Behavior-Based Algorithmic Pricing

Antoine Dubus

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Abstract

This article studies the impact of algorithmic pricing on market competition when firms collect data to charge personalized prices to their past customers. Pricing algorithms offer to each firm a rich set of pricing strategies combining first and third-degree price discrimination: they can choose for each of their past customers whether to charge them personalized or homogeneous prices. The optimal targeting strategy of each firm consists in charging personalized prices to past customers with the highest willingness to pay and a homogeneous price to the remaining consumers, including past customers with a low valuation on whom a firm has information. This targeting strategy maximizes rent extraction while softening competition between firms compared to classical models where firms target all past customers. In turn, price-undercutting and poaching practices are not sustainable with behavior-based algorithmic pricing, resulting in greater industry profits.

Keywords: Algorithmic Pricing; Data Collection; Behavior-based price discrimination.

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1 Introduction

With the advances in information technologies, companies are developing sophisticated pricing strategies based on the large amounts of data that they collect on their customers (Hinz et al., 2011; DalleMule and Davenport, 2017). Firms have now their data-management functions and chief data officers, and they are increasingly using practices of behavior-based price discrimination (BBPD), under which they collect data on their customers to propose them personalized offers and prices. Practices of BBPD are especially becoming common on the Internet (Gorodnichenko et al., 2018), where a firm such as Amazon can collect data on search behavior, GPS localization, and any type of personal information to feed machine-learning algorithms to personalize ads, products, and prices to the needs of its customers (Shiller et al., 2013). Recent studies document practices of BBPD in various industries such as newspapers (Asplund et al., 2008), credit markets (Ioannidou and Ongena, 2010) and mortgage markets (Thiel, 2019) among many others.

The widespread adoption of algorithmic pricing techniques has transformed the way companies use consumer data and design their pricing strategies (Calvano et al., 2020; Wang et al., 2022; Peiseler et al., 2022). Behavior-based algorithmic pricing, under which firms train pricing algorithms using data collected on past customers, brings two new elements to the classical models of BBPD analyzed by the literature (Fudenberg and Tirole, 2000; Acquisti and Varian, 2005; Fudenberg and Villas-Boas, 2006).[1] On the one hand, firms can design sophisticated pricing strategies using algorithms, combining techniques of first and third-degree price discrimination to enhance their profits (McSweeny and O'Dea, 2017; Gautier et al., 2020). While a classical assumption of the literature is that firms use all the information that they have collected on their customers, they are not constrained to do so once they adopt algorithmic pricing techniques. Instead, with algorithmic price discrimination, firms can choose for each past customer whether to charge a personalized price or to use a flexible third-degree price discrimination by pool-

[1] See also Chen and Pearcy (2010) who allow firms to reward loyal consumers, and Esteves et al. (2022) who consider general distributions of consumer preferences.

ing different consumers who are charged homogeneous prices. On the other hand, there is a growing recognition in the literature that firms can use algorithms as devices to commit to a specific pricing strategy (Salcedo, 2015; Klein, 2018; Bisceglia and Padilla, 2023; Brown and MacKay, 2023; Loots and den Boer, 2023).

This article embeds these two new elements in a model of behavior-based algorithmic pricing, to analyze how the ability for firms to target past customers strategically shapes their decision to collect consumer data, and impacts the competitive structure of data-driven industries. By doing so, it provides new theoretical evidences on the anti-competitive impact of pricing algorithms. It also derives important recommendations for companies willing to exploit at best the potential of their customer data bases, now that they have access to efficient algorithms that allow them to fine-tune their pricing strategies.

The analysis considers a theoretical framework where competing firms collect data on their consumers, and then use an algorithm to design their pricing strategy. It builds on the flexible model of Choe et al. (2018), who consider a two-period competition framework à la Fudenberg and Tirole (2000). In the first period, firms have no information on their customers and charge a homogeneous price to the whole market. At the end of the first period, each firm can perfectly learn the willingness to pay of its customers for its product. In the second period of the framework of Choe et al. (2018), and in line with the literature, each firm uses all available information to price discriminate past customers. Yet, using all available information may not be profit-maximizing for a firm, as information has two opposite effects on its profits. On the one hand, targeting consumers increases the profit of a firm through a better extraction of consumer surplus. On the other hand, information also intensifies competition, which reduces the profits of both firms. Indeed, when both firms target all their past customers, they price aggressively to poach consumers located far away from their locations. This increases the intensity of competition between firms and limits their ability to extract surplus from targeted consumers (Thisse and Vives, 1988). In this sense, firms targeting all past customers are 'non-strategic', and to the best of my knowledge, new practices of information design by firms using pricing algorithms have not yet been analyzed by previous literature, neither have their implications for firms and consumers.

The novelty of this article is to introduce pricing algorithms allowing firms to choose among a rich set of pricing strategies in the second competition period: a firm that has information on a group of past customers can choose to which consumers among this group it charges personalized prices or homogeneous prices. Such strategic targeting allows a firm to commit to a price structure that maximizes the surplus-extraction effect of information while softening its competitive effect.

Using this framework, I characterize the optimal strategy of a firm, which consists in targeting consumers with the highest willingness to pay for its product, and to charge a homogeneous price to a large share of low-valuation consumers – including past customers on whom the firm has collected data – to soften the intensity of competition. Hence, in equilibrium, firms do not use all available information, but charge personalized prices only to high-valuation consumers.

Central to the design of a targeting strategy is the ability for firms to commit to their strategy, and for this reason, this article also contributes to the literature on firms' commitment not to price discriminate consumers. While there are clear benefits from charging uniform prices to all customers and softening the intensity of competition between firms, the literature has also highlighted the difficulties of implementing such commitment. As Corts (1998) argues, even when competing firms can commit not to price discriminate consumers "the prisoner's-dilemma nature of the payoffs [...] dictates a unique equilibrium in which both firms discriminate" (p. 319).[2] Indeed in our framework, a non-strategic firm – that targets either all past customers or charges them a uniform price – has a unilateral incentive to price discriminate all customers and will never commit to charge uniform prices. An important contribution of this article is therefore to show that strategic targeting provides firms with incentives to unilaterally commit not to use all consumer information.

This new result has important implications for industries willing to implement strategic consumer targeting, and raises the question of how firms can credibly

[2]The author then provides conditions on firm asymmetry for a commitment not to price discriminate to be sustainable.

commit not to target all past customers. Besides allowing firms to design sophisticated pricing strategies, pricing algorithms also offer a simple way for firms to implement such a strategic commitment. The use of pricing algorithms as devices to commit a specific strategy is indeed the topic of a growing literature (Salcedo, 2015; Klein, 2018; Bisceglia and Padilla, 2023; Brown and MacKay, 2023; Loots and den Boer, 2023). In these models, the use of a pricing algorithm is observable by a firm's competitor and provides credible information on the ability of the firms to commit to a pricing strategy. Hence, a company that is transparent on the type of pricing algorithms that it uses sends a valuable signal to its competitors, which can be used as a commitment device to engage into strategic targeting.[3] For instance, Uber has publicly adopted route-based pricing techniques for the rides of its users,[4] and United Airlines and Delta have also made public their adoption of pricing algorithms for frequent users.[5] This result has important managerial implications, as it emphasizes the role of algorithmic pricing techniques on the sustainability of pricing strategies.

The remainder of this article is organized as follows. Section 2 describes the model. Section 3 characterizes the optimal targeting strategies of the firms, as well as the equilibrium in both competition periods. The case of forward-looking consumers who anticipate the targeting strategies of the firms is considered in Section 4. Section 5 discusses the main findings and concludes.

2 Description of the Model

Two horizontally differentiated firms – Firm A and Firm B – compete in a product market. There are two competition periods $s = 1, 2$, in which firms sell non-durable goods.[6] In the first period, firms have no information on consumers and

[3] Bertini and Koenigsberg (2021) also discusses how the adoption of algorithmic dynamic pricing can be used as a commitment to consumers to charge them fair prices, compared with "hand made" dynamic pricing.

[4] Is Uber Really Charging Frequent Users Higher Fares?; March 30, 2018

[5] United follows Delta in bringing dynamic pricing model for loyalty program reward redemption, Corporate Travel Community, April 19, 2019.

[6] Other models also consider an infinite number of competition periods (Villas-Boas, 1999, 2004). The results of this article are not affected by these different timing structures, and we focus on a two-stage framework for simplicity.

compete by setting homogeneous prices. Firms then learn the willingness to pay of each of their customers for their product, and in period 2, firms use a pricing algorithm allowing them to charge targeted prices to some of their past customers. Both firms incur the same marginal cost of production, which is normalized to zero, and in each period consumers have unit demands.

2.1 Consumers

Consumers are uniformly distributed on a unit line $[0, 1]$,[7] and in each period s they can buy one product at a price p_{As} from Firm A located at 0, or p_{Bs} from Firm B located at 1.[8] Consumers located at $x \in [0, 1]$ derive a utility V from purchasing the product. They incur a transportation cost $t > 0$ so that buying from Firm A (resp. from Firm B), has a total cost tx (resp. $t(1 - x)$). In each period, consumers purchase the product for which they have the highest utility, and in period 2, different consumers may pay different prices as firms know at this point the willingness to pay of their past customers.

In period $s = 1, 2$, consumers located at x have a utility function defined by:

$$u_s(x) = \begin{cases} V - p_{As} - tx, & \text{if they buy from Firm A,} \\ \\ V - p_{Bs} - t(1 - x), & \text{if they buy from Firm B.} \end{cases} \tag{1}$$

Consumers are assumed to be myopic in the baseline model, and they maximize their utility at each consumption period.[9] Moreover, we assume that there is no cost to switch from one product to the other after the first period. While switching costs in BBPD models is the topic of intense research (Mehra et al., 2012), we will see that they do not impact the outcome in this framework, as even at no cost, switching will not occur in equilibrium.

[7] Uniform consumer distribution is a standard specification of the literature, whose limits have recently been discussed by Esteves et al. (2022).

[8] The market is assumed to be covered, which is a standard consideration of the literature. See for instance Thisse and Vives (1988), Liu and Serfes (2004), Stole (2007), Ulph and Vulkan (2000), Montes et al. (2019), and Bounie et al. (2021).

[9] We relax this assumption by considering forward-looking consumers in Section 4.

2.2 Firms

This section describes the set of targeting strategies available to firms using pricing algorithms in the second period. It then provides the profits of the firms in each period of the game, as well as their objective functions.

Firms first compete in period 1 – the information acquisition period – in which each firm collects perfect information on its customers.[10] Let us denote by $\tilde{x}_1$ the consumer indifferent between buying from Firm A and Firm B in the first period, such that Firm A serves consumers on $[0, \tilde{x}_1]$ and Firm B serves consumers on $[\tilde{x}_1, 1]$.

2.2.1 Targeting Strategies

In period 2 – the targeting period – firms use pricing algorithms that simultaneously choose which of their past customers they price-discriminate. By using these algorithms, firms publicly commit to their strategy, and they set prices accordingly.[11] A pricing algorithm allows firms to target two different types of intervals of the consumer demand:[12]

(a) On the first type of intervals a firm charges personalized prices to all past customers.

(b) A firm can charge a homogeneous price to customers in the second type of intervals.

Figure 1 illustrates these two types of intervals for Firm A.

