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The case of Textbooks in Mathematics Teaching

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CHAP 5. THE CASE OF TEXTBOOKS IN MATHEMATICS TEACHING

INTRODUCTION

The mathematics textbook is, in France, a central object to teachers' professional practices (and more markedly among beginners), and in students' daily work, whether in class or in their personal work. Paradoxically though, it is rarely studied for itself in the education research in France (Bruillard, 2005) and especially in the mathematics education¹ research.

We first need to point out that in France, the design of the textbooks is the task of private publishers who, each, suggest their textbooks to the teachers. Hence, the choice of the textbook given to the students is done by the team of teachers in each school, the financing is provided by the administration.

A textbook is the result of a complex interaction between multiple constraints. We are interested as a first step in describing the perception of that interaction which we have developed through an experience of writing and editing a textbook. We make it a point to underline the resulting leeways.

Taking into account this description, it seemed interesting to evaluate the homogeneity of the textbook, to research whether the authors benefit from this leeway while writing, mainly when writing exercise panels which they have suggested. We have thus analyzed the exercises of four textbooks (covering the same theme) in terms of the knowledge at stake and the activities that they would generate for the students.

CONSTRAINTS AND LEEWAYS IN THE CONCEPTION OF A TEXTBOOK, A POINT OF VIEW BASED ON EXPERIENCE

This first section attempts to describe the interactions of the constraints that weigh on the writing, and in a less central manner on the use of textbooks. It is based on the experience of Christophe Hache as the person in charge of the Domino collection at the time of the publishing of the textbooks for grades six² (Hache, 2005) and seven³ (Hache 2006).

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*F. Vandrebrouck (ed) Activités des élèves et pratiques des enseignants en classe de mathématique.
Ouvrage collectif.*

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The textbook in question is that “of the student”. This textbook is given to the student at the beginning of the year and then taken from him at the end of the year. The constraints imposed by the fact that, materially speaking the textbook is a book, concern not only the users, of course, but the designers of the textbook as well.

As a first example, the fact that the student has at disposition the whole textbook complicates matters, and affects the statement of any discover activity of a notion (the entire chapter about a certain notion is in fact not very far), having the entire set of questions of a problem can interfere in the genuine research process while solving the first few questions. Similarly, the position of an exercise in the textbook is an indication de facto with respect to its solving.

The teachers can have access to a « teachers’ textbook ». However, if we exclude the re-transcription of the official programs and the exercise corrections from the textbook, their content turns out to be poor in general. In middle school⁴, in mathematics, teachers seldom use this teachers’ textbook⁵ (things are less clear at the primary level⁶ for example, where the content is much more developed, and where teachers are generally not specialized in mathematics).

Therefore, group work sessions, long research sessions, or sessions where arrangements for special modalities need to be made (exchanges between students, debates...) are hardly accounted for in the textbooks.

The paper format makes the appropriation or modification of the text by the teachers less natural (adaptation to the chosen approach, adaptation to students...). Regarding this point, we can mention the novelty of the Sésamath textbook (Sésamath, 2006): the textbook is sold in paper form but is also entirely available in digital form with a format that allows all modifications ([Sésamath](#) website).

A third constraint is related to the stability of the practices related to textbooks (writing, use, manipulation...). The expectations of the teachers (who choose the textbooks), the representations of the authors or publishers regarding what should be included in a textbook, and the way to present things are strongly influenced by the omnipresence and homogeneity of the existing or previous textbooks. We can, for example, refer to the Sésamath textbook experience (Sésamath, 2006) which was designed by a significantly large group of volunteer teachers without any intervention from a publisher. We will show that the non-commercial aim of the project and the absence of publisher during the writing eliminate a large number of constraints; the textbook however remains mostly classical. The project was not to produce a revolutionary textbook, but the proximity of the results and the standards shows well an important stability in the representations and expectations.

Last but not least, there is a time constraint to the writing of the textbook. The changes in official programs usually follow an age group: the programs are for instance changed for grade six at the beginning of the 2005 school year, then for

grade seven at the beginning of the 2006 school year, and so on until 2008 when changes are applied for grade nine. These official program changes go hand in hand with renewing the textbooks. The author teams are usually not overloaded⁷ and hence work on one textbook at a time. They therefore have around a year to conceive the textbooks based on the official program. This one-year deadline is one the heaviest constraint applied to the conception of a textbook: not much back stepping or possibilities to go backwards, not much proofreading, very few experimentations, a lot of urgent work, external collaborations and consultations are difficult to set up...

