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# Behavior-Based Price Discrimination with Strategic Customer Targeting* 

Antoine Dubus ${ }^{\dagger}$

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#### Abstract

Two competing firms collect information that they use to charge targeted prices to their past customers, in a two-period framework of behavior-based price discrimination (BBPD). Firms can target consumers strategically: they can choose for each of their past customers whether to charge them personalized or homogeneous prices. I derive the optimal targeting strategy of each firm, which consists in charging personalized prices to past customers with the highest willingness to pay and a homogeneous price to the remaining consumers, including past customers with a low valuation on whom a firm has information. This targeting strategy maximizes rent extraction while softening competition between firms compared to nonstrategic targeting under which firms target all their past customers. In turn, price-undercutting and poaching practices are not sustainable with strategic targeting, resulting in a lower consumer surplus compared with previous frameworks of BBPD.


Keywords: Strategic Targeting; Data Collection; Behavior-based price discrimination.

[^0]
## 1 Introduction

In the past decades, companies have increasingly adopted the services of thirdparty data brokers and marketing companies such as Equifax or Nielsen, specialized in consumer data collection and analytics for marketing campaigns. For instance, in 1979 Nielsen launched its Scantrack service providing retailers with business analytics on consumer demands. The quality of these personalization services has increased sharply with the rise of digital technologies, and Nielsen became dominant in the marketing industry by the mid-nineties. ${ }^{1}$

An essential factor of this success is that data brokers such as Nielsen benefit from important economies of scale in data processing, and they have developed sophisticated targeting strategies that allow firms to increase their profits using fine-grained information on consumer demand (Varian, 2018). With the quality of their information and their business analytics, data brokers have managed to secure the position of information gatekeeper.

This situation has recently started to change with the rising concerns of competition authorities on the impacts of third-party data brokers on competition in product markets. In particular, the Finnish competition authority recently found that Nielsen's Scantrack service led food retailers in Finland to soften competition and increase prices (Koski, 2018).

Moreover, with the advances in information technologies, companies are increasingly developing their own sophisticated marketing strategies based on the large amounts of data that they collect on their customers (DalleMule and Davenport, 2017). Such information, directly collected by a firm, is referred to as "first-party data", as opposed to "third-party data" acquired from data brokers. After terminating their agreements with Nielsen, companies such as Kesko, a major food retailer in Finland, have indeed developed their own data analytics and personalization services based on first-party customer data. ${ }^{2}$

Overall, firms have now their data-management functions and chief data officers, and they are increasingly using practices of behavior-based price discrimina-

[^1]tion (BBPD), under which they collect data on their customers to propose them personalized offers and prices. Practices of BBPD are especially becoming common on the Internet (Gorodnichenko et al., 2018), where a firm such as Amazon can collect data on search behavior, GPS localization, and any type of personal information to feed machine-learning algorithms to personalize ads, products, and prices to the needs of its customers (Shiller et al., 2013). Recent studies document practices of BBPD in various industries such as newspapers (Asplund et al., 2008), credit markets (Ioannidou and Ongena, 2010) and mortgage markets (Thiel, 2019) among many others.

The theoretical literature has for long analyzed BBPD by firms using their own (first-party) data collected on their customers (Fudenberg and Tirole, 2000; Acquisti and Varian, 2005; Fudenberg and Villas-Boas, 2006). ${ }^{3}$ In particular, Choe et al. (2018) provide an important contribution to this literature by considering a two-period competition framework à la Fudenberg and Tirole (2000). In the first period, firms have no information on their customers and charge a homogeneous price to the whole market. At the end of the first period, each firm can perfectly learn the willingness to pay of its customers for its product. In the second period, each firm uses this information to charge personalized prices to all past customers. Using this framework, Choe et al. (2018) show that BBPD with perfect information on past customers increases the intensity of market competition as firms engage in poaching strategies. In line with Fudenberg and Tirole (2000), some consumers switch in the second period, resulting in an inefficient market outcome. Moreover, Choe et al. (2018) show that this model of BBPD yields an asymmetric equilibrium under which one of the firms serves a large share of the consumer demand. This result has important implications for competition policy as it shows that digital markets where firms increasingly collect data to target consumers are likely to yield asymmetric competition where a firm dominates its competitor.

In this article, I argue that a central element of these practices of BBPD is the increasing ability of firms to target consumers strategically. Consider the model of Choe et al. (2018). In the second period, firms use all available information to

[^2]price discriminate past customers. However, this may not be optimal for firms, as there are two opposite effects of information on their profits. On the one hand, targeting consumers increases the profit of a firm through a better extraction of consumer surplus. On the other hand, information also intensifies competition, which reduces the profits of both firms. Indeed, when both firms target all their past customers, they price aggressively to poach consumers located far away from their locations. This increases the intensity of competition between firms and limits their ability to extract surplus from targeted consumers (Thisse and Vives, 1988). In this sense, firms targeting all past customers are 'non-strategic', and to the best of my knowledge, new practices of information design by firms that have developed their own strategies based on first-party data have not yet been analyzed by previous literature, neither have their implications for firms and consumers.