[10] Choe et al. (2018) also adopt a model where data allows firms to perfectly learn the location of each customer that it serves. This specification is required for firms to have sophisticated targeting strategies, which is the focus of this article.

[11] We discuss in detail how firms can implement such commitment in practice Section 5.

[12] These possible targeting strategies are in line with recent literature on information design in models with horizontal differentiation (Bounie et al., 2021).

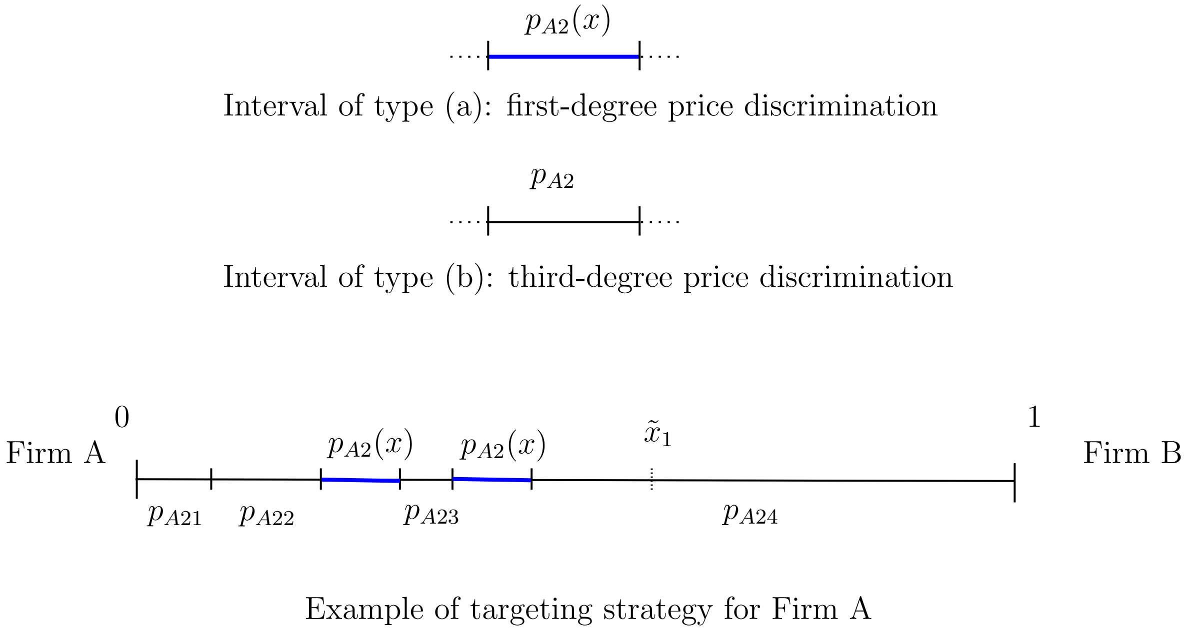

Figure 1: Possible types of intervals when Firm A targets consumers strategically.

On intervals of type (a) (in blue) Firm A charges to each customer a personalized price $p_{A2}(x)$. Firm A charges a homogeneous price p_{A2i} to all consumers in the ith intervals of type (b) (starting from the left).

A pricing algorithm allows for any combination of type (a) and (b) intervals. For instance, the last line of Figure 1 displays from the left to the right two type (b) intervals where Firm A charges respectively prices p_{A21} and p_{A22}. Then Firm A charges personalized prices to each consumer on an interval of type (a) represented in blue. Firm A then charges a homogeneous price p_{A23} to consumers in a third type (b) interval, personalized prices in a second (blue) type (a) interval, and finally, a homogeneous price p_{A24} for consumers on the rest of the line.

Remember that $\tilde{x}_1$ denotes the indifferent consumers in the first competition period, so that Firm A has no information on consumers located at the right of $\tilde{x}_1$ and must charge them a homogeneous price. In Figure 1, these consumers are pooled with some consumers located to the left of $\tilde{x}_1$ even though Firm A has information about them, and they are charged price p_{A24}. Similarly, Firm B has no information on consumers located to the left of $\tilde{x}_1$ and also charges them a homogeneous price. A novel result of this analysis is that, in equilibrium, firms charge the same price to consumers on whom they have no information and to some of their past customers even though they have information about them, as doing so softens the intensity of competition.

In period 2, Firm A's pricing algorithm can use any potential combination of type (a) and type (b) intervals to maximize profits. Let us denote by $\Psi_A(\tilde{x}_1)$ the continuous set of all possible partitions of the unit line into type (a) and type (b) intervals. This set depends on the share of customers on whom Firm A has collected data in period 1: $[0, \tilde{x}_1]$. Similarly, Firm B can choose any partition in the set $\Psi_B(\tilde{x}_1)$ generated by the information it has on customers in $[\tilde{x}_1, 1]$.

When choosing its targeting strategy, the algorithm of Firm θ ($\theta = A, B$) selects the partition $\mathcal{X}_\theta \in \Psi_\theta(\tilde{x}_1)$ of consumers to price discriminate. For a given partition $\mathcal{X}_\theta$, let us denote by $\mathcal{X}_\theta^a$ the subset of type (a) segments, and by $\mathcal{X}_\theta^b$ the subset of type (b) segments, such that $\mathcal{X}_\theta = \mathcal{X}_\theta^a \cup \mathcal{X}_\theta^b;\ \mathcal{X}_\theta^a \cap \mathcal{X}_\theta^b = \emptyset$.

Note that this strategy space includes as special cases the models of Fudenberg and Tirole (2000) where firms charge a homogeneous price to all past customers and another price to customers of the competitors, and of Choe et al. (2018) where firms target all past customers, and Firm A uses only one type (a) interval on $[0, \tilde{x}_1]$ and similarly for Firm B on $[\tilde{x}_1, 1]$.

While previous literature has focused on firms that price-discriminate all consumers that they have identified, this article considers strategic targeting as using all available information may not be optimal for a firm. There are indeed two opposite effects of information on the profit of a firm. On the one hand, targeting consumers increases the profit of a firm through a better extraction of consumer surplus. On the other hand, information also increases competition, which reduces the profits of both firms. Indeed, when both firms target all their past customers, they price aggressively to poach consumers located far away from their locations. This increases the intensity of competition between firms and limits their ability to extract surplus from targeted consumers. Section 3.1.1 characterizes the optimal information structure for each firm, which balances these two effects of information on the profits of the firms.

2.2.2 Profits

Profits in period 1.

This analysis focuses on Subgame Perfect Nash Equilibria.[13] At the beginning of period 1, firms only know that consumers are uniformly distributed on the unit line. The objective of Firm θ is to set a homogeneous price – denoted by $p_{\theta 1}$ and characterized in Section 3 – to maximize its total profits, composed of the sum of its profits in both periods by discounting period 2 with factor δ.[14] As is standard in Hotelling competition with uniform prices, the resulting demand can be written $d_{\theta 1} = \frac{p_{-\theta 1} - p_{\theta 1} + t}{2t}$, where $d_{A1} = \tilde{x}_1$ and $d_{B1} = 1 - \tilde{x}_1$. The profit of Firm θ in period 1 can be written $\pi_{\theta 1} = d_{\theta 1} p_{\theta 1}$.

Profits in period 2.

In period 2, the pricing algorithms first determine the targeting strategies of the firms, and then firms charge prices accordingly. The targeting strategies are chosen as simultaneous best responses and we will show that they constitute the unique pure strategy equilibrium of the game. Hence, in period 1, firms anticipate the equilibrium of period 2 and charge prices $p_{\theta 1}$ and $p_{\theta 2}$ accordingly.

For a given partition $\mathcal{X}_\theta = \mathcal{X}_\theta^a \cup \mathcal{X}_\theta^b$ prices are set as follows. In the set $\mathcal{X}_\theta^a$, prices $p_{\theta 2}(x)$ are set as high as possible under the competitive constraint exerted by price Firm $-\theta$. Firm θ charges homogeneous prices $p_{\theta 2i}$ on each segment in $\mathcal{X}_\theta^b$, with $i = 1, .., n$ and where n corresponds to the total number of segments in $\mathcal{X}_\theta^b$. In each segment in $\mathcal{X}_\theta^b$, prices $p_{\theta i2}$ yield corresponding demands $d_{\theta i2}$. Let us denote by $\mathbf{p}_{\theta 2} = (p_{\theta 12}, .., p_{\theta n2})$ the vector composed of all prices charged in the different segments of $\mathcal{X}_\theta^b$. In period 2, Firm θ sets prices in order to maximize the following profit function:

$$\pi_{\theta 2}(p_{\theta 2}(x), \mathbf{p}_{\theta 2}) = \int_{\mathcal{X}_\theta^a} p_{\theta 2}(x) dx + \sum_{i=1}^{n} p_{\theta i2} d_{\theta i2}. \tag{2}$$

[13]The equilibrium concept is discussed in detail in Section 2.4.

[14]Considering a discounted future for the firms is in line with the literature (Fudenberg and Villas-Boas, 2006). The model includes the limit case where the second period is not discounted when $\delta = 1$.

Moreover, the two following specifications are adopted regarding the targeting and pricing decisions of the firms, as well as sequential pricing.

Sequential targeting and pricing decisions.

The algorithms first choose their pricing strategies, and then firms implement them and charge prices. This timing is common in the literature on algorithmic pricing where firms first calibrate the properties of their algorithm, which then determines a pricing strategy based on the characteristics of the market and of competing firms among other (Hansen et al., 2021; Eschenbaum et al., 2022). It is also used in the theoretical literature on targeted advertising, where firms first choose to which consumers they send an ad, and then set prices accordingly (Anderson and Renault, 2009).

Our focus on this timing is also supported by managerial practices. As Du et al. (2021) emphasize, data analytics teams – in charge of the targeting strategy – and marketing decision-makers – in charge of setting prices – are frequently at arm's length in centralized organizations.

Sequential pricing.

Considering the pricing decisions of the firms, it is necessary to compute demands and prices on each consumer segment to obtain the profits of the firms. When a firm has no information, it sets a uniform price on the whole interval $[0, 1]$. On the contrary, a firm that uses a partition $\mathcal{X}_\theta$ can personalize prices. For each consumer in $\mathcal{X}_\theta^a$, Firm θ will charge a personalized price as a monopolist constrained by the homogeneous price charged by Firm $-\theta$. In the set $\mathcal{X}_\theta^b$, after firms set their prices, there are two types of segments to analyze: segments on which both firms have a strictly positive demand, and segments on which Firm θ is a monopolist.

The model adopts the additional assumption that, after having charged prices on the different segments, each Firm θ can reset its prices on the segments where it is a monopolist. Hence, Firm θ first sets prices $p_{\theta i2}$ on all segments of $\mathcal{X}_\theta^b$, as well as prices $p_{\theta 2}(x)$ for targeted consumers. Then it resets the monopoly prices $p_{\theta 2}(x)$ for each targeted consumer in $\mathcal{X}_\theta^a$ to which it sells its product, and prices $p_{\theta i2}$ on the segments of $\mathcal{X}_\theta^b$ where Firm θ is a monopolist. Consumers observe prices and make their consumption decision after this price reset.

