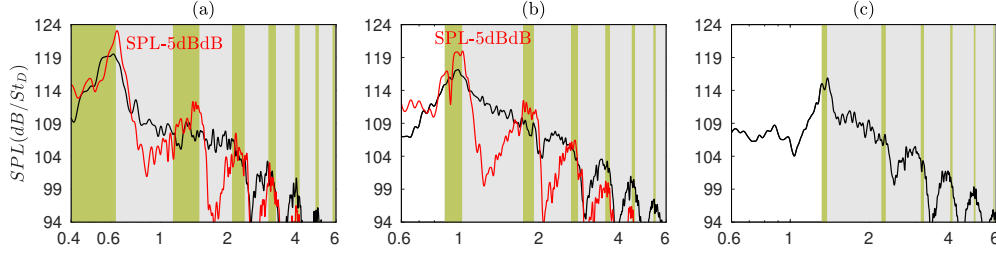


FIGURE 26. Sound pressure levels obtained at  $z = 0$  and  $r = 1.5r_0$  for the jets at  $M = 0.75$  — with tripped boundary layers and — with untripped boundary layers with  $\delta_{BL} = 0.2r_0$  for (a)  $n_\theta = 0$ , (b)  $n_\theta = 1$  and (c)  $n_\theta = 2$  as a function of  $St_D$ ; allowable ranges for the upstream-propagating guided jet waves (greenish grey) between the Strouhal numbers at points L and I on the dispersion curves of the modes and (light grey) otherwise.

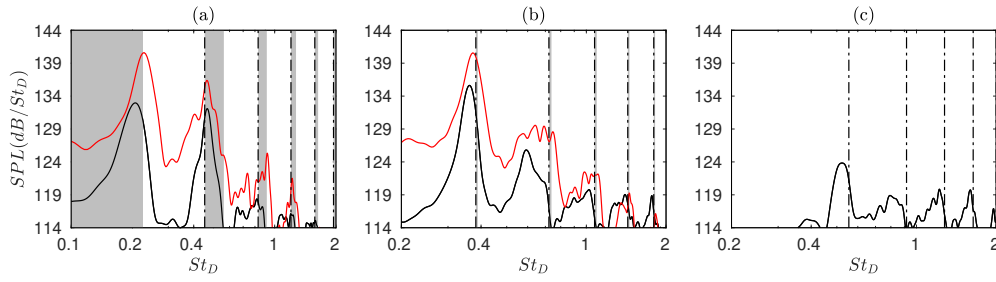


FIGURE 27. Sound pressure levels obtained at  $z = 0$  and  $r = 1.5r_0$  for the jets at  $M = 1.30$  — with tripped boundary layers and — with untripped boundary layers with  $\delta_{BL} = 0.2r_0$  for (a)  $n_\theta = 0$ , (b)  $n_\theta = 1$  and (c)  $n_\theta = 2$  as a function of  $St_D$ ; (dark grey) allowable ranges for the upstream-propagating guided jet waves, - · - · Strouhal numbers at points L on the dispersion curves.

969 1.30 with tripped boundary layers and with untripped boundary layers with  $\delta_{BL} =$   
 970  $0.2r_0$  are gathered in figures 27(a-c). The frequency ranges for the upstream-propagating  
 971 guided jet waves according to the vortex-sheet model are displayed in grey. Downstream-  
 972 propagating waves are permitted in the bands. The Strouhal numbers at the points L  
 973 of the dispersion curves of the waves on  $k = -\omega/c_0$  are indicated by dash-dotted lines.  
 974 They allow us to roughly locate the frequencies of the upstream-propagating guided  
 975 jet waves when these waves are not predicted by the vortex-sheet model, refer to the  
 976 frequency-wavenumber spectra in figures 23(g,h,i). In figure 27, the peaks are typically  
 977 one order of magnitude higher for  $n_\theta = 0$  and 1 than for  $n_\theta = 2$ . They are all near the  
 978 grey bands, when available, or the dash-dotted lines, otherwise, despite the relatively  
 979 poor agreement between the dispersion curves given by the model and the simulations  
 980 in figures 21(d-i) for the jets at  $M = 1.10$  and 2. Therefore, for supersonic jets, the near-  
 981 nozzle peaks can also be related to the upstream-propagating guided jet modes. In this  
 982 case, interactions are possible between free-stream upstream-propagating and duct-like  
 983 downstream-propagating waves close to the cut-off frequencies of the modes, as suggested  
 984 by figures 23(d-i).