This article studies the adoption of strategic customer targeting by firms and analyzes its implications for consumers and market competition. The analysis builds on the flexible framework of behavior-based price-discrimination proposed by Choe et al. (2018), and allows firms to choose among a rich set of targeting strategies: a firm that has information on a group of past customers can choose to which consumer among this group it charges personalized prices or homogeneous prices. Such strategic targeting allows a firm to choose a price structure that maximizes the surplus extraction effect of information while softening its competitive effect. This approach has been recently used by Bounie et al. (2021) in the case of a seller of (third-party) information. Including it in a model of BBPD allows to explicitly model the strategy of information acquisition of each firm, and allows in turn to analyze strategic targeting by firms using this first-party data.

The main contribution of this article is to show that the optimal strategy of a firm consists in targeting consumers with the highest willingness to pay for their product, and to charge a homogeneous price to a large share of low-valuation consumers - including past customers on whom the firm has collected data - to soften the intensity of competition. This result challenges previous assumptions of the literature where firms use all available information to price discriminate consumers. Such a targeting practice is sub-optimal for a firm, as the resulting intensity of competition drastically reduces the profits of the firms. A firm can
reach higher profits by strategically keeping some of its past customers untargeted. Previous literature has emphasized the benefits for firms to commit not to price discriminate, and by doing so to soften the intensity of competition (Corts, 1998; Dobson and Waterson, 2008). However, to the best of my knowledge, this article is the first to prove that without the need of commitment, firms unilaterally benefit from not using part of their consumer data. This new result has important implications for policymakers willing to assess the competitive impacts of new data-driven targeting practices.

Motivated by the sharp development of the data brokerage industry, and by the increasing ability of firms to target their customers, this article then compares consumer surplus and firms' profits with strategic targeting and with two other regimes of information acquisition and targeting: (1) BBPD when firms use all available information, and are therefore 'non-strategic' (Choe et al., 2018); (2) price-discrimination when firms acquire information from a third-party data broker (Bounie et al., 2021).

The comparison of competition and surplus under these different regimes contributes to the literature on two main points. First, when firms are non-strategic and target all past customers, Choe et al. (2018) have shown that the equilibrium is asymmetric and yields a strong intensity of competition in both periods: in the first period one firm prices aggressively to limit the share of consumers served by its competitor and this way, limit the number of consumers on whom it collects data; the firm that has served the larger share of consumers in the first period has also collected data on these consumers, and thus can serve a larger consumer demand than its competitor in the second period.

Such constraining strategies are not sustainable when firms use their data to target consumers strategically, and the resulting equilibrium is symmetric. As strategic firms charge a homogeneous price to a large share of the consumer demand - including some of their past customers on whom they have information they can soften the competitive effect of targeting in the second period. Consequently, in the first period strategic firms do not have interest to undercut prices to acquire information on low-valuation consumers who will not be targeted, and
they compete as in the standard Hotelling framework. The resulting competitive equilibrium with strategic targeting is symmetric, and overall, strategic targeting softens the competitive effect of BBPD in both competition periods.

Secondly, strategic targeting using first-party data increases market competition compared with information purchased from third-party data brokers. Data brokers can internalize the business stealing effect of information by keeping a large share of consumers unidentified from firms, and by selling information to only one of the firms, keeping the other uninformed (Bounie et al., 2021). Such exclusionary practice does not take place when firms collect and treat their own first-party data. This article shows that in equilibrium, firms using strategic targeting based on first-party data will target more consumers than a data broker, which increases competition and consumer surplus compared to third-party data.

These results have important implications for firms and for competition authorities, which depend on the level of reliance of industries on data brokers. For instance, retailers heavily rely on data brokers for their marketing and personalization campaigns (Crain, 2018), and a move of firms toward first-party data is likely to increase competition. In particular, first-party targeting would prevent exclusionary practices that are analyzed by the theoretical literature (Montes et al., 2019; Bounie et al., 2021), and which have been implemented for instance by Facebook. ${ }^{4}$

The remainder of this article is organized as follows. Section 2 describes the model. Section 3 characterizes the optimal targeting strategies of the firms, as well as the equilibrium in both competition periods. The welfare implications of strategic versus non-strategic targeting, and of third-party versus first-party data are discussed in Section 4. Section 5 analyzes forward-looking consumers who anticipate targeting and may change their purchasing behavior accordingly. Section 6 discusses the main findings and concludes.

[^3]
## 2 Description of the Model

Two horizontally differentiated firms - Firm A and Firm B - compete in a product market. There are two competition periods $s=1,2$, in which firms sell nondurable goods. ${ }^{5}$ In the first period, firms have no information on consumers and compete by setting homogeneous prices. Firms then learn the willingness to pay of each of their customers for their product, and in period 2 , firms can charge targeted prices to some of their past customers. Both firms incur the same marginal cost of production, which is normalized to zero, and in each period consumers have unit demands.