Sequential pricing decision is necessary to avoid the non-existence of Nash equilibrium in pure strategies. Consider indeed the case where Firm A sets prices simultaneously in two different segments – p_{A12} in segment 1 and p_{A22} in segment 2 – and Firm B charges a homogeneous price p_{B2}. The equilibrium prices p_{A12}, p_{A22}, and p_{B2} are chosen as simultaneous best responses. In the case where Firm A is a monopolist on segment 1, the price p_{A12} taken as the best response to p_{B2} is not profit maximizing: an increase in p_{A12} increases the profits of Firm A as long as Firm B does not reach a positive demand on the segments of p_{A12}. Let us denote $\hat{p}_{A12}$ this limit price. The situation where Firm A sets $\hat{p}_{A12}$ and Firm B charges p_{B2} is not an equilibrium either as Firm B has now an additional incentive to increase p_{B2} and reach a positive demand on both segments where Firm A charges $\hat{p}_{A12}$ and p_{A22}, and there is no Nash equilibrium in pure strategy in this case. Under sequential pricing, when firms reset their monopoly prices they do not change their competitive prices anymore, and the resulting prices constitute a pure strategy Nash equilibrium.

For this reason, the decision of firms to set prices in two stages is a standard consideration of the literature on price personalization. For instance, Choudhary et al. (2005), Jentzsch et al. (2013), Matsumura and Matsushima (2015), Chen et al. (2020), Belleflamme et al. (2020) and Bounie et al. (2021) focus on sequential pricing where a higher personalized price is charged to identified consumers after a firm sets a uniform price. We will see that in equilibrium, this assumption boils down to having firms set first their homogeneous prices, and then targeted prices for consumers that they price discriminate.

Sequential pricing is also common in managerial practices. Recently, Amazon has been accused of showing higher prices for Amazon Prime subscribers – who pay an annual fee for unlimited shipping services – than for non-subscribers (Lawsuit alleges Amazon charges Prime members for "free" shipping, Consumer affairs, August 29, 2017). Thus Amazon first sets a uniform price and then increases prices for high-valuation consumers who are better identified when they join the Prime program.

Objective functions of the firms.

Demands in the information acquisition period have an impact on the targeting strategies of the firms in the targeting period. To emphasize the impact of prices in period 1 on market outcome in period 2, the location of the indifferent consumer can be written as a function of prices in period 1: $\tilde{x}_1(p_{A1}, p_{B1})$. Overall the objective functions of the firms at the beginning of the game are:

$$
\begin{aligned}
&\text{For Firm A:} \quad \max_{p_{A1}} \{\pi_{A1}(p_{A1}, p_{B1}) + \delta \pi_{A2}(p_{A2}(x), \mathbf{p}_{A2}, \tilde{x}_1(p_{A1}, p_{B1}))\} \\
&\text{For Firm B:} \quad \max_{p_{B1}} \{\pi_{B1}(p_{B1}, p_{A1}) + \delta \pi_{B2}(p_{B2}(x), \mathbf{p}_{B2}, \tilde{x}_1(p_{A1}, p_{B1}))\}
\end{aligned}
\tag{3}
$$

2.3 Timing

This section summarizes the timing of the game. In period 1, firms compete and collect data on their customers. In period 2, the pricing algorithms choose the partitions $\mathcal{X}_A$ and $\mathcal{X}_B$ of consumers that they target. Then firms set prices on the different segments, and in the last stage, firms reset prices on their monopoly segments. The timing of the game is the following:

- Period 1:
  - Firms compete by setting prices p_{A1} and p_{B1} and learn the location of their customers on the unit line.
- Period 2:
  - Stage 1: Each Firm θ chooses the partition $\mathcal{X}_\theta$ of consumers to price-discriminate and publicly commits to this strategy.
  - Stage 2: Each Firm θ sets prices on the different segments of $\mathcal{X}_\theta$.
  - Stage 3: Firms reset prices $p_{\theta 2}(x)$ on consumers that they price-discriminate in $\mathcal{X}_\theta^a$, as well as prices on the monopoly segments of $\mathcal{X}_\theta^b$.
  - Stage 4: Consumers observe prices and make their consumption decisions.

2.4 Equilibrium Concept and the Role of Commitment

Throughout the analysis, the focus is on subgame perfect Nash equilibria. In period 1, firms anticipate that the information they collect on consumers can be used in period 2. Firms therefore set prices according to two different forces. On the one hand, they want to maximize profits in the first competition period. On the other hand, they may also want to serve a large consumer demand to achieve higher prices in period 2.[15]

In period 2, each firm first determines its targeting strategy, using the information collected in period 1, and commits to this strategy. Then, firms set prices in two stages, and finally consumers observe prices and make their consumption decisions.

Firms commitment to a targeting strategy.

As is standard with Nash equilibria, a firm does not need to observe the targeting strategy of its competitors, but only to know its equilibrium best response. Hence, at the beginning of period 2, the partitions $\mathcal{X}_A$ and $\mathcal{X}_B$ are chosen as simultaneous best responses constituting the unique pure strategy Nash equilibrium of this stage.

Nevertheless, for each firm to have interest not to use all available information, it is necessary that firms can reveal their targeting strategy at the end of stage 1 of period 2, and that the following prices are set according to the strategies of both firms. When the targeting strategy of a firm cannot be observed by its competitor, it has interest to target all customers and engage into all-out competition (Corts, 1998). Hence, for each firm the revelation of its targeting strategies has the value of a credible commitment to use this strategy in the remaining of the game.

This specification is increasingly common in a vast strand of the literature on algorithmic pricing (Klein, 2018; Bisceglia and Padilla, 2023; Loots and den Boer,

[15] The contribution of this article will be to show that, contrary to previous literature, when firms target consumers strategically in period 2, they do not fight for information acquisition in period 1. Hence firms set prices in period 1 only accounting for their present profits, and competition is identical to the standard Hotelling framework.

2023).[16] Overall, these articles argue that, for most algorithmic pricing systems, the learning process and the design of the optimal strategy occur offline, as it is assumed in this article since the firms first design the optimal targeting strategy, and then engage into targeted pricing online. Hence, in the articles above, the adoption of pricing algorithms by firms allows them to commit to a pricing strategy, at least in the short run. This point is also made by Calvano et al. (2020) and Asker et al. (2023) and is well understood in the computer science literature and among industry practitioners. In this spirit, Harrington Jr (2022) considers firms using the algorithm developed by a third party, and delegation allows for strategic commitment. We discuss in Section 5 the different ways through which firms can implement this commitment in practice, which allows us to derive important managerial implications.

3 Analysis

As usual, the analysis proceeds backward. First, the optimal targeting strategies of firms in the second period are characterized in Section 3.1. Section 3.2 analyzes competition in period 1 and information acquisition by firms.

For clarity we summarize here the main results that are then proved and discussed in the following sections.

(a) In equilibrium, firms target only part of their past customers:

- Each firm charges personalized prices to high-valuation consumers.
- Remaining consumers are untargeted, including low-valuation customers on whom firms have information.

(b) The unique equilibrium of the game is symmetric.

(c) Firms do not engage in consumer poaching during the targeting period.

(d) Firms do not undercut prices during the information acquisition period.

[16] See also Peiseler et al. (2022) on collusion when firms use algorithms to price discriminate consumers.

3.1 Period 2: Strategic Targeting

This section characterizes the optimal targeting strategies determined by the pricing algorithms of the firms in period 2 when they can price discriminate their past customers. It shows that strategic firms optimally target close-by consumers with the highest willingness to pay for their products and charge a homogeneous price to all remaining consumers, including some consumers on whom they have information. Such information structure maximizes surplus extraction from consumers with the highest willingness to pay while softening the competitive effect of information.

3.1.1 Optimal Information Structure

In period 2 the firms use pricing algorithms to choose the partitions $\mathcal{X}_A$ and $\mathcal{X}_B$ of past customers that each firm price-discriminates. Proposition 1 characterizes the optimal partitions.

Proposition 1

There exist $x_A \in [0, \tilde{x}_1]$ & $x_B \in [0, 1 - \tilde{x}_1]$ such that in equilibrium:

- *Firm A targets all consumers on $[0, x_A]$ and charges a homogeneous price on consumers on $[x_A, 1]$.*
- *Firm B targets all consumers on $[1 - x_B, 1]$ and charges a homogeneous price on consumers on $[0, 1 - x_B]$.*

Proof: see Appendix A.1.

The proof proceeds in the following way. Considering any information structure for each firm, it shows that, for any targeting strategy adopted by Firm B, Firm A finds it profitable to re-order segments so that Firm A first-degree price discriminates consumers closest to its location, and charges a homogeneous price to the rest of the unit line. Focusing on close-by consumers allows Firm A to extract surplus from customers with the highest willingness to pay, while limiting the competitive effect of information by leaving a large share of consumers who are charged a homogeneous price. Applying this reasoning to Firm B allows us to show that it also has interest to first-degree price discriminate close-by consumers only.

Proposition 1 is a central result of this article and makes an important contribution to the literature, which usually assumes that a firm uses all available information to target consumers. An optimal information partition maximizes the profit of a firm by dividing the unit line into two intervals. Firm A charges targeted prices to consumers in the first interval on $\mathcal{X}_A^a = [0, x_A]$, which is referred to as the share of targeted consumers, who have the highest willingness to pay for Firm A's product. Firm A does not target consumers on $\mathcal{X}_A^b = [x_A, 1]$ – with a lower willingness to pay –, and charges a uniform price on this second interval, referred to as the share of untargeted consumers. Similarly, Firm B optimally targets high-valuation consumers belonging to $\mathcal{X}_B^a = [1 - x_B, 1]$ and charges a homogeneous price to low-valuation consumers on $\mathcal{X}_B^b = [0, 1 - x_B]$. Consumers on $[x_A, 1 - x_B]$ are targeted by none of the firms in period 2. By leaving a share of consumers untargeted by firms, these optimal targeting strategies balance the rent extraction and the competition effects of information.[17]

Figure 2 illustrates the targeting strategies of Firm A and Firm B. The thick lines represent consumers who are targeted by Firm A on $[0, x_A]$, and by Firm B on $[1 - x_B, 1]$. Consumers on segments $[x_A, 1]$ and $[0, 1 - x_B]$ are charged a homogeneous price by Firm A and Firm B respectively. Figure 2 also displays $\tilde{x}_1$, the location of the indifferent consumer in period 1.

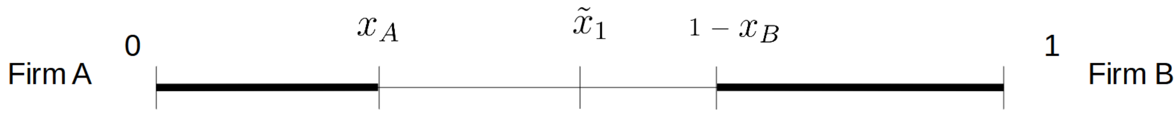

Figure 2: Targeting strategies of firms in period 2.

Our focus in the first stage of period 2 is on pure strategy Nash equilibria when firms can commit to their targeting strategy, characterized by the (unique) equilibrium values of x_A and x_B. As it is assumed that firms choose x_A and x_B

[17]Proposition 1 also generalizes the results of Bounie et al. (2021) by characterizing the optimal targeting strategies of firms using first-party data.

simultaneously, their equilibrium values will be derived by computing x_A as a simultaneous best response to x_B and reciprocally.