985 The Strouhal numbers of the first three peaks associated with the guided jet modes in  
 986 the near-nozzle spectra are represented as a function of the Mach number in figures 28(a-  
 987 c) for the tripped jets for  $n_\theta = 0, 1$  and 2 and in figures 29(a,b) for the untripped jets  
 988 for  $n_\theta = 0$  and 1. For the latter jets, the Strouhal numbers of the peaks obtained for  
 989  $M \leq 0.65$  close to the vortex-pairing frequency, as documented in appendix B, are also  
 990 shown. The allowable frequency bands for the upstream-propagating guided jet waves,



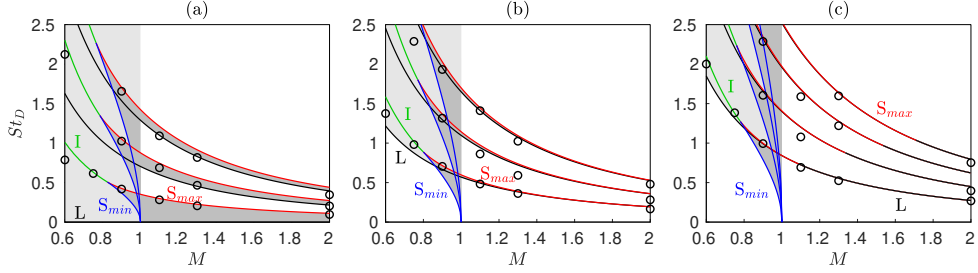


FIGURE 28. Mach number variations of  $\circ$  the Strouhal numbers of the first three peaks in the spectra of pressure fluctuations at  $z = 0$  and  $r = 1.5r_0$  for the jets with tripped boundary layers for (a)  $n_\theta = 0$ , (b)  $n_\theta = 1$  and (c)  $n_\theta = 2$ ; allowable ranges for the upstream-propagating guided jet waves (dark grey) with and (light grey) without downstream-propagating guided waves, points — L, —  $S_{max}$ , —  $S_{min}$  and — I.

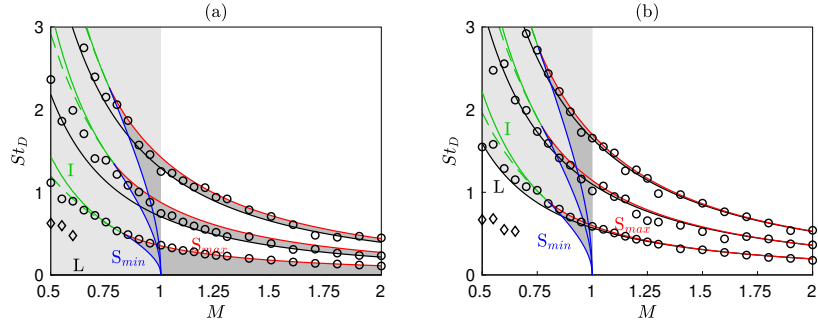


FIGURE 29. Mach number variations of the peak Strouhal numbers in the spectra of pressure fluctuations at  $z = 0$  and  $r = 1.5r_0$  for the jets with untripped boundary layers with  $\delta_{BL} = 0.2r_0$  for (a)  $n_\theta = 0$  and (b)  $n_\theta = 1$ :  $\circ$  peaks associated with the first three guided jet modes and  $\diamond$  peaks at the vortex-pairing frequencies; allowable ranges for the upstream-propagating guided jet waves (dark grey) with and (light grey) without downstream-propagating guided waves, points — L, —  $S_{max}$ , —  $S_{min}$ , — I and - - - I'.