### 2.1 Consumers

Consumers are uniformly distributed on a unit line $[0,1],{ }^{6}$ and in each period $s$ they can buy one product at a price $p_{A s}$ from Firm A located at 0 , or $p_{B s}$ from Firm B located at $1 .{ }^{7}$ Consumers located at $x \in[0,1]$ derive a utility $V$ from purchasing the product. They incur a transportation cost $t>0$ so that buying from Firm A (resp. from Firm B), has a total cost $t x$ (resp. $t(1-x)$ ). In each period, consumers purchase the product for which they have the highest utility, and in period 2, different consumers may pay different prices as firms know at this point the willingness to pay of their past customers.

In period $s=1,2$, consumers located at $x$ have a utility function defined by:

$$
u_{s}(x)=\left\{\begin{array}{l}
V-p_{A s}-t x, \text { if they buy from Firm A, }  \tag{1}\\
V-p_{B s}-t(1-x), \text { if they buy from Firm B. }
\end{array}\right.
$$

Consumers are assumed to be myopic in the baseline model, and they max-

[^4]imize their utility at each consumption period. ${ }^{8}$ Forward-looking consumers are considered in Section 5.

### 2.2 Firms

This section describes the set of targeting strategies available to firms in the second period. It then provides the profits of the firms in each period of the game, as well as their objective functions.

Firms first compete in period 1 - the information acquisition period - in which each firm collects perfect information on its customers. ${ }^{9}$ Let's denote by $\tilde{x}_{1}$ the consumer indifferent between buying from Firm A and Firm B in the first period, such that Firm A serves consumers on $\left[0, \tilde{x}_{1}\right]$ and Firm B serves consumers on $\left[\tilde{x}_{1}, 1\right]$.

### 2.2.1 Targeting Strategies

In period 2 - the targeting period - firms choose which of their past customers they price-discriminate. Each firm can target two different types of intervals of the consumer demand: ${ }^{10}$
(a) On the first type of intervals a firm charges personalized prices to all past customers.
(b) A firm can charge a homogeneous price to customers in the second type of intervals.

Figure 1 illustrates these two types of intervals for Firm A when Firm B sets a homogeneous price $p_{B 2}$.

[^5]Interval of type (a): first-degree price discrimination


Figure 1: Possible types of intervals when Firm A targets consumers strategically.

On intervals of type (a) - displayed in Figure 1 (a) - Firm A charges each customer a personalized price $p_{A 2}(x)$. Firm A charges a homogeneous price $p_{A 2 i}$ on the $i$ th intervals of type (b) (starting from the left) illustrated in Figure 1 (b).

The model allows for any combination of type (a) and (b) intervals. For instance, the last line of Figure 1 displays from the left to the right a type (a) interval where Firm A charges price $p_{A 2}(x)$ to a consumer located at $x$, then two type (b) intervals respectively with prices $p_{A 2 i}$ and $p_{A 2 i+1}$, and a second type (a) interval and so on. Firm B has no information on these consumers and charges a homogeneous price $p_{B 2}$.

In period 2, Firm A can use any potential combination of type (a) and type (b) intervals to maximize its profits. Let's denote by $\Psi_{A}\left(\tilde{x}_{1}\right)$ the continuous set of all possible partitions of the unit line into type (a) and type (b) intervals. This set
depends on the share of customers on whom Firm A has collected data in period 1: $\left[0, \tilde{x}_{1}\right]$. Similarly, Firm B can choose any partition in the set $\Psi_{B}\left(\tilde{x}_{1}\right)$ generated by the information it has on customers in $\left[\tilde{x}_{1}, 1\right]$.

When choosing its targeting strategy, Firm $\theta(\theta=A, B)$ selects the partition $X_{\theta} \in \Psi_{\theta}\left(\tilde{x}_{1}\right)$ of consumers to price discriminate. For a given partition $X_{\theta}$, let's denote by $X_{\theta}^{a}$ the subset of type (a) segments, and by $X_{\theta}^{b}$ the subset of type (b) segments, such that $X_{\theta}=X_{\theta}^{a} \cup X_{\theta}^{b} ; X_{\theta}^{a} \cap X_{\theta}^{b}=\emptyset$.

While previous literature has assumed that firms price-discriminate all consumers that they have identified, ${ }^{11}$ this article considers strategic targeting as using all available information may not be optimal for a firm. There are indeed two opposite effects of information on the profit of a firm. On the one hand, targeting consumers increases the profit of a firm through a better extraction of consumer surplus. On the other hand, information also increases competition, which reduces the profits of both firms. Indeed, when both firms target all their past customers, they price aggressively to poach consumers located far away from their locations. This increases the intensity of competition between firms and limits their ability to extract surplus from targeted consumers. Section 3.1.1 characterizes the optimal information structure for each firm, which balances these two effects of information on the profits of the firms.