Lee et al. (2011) have been the first ones to consider the possibility for firms to use such information partitions to price discriminate consumers, analyzing competition for exogenous values of x_A and x_B and focusing on consumer privacy.[18] The present article contributes to their analysis by formally characterizing the optimal information partitions chosen by firms using collected data, and by providing a proof of the optimality of these information structures.

Moreover, our results also contribute to the literature on firms' commitment not to use personalized pricing. In particular, Corts (1998) has emphasized the difficulties for firms to commit not to price discriminate consumers. Indeed, in the present model, we can show that firms price discriminate all customers when they are not strategic, even when they can commit not to use information. A unilateral deviation to use information is always profitable for a firm. Proposition 1 shows that the ability of firms to target customers strategically gives them incentives to commit not to use all information, which softens the intensity of competition.

Profits in period 2.

In period 2, each firm charges personalized prices $p_{\theta 2}(x)$ to targeted consumers and charges price $p_{\theta 2}$ on the rest of the unit line. Prices $p_{\theta 2}(x)$ are set as high as possible under the competitive constraint exerted by price $p_{-\theta 2}$. Hence, firms set prices in period 2 in order to maximize the following profit functions:

$$\begin{aligned} \pi_{A2}(p_{A2}(x), p_{A2}) &= \int_0^{x_A} p_{A2}(x)dx + p_{A2}d_{A2}, \\ \pi_{B2}(p_{B2}(x), p_{B2}) &= \int_{1-x_B}^{1} p_{B2}(x)dx + p_{B2}d_{B2}. \end{aligned} \tag{4}$$

Strategic interaction between the two periods.

The targeting strategies of Firm A and Firm B are characterized by their choices of x_A and x_B, which can be constrained by the number of consumers on whom firms

[18]See also more recently Chen et al. (2020).

have acquired information. In period 1, Firm A collects information on $[0, \tilde{x}_1]$ and Firm B collects information on $[\tilde{x}_1, 1]$. Therefore, the targeting strategy of each firm must verify: $x_A \leq \tilde{x}_1$ and $x_B \leq 1 - \tilde{x}_1$. Hence, competition in period 1 can have an impact on the targeting strategies of the firms. Note that the situation where each firm targets all its past customers is a special case of this approach, where $x_A = 1 - x_B = \tilde{x}_1$.

3.1.2 Equilibrium Targeting

This section characterizes the optimal number of customers that each firm targets in period 2. Firm A price discriminates consumers on $[0, x_A]$, and charges a homogeneous price to consumers on $[x_A, 1]$. Similarly, Firm B price discriminates consumers on $[1 - x_B, 1]$, and charges a homogeneous price to consumers on $[0, 1 - x_B]$. The choices of x_A and x_B correspond to the targeting strategies of Firm A and Firm B, and their optimal values x_A^* and x_B^* are characterized in this section.

Each firm can target in period 2 customers that it has served in period 1, and the value of $\tilde{x}_1$ may constrain the targeting strategy of firms in period 2. Indeed, if $\tilde{x}_1 \in [x_A^*, 1 - x_B^*]$, both firms can target their optimal number of consumers in period 2. On the contrary, if $\tilde{x}_1 \in [0, x_A^*]$ or if $\tilde{x}_1 \in [1 - x_B^*, 1]$, respectively Firm A or Firm B cannot target their optimal number of consumers, and are constrained in their targeting strategy.

Proposition 2 characterizes the equilibria when $\tilde{x}_1 \in [x_A^*, 1 - x_B^*]$ and firms are not constrained on their targeting strategies, and when $\tilde{x}_1 \in [0, x_A^*]$ and Firm A is constrained (the case where Firm B is constrained is identical).

Proposition 2

(a) *In period 2 when $\tilde{x}_1 \in [\frac{1}{3}, \frac{2}{3}]$ the subgame perfect equilibrium is unconstrained and firms target symmetric shares of consumers:*

$$x_A^* = x_B^* = \frac{1}{3}, \qquad \tilde{x}_2 = \frac{1}{2}.$$

(b) In period 2 when $\tilde{x}_1 \in [0, \frac{1}{3}]$ the subgame perfect equilibrium is constrained and firms target asymmetric shares of consumers:

$$x_A^* = \tilde{x}_1, \qquad x_B^* = \frac{3}{7} - \frac{2\tilde{x}_1}{7}, \qquad \tilde{x}_2 = \frac{6\tilde{x}_1 + 5}{14}.$$

Proof: see Appendix A.2.

In the unconstrained equilibrium characterized by Proposition 2 (a), both pricing algorithms design the same strategies in which firms target only part of their past customers. In period 2 Firm A and Firm B have information on consumers respectively in $[0, \tilde{x}_1]$ and $[\tilde{x}_1, 1]$ (with $\tilde{x}_1 \in [\frac{1}{3}, \frac{2}{3}]$), but they charge a homogeneous price on $[\frac{1}{3}, 1]$ and $[0, \frac{2}{3}]$ to soften competition. We analyze in the next section how strategic targeting impacts the first period of competition.

Proposition 2 (b) characterizes the equilibrium when Firm A is constrained on its targeting strategy and price-discriminates fewer consumers than its unconstrained optimum. This relaxes the competitive pressure on Firm B, which targets more consumers and makes higher profits than in the symmetric equilibrium. Hence it is profitable for a firm to face a competitor constrained on targeting. This can be achieved by undercutting prices in period 1. Indeed, the value of $\tilde{x}_1$ in period 1 depends on the prices set by the firms, and the next section analyzes whether a firm has interest to undercut prices in period 1 in order to constrain the targeting strategy of its competitor in period 2.

An important element of the analysis is the locations of the indifferent consumers $\tilde{x}_1$ in period 1 and $\tilde{x}_2$ in period 2. Indeed, the literature usually finds that BBPD results in poaching practices: some consumers purchase from one firm in period 1, and then from its competitor in period 2. Poaching is considered beneficial for consumers as it results in a more competitive market in period 2 but yields overall inefficiency as some consumers do not buy the product closest to their taste. The next section shows that in the unique equilibrium of the game with strategic targeting, $\tilde{x}_1 = \tilde{x}_2$ and consumers do not switch from firms across periods, so that consumer poaching does not take place.

3.2 Period 1: Information Acquisition

This section analyzes competition in the information acquisition period. In the symmetric equilibrium, firms maximize profits in period 1 and market equilibrium is identical to standard Hotelling competition without data collection. Proposition 3 states that the unique equilibrium is symmetric, and that price undercutting in period 1 to constrain a firm's competitor is not sustainable.

Proposition 3

- *The unique pure strategy equilibrium of the game is symmetric.*
- *Firms do not engage in price-undercutting strategies.*
- *In both periods:*
  - *Consumers on* $[0, \frac{1}{2}]$ *purchase from Firm A.*
  - *Consumers on* $[\frac{1}{2}, 1]$ *purchase from Firm B.*
  - *Poaching does not take place.*

Proof: see Appendix A.3.

Firms do not have interest to engage in constraining strategies: to constrain their competitor in period 2, firms must undercut prices so that the indifferent consumer $\tilde{x}_1$ is very close to the competitor's location in period 1. For instance, if Firm B wants to constrain Firm A, it must be that $\tilde{x}_1 < \frac{1}{3}$. Reaching such a constraining market outcome induces an important loss in period 1 for a firm, and the increase in profits of period 2 is not sufficient to cover this loss.

The literature on BBPD classically finds that competition in the first period is driven by two main forces: firms want to reach high profits in this first period, but they also anticipate the second competition period in which they have information on their past customers. Hence, competition is fiercer than in the standard Hotelling model without information, as a firm has an additional incentive to serve a larger customer demand in period 1 to charge targeted prices to more consumers in the second period (Choe et al., 2018). This second dimension can be interpreted as a 'competition for information acquisition' in period 1. When firms use algorithmic pricing techniques, Proposition 3 shows that they do not charge targeted prices to consumers in the middle of the line, and thus they do not make additional benefits from identifying these consumers in period 1: firms do not compete to acquire consumer information.[19]

Moreover, the same consumer demands are served in both competition periods $\tilde{x}_1 = \tilde{x}_2$. This is a natural consequence of strategic targeting under which firms do not have interest to undercut prices and poach consumers, but soften competition by keeping a large share of consumers untargeted. This result contrasts with previous literature in which poaching occurs in period 2 and some consumers switch products (Fudenberg and Tirole, 2000). Hence, Proposition 3 also contributes to the literature by showing that the adoption of strategic targeting by firms allows them to avoid poaching and price undercutting in the second competition period.

Proposition 3 presents interesting connections with the results of Choe et al. (2018), who show in a similar setting without strategic targeting that the only equilibrium is asymmetric, and poaching occurs in period 2. Proposition 3 states that when firms use pricing algorithms, the equilibrium of the game is symmetric. Pricing algorithms give firms the ability to commit to their targeting strategy, a central precondition for partial consumer targeting to take place. In the case where firms cannot commit, they charge personalized prices to all their past customers and the equilibrium is identical to Choe et al. (2018). Hence, firms' commitments to their targeting strategies have important managerial implications, which are discussed in Section 5.

4 Forward-looking Consumers

Consumers may anticipate in period 1 that firms collect their information to charge targeted prices in period 2 (Li and Jain, 2016). This ability of consumers to anticipate BBPD and purchase products accordingly is a classical consideration

[19]Interestingly, Fudenberg and Tirole (2000) also find that competition in the first period is identical to the standard Hotelling framework when consumers are myopic. In our model, we will see in Section 4 that this result holds also with forward-looking consumers.

of the literature that usually finds a reduction of competition in the information acquisition period as demand becomes less price-sensitive (Fudenberg and Tirole, 2000).

This section considers consumers located in $[0, \frac{1}{3}]$ who anticipate that firms use pricing algorithms and that they will be charged a targeted price in period 2. It analyzes whether these consumers have interest to purchase from Firm B in period 1 in order to remain hidden from Firm A and pay a homogeneous price in period 2. When choosing which product to purchase in the first period, consumers maximize the sum of utilities over both periods by discounting period 2 with a factor δ_c.

Proposition 4

- *The unique equilibrium in both competition periods is identical to the case with myopic consumers.*
- *Strategic consumers purchase from the same firm in both competition periods and do not engage in hiding strategies.*

Proof: see Appendix A.4.

Proposition 4 states that consumers do not have interest to hide from firms, and the equilibrium is identical to the baseline framework with myopic consumers. This result contributes to previous literature that has shown that consumers have interest to change their consumption behavior when they are relatively indifferent between the products of both firms (Fudenberg and Tirole, 2000).

When firms engage in behavior-based algorithmic pricing, only consumers with high valuations for a firm's product are targeted in the second period and may have interest to purchase their least preferred product in the first period. Of course, in practice consumers switch between different brands, which can be explained by many factors. For instance, consumers may discover their true taste for a product after its consumption, or firms can engage in product personalization changing the willingness to pay of consumers for their products. Nevertheless, this analysis suggests that switching is less likely to occur when firms use customer information strategically. Purchasing Firm B's product in period 1 would induce an important opportunity cost for a consumer close to Firm A, which is not recovered in period 2, and consumers purchase their preferred product in both periods. Hence, consumers have no interest to hide when firms use strategic targeting.[20]

5 Discussion and Implications

Committing to a targeting strategy.