The relationships between publishers of a textbook⁸ and the authors are continuous: the publisher is present from the start of the work and follows the writing process in an uninterrupted⁹ manner. The structuring of the book, the layout of the chapters, the exercise sections... are also elements which interfere at a very early stage in the conception of the textbooks. They link content (which is mainly the responsibility of the authors) and form (which is mainly the responsibility of the publisher). Thus the work of the authors cannot be done without taking into account the constraints of publishing (and vice versa). The publisher has, in a central way, a commercial objective: s/he designs products, looks for clients (tries to seduce them despite the competition) and wants to sell. The quantity of sold products determines the profit made. We can for example refer to the fact that in 2008, there were no textbooks for the grade 12¹⁰ humanities track because the market was “too small”. From a commercial point of view, the “client” is easily defined: it’s the teacher who, within his/her team and once having received all the published textbooks, must choose the one that will be bought by the students of his/her school the next year. The “client” expectations are however difficult to define, the choice criteria not being explicit nor totally rational. “The publishers naturally seek, as a priority, the support of prescribing teachers; the textbooks are hence conceived in terms of the wishes of the teachers more than those of the institutions or the needs of the students. (...) If the publishers continue to propose, as they constantly claim, the product desired by the teachers, and if these same teachers prefer a safe product which reassures them in their habits, then how can pedagogical innovation be made? (...) Reflecting on the textbook and its necessary evolutions must start from the needs of the students and not the wishes of the teachers.” (Borne, IGEN¹¹ 1998)

Appeal to a maximum number of teachers (regardless of their practices) and eventually get the students interested (which is reflected by “show the teacher that the textbook will be interesting to the students”) goes through a certain number of rules, as far as the publisher is concerned. An example: the principle of elaborating the most omnipresent textbook, and this for all levels, is that the principle of regularity. In order for the textbook to be “well done”, in order for it to be “user-friendly to the students”, in order to reassure the teachers, all chapters “must” be structured in the same way (sections, number of pages...). The irregularity of the types of notions considered is not really taken into account.

The mathematical content of the textbooks is framed by the official programs. These are commented and underlined through the “program accompanying

documents” or the “program implementation documents”. Note that, in general, the official programs are issued one year before being implemented, and these accompanying or implementation documents¹² are issued at a later stage, commonly after the publishing of the textbooks. These are not the only constraints linked to contents of teaching. In fact, the mathematical contents impose layout and organization constraints; this is one subject of research in mathematics didactics. The choice of the type of task suggested, their declination into exercises, and the balance in volume between the different types of expected activities are also elements constrained by the addressed content.

The work on the mathematic contents is mainly the task of the authors. Usually, the publisher’s influence is indirect, in particular through the rigidity of the layout or the time constraint. It could happen that the publisher interferes at the level of a exercise, or less commonly a chapter, refusing a certain way of presenting a notion (the official instructions argument is then negligible compared to, for example, that of the teachers being used to a certain way of doing, or being ready, or supposedly ready, or not, to accept certain changes).

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Leeway is given to authors (and then users) of textbooks. During the elaboration of the textbook, the setting up of the team of authors (and the team of reviewers), and macroscopic editorial choices (ways of approaching programs, chapter division...) are aspects for which constraints seem less pressing (excluded, always, the time constraints for reflection, experimentation...).

Leeway during the writing phase is also important, especially when it comes to the “exercises” section of the book. In fact, the publisher focuses on the first parts of the chapters: introductory activities, lesson pages, solved exercises... We can put forward the following explanations:

- The publishers refer to the teachers’ declarations, who say that a significant attention is given to the lesson pages and solved exercise (rigor, accessibility for the students...) in the choice of their textbooks¹³,
- The publishers (and teachers) have very few tools to analyze and evaluate an exercise or set of exercises.

Writing the exercises is hence essentially dictated by quantitative constraints: size of exercise given, number of exercises, and sometimes the possibility of illustration.

Of course, beyond the making-of of the textbook, teachers are allowed significant leeway. The work of Sara Arditì (Arditì, 2011) explores precisely the variability of practices of teachers using the same textbook. The job habits don’t leave much choice with respect to buying or not a textbook, but it can be noted that certain teachers don’t use a textbook; they are self-produce the pedagogical material needed, or they use equality different textbooks , or other resources at their disposal, and the photocopier (Ben Salah Breigeat, 2001). The choice of textbook is an important aspect for which the criteria are rarely explicitly stated (there is no research about this topic nor any training, or only little...). This is where the teacher plays his/her role of client for the publisher.