### 2.2.2 Profits

## Profits in period 1.

This analysis focuses on Subgame Perfect Nash Equilibria. ${ }^{12}$ At the beginning of period 1 , firms only know that consumers are uniformly distributed on the unit line. The objective of Firm $\theta$ is to set a homogeneous price - denoted by $p_{\theta 1}$ and characterized in Section 3 - to maximize its total profits, composed of the sum of its profits in both periods by discounting period 2 with factor $\delta .{ }^{13}$ As is standard in Hotelling competition with uniform prices, the resulting demand can be written

[^6]$d_{\theta 1}=\frac{p_{-\theta 1}-p_{\theta 1}+t}{2 t}$, where $d_{A 1}=\tilde{x}_{1}$ and $d_{B 1}=1-\tilde{x}_{1}$. The profit of Firm $\theta$ in period 1 can be written $\pi_{\theta 1}=d_{\theta 1} p_{\theta 1}$.

## Profits in period 2.

In period 2, firms first determine their targeting strategies and then charge prices accordingly. The targeting strategies are chosen as simultaneous best responses and we will show that they constitute the unique pure strategy equilibrium of the game. Hence, in period 1, firms anticipate the equilibrium of period 2 and charge prices $p_{\theta 1}$ and $p_{\theta 2}$ accordingly.

For a given partition $X_{\theta}=X_{\theta}^{a} \cup X_{\theta}^{b}$ prices are set as follows. In the set $X_{\theta}^{a}$, prices $p_{\theta 2}(x)$ are set as high as possible under the competitive constraint exerted by price Firm $-\theta$. Firm $\theta$ charges homogeneous prices $p_{\theta 2 i}$ on each segment in $X_{\theta}^{b}$, with $i=1, . ., n$ and where $n$ corresponds to the total number of segments in $X_{\theta}^{b}$. In each segment in $X_{\theta}^{b}$, prices $p_{\theta i 2}$ yield corresponding demands $d_{\theta i 2}$. Let us denote by $\mathbf{p}_{\theta 2}=\left(p_{\theta 12}, . ., p_{\theta n 2}\right)$ the vector composed of all prices charged in the different segments of $\mathcal{X}_{\theta}^{b}$. In period 2 , Firm $\theta$ sets prices in order to maximize the following profit function:

$$
\begin{equation*}
\pi_{\theta 2}\left(p_{\theta 2}(x), \mathbf{p}_{\theta 2}\right)=\int_{x_{\theta}^{a}} p_{\theta 2}(x) d x+\sum_{i=1}^{n} p_{\theta i 2} d_{\theta i 2} \tag{2}
\end{equation*}
$$

Moreover, the two following specifications are adopted regarding the targeting and pricing decisions of the firms, as well as sequential pricing.

## Sequential targeting and pricing decisions.

It is assumed that firms first choose the share of consumers that they want to target, and then charge prices. This specification is common in the theoretical literature on targeted advertising, where firms first choose to which consumers they send an ad, and then set prices accordingly (Anderson and Renault, 2009).

This assumption is also supported by managerial practices. As Du et al. (2021) emphasize, data analytics teams - in charge of the targeting strategy - and mar-
keting decision-makers - in charge of setting prices - are frequently at arm's length in centralized organizations. ${ }^{14}$

## Sequential pricing.

Turning to the pricing decisions of the firms, it is necessary to compute demands and prices on each consumer segment to obtain the profits of the firms. When a firm has no information, it sets a uniform price on the whole interval $[0,1]$. On the contrary, a firm that uses a partition $X_{\theta}$ can personalize prices. For each consumer in $X_{\theta}^{a}$, Firm $\theta$ will charge a personalized price as a monopolist constrained by the homogeneous price charged by Firm $-\theta$. In the set $X_{\theta}^{b}$, there are two types of segments to analyze: segments on which both firms have a strictly positive demand, and segments on which Firm $\theta$ is a monopolist.

The model adopts the additional assumption that Firm $\theta$ sets prices in two stages. First, it sets prices $p_{\theta i 2}$ on the segments where both firms compete and make positive profits. Then it sets monopoly prices $p_{\theta 2}(x)$ for each targeted consumer in $X_{\theta}^{a}$, and prices $p_{\theta i 2}$ on the segments of $X_{\theta}^{b}$ where Firm $\theta$ is a monopolist.

Sequential pricing decisions avoid the non-existence of Nash equilibrium in pure strategies and are common in the literature supported by managerial practices. For instance, Jentzsch et al. (2013), Belleflamme et al. (2020) and Bounie et al. (2021) focus on sequential pricing where a higher personalized price is charged to identified consumers after a firm sets a uniform price. Sequential pricing is also common in business practices. Recently, Amazon has been accused of showing higher prices for Amazon Prime subscribers, who pay an annual fee for unlimited shipping services, than for non-subscribers (Lawsuit alleges Amazon charges Prime members for "free" shipping, Consumer affairs, August 29, 2017). Thus Amazon first sets a uniform price and then increases prices for high-valuation consumers who are better identified when they join the Prime program.