The results derived in this analysis rely on the possibility for the firms to commit to a targeting strategy. As mentioned earlier, the possibility of such a strategic commitment is a growing consideration of the literature on algorithmic pricing (Salcedo, 2015; Klein, 2018; Bisceglia and Padilla, 2023; Loots and den Boer, 2023; Brown and MacKay, 2023). Scholars and practitioners have identified two main ways for firms to implement such a commitment in practice.

On the one hand, strategic commitment could be attained through the public adoption of a pricing software by firms. The number of pricing software has indeed increased sharply in the past years, companies such as Vendavo, Glew, Pricemoov or Price2spy now sell their software to major companies in any type of industry, and the adoption of one of these services by a company is usually public.[21] For the commitment to be deviation-proof, firms must not be able to use another pricing algorithm allowing to identify all past customers. This could be guaranteed for large companies as any information disclosed to shareholders is legally binding: the firm cannot lie to its shareholders, in particular when it comes to its technological and market strategies. A firm claiming to use a pricing software while it does not use it would be exposed to legal actions.

On the other hand, as already discussed in the introduction, firms are also increasingly making public statements when they adopt new algorithmic pricing techniques (Salcedo, 2015). While there can be several reasons for a firm to disclose its pricing technology, such as to convince stakeholders of its innovation efforts,

[20] This result holds when one of the firms engages in a constraining strategy and undercuts prices in the first competition period (the proof is available upon request).

[21] See for instance Glew customers, last accessed, March 14, 2023.

this analysis suggests that these public statements could also have the value of a commitment to use a specific targeting strategy, which in turn could be used to relax price competition.

There is an additional way for firms to engage into a strategic commitment. Indeed, firms can disclose their targeting strategy to their competitors by developing their pricing algorithms in open source. In this case, the observability of the pricing decision could have the value of commitment to the other firms. For instance, Airbnb has published an open-source machine-learning package providing its competitors with the core decision factors used by its pricing algorithm.[22] While the literature has for long acknowledged how open source can be used by developers to signal their coding abilities (Lerner and Tirole, 2002; Lee et al., 2003), to the best of my knowledge, this article is the first to highlight the possibility for firms to use open source pricing algorithms to commit to a targeting strategy.

Data intermediation vs. data collection.

These results also have important implications for firms that do not directly collect consumer information, but that purchase data from third parties. For instance, retailers heavily rely on data brokers for their marketing and personalization campaigns (Crain, 2018). Montes et al. (2019), Bounie et al. (2021), and Abrardi et al. (2022) find that data brokers have interest to sell information exclusively to one firm and prevent its competitor from acquiring information.[23] Our results have shown that firms that directly collect data on consumers compete on a level playing field and are not exposed to excluding practices.

Further research.

This simple two-period competition framework could be extended to account for positive data collection costs. Collecting, treating, and storing data is indeed costly, and may reduce the profitability of consumer targeting for firms. In par-

[22] A Look at Airbnb's Pricing Algorithm and Open Source Machine Learning Tool, Dataversity, September 1, 2015.

[23] Delbono et al. (2021) and Bounie et al. (2023) analyze settings in which this exclusivity assumption falls, respectively when data brokers can commit to sell data to a coalition of firms and when they compete.

ticular, asymmetric data collection costs can provide a firm with a significant competitive advantage and could restore asymmetric competition.

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A Online Appendix

A.1 Proof of Proposition 1

This section characterizes the optimal targeting strategy of Firm A. For any given partition used by Firm B, I show that the optimal partition for Firm A is composed of one type (a) segment closest to its location where all consumers are charged personalized prices, and one type (b) segment on the rest of the line where consumers are charged a uniform price. The proof of the optimal partition for Firm B follows the same reasoning.

Firm A can choose any partition $\mathcal{X}_A \in \Psi_A(\tilde{x}_1)$ for a given $\tilde{x}_1$. There are three types of segments to consider:

- Segments α, where Firm A is in constrained monopoly;
- Segments β, where Firms A and B compete.
- Segments γ, where Firm A makes zero profit.

All segments in $\mathcal{X}_A^a$ are necessarily of type α, while segments in $\mathcal{X}_A^b$ may be of type α, β, and γ.

To find the partition that maximizes the profits of Firm A, the proof proceeds in three steps. Step 1 analyzes type α segments, and shows that it is optimal for Firm A to first-degree price discriminate all consumers in these segments. Step 2 shows that all segments of type α are located closest to Firm A. Step 3 analyzes segments of type β and shows that it is always more profitable to target a union of such segments. Therefore, there is only one segment of type β, located furthest away from Firm A, and of size $1-x_A$. Finally, segments of type γ can be discarded because information on consumers on these segments does not increase profits.

Step 1: I analyze segments of type α where Firm A is in constrained monopoly, and show that targeting all consumers is optimal.

Consider any segment $I = [i, i+l]$ of type α with l, i verifying $i + l \leq 1$, such that Firm A is in monopoly on this segment, constrained by Firm B charging price p_B. I compare profits with first and third-degree price discrimination and I show that the former is more profitable for Firm A. I write π_A^{third} and π_A^{first} the profits of Firm A on I with third-degree and first-degree price discrimination.

To prove this claim, I establish that π_A^{first} is greater than π_A^{third}. First, profits with first-degree price discrimination are: $\pi_A^{first} = \int_i^{i+l} p_A(x)dx$. The demand is l as Firm A gets all consumers by assumption.

$$V - tx - p_A(x) = V - t(1-x) - p_B \implies p_A(x) = t - 2tx + p_B.$$

Note that price p_B is only affected by strategic interactions on the segments where firms compete, and therefore does not depend on the pricing strategy of Firm A on type α segments. I write the profit function for any p_2, replacing p_A:

$$\pi_A^{third} = l(t + p_B - 2(l+i)t).$$

Secondly, using a similar argument, I show that the profit with first-degree price discrimination is:

$$\pi_A^{first} = \int_i^{i+l} (t - 2tx + p_B)\, dx.$$

Comparing π_A^{third} and π_A^{first} shows that the profit of Firm A using the first-degree price discrimination is higher than under third-degree price discrimination, which establishes the claim. Therefore, in equilibrium, there is no type α segment in the set $\mathcal{X}_\theta^b$.

Step 2: I show that all segments of type α are closest to Firm A (located at 0 on the unit line by convention).

Going from left to right on the Hotelling line, look for the first time where a type β interval, $J = [i; i+l]$ of length l, is followed by an interval $I = [i+l, i+l+\epsilon]$ of type α.

A simple comparison allows to show that a reordering of the overall interval $J \cup I = [i, i+l+\epsilon]$ in two intervals $I' = [i; i+\epsilon]$ and $J' = [i+\epsilon, i+l+\epsilon]$ increases the profit of Firm A. Indeed, after the re-ordering, full surplus is extracted from consumers in I', who have the highest valuation on this interval. All consumers located on J' are charged a homogeneous price, which softens the competitive pressure on Firm B compared with $J \cup I$ and increases the competitive price charged by Firm B. By iteration, I conclude that type α segments are always at the left of type β segments.

Step 3: I now analyze segments of type β where firms compete. For Firm A, starting from any partition with at least two segments of type β, I show that a coarser partition always increases the profits of Firm A.

The two previous steps have shown that an optimal partition must be composed for each firm of one type α segment closest to its location and potentially several type β segments on the rest of the line, as illustrated in Figure 3.

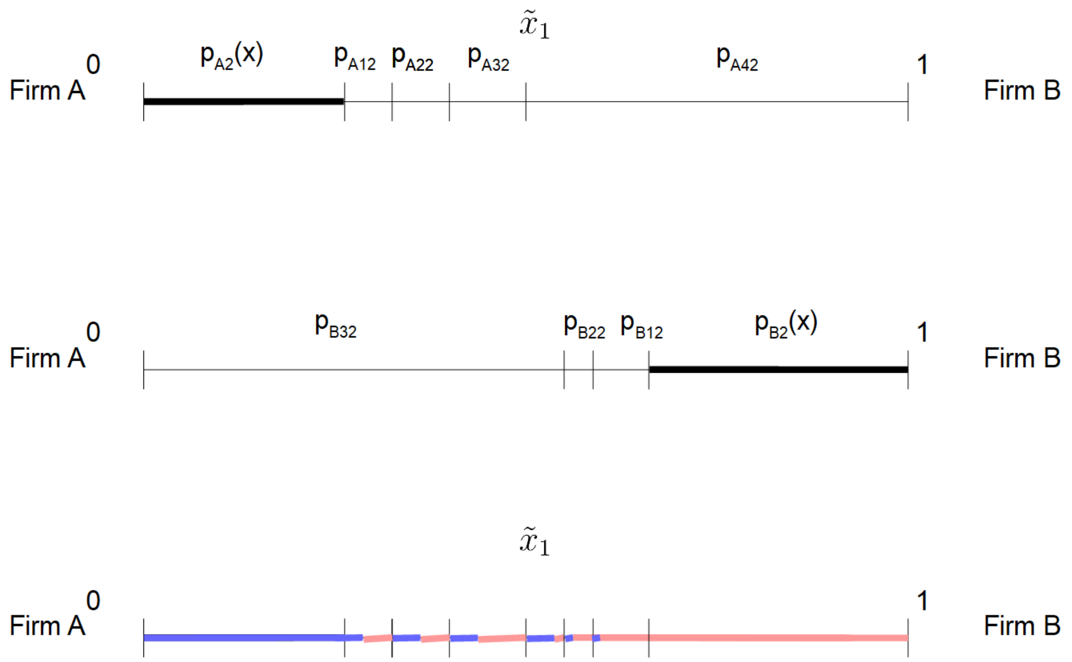

Figure 3: Structures of the optimal partitions for each firm after applying steps 1 and 2.

The first two lines represent respectively the partitions used by Firm A and by Firm B. The thick black lines correspond to the consumers who are charged a personalized price by the closest firm. On each remaining segment of the first line, Firm A charges a homogeneous price: p_{A12} on the first segment starting from the left, p_{A22} on the second segment, and so on. Similarly on the second line, Firm B charges homogeneous price p_{B12} on the first segment after the thick line, starting from the right, price p_{B22} on the second segment and so on.

The third line represents the resulting consumer demands, with in blue the demand of Firm A and in red the demand of Firm B. In each segment where a homogeneous price is charged by both firms, both firms reach a positive consumer demand, or else, one of the firms would charge personalized prices following step 1.

We want to show that the optimal partition has only one type β segment where firms charge a homogeneous price (contrary to four segments for Firm A and three segments for Firm B in the above example).

To do so, we show that if Firm A has a partition $\mathcal{X}_1$ with at least two segments where it competes with Firm B, a coarser partition $\mathcal{X}_1'$ where the two adjacent segments located closest to Firm A are merged yields higher profits.

I compute the profits of Firm A on all the segments where firms compete. There are three types of segments to consider:

1. segments of type α that with partition $\mathcal{X}_1$ that remain of type α with partition $\mathcal{X}_1'$.

2. segments of type β with partition $\mathcal{X}_1$ that become of type α with partition $\mathcal{X}_1'$.

3. segments of type β with partition $\mathcal{X}_1$ that remain of type β with partition $\mathcal{X}_1'$.