991 as well as the points L,  $S_{max}$ ,  $S_{min}$ , I and I' on the dispersion curves, defined in section 3  
 992 based on the vortex-sheet model, are displayed. For both tripped and untripped jets, for  
 993 all azimuthal modes, the circles remarkably follow the variations of the guided jet modes  
 994 over the entire Mach number range. They are found between points L and I for  $M \leq 0.75$   
 995 and points L and  $S_{max}$  for  $M \geq 0.80$ . Thus, they all lie within the frequency ranges  
 996 over which free-stream upstream-propagating guided jet waves are possible according to  
 997 figure 6. This further supports that the presence of the near-nozzle peaks is mainly due to  
 998 a filtering of the upstream-travelling sound waves by the guided jet modes, the amplitude  
 999 of the waves with frequencies in specific ranges being preserved while that of the other  
 1000 waves decreases because of their evanescent nature.

1001 The intensities, degrees of emergence and full widths at half maximum of the spectral  
 1002 peaks at  $z = 0$  and  $r = 1.5r_0$  are represented as a function of the Mach number in  
 1003 figure 30 for the tripped jets and in figure 31 for the untripped jets. For the tripped jets,  
 1004 the peaks are those associated with the first radial guided jet modes  $n_r = 1$  for  $n_\theta = 0$ ,  
 1005 1 and 2, while for the untripped jets, modes  $n_r = 1$  and 2 are both considered for  $n_\theta = 0$   
 1006 and 1. For the latter jets, zoomed views between  $M = 0.75$  and  $0.85$  are also available in  
 1007 appendix A.

1008 In figures 30(a) and 31(a,d), the peak levels increase roughly as  $M^8$  for  $M \leq 1$  and  
 1009 as  $M^3$  for  $M \geq 1$ , following the typical scaling laws of aerodynamic noise for subsonic

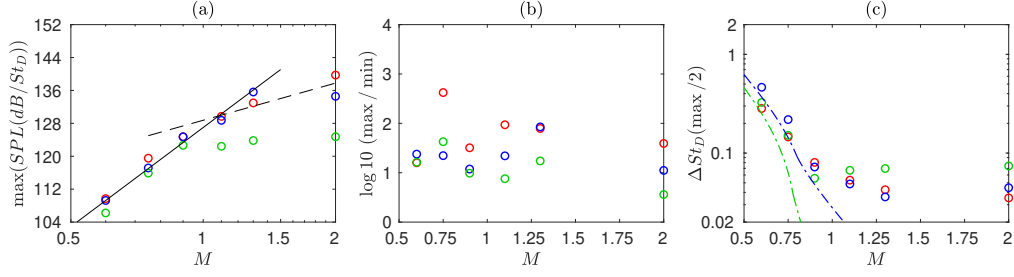


FIGURE 30. Near-nozzle peaks associated with the radial modes  $n_r = 1$  of the guided jet waves for  $\circ n_\theta = 0$ ,  $\circ n_\theta = 1$  and  $\circ n_\theta = 2$  for the jets with tripped boundary layers: (a) peak levels, (b) ratios between the peak levels and the minimum values for higher  $St_D$  and (c) peak widths as a function of  $M$ ; ———  $M^8$ , - - -  $M^3$ , (dash-dotted lines)  $\Delta St_D$  between points I and L or between  $S_{max}$  and L using the same colours for  $n_\theta$  as for the circles.