[^7]
## Objective functions of the firms.

Demands in the information acquisition period can have an impact on the targeting strategies of the firms in the targeting period. To emphasize the impact of prices in period 1 on market outcome in period 2 , the location of the indifferent consumer can be written as a function of prices in period 1: $\tilde{x}_{1}\left(p_{A 1}, p_{B 1}\right)$. Overall the objective functions of the firms at the beginning of the game are:

$$
\begin{aligned}
& \text { For Firm A: } \max _{p_{A 1}}\left\{\pi_{A 1}\left(p_{A 1}, p_{B 1}\right)+\delta \pi_{A 2}\left(p_{A 2}(x), \mathbf{p}_{A 2}, \tilde{x}_{1}\left(p_{A 1}, p_{B 1}\right)\right)\right\} \\
& \text { For Firm B: } \max _{p_{B 1}}\left\{\pi_{B 1}\left(p_{B 1}, p_{A 1}\right)+\delta \pi_{B 2}\left(p_{B 2}(x), \mathbf{p}_{B 2}, \tilde{x}_{1}\left(p_{A 1}, p_{B 1}\right)\right)\right\}
\end{aligned}
$$

### 2.3 Timing

This section summarizes the timing of the game. In period 1 , firms compete and collect data on their customers. In period 2 , firms choose the partitions $\mathcal{X}_{A}$ and $\mathcal{X}_{B}$ of consumers that they target. Then firms set prices on the competitive segments, and in the last stage, firms set targeted prices on their monopoly segments. The timing of the game is the following:

## - Period 1:

- Firms compete by setting prices $p_{A 1}$ and $p_{B 1}$, and learn the location of their customers on the unit line.
- Period 2:
- Stage 1: Each Firm $\theta$ chooses the partition $X_{\theta}$ of consumers to pricediscriminate.
- Stage 2: Firms set prices on the competitive segments of $X_{\theta}^{b}$.
- Stage 3: Firms set prices $p_{\theta 2}(x)$ on consumers that they price-discriminate in $X_{\theta}^{a}$, as well as prices on the monopoly segments of $X_{\theta}^{b}$.


### 2.4 Equilibrium Concept

Throughout the analysis, the focus is on subgame perfect Nash equilibria. In period 1, firms anticipate that the information they collect on consumers can be used in period 2. Firms therefore set prices according to two different forces. On the one hand, they want to maximize profits at the first competition period. On the other hand, they may also want to serve a large consumer demand to achieve higher prices in period $2 .{ }^{15}$

In period 2, each firm first determines its targeting strategy, using the information collected in period 1 . Then, firms set their homogeneous prices and finally they set their targeted prices.

As is standard with Nash equilibria, a firm does not need to observe the targeting strategy of its competitors, but only to know its equilibrium best response. Hence, at the beginning of period 2 , the partitions $X_{A}$ and $X_{B}$ are chosen as simultaneous best responses constituting the unique pure strategy Nash equilibrium of this stage.

## 3 Analysis

Theorem 1 states the main results of the analysis, which are proved in the following sections.

## Theorem 1

(a) In equilibrium, firms target only part of their past customers:

- Each firm charges personalized prices to high-valuation consumers.
- Remaining consumers are untargeted, including low-valuation customers on whom firms have information.
(b) The unique equilibrium of the game is symmetric.

[^8](c) Firms do not engage in consumer poaching during the targeting period.
(d) Firms do not undercut prices during the information acquisition period.

Theorem 1 (a) is a central result of this article, and makes an important contribution to the literature, which assumes that a firm uses all available information to target consumers. We will show in Section 3.1.1 that, starting from a situation where a firm targets all past customers, it is always more profitable for each firm to charge a homogeneous price to its past customers with a low willingness to pay to soften the competitive effect of information.

Theorem 1 (b) contributes to a recent literature arguing that when firms can collect precise information on consumers, the two-period game of price discrimination necessarily results in an asymmetric outcome under which one firm serves a large share of consumers. A central contribution of this article is to show that such equilibrium does not take place when firms target consumers strategically. The mechanisms underlying this result are analyzed in Section 3.2.

Theorem 1 (c) relates to a central result of the literature on BBPD: firms have incentives to charge low prices to poach the customers of their rivals after the first consumption period (Fudenberg and Tirole, 2000). This article contributes to this literature by showing that poaching does not take place when firms engage in strategic targeting, as shown in Section 3.2.

Finally, Fudenberg and Tirole (2000) and Choe et al. (2018) have also shown that firms compete more fiercely in period 1 than in standard models of Hotelling competition when they can collect data on their customers, as firms can use this data to compete more efficiently in the following period. Theorem 1 (d) states that, when firms use data to target consumers strategically, they do not compete for information acquisition in period 1, and we will see in section Section 3.2 that competition is identical to the standard Hotelling without information acquisition.