Note that there exists $\tilde{x}_B$ such that Firm B charges a homogeneous price on segment $[0, \tilde{x}_B]$ with $\tilde{x}_B > \tilde{x}_1$. Throughout the resolution we denote by $\hat{p}_B$ this price under partition $\mathcal{X}_1$ and $\hat{p}_B'$ with partition $\mathcal{X}_1'$. Similarly, there exists $\tilde{x}_A$ such that Firm A charges a homogeneous price on segment $[\tilde{x}_A, 1]$ with $\tilde{x}_A < \tilde{x}_1$. Throughout the resolution we denote by $\hat{p}_A$ this price under partition $\mathcal{X}_1$ and $\hat{p}_A'$ with partition $\mathcal{X}_1'$. Moreover, for simplicity we denote by $\hat{p}_{Ai}$ and $\hat{p}_{Bi}$ the homogeneous prices charged by Firm A and Firm B on their type β segments.

Profits always increase on segments that are of type α with partitions $\mathcal{X}_1$ and $\mathcal{X}'_1$. Indeed, I will show that $\hat{p}'_B$ with partition $\mathcal{X}'_1$ is higher than $\hat{p}_B$ with partition $\mathcal{X}_1$, and thus the profits of Firm A on type α segments increase.

It will also be useful to introduce the following notations. On interval $[0, \tilde{x}_A]$, there are n segments where firms compete. Among them, there are m segments which are type β in partition $\mathcal{X}_1$, but are no longer necessarily of type β in partition $\mathcal{X}_1$ (and are therefore of type α). There are $n+1-m$ segments of type β with partition $\mathcal{X}_1$ that remain of type β with partition $\mathcal{X}'_1$. I compute prices and profits on these $n+1+m$ segments.

On interval $[\tilde{x}_A, \tilde{x}_B]$, withe partition $\mathcal{X}_1$ ($\mathcal{X}'_1$) Firm A charges price $\hat{p}_A$ ($\hat{p}'_A$) and Firm B charges price $\hat{p}_B$ ($\hat{p}'_B$).

To compare the profits of the informed firm under both partitions, I first characterize type β segments. A segment of type β is non null, if the following restrictions imposed by the structure of the model, are met: respectively positive demand and the existence of competition on segments of type β. In order to characterize type α and type β segments, it is useful to consider the following inequality:

$$\forall \;\; i, l \in [0,1] \;\; s.t. \;\; 0 \leq l \leq 1-i, i \leq \frac{\hat{p}_B + t}{2t} \;\; \text{and} \;\; \frac{\hat{p}_B + t}{2t} - l \leq i + l. \tag{5}$$

In particular, I use the relation that Eq. 5 draws between price $\hat{p}_B$ and segments endpoints i and $i+l$ to compare the profits of Firm A with partitions $\mathcal{X}_1$ and $\mathcal{X}'_1$. Without loss of generality, I rewrite the notation of type α and β segments. The segment of type α is of size ϵ and is located at $u_i - \epsilon$, and segments of type β are located at s_i and are of size l_i.[24] On interval $[0, \tilde{x}_B]$, here are $n \in \mathbb{N}$ segments of type β, where prices are noted $\hat{p}^{\beta}_{Ai}$. On interval $[\tilde{x}_B, 1]$, there are n' segments where firms compete with Firm A charging price $\hat{p}_A$ and Firm B charging prices $\hat{p}_B$ and $\hat{p}^{\beta}_{Bi}$.

Our next point will be to show that profits on interval $[0, \tilde{x}_B]$ where Firm B charges a homogeneous price increase with partition $\mathcal{X}'_1$. Profits on interval

[24]With u_i and s_i lower than 1.

$[\tilde{x}_B, 1]$ clearly increase, as we will show that $\hat{p}_B$ increases, relaxing the competitive pressure on Firm A in this interval.

I find the demand for Firm A on segments in $[0, \tilde{x}_B]$ using the location of the indifferent consumer:

$$d_{Ai} = x - s_i = \frac{\hat{p}_B - \hat{p}^{\beta}_{Ai} + t}{2t} - s_i.$$

I can rewrite the profits of Firm A as the sum of three terms. The first term represents the profits on segments of type α. The second term represents the profits on segments of type β on interval $[0, \tilde{x}_B]$, the third term represents the profits on segments of type β on interval $[0, \tilde{x}_B]$.

$$\begin{aligned}\pi_A(\mathcal{X}_1) = &\int_0^{s_1} \hat{p}^{\alpha}_A(x)dx + \sum_{i=1}^{n} \hat{p}^{\beta}_{Ai}[\frac{\hat{p}_B - \hat{p}^{\beta}_{Ai} + t}{2t} - s_i] \\ &+ \hat{p}_A[\frac{\hat{p}_A - \hat{p}_B + t}{2t} - \tilde{x}_A] + \sum_{i=1}^{n'} \hat{p}_A[\frac{\hat{p}^{\beta}_{Bi} - \hat{p}_A + t}{2t} - s_i].\end{aligned}$$

The price $\hat{p}_B$ is chosen by Firm B to maximize local profits generated on segments of type β only, where the demand for Firm B is:

$$d_{Bi} = s_i + l_i - x = \frac{\hat{p}^{\beta}_{Ai} - \hat{p}_B - t}{2t} + s_i + l_i.$$

Firm B sets price $\hat{p}_B$ to maximize profits on interval $[0, \tilde{x}_B]$, which can be written as:

$$\pi_{Bl}(\mathcal{X}_1) = \sum_{i=1}^{n} \hat{p}_B[\frac{\hat{p}^{\beta}_{Ai} - \hat{p}_B - t}{2t} + s_i + l_i]. \tag{6}$$

Firm A maximizes profits $\pi_A(\mathcal{X}_1)$ with respect to $\hat{p}^{\alpha}_A(x)$, $\hat{p}^{\beta}_{Ai}$ and $\hat{p}_A$, and Firm B maximizes $\pi_{Bl}(\mathcal{X}_1)$ with respect to $\hat{p}_B$, both profits are strictly concave.

Equilibrium prices are:

$$\begin{aligned}\hat{p}_B &= -\frac{t}{3} + \frac{4t}{3(n+1)} \sum_{i=1}^{n+1} [\frac{s_i}{2} + l_i], \\ \hat{p}^{\beta}_{Ai} &= \frac{\hat{p}_B + t}{2} - s_i t \\ &= \frac{t}{3} + \frac{2t}{3(n+1)} \sum_{i=1}^{n+1} [\frac{s_i}{2} + l_i] - s_i t.\end{aligned}$$

Let $\hat{p}^{\beta}_{1s}$ and $\hat{p}^{\beta}_{1s+l}$ be the prices on the first two segments when the partition is $\mathcal{X}_1$.

$$\hat{p}^{\beta}_{1s} = \frac{\hat{p}_B + t}{2} - \frac{st}{k},$$
$$\hat{p}^{\beta}_{1s+l} = \frac{\hat{p}_B + t}{2} - \frac{s+l}{k}t,$$

$\hat{p}'_B$ is the price set by Firm B with partition $\mathcal{X}'_1$, and $\hat{p}^{\beta'}_{1s}$ is the price set by Firm A on the last segment of partition $\mathcal{X}'_1$.

Inequalities in Eq. 5 might not hold as price $\hat{p}_B$ varies depending on the partition acquired by Firm A. This implies that segments which are of type β with partition $\mathcal{X}_1$ are then of type α with partition $\mathcal{X}'_1$. This is because the coarser the partition, the higher $\hat{p}_B$. I note $\tilde{s}_i$ the m segments where it is the case. I then have:

$$\begin{aligned}
\hat{p}'_B &= \frac{4t}{3(n-m)}[-\frac{n-m}{4} + \sum_{i=1}^{n}[\frac{s_i}{2} + l_i] - \sum_{i=1}^{m}\frac{\tilde{s}_i}{2}] \\
&= \frac{4t}{3(n-m)}[-\frac{n+1}{4} + \sum_{i=1}^{n+1}[\frac{s_i}{2} + l_i] + \frac{m+1}{4} - \sum_{i=1}^{m}\frac{\tilde{s}_i}{2} - \frac{s+l}{2}] \\
&= \hat{p}_B + \frac{4t}{3(n-m)}[\frac{3(m+1)\hat{p}_B}{4t} + \frac{m+1}{4} - \sum_{i=1}^{m}\frac{\tilde{s}_i}{2} - \frac{s+l}{2}] \\
&\geq \hat{p}_B + \frac{4t}{3(n-m)}[\frac{3}{4t}\hat{p}_B + \frac{m\hat{p}_B}{2t} + \frac{1}{4} - \frac{s+l}{2}], \\
\hat{p}^{\beta'}_{1s} &= \frac{\hat{p}_B + t}{2} - \frac{st}{k},
\end{aligned}$$

$$\begin{aligned}
\pi_A(\mathcal{X}_1) = &\sum_{i=1, s_i \neq \tilde{s}_i}^{n} p_{Ai}[\frac{\hat{p}_B + t}{4t} - \frac{s_i}{2}] \\
&+ \sum_{i=1}^{m}\hat{p}^{\beta}_{Ai}[\frac{\hat{p}_B + t}{4t} - \frac{\tilde{s}_i}{2}] + \hat{p}^{\beta}_{1s+l}[\frac{\hat{p}_B + t}{4t} - \frac{s+l}{2}] + \sum_{i=1}^{n'}\hat{p}_A[\frac{\hat{p}^{\beta}_{Bi} - \hat{p}_A + t}{2t} - s_i] \\
\pi_A(\mathcal{X}'_1) = &\sum_{i=1, s_i \neq \tilde{s}_i}^{n}\hat{p}^{\beta'}_{Ai}[\frac{\hat{p}'_B + t}{4t} - \frac{s_i}{2}] + \sum_{i=1}^{m}\frac{\tilde{l}_i}{k}[\hat{p}'_B + t - 2t\frac{\tilde{s}_i + \tilde{l}_i}{k}] + \sum_{i=1}^{m'}\hat{p}'_A[\frac{\hat{p}^{\beta'}_{Bi} - \hat{p}'_A + t}{2t} - s_i].
\end{aligned}$$

I compare the profits of Firm A in both cases in order to show that $\mathcal{X}'_1$ induces higher profits. Clearly, because $\hat{p}'_B > \hat{p}_B$, we have that:

$$\hat{p}_A[\hat{p}_B-\frac{\hat{p}_A+t}{2t}-\tilde{x}_A]+\sum_{i=1}^{n'}\hat{p}_A[\hat{p}^{\beta}_{Bi}-\frac{\hat{p}_A+t}{2t}-s_i] > \hat{p}'_A[\hat{p}'_B-\frac{\hat{p}'_A+t}{2t}-\tilde{x}_A]+\sum_{i=1}^{m'}\hat{p}'_A[\frac{\hat{p}^{\beta'}_{Bi}-\hat{p}'_A+t}{2t}-s_i].$$