The main freedom for the teacher with respect of the textbook remains of course that of organizing his/her teaching. The way of using a textbook is beyond any doubt very varied. During the design of a teaching session or a sequence of sessions, the teacher takes ownership of the textbook, and reconstructs what s/he needs from what is suggested. During the teaching process, when s/he uses the textbook, the teacher also adapts the content of the textbook to the flow of the session, whether consciously or not.

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Several fields of questions come to mind based on the notions discussed above. We can first wonder about the different effects of these constraints: is there a strong uniformity in the content of textbooks? Can we see some variations? We can wonder about the things that do not appear in the textbooks because of these constraints. Reading between the lines is always difficult, but it is always possible to confront, from this point of view, the content of textbooks with the official programs. Identifying and describing a potential uniformity, or at least similarity, of the textbooks brings out the question of how to describe, or characterize the textbooks.

Considering what precedes, we can question in what way the textbook can be an efficient tool to diffuse research results and new teaching approaches: what textbook allows such transposition? What type of organization is required while writing? ...

Several questions are laid forward in what precedes: how to organize teaching contents over the year (as the author of a textbook, but also in a broader sense, to teach in class)? How to analyze teachers' expectations while they choose a textbook, and then while they use one? Are teachers' practices linked to the used textbook? How do students use the textbooks inside and outside the classroom? ...

The idea that mathematics textbooks are very similar to each other for a same class level and a specific time period is widely spread. The layout, number of chapters (typically between 15 and 20 chapters in middle school, each tackling a specific notion), and the division of each chapters into sections (typically "activities"¹⁴, "lesson", "solved exercises", "direct application exercises", "challenging/practice exercises", all on around 20 pages) reinforce that impression. Moreover, we saw in the previous paragraph that several elements seem to concur with this standardization.

These declarations can be opposed by the fact that teachers mainly use the textbooks as a source of exercises, and that the elements mentioned above (layout, number of chapters, division of chapters...) are essentially linked to the form: the choice of textbook can keep its meaning if we only rely on the content (choice of approach to notions, suggested exercises...). It is important to note that the form is of course not without any link to the content: the division of chapters and the allocation of weight to a notion in a chapter does not allow for example, and very generally, the valuing of the work done on multiple notions (except in the last chapters, if the notions have been discussed in the order suggested by the textbook). Changing frames becomes rather rare, and when it happens, they seldom link two new notions. It is not rare that once a chapter is covered, the studied

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notion is only evoked once or twice in the rest of the textbook. What can be said then of the structure and organization of the student knowledge?

ANALYSIS AND CHARACTERIZATION OF EXERCISES IN TEXTBOOKS

In what follows, we propose to present a work methodology to tackle the question of similarity of textbooks, in a second time we present some of the results obtained (a more detailed account of the corresponding research can be read in Hache 2008). We wondered, while limiting ourselves to the set of exercises suggested in comparable chapters, to what extent do the constraints described in the first section of our chapter have a unifying effect on the content of the exercises in the textbooks.

Our choice to focus on the exercises has several justifications:

- The teachers use several textbooks as a database for exercises (the textbook distributed to students as well as other textbooks that they have at their disposal).
- We have seen that writing the exercises is one of the parts of the textbook where constraints imposed to authors are the least weighing, hence this is where most of the diversity should be found, a priori.

We studied a given chapter in four textbooks. The methodological choices (choice of textbooks, choice of studied chapter, and as mentioned earlier choice of analyzing exercises in this chapter) have been made such that we increase the chances of finding differences between the textbooks. The notion studied through the analyzed exercises, proportionality, was chosen according to the following criteria. It is one of the notions traditionally studied in middle school, but the new programs give it a much more central position than before. Its approach has evolved with respect to the old programs, its study starts now more explicitly as of grade six. The organization of the chapter (the grouping of exercises for example, as well as their content) had to be “reinvented” by the authors: for example, the cross multiplication is not studied until grade eight although it used to be seen in grade seven. Since the position of the notion changing in the programs, the way it is handled in the textbooks cannot conform to pre-established canons.