The model is solved by backward induction. First, the optimal targeting strategies of firms in the second period are characterized in Section 3.1. Section 3.2 analyzes competition in period 1 and information acquisition by firms, and shows that the unique equilibrium of the game is symmetric.

### 3.1 Period 2: Strategic Targeting

This section characterizes the optimal targeting strategies of the firms in period 2 when they can price discriminate their past customers. It shows that strategic firms optimally target close-by consumers with the highest willingness to pay for their products and charge a homogeneous price to all remaining consumers, including some consumers on whom they have information. Such information structure maximizes surplus extraction from consumers with the highest willingness to pay while softening the competitive effect of information.

### 3.1.1 Optimal Information Structure

In period 2 firms choose the partitions $X_{A}$ and $X_{B}$ of their past customers that they price-discriminate. Proposition 1 characterizes the optimal partitions.

## Proposition 1

There exist $x_{A} \in\left[0, \tilde{x}_{1}\right] \& x_{B} \in\left[0,1-\tilde{x}_{1}\right]$ such that in equilibrium:

- Firm A targets all consumers on $\left[0, x_{A}\right]$ and charges a homogeneous price on consumers on $\left[x_{A}, 1\right]$.
- Firm $B$ targets all consumers on $\left[1-x_{B}, 1\right]$ and charges a homogeneous price on consumers on $\left[0,1-x_{B}\right]$.

Proof: see Appendix A.1.
The proof proceeds in the following way. Considering any information structure for each firm, it shows that, for any targeting strategy adopted by Firm B, Firm A finds profitable to re-order segments so that Firm A first-degree price discriminates consumers closest to its location, and charges a homogeneous price to the rest of the unit line. Focusing on close-by consumers allows Firm A to extract surplus from customers with the highest willingness to pay, while limiting the competitive effect of information by leaving a large share of consumers who are charged a homogeneous price. Applying this reasoning to Firm B allows to show that it also has interest to first-degree price discriminate close-by consumers only.

Proposition 1 is a central result of this article and makes an important contribution to the literature, which usually assumes that a firm uses all available information to target consumers. An optimal information partition maximizes the profit of a firm by dividing the unit line into two intervals. Firm A charges targeted prices to consumers in the first interval on $X_{A}^{a}=\left[0, x_{A}\right]$, which is referred to as the share of targeted consumers, who have the highest willingness to pay for Firm A's product. Firm A does not target consumers on $X_{A}^{b}=\left[x_{A}, 1\right]$ - with a lower willingness to pay -, and charges a uniform price on this second interval, referred to as the share of untargeted consumers. Similarly, Firm B optimally targets high-valuation consumers belonging to $X_{B}^{a}=\left[1-x_{B}, 1\right]$ and charges a homogeneous price to low-valuation consumers on $X_{B}^{b}=\left[0,1-x_{B}\right]$. Consumers on $\left[x_{A}, 1-x_{B}\right]$ are targeted by none of the firms in period 2 . By leaving a share of consumers untargeted by firms, these optimal targeting strategies balance the rent extraction and the competition effects of information. ${ }^{16}$

Figure 2 illustrates the targeting strategies of Firm A and Firm B. The thick lines represent consumers who are targeted by Firm A on $\left[0, x_{A}\right]$, and by Firm B on $\left[1-x_{B}, 1\right]$. Consumers on segments $\left[x_{A}, 1\right]$ and $\left[0,1-x_{B}\right]$ are charged a homogeneous price by Firm A and Firm B respectively. Figure 2 also displays $\tilde{x}_{1}$, the location of the indifferent consumer in period 1.


Figure 2: Targeting strategies of firms in period 2.

When analyzing the targeting strategies of the firms, our focus will be on pure strategy Nash equilibria, characterized by the (unique) equilibrium values of $x_{A}$ and $x_{B}$. As it is assumed that firms choose $x_{A}$ and $x_{B}$ simultaneously, their equilibrium values will be derived by computing $x_{A}$ as a simultaneous best response to $x_{B}$ and reciprocally.

[^9]Recent articles have considered the possibility for firms to use such information partitions to price discriminate consumers, analyzing competition for exogenous values of $x_{A}$ and $x_{B}$, and without proving the optimality of these information structures (Lee et al., 2011; Chen et al., 2020). To the best of my knowledge, this is the first article that formally characterizes the optimal information partitions chosen by firms using collected data.

Moreover, while the literature has emphasized the benefits for firms to commit to not price discriminate consumers (Corts, 1998; Dobson and Waterson, 2008), it is important to stress that in this model, firms do not commit not to use information, but it is unilaterally profitable for each firm to keep a large share of consumers that they do not target to soften the competitive effect of information. This is an important contribution of this article that proves that, without the need of commitment, firms do not have interest to use all available consumer information.