Hence, we focus on the rest of the expression in the remaining of the proof:

$$\begin{aligned}
\Delta\pi_A &= \pi_A(\mathcal{X}'_1)-\pi_A(\mathcal{X}_1)\\
&\geq \sum_{i=1,s_i\neq\tilde{s}_i}^{n}\hat{p}^{\beta'}_{Ai}[\frac{\hat{p}'_B+t}{4t}-\frac{s_i}{2}]-\sum_{i=1,s_i\neq\tilde{s}_i}^{n}\hat{p}^{\beta}_{Ai}[\frac{\hat{p}_B+t}{4t}-\frac{s_i}{2}]\\
&+\sum_{i=1}^{m}\frac{\tilde{l}_i}{k}[\hat{p}'_B+t-2t\frac{\tilde{s}_i+\tilde{l}_i}{k}]-\sum_{i=1}^{m}\hat{p}^{\beta}_{Ai}[\frac{\hat{p}_B+t}{4t}-\frac{\tilde{s}_i}{2}]-\hat{p}^{\beta}_{1s+l}[\frac{\hat{p}_B+t}{4t}-\frac{s+l}{2}]\\
&=\frac{t}{2}\sum_{i=1,s_i\neq\tilde{s}_i}^{n}[\frac{\hat{p}'_B+t}{2t}-s_i]^2-\frac{t}{2}\sum_{i=1,s_i\neq\tilde{s}_i}^{n}[\frac{\hat{p}_B+t}{2t}-s_i]^2\\
&+\frac{t}{2}\sum_{i=1}^{m}\frac{\tilde{l}_i}{k}[2\frac{\hat{p}'_B+t}{t}-4\frac{\tilde{s}_i+\tilde{l}_i}{k}]-\frac{t}{2}\sum_{i=1}^{m}[\frac{\hat{p}_B+t}{2t}-\frac{\tilde{s}_i}{2}]^2-\frac{t}{2}[\frac{\hat{p}_B+t}{2t}-\frac{s+l}{k}]^2.
\end{aligned}$$

I consider the terms separately. First,

$$\begin{aligned}
&\frac{t}{2}\sum_{i=1,s_i\neq\tilde{s}_i}^{n}[\frac{\hat{p}'_B+t}{2t}-s_i]^2-\frac{t}{2}\sum_{i=1,s_i\neq\tilde{s}_i}^{n}[\frac{\hat{p}_B+t}{2t}-s_i]^2\\
=&\frac{t}{2}\sum_{i=1,s_i\neq\tilde{s}_i}^{n}[[\frac{2}{3(n-m)}[\frac{3}{4t}\hat{p}_B+\frac{m\hat{p}_B}{2t}+\frac{1}{4}-\frac{s+l}{2}]]^2\\
&+[\frac{\hat{p}_B+t}{2t}-s_i][\frac{4}{3(n-m)}[\frac{3}{4t}\hat{p}_B+\frac{m\hat{p}_B}{2t}+\frac{1}{4}-\frac{s+l}{2}]]]\\
\geq&\frac{t}{2}[\frac{\hat{p}_B+t}{2t}-\frac{s+l}{k}]\frac{4}{3}[\frac{3}{4t}\hat{p}_B+\frac{m\hat{p}_B}{2t}+\frac{1}{4}-\frac{s+l}{2}].
\end{aligned}$$

Secondly, on segments of type β with partition $\mathcal{X}_1$ that are of type α with partition $\mathcal{X}'_1$:

$$\frac{t}{2}\sum_{i=1}^{m}\frac{\tilde{l}_i}{k}[2\frac{\hat{p}'_B+t}{t}-4\frac{\tilde{s}_i+\tilde{l}_i}{k}]-\frac{t}{2}\sum_{i=1}^{m}[\frac{\hat{p}_B+t}{2t}-\frac{\tilde{s}_i}{2}]^2.$$

On these m segments, inequalities in Eq. 5 hold for price $\hat{p}'_B$ but not for $\hat{p}_B$. Thus I can rank prices according to $\tilde{s}_i$ and $\tilde{l}_i$:

$$\frac{\tilde{s}_i+\tilde{l}_i}{k}\geq\frac{\hat{p}_B+t}{2t}-\frac{\tilde{l}_i}{k}\quad\text{and}\quad\frac{\hat{p}'_B+t}{2t}-\frac{\tilde{l}_i}{k}\geq\frac{\tilde{s}_i+\tilde{l}_i}{k}.$$

thus:

$$2\frac{\tilde{l}_i}{k} \geq \frac{\hat{p}_B + t}{2t} - \tilde{s}_i \quad \text{and} \quad \frac{\hat{p}'_B + t}{2t} - 2\frac{\tilde{l}_i}{k} \geq \tilde{s}_i.$$

By replacing $\tilde{s}_i$ by its upper bound value and then $\tilde{l}_i$ by its lower bound value I obtain:

$$\frac{t}{2}\sum_{i=1}^{m}\frac{\tilde{l}_i}{k}[2\frac{\hat{p}'_B + t}{t} - 4\frac{\tilde{s}_i + \tilde{l}_i}{k}] - \frac{t}{2}\sum_{i=1}^{m}[\frac{\hat{p}_B + t}{2t} - \frac{\tilde{s}_i}{2}]^2 \geq 0.$$

Getting back to the profits difference, I obtain:

$$\begin{aligned}\Delta\pi_A &\geq \frac{t}{2}[\frac{\hat{p}_B + t}{2t} - \frac{s+l}{k}]\frac{4}{3}[\frac{3}{4t}\hat{p}_B + \frac{m\hat{p}_B}{2t} + \frac{1}{4} - \frac{s+l}{2}] - \frac{t}{2}[\frac{\hat{p}_B + t}{2t} - \frac{s+l}{k}]^2 \\ &\geq \frac{t}{2}[\frac{\hat{p}_B + t}{2t} - \frac{s+l}{k}][\frac{\hat{p}_B}{2t} + \frac{s+l}{3k} - \frac{1}{6}].\end{aligned} \tag{7}$$

The first bracket of Equation 7 is positive given Eq. 5. The second bracket is positive if $\frac{\hat{p}_B}{2t} + \frac{s+l}{3k} \geq \frac{1}{6}$. A sufficient condition for this result to hold is $\hat{p}_B \geq \frac{t}{3}$. I prove that this inequality is always satisfied by showing that the partition that contains all segments minimizes the price and profit of Firm B, and that in this case, $\hat{p}_B \geq \frac{t}{2}$.[25] And as this price is greater than $\frac{1}{6}$, the second bracket of Equation 7 is positive. This proves that $\Delta\pi_A \geq 0$.

At this point of the step, we have shown that it is always optimal for each firm to use a partition with the following shape: each firm uses a partition with at most two segments of type β located furthest from its location. If both firms have only one such segment in the middle of the line, the proof is completed. If one firm has two segments of type β and the other firm has one segment of type β, the step above directly applies, and the transformation consisting in merging both type β segments is profit increasing for the firm.

If both firms have two type β segments, we must show that such partition - denoted X_A - always yields lower profits than the same partition but with the two type β segments merged into one type β segment - denoted X'_A (we denote by X_B the partition of Firm B). This is the last part of the proof, which is established now. Such partition is depicted on Figure 4.

[25] As shown in Liu and Serfes (2004).

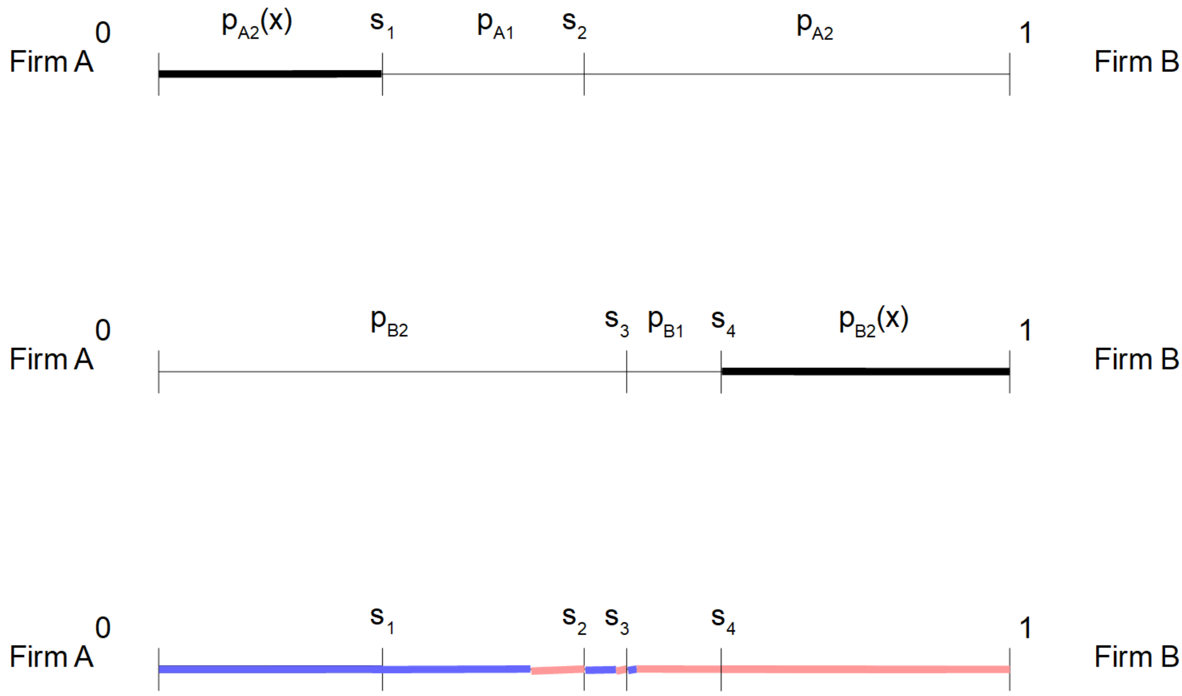

Figure 4: Structures of the optimal partitions for each firm after applying steps 1, 2, and the previous part of step 3.

The profits of Firm A can be written:

$$\begin{aligned}\pi_A(X_A) &= \int_0^{s_1} \hat{p}_A^{\alpha}(x)dx + \hat{p}_{A1}[\frac{\hat{p}_{B2}-\hat{p}_{A1}+t}{2t} - s_1] \\ &+ \hat{p}_{A2}[\frac{\hat{p}_{B2}-\hat{p}_{A2}+t}{2t} - s_2] + \hat{p}_{A2}[\frac{\hat{p}_{B1}-\hat{p}_{A2}+t}{2t} - s_3]. \\ \pi_B(X_B) &= \int_{s_4}^{1} \hat{p}_B^{\alpha}(x)dx + \hat{p}_{B1}[s_4 - \frac{\hat{p}_{B1}-\hat{p}_{A2}+t}{2t}] \\ &+ \hat{p}_{B1}[s_3 - \frac{\hat{p}_{B1}-\hat{p}_{A2}+t}{2t}] + \hat{p}_{B2}[s_2 - \frac{\hat{p}_{B1}-\hat{p}_{A2}+t}{2t}].\end{aligned}$$

We can show that the profits of Firm A are lower than when merging the two type β segments and choosing an optimal s_1^* such that all consumers are charged a targeted price on $[0, s_1^*]$ and one homogeneous price on $[s_1^*, 1]$. We need to consider two cases. Either Firm A makes positive profits on $[s_3, s_4]$ and makes profits as follows:

$$\pi_A(X'_A) = \int_0^{s_1^*} \hat{p}_A^\alpha(x)dx + \hat{p}'_{A1}[\frac{\hat{p}_{B2} - \hat{p}'_{A1} + t}{2t} - s_1^*] + \hat{p}'_{A1}[\frac{\hat{p}_{B1} - \hat{p}'_{A1} + t}{2t} - s_3].$$
$$\pi_B(X'_B) = \int_{s_4}^{1} \hat{p}_B^\alpha(x)dx + \hat{p}_{B1}[s_4 - \frac{\hat{p}_{B1} - \hat{p}'_{A1} + t}{2t}] + \hat{p}_{B2}[s_3 - \frac{\hat{p}_{B2} - \hat{p}'_{A1} + t}{2t}].$$

Or Firm A does not make positive profits on $[s_3, s_4]$ and makes profits as follows:

$$\pi_A(X'_A) = \int_0^{s_1^*} \hat{p}_A^\alpha(x)dx + \hat{p}'_{A1}[\frac{\hat{p}_{B2} - \hat{p}'_{A1} + t}{2t} - s_1^*].$$
$$\pi_A(X_B) = \int_{s_4}^{1} \hat{p}_B^\alpha(x)dx + \hat{p}_{B1}[s_4 - \frac{\hat{p}_{B1} - \hat{p}'_{A1} + t}{2t}]$$
$$+ \hat{p}_{B1}[s_3 - \frac{\hat{p}_{B1} - \hat{p}'_{A1} + t}{2t}] + \hat{p}_{B2}[s_2 - \frac{\hat{p}_{B2} - \hat{p}'_{A1} + t}{2t}].$$

Having stated this last step of the problem, I keep its resolution available upon request to keep the proof as concise as possible. Overall, the last transformation increases the profits of Firm A.