The chosen textbooks are the following: Phare 5ème (Brault et al., 2006), Domino 5ème (Hache, 2006), Transmath 5ème (Malaval, 2006) et Sésamath 5ème (Sésamath Ass., 2006). The choice was also made here so that it favors heterogeneity. Phare is a recent collection, for which the grade six edition was a major success among the teachers in 2005. Domino is also a recent collection written by a team of authors, among which some are researchers in mathematics didactics (but it had little success among grade six teachers). Transmath is one of the oldest existing collections, the grade six textbook was also a big success in 2005. Sésamath is a new collection (no grade six textbook in 2005), the textbook is written by a team of around 30 teachers, it is available online for free (and sold in paper form, the publisher is not a school publisher but a publisher of educational and cultural CDs and software, the paper form of the textbook sold well in 2006).

At this point of our work, we have four complete sets of exercises from the chapter about proportionality, from four textbooks of grade seven edited in 2006. We will hence present the analysis method that has been set up in order to better understand the similarities, or on the contrary the differences, between these textbooks.

We analyze the proposed tasks in the textbooks with respect to the activities that we believe they generate for the students. In order to do so, we place ourselves in a theoretical situation where the student masters his/her lesson (in case of hesitation about the content of the lesson, we can't refer to the teacher's lesson, we refer to the lesson in the studied textbook), where he has taken ownership of the question, and where he wants to solve the exercise...

In order to describe the possible student activities from the exercise given, we have used the classification of adaptations introduced by Robert (chapter 2). For each studied given, we have detailed the different necessary adaptations for the expected answer, while specifying the knowledge at stake. Hence, it seems important to specify and classify the different types of knowledge related to proportionality used in grade seven (whether they are directly related to this notion or used in parallel).

1. Analysis of tasks: mathematical content

In this paragraph, we detail the way we analyze the mathematical content to be taught. This step is a prerequisite to the analysis of exercises.

Excerpts for national programs :

Programs of grade six 2004 (Official bulletin special issue n°5 on Sept. 9th 2004):

“Solving proportionality problems has already been done in elementary school. It is carried on in grade six, with new tools”

“Handling “proportionality” problems, using appropriate reasoning, in particular:

- using the image of one unity;
- using a linearity ratio expressed, if needed, in the form of a quotient;
- using a proportionality coefficient expressed, if needed, in the form of a quotient.

Recognizing situations which are related to proportionality and those that are not related. Applying a percentage rate”

Programs of grade seven 2005 (Official bulletin n°5 on Aug. 15th 2005):

“Proportionality still has a central position”

“Completing a table of numbers whose values are only partially provided, which represents a proportionality relation. In particular, determine a fourth proportional.

Recognize if a complete table of numbers is, or isn't, a proportionality table.

- Use proportionality in the following case: compare proportions,
- Calculate and use a percentage,

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- Calculate and use the scale of a map or a sketch, Recognize a uniform movement from the existence of a proportionality relation between duration and distance, use this proportionality”

The skills studied in grade seven, through the filter of the textbooks, have been classified as follows.

Recognizing a situation/ a proportionality table

Many things in the textbooks remain as hidden knowledge of this domain. When this is the case, the fact that the situation is a proportionality situation by choice, by modeling, in order to make calculations, is always implicit. This will be a problem in statements where proportionality is assumed whereas it could also be questioned (which is in general done in one or two exercises in each textbook).

An example of a situation where proportionality doesn't go without saying:

What distance does a snail travel in 2 hours 30 minutes at a speed of 14 meter per hour? (Transmath, n°56 p99)¹⁵

An example of calling into question:

“Currently, the length of a day (between sunrise and sunset) decreases by 3 minutes every day. What would be the total decrease within 30 days?” What is implied by this statement? Is it correct or wrong and why? (Transmath, n°46 p99)

Similarly, the distinction between proportionality situation and proportionality table is sometimes blurry:

A snail is moving on a branch.

The total duration of its trip, in minutes, and the distance covered, in centimeters, are given in the table below.

Duration (in min)	4	6	9
Distance (in cm)	26	39	58.5

- 1) a. Show that the movement of the snail appears to be uniform.
b. Specify the proportionality coefficient of this situation.
- 2) (...)
b. What distance can the snail travel in one hour if its movement remains uniform? (...) (Phare, n°70 p130)

The use of the expression “appears to be” seems to indicate that the statement is emphasizing the fact that the proportional character of the table does not allow to prove the fact that the movement is uniform. The 2) b. questions seem to imply the opposite.

Similarly:

For exercises 14 through 19, state whether the table corresponds to a proportionality situation or not.