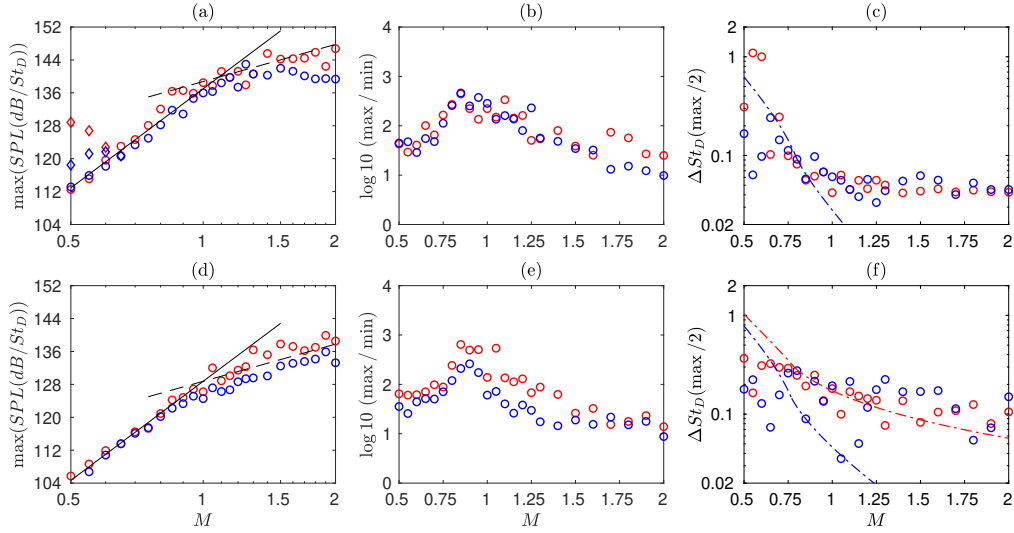


FIGURE 31. Near-nozzle peaks (circles) associated with the guided jet modes (a-c)  $n_r = 1$  and (d-f)  $n_r = 2$  and (diamonds) at the vortex-pairing frequencies for (red)  $n_\theta = 0$  and (blue)  $n_\theta = 1$  for the jets with untripped boundary layers with  $\delta_{BL} = 0.2r_0$ : (a,d) peak levels, (b,e) ratios between the peak levels and the minimum values for higher  $St_D$  and (c,f) peak widths as a function of  $M$ ; ———  $M^8$ , - - -  $M^3$ , (dash-dotted lines)  $\Delta St_D$  between points I and L or between  $S_{max}$  and L using the same colours for  $n_\theta$  as for the symbols.

1010 jets (Lighthill 1952) and supersonic jets (Ffowcs Williams 1963). This indicates, unsurprisingly, that the pressure waves propagating up to the near-nozzle region are acoustic waves generated by the jets. In most cases, there are no significant deviations from the  $M^8$  law between  $M = 0.5$  and 1, suggesting that the acoustic resonances which can occur in the jet potential core for  $0.80 \leq M \leq 1$  have a rather limited effect on the near-nozzle peak intensities. Given the 3-5 dB excess observed between the red circles and the trend line around  $M = 0.85$  in figure 31(a), this may, however, not be true for the untripped jets for the peaks associated with the first radial axisymmetric guided jet mode. In figure 31(a), the levels of the peaks obtained for the untripped jets at low Mach numbers at the vortex-pairing frequency  $St_D \simeq 0.70$  are also represented. For the axisymmetric mode, they strongly increase as the Mach number decreases. A tone can even be observed at  $St_D \simeq 0.70$  in the near-nozzle spectrum for the jet at  $M = 0.50$ , provided in

1022 figure 35(a) of appendix B. This may result from a feedback loop establishing between  
 1023 the Kelvin-Helmholtz instability waves and the sound waves generated by the vortex  
 1024 pairings.

1025 Regarding the peak emergence, it is difficult to identify a clear trend in figures 30(b) for  
 1026 the tripped jets. For the untripped jets, however, the peak emergence gradually increases  
 1027 from  $M = 0.50$  up to  $M \simeq 0.85$  and then decreases with the Mach number for  $M \geq 0.85$   
 1028 in figures 31(b,e). The increase may be linked to the steepening of the filters induced by  
 1029 the guided jet modes at their upper cut-off frequencies, shown in figure 6(a-c), providing  
 1030 lower noise levels just above these frequencies. As for the decrease, it may be due to  
 1031 the fact that at a higher Mach number the bands of the filters are closer to each other,  
 1032 leading to smaller bands without upstream-propagating guided jet waves as illustrated  
 1033 in the spectra of figures 23(f,i).