Conclusion

This result allows us to establish that it is always more profitable for Firm A to use a partition with one segment of type β than to use a partition with several segments of type β.

These three steps prove that the optimal partition for each firm includes two intervals: Firm A first-degree price discriminates consumers on interval $[0, x_A]$, and charges a homogeneous price on the second interval located at $[x_A, 1]$. By symmetry, it is optimal for Firm B to target all consumers on interval $[1 - x_B, 1]$ and to charge a homogeneous price to consumers on $[0, 1 - x_B]$.

A.2 Proof of Proposition 2

I characterize the optimal targeting strategies of the firms in period 2. I first compute prices and demands in period 2 when firms target consumers strategically. Firm A chooses the value of x_A such that it price discriminates consumers on $[0, x_A]$, and charges consumers on $[x_A, 1]$ a homogeneous price. Similarly, Firm B price discriminates consumers on $[1 - x_B, 1]$, and charges a homogeneous price to consumers on $[0, 1-x_B]$. The choices of x_A and x_B correspond to the targeting strategies of Firm A and Firm B. I will provide prices and profits in period 2, and I will characterize x_A and x_B.

Prices and demand.

Firm A sets a price $p_{A2}(x)$ for consumers located at $[0, x_1]$. Similarly, Firm B sets a price $p_{B2}(x)$ for consumers located at $[1-x_B, 1]$. Firm θ then sets a unique price $p_{\theta 2}$ on the rest of the unit line. The price charged to targeted consumers by Firm A satisfies:

$$\begin{aligned}
&V - tx - p_{A2}(x) = V - t(1-x) - p_{B2} \\
\implies &x = \frac{p_{B2} - p_{A2}(x) + t}{2t} \\
\implies &p_{A2}(x) = p_{B2} + t - 2tx.
\end{aligned}$$

Firm B charges homogeneous price p_{B2} on interval $[0, 1-x_B]$, and charges targeted prices on $[1-x_B, 1]$:

$$p_{B2}(x) = p_{A2} + t - 2tx.$$

Let denote d_{A2} the demand for Firm A (resp. d_{B2} the demand for Firm B) where firms compete. d_{A2} is determined by the indifferent consumer $\tilde{x}_2$:

$V - t\tilde{x}_2 - p_{A2} = V - t(1-\tilde{x}_2) - p_{B2} \implies \tilde{x}_2 = \frac{p_{B2}-p_{A2}+t}{2t}$ and $d_{A2} = \tilde{x}_2 - x_A = \frac{p_{B2}-p_{A2}+t}{2t} - x_A$ (resp. $d_{B2} = 1 - x_B - \frac{p_{B2}-p_{A2}+t}{2t}$).

Profits of the firms.

The profits of the firms are:

$$\begin{aligned}
\pi_{A2} &= \int_0^{x_A} p_{A2}(x)dx + d_{A2}p_{A2} = \int_0^{x_A} (p_{B2} + t - 2tx)dx + (\frac{p_{B2} - p_{A2} + t}{2t} - x_A)p_{A2}, \\
\pi_{B2} &= \int_{1-x_B}^{1} p_{B2}(x)dx + d_{B2}p_{B2} = \int_{1-x_B}^{1} (p_{A2} + t - 2tx)dx + (\frac{p_{A2} - p_{B2} + t}{2t} - x_B)p_{B2}.
\end{aligned}$$

Prices and demands in equilibrium.

I now compute the optimal prices and demands, using first-order conditions on π_θ with respect to p_θ. Prices in equilibrium are:

$$p_{A2} = t[1 - \frac{2}{3}x_B - \frac{4}{3}x_A],$$
$$p_{B2} = t[1 - \frac{2}{3}x_A - \frac{4}{3}x_B].$$

I rule out negative prices from the analysis: $p_{\theta 2}$ is taken equal to zero in case its above expression is negative.

Replacing these values in the above demands and prices gives:

$$p_{A2}(x) = 2t - \frac{4t}{3}x_B - \frac{2t}{3}x_A - 2tx,$$
$$p_{B2}(x) = 2t - \frac{4t}{3}x_A - \frac{2t}{3}x_B - 2tx$$

Demands in equilibrium are as follows:

$$d_{A2} = \frac{1}{2} - \frac{2}{3}x_A - \frac{1}{3}x_B,$$
$$d_{B2} = \frac{4}{3}x_B - \frac{1}{2} - \frac{1}{3}x_A.$$

Profits in equilibrium.

I compute profits by replacing prices and demands by their equilibrium values:

$$\pi_{\theta 2} = \frac{t}{2} - \frac{7}{9}x_\theta^2 t + \frac{2}{9}x_{-\theta}^2 t - \frac{4}{9}x_\theta x_{-\theta} t + \frac{2}{3}x_\theta t - \frac{2}{3}x_{-\theta} t.$$

Profits are strictly concave functions with respect to x_A and x_B, and they have a unique maximum.

Optimal targeting strategies: unconstrained.

I derive the optimal targeting strategies x_A^* and x_B^* of Firm A and Firm B. The targeting strategies x_A^* and x_B^* are chosen as simultaneous best responses. I apply the first-order condition on π_{A2} with respect to x_A and to π_{B2} with respect to x_B, and I find:

$$x_A^* = x_B^* = \frac{1}{3}.$$

As $p_{A2}^* = p_{B2}$, the indifferent consumer in period 2 is located at $\tilde{x}_2 = \frac{1}{2}$.

By replacing x_A^* and x_B^* into π_{A2} and π_{B2} I obtain:

$$\pi^*_{\theta 2} = \frac{7t}{18}.$$

Optimal targeting strategies: constrained.

I derive the optimal targeting strategy x^*_B of Firm B when Firm A is constrained and $x^*_A = \tilde{x}_1$.

x^*_B is chosen as a best response to $x^*_A = \tilde{x}_1$. The profits of the firms are the following:

$$\pi_{A2} = \frac{t}{2} - \frac{7}{9}\tilde{x}_1^2 t + \frac{2}{9}x_B^2 t - \frac{4}{9}\tilde{x}_1 x_B t + \frac{2}{3}\tilde{x}_1 t - \frac{2}{3}x_B t.$$

$$\pi_{B2} = \frac{t}{2} - \frac{7}{9}x_B^2 t + \frac{2}{9}\tilde{x}_1^2 t - \frac{4}{9}x_B \tilde{x}_1 t + \frac{2}{3}x_B t - \frac{2}{3}\tilde{x}_1 t.$$

I apply first-order conditions on π_{B2} with respect to x_B:

$$x^*_B = \frac{3}{7} - \frac{2\tilde{x}_1}{7}.$$

Replacing x^*_A and x^*_B into p^*_{A2} and p^*_{B2}, I find that the indifferent consumer in period 2 is located at $\tilde{x}_2 = \frac{6\tilde{x}_1+5}{14}$.

By replacing x^*_A and x^*_B by their expressions into π_{A2} and π_{B2} I obtain:

$$\pi^*_{A2} = \frac{25t}{98} + \frac{30t}{49}\tilde{x}_1 - \frac{31t}{49}\tilde{x}_1^2,$$

$$\pi^*_{B2} = \frac{9t}{14} - \frac{6t}{7}\tilde{x}_1 + \frac{2t}{7}\tilde{x}_1^2.$$

A.3 Proof of Proposition 3

I show that the only equilibrium of the game is symmetric. In the previous proof, prices and targeting strategies in the symmetric case are computed as simultaneous best responses, deviation is not profitable and these strategies constitute an equilibrium.

I now show that the constrained, asymmetric strategy when Firm B undercuts prices is not sustainable, as a firm willing to constrain its competitor by undercutting prices always benefits from deviating to the symmetric equilibrium.

Profits in the symmetric equilibrium are equal to $\frac{7t}{18}$ in period 2 and $\frac{t}{2}$ in period 1 and overall, the payoff of firms in the symmetric equilibrium is $\delta\frac{7t}{18} + \frac{t}{2}$.

I provide an upper bound to the profits of a firm adopting a constraining strategy. The maximal profit in period 2 in the constrained case is reached when $\tilde{x}_1 = 0$ (when Firm A is constrained) and is equal to $\frac{9t}{14}$.

For Firm B to constrain Firm A in period 1, it must be the case that $\tilde{x}_1 \leq \frac{1}{3}$. Let us consider the least constraining case where $\tilde{x}_1 = \frac{1}{3}$, which leads to the highest profits of Firm B among the set of constraining strategies in period 1.

It is easy to show that to obtain $\tilde{x}_1 = \frac{1}{3}$, Firm B must charge $p_{B1} = \frac{t}{3}$ yielding profits in period 1 equal to $\frac{2t}{9}$.

Thus the sum of profits over both periods in the constraining case is therefore lower than $\frac{2t}{9} + \delta\frac{9t}{14} < \frac{t}{2} + \delta\frac{7t}{18}$, and profits are higher in the symmetric equilibrium.

Hence deviation is profitable, asymmetric pricing is not sustainable, and the only equilibrium of the game is symmetric.

A.4 Proof of Proposition 4

I compare the utility of consumers located at $x \in [0, \frac{1}{3}]$ when purchasing their preferred product with their utility from purchasing Firm B's product in period 1 and paying the homogeneous price in period 2.

The utility when purchasing from Firm A is $u_1(x) = V - tx - t$ in period 1 and $u_2(x) = V - tx - \frac{4t}{3} + 2tx$ in period 2.

The utility when purchasing from Firm B in period 1 is $u_{1B}(x) = V - t(2 - x)$ and $u_{2A}(x) = V - tx - \frac{t}{3}$ when paying the homogeneous price in period 2.

For all $\delta_c \leq 1$, $u_1(x) + \delta_c u_2(x) \geq u_{1B}(x) + \delta_c u_{2A}(x)$ and consumers do not hide.

We can apply a similar reasoning to show that even if a firm engages in a constraining strategy, forward-looking consumers do not have interest to deviate and purchase their preferred product at each stage. (The proof is available upon request.)