If it does, specify the proportionality coefficient. (...) (Phare, p124)

We have labeled five skills related to identifying a proportionality situation (or table):

- Being able to recognize the existence of a proportionality situation, or the strong presumption of existence (coded Rr);
- Being able to identify (or propose) proportional magnitudes (coded Rg);
- Being able to admit the proof of proportionality (it's the case in many concrete situations) (coded Ra);
- Being able to prove proportionality (calculation of quotients for a table, existence of a formula in a situation, existence of a uniform movement, of a scale...) (coded Rp);
- Being able to prove non-proportionality (coded Rnp).

Example:

On the label of a one liter fruit juice bottle we read:

Proteins	0.4 g / 100 mL
Average energetic value : 50 Kcal	

Copy then complete the following table:

Orange juice volume	1 L	0.25 L	1.5 L	2 L
Proteins				
Energetic value				

(Sésamath, n°5 p76)

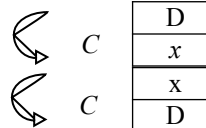
This exercise requires, among other things, recognizing in this situation several proportionality situations (Rr), the case of lipids (upper bound) and of energetic value (50 Kcal / 100mL or 50 Kcal / L ?) are particularly delicate, they could be coded Rg. In all cases, the proportionality is eventually admitted (Ra). The last two categories are linked to proofs.

Exploring a proportionality situation

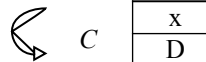
Proportionality whether given, admitted or proven, several explorations can be considered. We have distinguished the following cases:

- The coefficient (or scale, or speed, or percentage...) being known, as well as one more value given, calculate the missing value

° [Cmu] We multiply by the coefficient



° [Cdi] We divide by the coefficient



- Similarly, given a formula, knowing how to use it:

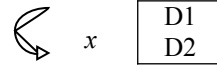
° [Fmu] directly,

° [Fdi] begin by reversing it.

The use of the coefficient alone rarely appears in the textbooks.

The use of formulas is not considered. Therefore, the work is focused on the following three skills:

- [Coef] Two values are known, knowing how to calculate the coefficient



- [QP] Fourth proportional. Three values are known, knowing how to calculate the fourth. Usually, several methods are possible : calculation (then use) of the coefficient, using the image of one unity, multiplication of one “column” to obtain the other (the method can be imposed or not).

D1	D13
D2	

- [DonInc] Values to be completed : the known values are not enough to calculate the missing value (either they are “too many” values, or there are several missing values). A new skill emerges, for example: being able to use addition of “columns” (the method can be imposed or not).

Other practiced skills

In the chapters where the notion of proportionality is practiced, other skills are used or practiced. We retain the following groups of skills:

- Graphs, tables

Proportionality is practiced in grade 6 (and before) through the concept of proportionality between two magnitudes. The tables and graphs are also tools which are practiced in grade six (reading and interpretation, some constructions are considered). In grade seven, the proportionality table is introduced as well as how it is related to previously studied proportionality situations.

The link between the alignment of points with the origin of a coordinate system and proportionality can be seen without being required, it will be explicitly studied in grade eight. However, there exists a strong link between graphical representations of data and proportionality (proportionality between frequency and height in a bar diagram, between frequency and angles in a circular diagram...)

- Skills related to magnitudes

We have distinguished between the skills related to converting different magnitudes (lengths, areas, weights, volumes, durations, speed), from the skills related to the notion of magnitude itself (recognize the magnitude at play, compare magnitudes without measurement or with an ad hoc unit).

Regarding this last point, it should be noted that the first magnitudes and their measures are studied starting elementary school (length, weight, capacity and duration are for example studied in cycle 2), angles and volumes are studied in cycle 3, and their measure is studied in grade six. The conversions of volume measures are skills in the process of being acquired during grade seven.

Speed has a special status: it is not studied on its own, only the idea of “uniform speed” is covered (as an example of a situation of proportionality relation between duration and distance).

The quotient magnitudes are introduced by the program in grade eight.

- Skills related to context

Many skills are used in solving exercises linked to proportionality. In addition to those mentioned above (related to magnitudes, graphs and tables) several old skills related to geometry are used (mainly drawings, measures, formulas, eventually in space geometry). In addition, manipulations of rounding, orders of magnitude, fraction notion, and percentages are carried out (notions studied in grade six).

- Others

Some exercises attempt to link the work on proportionality to the beginning of algebraic manipulations in grade seven. A textbook suggests some exercises using the scientific notation of numbers, which is a notion studied in grade eight.