## Profits in period 2.

In period 2, each firm charges personalized prices $p_{\theta 2}(x)$ to targeted consumers and charges price $p_{\theta 2}$ on the rest of the unit line. Prices $p_{\theta 2}(x)$ are set as high as possible under the competitive constraint exerted by price $p_{-\theta 2}$. Hence, firms set prices in period 2 in order to maximize the following profit functions:

$$
\begin{align*}
& \pi_{A 2}\left(p_{A 2}(x), p_{A 2}\right)=\int_{0}^{x_{A}} p_{A 2}(x) d x+p_{A 2} d_{A 2} \\
& \pi_{B 2}\left(p_{B 2}(x), p_{B 2}\right)=\int_{1-x_{B}}^{1} p_{B 2}(x) d x+p_{B 2} d_{B 2} \tag{4}
\end{align*}
$$

## Strategic interaction between the two periods.

The targeting strategies of Firm A and Firm B are characterized by their choices of $x_{A}$ and $x_{B}$, which can be constrained by the number of consumers on whom firms have acquired information. In period 1 , Firm A collects information on $\left[0, \tilde{x}_{1}\right]$ and Firm B collects information on $\left[\tilde{x}_{1}, 1\right]$. Therefore, the targeting strategy of each firm must verify: $x_{A} \leq \tilde{x}_{1}$ and $x_{B} \leq 1-\tilde{x}_{1}$. Hence, competition in period 1 can
have an impact on the targeting strategies of the firms. Note that the situation where each firm targets all its past customers is a special case of this approach, where $x_{A}=1-x_{B}=\tilde{x}_{1}$.

### 3.1.2 Equilibrium Targeting

This section characterizes the optimal number of customers that each firm targets in period 2. Firm A price discriminates consumers on $\left[0, x_{A}\right]$, and charges a homogeneous price to consumers on $\left[x_{A}, 1\right]$. Similarly, Firm B price discriminates consumers on $\left[1-x_{B}, 1\right]$, and charges a homogeneous price to consumers on $\left[0,1-x_{B}\right]$. The choices of $x_{A}$ and $x_{B}$ correspond to the targeting strategies of Firm A and Firm B, and their optimal values $x_{A}^{*}$ and $x_{B}^{*}$ are characterized in this section.

Each firm can target in period 2 customers that it has served in period 1, and the value of $\tilde{x}_{1}$ may constrain the targeting strategy of firms in period 2. Indeed, if $\tilde{x}_{1} \in\left[x_{A}^{*}, 1-x_{B}^{*}\right]$, both firms can target their optimal number of consumers in period 2. On the contrary, if $\tilde{x}_{1} \in\left[0, x_{A}^{*}\right]$ or if $\tilde{x}_{1} \in\left[1-x_{B}^{*}, 1\right]$, respectively Firm A or Firm B cannot target their optimal number of consumers, and are constrained in their targeting strategy.

Proposition 2 characterizes the equilibria when $\tilde{x}_{1} \in\left[x_{A}^{*}, 1-x_{B}^{*}\right]$ and firms are not constrained on their targeting strategies, and when $\tilde{x}_{1} \in\left[0, x_{A}^{*}\right]$ and Firm A is constrained (the case where Firm B is constrained is identical).

## Proposition 2

(a) In period 2 when $\tilde{x}_{1} \in\left[\frac{1}{3}, \frac{2}{3}\right]$ the subgame perfect equilibrium is unconstrained and firms target symmetric shares of consumers:

$$
x_{A}^{*}=x_{B}^{*}=\frac{1}{3}, \quad \tilde{x}_{2}=\frac{1}{2} .
$$

(b) In period 2 when $\tilde{x}_{1} \in\left[0, \frac{1}{3}\right]$ the subgame perfect equilibrium is constrained and firms target asymmetric shares of consumers:

$$
x_{A}^{*}=\tilde{x}_{1}, \quad x_{B}^{*}=\frac{3}{7}-\frac{2 \tilde{x}_{1}}{7}, \quad \tilde{x}_{2}=\frac{6 \tilde{x}_{1}+5}{14} .
$$

Proof: see Appendix A.2.
In the unconstrained equilibrium characterized by Proposition 2 (a), firms have the same optimal targeting strategies in which they target only part of their past customers. In period 2 Firm A and Firm B have information on consumers respectively in $\left[0, \tilde{x}_{1}\right]$ and $\left[\tilde{x}_{1}, 1\right]$ (with $\tilde{x}_{1} \in\left[\frac{1}{3}, \frac{2}{3}\right]$ ), but they charge a homogeneous price on $\left[\frac{1}{3}, 1\right]$ and $\left[0, \frac{2}{3}\right]$ to soften competition. Hence, by assuming that firms use all available information to price discriminate consumers, previous literature overestimates the competitive effect of information in period 2, as well as the incentives of firms to acquire information in period 1 . We analyze in the next section how strategic targeting impacts the first period of competition.