1034 Finally, in figures 30(c) and 31(c,f), the peak widths decrease significantly between  
 1035  $M = 0.50$  and 1, and then very slightly for  $M \geq 1$ . These variations can be explained by  
 1036 the narrowing of the allowable frequency bands for the free-stream guided jet modes as the  
 1037 Mach number increases. To demonstrate this, the band widths, estimated as the frequency  
 1038 differences between points L and I or points L and  $S_{max}$  on the dispersion curves obtained  
 1039 using the vortex-sheet model, are plotted. They are in fairly good agreement with the  
 1040 peak widths nearly up to  $M = 1$  in all cases. Above  $M = 1$ , except for the mode ( $n_\theta = 0$ ,  
 1041  $n_r = 2$ ) in figure 31(f), they are much smaller than the peak widths, which is not  
 1042 surprising considering the considerable discrepancies observed between the dispersion  
 1043 curves given by the model and the simulations near  $k = -\omega/c_0$  in figures 23(d-i) for  
 1044 supersonic Mach numbers.

## 1045 5. Conclusion

1046 In the present paper, the presence and properties of acoustic tones in the pressure  
 1047 spectra computed near the nozzle of jets have been investigated for isothermal round  
 1048 jets with different Mach numbers, nozzle-exit boundary-layer thicknesses and turbulence  
 1049 intensities using large-eddy simulations. For all jets, acoustic peaks appear in the near-  
 1050 nozzle spectra over the whole range of Mach numbers considered, from  $M = 0.50$  up to  
 1051  $M = 2$ , at frequencies which do not depend appreciably on the initial jet flow conditions.  
 1052 The peaks have a tonal shape above  $M \simeq 0.75$ , broaden for lower Mach numbers, and  
 1053 correspond to peaks in the spectra of the first azimuthal modes  $n_\theta = 0$  to  $n_\theta^{max}$ , with  
 1054  $n_\theta^{max}$  reaching a value of 8 for  $M = 0.75$  for instance. The peak levels increase roughly  
 1055 as the eighth power of the jet Mach number for  $M \leq 1$  and then as the third power  
 1056 for  $M \geq 1$ , following the typical scaling laws of aerodynamic noise. Their other proper-  
 1057 ties vary continuously with the Mach number, without spectacular changes around the  
 1058 threshold Strouhal numbers  $M \simeq 0.80$  for the downstream-propagating guided jet waves,  
 1059 and without stage jump for supersonic Mach numbers, contrary to the screeching modes  
 1060 observed for non ideally expanded jets. In the acoustic far field, the peaks can also be  
 1061 detected for large radiation angles  $\phi \geq 135^\circ$  and they are predominant in the upstream  
 1062 direction.

1063 The properties of the near-nozzle peaks and their links with the acoustic tones emerging  
 1064 in the jet potential core in some cases have been carefully examined by the computa-  
 1065 tions of frequency and frequency-wavenumber spectra inside and outside of the jet flow  
 1066 and comparisons with the dispersion relations and eigenfunctions of the guided jet waves  
 1067 predicted for a vortex-sheet model. The near-nozzle peaks are found to lie within the  
 1068 frequency bands of the upstream-propagating guided jet waves with a significant radial  
 1069 support outside of the jet shear layer. This suggests that they are mainly due to the filter-

1070 ing of the upstream-travelling sound waves by the guided jet modes. The sharpness and  
 1071 prominence of the peaks can thus be explained by the decay rate of the filtering transfer  
 1072 functions at their cut-off frequencies. Concerning the upstream-propagating guided jet  
 1073 waves possibly resonating in the jet potential core for high subsonic Mach numbers, only  
 1074 those close to the cut-off frequencies of the guided jet modes can contribute to the near-  
 1075 nozzle peak. Naturally, for impinging jets or supersonic non-ideally expanded supersonic  
 1076 jets, the upstream-propagating guided jet waves of the jets are likely to couple with the  
 1077 Kelvin-Helmholtz instability waves growing in the jet mixing layers to establish intense  
 1078 aeroacoustic feedback loops. For free, ideally-expanded jets, such a coupling may exist,  
 1079 but its strength can be expected to depend on the laminar or turbulent state of the  
 1080 boundary layers at the nozzle exit. This will be discussed in future studies.