2. Analysis of tasks: adaptations

These adaptations are grouped into eight categories (see chapter 2):

- SIT: simple and isolated tasks, these are tasks for which the student uses a skill without adaptation while solving. Ref. examples 1, 2 and 3.
- A1: the student must recognize the modalities of the application of a skill in order to be able to use it. Ref. examples 2, 6 and 8.
- A2: the student must introduce an intermediate item to be able to progress in his/her work. It could be placing a new point in geometry (or simply naming an existing point), doing intermediate calculation... Ref. examples 3, 4 and 5.
- A3: it's about being able to relate several skills to each other, make a change of frame, link the work on new skills with old ones. Ref. examples 3 and 6.
- A4: the given leaves it up to the students to introduce a new reasoning step. It is not uncommon that, for example, in exercises related to proportionality using a context, the hypotheses of proportionality be kept hidden, hence the student must theoretically decide or admit it. Ref. example 7.
- A5: the answer to a question requires the use of a result established in the previous questions.
- A6: choosing one method among many. Ref. example 8.
- A7: working with skills that have not yet been introduced as is at this level, not yet formalized. Ref. example 9.

Example 1 (it's a Multiple Choice Question):

Voici un tableau de proportionnalité.							
<table border="1" style="display: inline-table; vertical-align: middle;"> <tr><td style="padding: 2px 5px;">3</td><td style="padding: 2px 5px;">1,5</td></tr> <tr><td style="padding: 2px 5px;">4</td><td style="padding: 2px 5px;">2</td></tr> </table> } $x \dots$	3	1,5	4	2	$\frac{4}{3}$	$\frac{3}{4}$	0,5
3	1,5						
4	2						
Le coefficient de proportionnalité manquant est...							

(Transmath, n°60 p100)

Proportionality is explicitly stated in the given. The question is specific (calculating a coefficient) and refers to a notion in the lesson. We can think that the absence of context allows an application of the lesson without any particular difficulty.

The question was coded SIT-Coef.

Example 2 :

The price of a pair of sunglasses has increased by 3.2€.
Its initial price was 40€.

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What percentage of the initial price does this increase represent? (Phare, n°3 p122)

The necessary numerical values are provided in the given, the student must however adapt his/her knowledge to the situation. The question was coded A1-QP (in fact, it has been decided to coded percentage calculations QP: they are introduced in grade seven as a calculation of the fourth proportional).

The distinction between SIT and A1 is done according to the context. The existence of a casing, the existence of examples close to the exercise situation in the lesson, the position of the exercise in the chapter are all elements which can be taken into account while deciding of the necessary qualification and adaptation for the student.

Example 3 :

1st example: Give a possible scale for a doll house which is 50 cm high. (Domino, n°39p63)

It is necessary at this point to introduce the order of magnitude of the size of a house (as we imagine it), and this was coded A2-Order of magnitude. Two other activities are coded for this exercise: the change of unit (A3- Length conversion) and the scale calculation once the real height is chosen and the conversions computed (SIT-Coef).

Example 4:

1. Represent by a circular diagram Distribution of the population by the distribution of the population continent in 2003 by continent in 2003

Continent	Percentage
Africa	13.6%
America	13.7%
Asia	60.7%
Europe	11.5%
Antarctica	0.5%

2. What complimentary information would allow you to complete the study?

(Domino, n°45 p64)

To solve question 1), the student must introduce numerical data: in one way or another, s/he must introduce a line for the "Total". This part of the question is coded A2-Total.

Just like the below exercise, s/he must decide of which measure to make in question 1) (This part of the question is coded A2-Measures).

Example 5:

The sketch of an apartment is given, dimensions a, b, c and d are coded.

Length a is in reality equal to 17.2m.

Determine the scale of this sketch.

Determine the real lengths b, c and d. (Phare, n°8 p123)

Example 6:

During a cross Atlantic sailing trip, the skipper has noted the caps in order to trace back the route he sailed.

A broken line is drawn (6 segments, “Boston” on the left, “Lisbon” on the right).

What is, rounded to the nearest millimeter, the length of the broken line representing his route?

His drawing is done at a scale of 1/600 000 000. What distance did he sail? (Sésamath, n°14 p77)

The student will probably realize the usefulness of converting the length unit in order to answer question b., conversions of lengths being old skills (reviewed in grade 6). This part of the question is coded A3-Length conversion. The use of a scale in question b. was coded A1-Cdi. Note that this is one of the rare cases in the studied set of exercises where the coefficient is used with only one value (Cdi or Cmu).

Example 7:

In a salted swamp, for each 1000 g of evaporated water we get 32 g of salt.

a. Calculate the mass of salt obtained from 5000Kg of sea water.

b. How much water must be evaporated in order to obtain one ton of salt?

(Transmath, n°17 p96)

Even if undoubtedly the chosen model here is proportionality, it is still part of the student’s job to indicate that (and eventually be convinced of it). This part of the work was coded A4-Rr.

Example 8:

Below are the US Metric conversions:

For length: 1 yard = 0,9144 m 1 foot = 12 inches 1 yard = 3 feet 1 mile = 1760 yard	For mass: 1 pound = 16 ounces 1 pound = 0,4536 kg
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Express the height and weight of the students of the class using these units. (Domino, n°69 p67)

It is the student’s job to set up his work method in order to find the adapted units to the situation, and to make the corresponding calculations. This part of the question was coded A6-Magnitude.

It must be noted, that here also, the qualification as A6 can depend on the context. It is possible that while reasoning, a student might have to make method choices, the adaptation can hence be classified as A1 for instance if the choice is very local and is made explicit and detailed in the lesson. When an incomplete

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proportionality table is given, there exist, for example, several methods to complete it (calculating a coefficient, adding columns...). This type is generally classified as A1.

Example 9:

The speed of sound is 340 meters per second and that of light is 299 792 480 meters per second.

- a. Express these speeds in kilometers per hour.
(...) (Sésamath, n°28 p78)

The notion of speed and corresponding conversions of measurement units are studied in grade eight (coded A7-Speed).

CONCLUSIONS

In the work discussed above (Hache, 2008), we have tested, in the context of research in mathematics didactics, constraints and leeway perceived throughout the conception of a school textbook. We described in particular the methodology that was put in place to test the similarity between sets of exercises proposed in four French textbooks (for the chapter about proportionality in grade seven). For this purpose, we have analyzed each exercise in the textbook detailing the different adaptations needed for solving it, linking them to the mathematical skills at work.

The conclusions can be of different orders. We can first ask ourselves whether the analyses made reinforce the impression of uniformity in the content of the textbooks for the reader, and thus by this means, question the effectiveness of the constraints perceived during the design. We can furthermore draw conclusions regarding the processing of proportionality in the textbooks: the analyses conducted allow us to distinguish the types of activities proposed to students... and certain activities which are not proposed.

Before concluding about these issues, we need to highlight the gap that exists between the analyses of the exercise statements in a textbook and the student activities in the classroom. Three important filters intervene between the two:

- the teacher, when preparing his session, assuming that s/he picks exercises for the students only from the designated textbook, will not solve all of them. Quantitative analyses including the set of all exercises are thus left with only a restricted meaning, an indicative value: this set of exercises corresponds to the offer proposed to teachers, the offer would be more interesting for certain types of activities possible for the students, and less interesting for others. The teacher could (or not) differentiate, in a certain way, these relative important notions while selecting the exercises.

- On the other hand, the teacher can modify the exercise statement. S/he can do it consciously and a priori, before proposing it to the students, but s/he will above all (and necessarily) do it during the teaching session. We can see in other chapters of this book that these modifications of the proposed tasks are usually in the sense of

a simplification and a reduction of the adaptations which are left for the students. Refer to the work for Sara Arditì about this last point (Arditi, 2011).

- Lastly, the study of the gap between the possible activity inferred from the analysis of an exercise statement and the actual activity of a student in the classroom while working on this statement is a research filed still under development.

The analyzed exercises show that nevertheless, there are heavy and recurring tendencies while processing a chosen notion. We could allow ourselves to deduce some elements with respect to the teaching of proportionality. The objective of the study was more restricted and consisted in studying the proposed content in the textbooks, and estimate the similarity between textbooks.

Globally, even if the global “portrait” of the textbooks leaves some room for fluctuations (the perceived tendencies are well underlined), we can still observe some specificities for each textbook.

We can first detect that an important part (usually more than half¹⁶) of the possible student activities are about simple and isolated tasks, no adaptation is needed for the use of the intended skill. If we include the uses where students need to “simply” recognize the application modalities, we realize that in almost three quarters of the situations the skills are used without leaving any real initiative to the student: the work necessary for learning is thus mainly perceived as using skills without really taking any step back. This phenomenon is very stable in three of the textbooks and less clear in one of them (Sésamath), where the reasoning steps are, for example, left to the students (in fact, it’s mostly about introducing the idea that’s there’s actually proportionality).