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### 1094 Declaration of interests

1095 The author reports no conflict of interest.

### 1096 Appendix A. Near-nozzle acoustic tones for the untripped jets at 1097 Mach numbers varying from 0.75 to 0.85

1098 In this first appendix, the near-nozzle acoustic peaks obtained for the jets with un-  
 1099 tripped boundary layers at Mach numbers increasing from  $M = 0.75$  up to  $M = 0.85$  in  
 1100 increments of  $\Delta M = 0.01$  are examined in order to reveal their possible changes around  
 1101 the Mach numbers below which downstream-propagating guided waves are not permitted  
 1102 in the jets according to the vortex-sheet model.

1103 The spectrograms of pressure fluctuations calculated at  $z = 0$  and  $r = 1.5r_0$  for  
 1104  $n_\theta = 0$  and 1 are represented in figures 32(a,b) as a function of  $M$  and  $St_D$  using  
 1105 logarithmic scales. Lines indicating the Strouhal numbers at the limit points L on  $k =$   
 1106  $-\omega/c_0$ , the stationary points  $S_{max}$  and  $S_{min}$  and the inflexion points I on the dispersion  
 1107 curves of the guided jet modes according to the vortex-sheet model are also displayed.  
 1108 Significant energy is found inside the bands between points L and I and then points L  
 1109 and  $S_{max}$ , as expected given the eigenfunction magnitudes obtained in figure 6 for the  
 1110 waves travelling in the upstream direction outside of the jet flow. These are the bands of  
 1111 the free-stream guided jet waves. In particular, no notable change seems to occur around  
 1112 point  $S_{min}$ , which marks the cut-on frequency of the waves which can be involved in  
 1113 resonant mechanisms in the potential core.

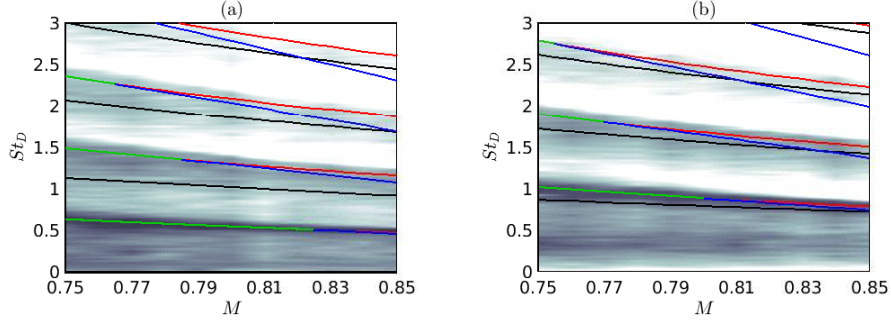


FIGURE 32. Power spectral densities of pressure fluctuations at  $z = 0$  and  $r = 1.5r_0$ , normalized by their peak values, for (a)  $n_\theta = 0$  and (b)  $n_\theta = 1$  for the jets with untripped boundary layers with  $\delta_{BL} = 0.2r_0$  as a function of  $(M, St_D)$ ; points — L, —  $S_{max}$ , —  $S_{min}$  and — I on the dispersion curves of the guided jet modes. The greyscale ranges logarithmically (a) from  $10^{-3}$  to 10 and (b) from  $10^{-2}$  to 10.