Moreover, it can be noted that the skills related to proportionality and used in the exercises are mainly linked to the exploration of a proportionality situation (calculating missing values), and are barely linked to the questioning about the proportional character of the situation (the ratio is almost two thirds to one third). In this case also one of the textbook (Domino) is shifted compared to the others: more than half the items are linked to the question of proportionality and not to the exploration of the situation of proportionality.

In these chapters about proportionality, many skills not directly related to proportionality are used: skills related to measures of magnitudes, to graphs, to tables, to geometry, to writing numbers... Globally, the proportion of skills linked to proportionality represents the majority, this phenomenon is found in three textbooks. In one of the textbooks (Domino), we find a more significant proportion (66% instead of 38% for the other three textbooks) of possible activities which use skills not directly related to proportionality.

The skills not directly related to proportionality vary from one textbook to the other depending on the explored situations. The contexts of exercises, the recommended tools, the skills used differ (scales, uniform speeds, percentages, graphical representation of data...).

Beyond these variations, which lead us to believe that the leeway given to authors is only used in minimal way¹⁷, the global portrait of the work about proportionality in the textbooks is very particular: using mainly calculations instead of

explorations and exercises which are almost systematically guided (the student only having to take punctual initiatives). Even we globally notice a relative variety of the tackled notions accompanying work done on proportionality, for each textbook the skills practiced are not many (even very few for Phare and Sésamath)... We detect through these analyses a relative absence of exercises allowing the student to take ownership of the question of proportionality for a given situation, to take the initiative in choosing the calculations to be done, the method to be followed, or the other skills that need to be mobilized.

Certain questions are asked with more acuity stemming from this work. Are the results found generally applicable? In particular, the small number of situations allowing the student to take the initiative for some adaptations would be reason to worry about the reality and nature of the possible learning from exercises. A study about several contents would allow us to make progress regarding this point.

¹ We can refer to the thesis of Ben Salah Breigeat (2001) which focuses on the implementation of the practices of young teachers, and analyzes among other things the similarity between their discourse and that of the textbook used. The work of Robert et Robinet (1989) studies the meta-cognitive representations of authors of textbooks through exercises they propose. In parallel, the APMEP (association of mathematics teachers in public teaching) has guided several reflections about the textbook as an object.

² Mainly 11 year old students.

³ Mainly 12 year old students.

⁴ School grades six, seven, eight and nine (mainly 11 to 14 years old students).

⁵ We can for example consider the revenue from selling these textbooks when they are proposed separately from the student textbook .

⁶ Mainly 6 to 10 years old students.

⁷ The Sésamath textbook is distinguished by the fact that it was designed by a group of around 50 teachers (regarding the grade seven textbook). Several teams work in parallel and the deadline for elaborating a textbook is so far around two years. It should also be noted that the e-version of the textbook (can be downloaded) can theoretically be modified even after the paper form is published.

⁸ The word «publisher» remains relatively blurry here. It designates at the same time a company, the people working in it, including but not exclusively those who work on a mathematics textbook....

⁹ Note that nevertheless among current textbooks commercial marketed in middle school, two are distinguished with respect to this point of view: the Sésamath textbook, the editor participates a bit in the design. And the venture math textbook (Busser et Massot, 2006) designed and edited by the team of a magazine (Tangente).

¹⁰ Mainly 17 year old students

¹¹ General Inspection of National Education.

¹² Official documents which complete, explain and enrich the texts of the programs.

¹³ Paradoxically the teachers claim at the same time using exclusively lists of exercise in their daily practice.

¹⁴ The word « activity » in the textbooks very generally refers to exercises that are to be done before the lesson to introduce it: revision, discovery and introductory exercises... This meaning is different than that of the theoretical framework of this book.

¹⁵ For more simplicity, we will name the textbooks by the collection names (and not by the author name), we will only indicate the grade level and year when it's not a grade seven class in 2006.

¹⁶ For more quantitative data see Hache 2008.

¹⁷ The notion and textbooks were chosen in a way that maximizes differences between the textbooks (see above, the paragraph about methodology).

LE CAS DES MANUELS DANS L'ENSEIGNEMENT DES MATHÉMATIQUES

It seems fundamental on the other hand to complete the above analyses in two directions: do the choices made by the teachers and the modifications made change the conclusions drawn above? In what sense? In what way, if need be, does the work of students on exercise statements, whether in the classroom or at home, compensate for the evoked gaps?

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