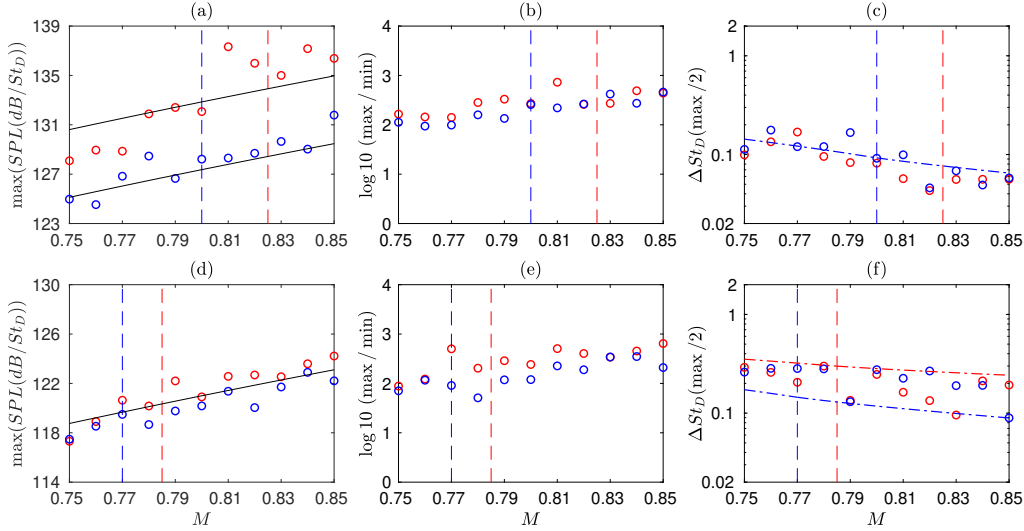


FIGURE 33. Near-nozzle peaks associated with the guided jet modes (a-c)  $n_r = 1$  and (d-f)  $n_r = 2$  for  $\circ n_\theta = 0$  and  $\circ n_\theta = 1$  for the jets with untripped boundary layers with  $\delta_{BL} = 0.2r_0$ : (a,d) peak levels, (b,e) ratios between the peak levels and the minimum values for higher  $St_D$  and (c,f) peak widths as a function of  $M$ ; —  $M^8$ , (dashed lines) Mach number thresholds for the downstream-propagating waves and (dash-dotted lines)  $\Delta St_D$  between points I and L or between  $S_{max}$  and L using the same colours for  $n_\theta$  as for the circles.

1114 In support of the preceding observations, some properties of the near-nozzle peaks  
 1115 associated with the guided jet modes  $n_r = 1$  and 2 for  $n_\theta = 0$  and 1 are plotted in  
 1116 figure 33 as a function of the Mach number. Over the whole Mach number range, the  
 1117 peak levels grow roughly as  $M^8$  in figures 33(a,d). The peak prominence increases with  
 1118 the jet velocity in figures 33(b,e) and the peak full widths at half maximum decrease in  
 1119 figures 33(b,e), in good agreement with the filter band widths estimated between points L  
 1120 and I and then points L and  $S_{max}$ . Except maybe for the intensity of the peaks for mode  
 1121 ( $n_\theta = 0, n_r = 1$ ) in figure 33(a), there are no significant variations near the threshold  
 1122 Mach numbers for the downstream-propagating guided jet waves.

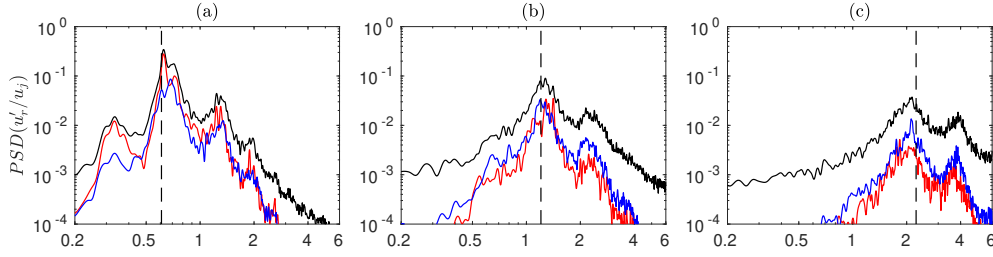


FIGURE 34. Power spectral densities of radial velocity fluctuations for the jets at  $M = 0.50$  with untripped boundary layers with (a)  $\delta_{BL} = 0.2r_0$ , (b)  $\delta_{BL} = 0.1r_0$  and (c)  $\delta_{BL} = 0.05r_0$ , at  $r = r_0$  and  $z = 3.6r_0$ ,  $2r_0$  and  $1.2r_0$ , respectively: — full spectra, —  $n_\theta = 0$  and —  $n_\theta = 1$ ; - - -  $St_\theta = f\delta_\theta(z=0)/u_j = 0.007$ .

## 1123 Appendix B. Near-nozzle tones at the vortex-pairing frequency for 1124 the untripped jets at low Mach numbers

1125 In this second appendix, results obtained for the three jets at  $M = 0.50$  with untripped  
1126 boundary layers of thickness  $\delta_{BL} = 0.2r_0$ ,  $0.1r_0$  and  $0.05r_0$  are provided in order to  
1127 investigate the origin of the peaks at  $St \simeq 0.70$  in the near-nozzle spectra for the untripped  
1128 jets with  $\delta_{BL} = 0.2r_0$  for  $M \leq 0.6$ .

1129 In order to determine the frequencies of the first vortex pairings in the mixing layers,  
1130 the spectra of radial velocity fluctuations computed for the three jets at  $r = r_0$  for  
1131  $z = 3.6r_0$ ,  $2r_0$  and  $1.2r_0$ , that is approximately at the positions of the vortex pairings,  
1132 are plotted in figures 34(a-c). The spectra for  $n_\theta = 0$  and  $n_\theta = 1$  are also depicted. For  
1133 all jets, peaks at harmonic frequencies are observed. The dominant ones are centered  
1134 around the vortex-pairing frequencies, providing  $St_\theta = f\delta_\theta(z=0)/u_j \simeq 0.007$  when  
1135 normalized by the nozzle-exit momentum thickness. This  $St_\theta$  value corresponds to half  
1136 of the frequency of the initial instability waves in laminar shear layers (Zaman & Hussain  
1137 1981; Gutmark & Ho 1983).

1138 The spectra of pressure fluctuations at  $z = 0$  and  $r = 1.5r_0$  for the full pressure signal  
1139 and for the first two azimuthal modes are reported in figures 35(a-c). The vortex-pairing  
1140 frequencies obtained from the velocity spectra are indicated by a mixed line. At these  
1141 frequencies, a tone clearly emerges in figure 35(a) for  $\delta_{BL} = 0.2r_0$ , a small hump is visible  
1142 in figure 35(b) for  $\delta_{BL} = 0.1r_0$ , but no specific components are found in figure 35(c) for  
1143  $\delta_{BL} = 0.05r_0$ . Therefore, the peaks in the near-nozzle spectra for the untripped jets with  
1144  $\delta_{BL} = 0.2r_0$  for low Mach numbers are related to the vortex-pairing process, and vanish  
1145 for a thinner boundary layer. In addition, the tonal shape of the peak for  $M = 0.50$  in  
1146 figure 35(a) suggests the establishment of an feedback loop between the growing Kelvin-  
1147 Helmholtz instability waves and the upstream-propagating sound waves generated by the  
1148 first stage of vortex pairings in the shear layers in that case. The possibility of such a  
1149 feedback mechanism in free jets, similar to that found in impinging jets (Ho & Nosseir  
1150 1981), was proposed forty years ago by Laufer & Monkewitz (1980) and Ho & Huang  
1151 (1982), for instance.

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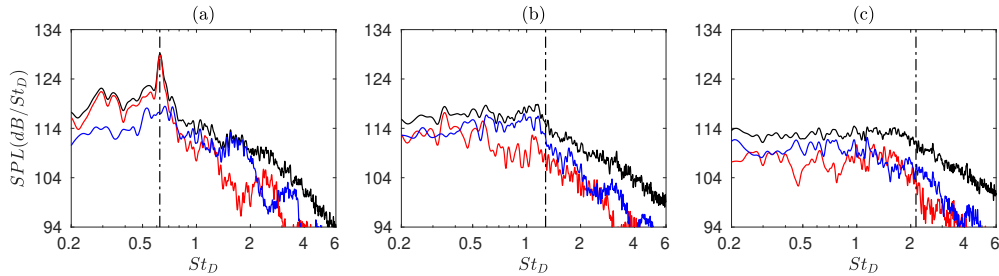


FIGURE 35. Sound pressure levels obtained at  $z = 0$  and  $r = 1.5r_0$  for the jets at  $M = 0.50$  with untripped boundary layers with (a)  $\delta_{BL} = 0.2r_0$ , (b)  $\delta_{BL} = 0.1r_0$  and (c)  $\delta_{BL} = 0.05r_0$ : — full spectra, —  $n_\theta = 0$  and —  $n_\theta = 1$ ; - - - vortex-pairing frequencies.

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