Proposition 2 (b) characterizes the equilibrium when Firm A is constrained on its targeting strategy and price-discriminates fewer consumers than its unconstrained optimum. This relaxes the competitive pressure on Firm B, which targets more consumers and makes higher profits than in the symmetric equilibrium. Hence it is profitable for a firm to face a competitor constrained on targeting. This can be achieved by undercutting prices in period 1 . Indeed, the value of $\tilde{x}_{1}$ in period 1 depends on the prices set by the firms, and the next section analyzes whether a firm has interest to undercut prices in period 1 in order to constrain the targeting strategy of its competitor in period 2 .

An important element of the analysis is the locations of the indifferent consumers $\tilde{x}_{1}$ in period 1 and $\tilde{x}_{2}$ in period 2 . Indeed, the literature usually finds that BBPD results in poaching practices: some consumers purchase from one firm in period 1, and then from its competitor in period 2. Poaching is considered beneficial for consumers as it results in a more competitive market in period 2, but yields overall an inefficiency as some consumers do not buy the product closest to their taste. The next section shows that in the unique equilibrium of the game with strategic targeting, $\tilde{x}_{1}=\tilde{x}_{2}$ and consumer poaching does not take place.

### 3.2 Period 1: Information Acquisition

This section analyzes competition in the information acquisition period. In the symmetric equilibrium, firms maximize profits in period 1 and market equilibrium
is identical to standard Hotelling competition without data collection. Proposition 3 states that the unique equilibrium is symmetric, and that price undercutting in period 1 to constrain a firm's competitor is not sustainable.

## Proposition 3

- The unique pure strategy equilibrium of the game is symmetric.
- Firms do not engage in price undercutting strategies.
- In both periods:
- Consumers on $\left[0, \frac{1}{2}\right]$ purchase from Firm A.
- Consumers on $\left[\frac{1}{2}, 1\right]$ purchase from Firm B.
- Poaching does not take place.

Proof: see Appendix A.3.
Firms do not have interest to engage in constraining strategies: to constrain their competitor in period 2, firms must undercut prices so that the indifferent consumer $\tilde{x}_{1}$ is very close to the competitor's location in period 1. For instance, if Firm B wants to constrain Firm A, it must be that $\tilde{x}_{1}<\frac{1}{3}$. Reaching such a constraining market outcome induces an important loss in period 1 for a firm, and the increase in profits of period 2 is not sufficient to cover this loss.

The literature on BBPD classically finds that competition in the first period is driven by two main forces: firms want to reach high profits in this first period, but they also anticipate the second competition period in which they have information on their past customers. Hence, competition is fiercer than in the standard Hotelling model without information, as a firm has an additional incentive to serve a larger customer demand in period 1 to charge targeted prices to more consumers in the second period (Choe et al., 2018). This second dimension can be interpreted as a 'competition for information acquisition' in period 1. When firms use strategic targeting, Proposition 3 shows that they do not charge targeted prices to consumers in the middle of the line, and thus they do not make
additional benefits from identifying these consumers in period 1: firms do not compete to acquire consumer information.

Moreover, the same consumer demands are served in both competition periods $\tilde{x}_{1}=\tilde{x}_{2}$. This is a natural consequence of strategic targeting under which firms do not have interest to undercut prices and poach consumers, but soften competition by keeping a large share of consumers untargeted. This result contrasts with previous literature in which poaching occurs in period 2 and some consumers switch products (Fudenberg and Tirole, 2000). Hence, Proposition 3 also contributes to the literature by showing that the adoption of strategic targeting by firms allows them to avoid poaching and price undercutting in the second competition period.

Proposition 3 presents interesting connections with the results of Choe et al. (2018), who show in a similar setting without strategic targeting that the only equilibrium is asymmetric, and poaching occurs in period 2. Proposition 3 states that when firms target consumers strategically, the equilibrium of the game is symmetric. This has important implications for the profits of the firms and for consumer surplus, which are discussed in the next section.

## 4 Welfare

Using the characterization of the equilibrium with strategic targeting, we can now compare the profits of the firms and consumer surplus within this framework where firms target consumers strategically with those in Choe et al. (2018) - in which firms do not target consumers strategically - and in Bounie et al. (2021) - where strategic targeting occurs but firms acquire third-party data from a data broker. An important element of the analysis is that profits, consumer surplus, and the intensity of competition depend on the number of consumers targeted by each firm, which changes in the three regimes.

### 4.1 Strategic vs non-Strategic Targeting

This section compares the profits of the firms and consumer surplus when firms use strategic targeting with those when firms target all past customers, analyzed by

Choe et al. (2018). When firms target all past customers, in the targeting period Firm A charges personalized prices on $\left[0, \tilde{x}_{1}\right]$ and a homogeneous price on $\left[\tilde{x}_{1}, 1\right]$, where $\tilde{x}_{1}$ is the location of the indifferent consumer in period 1. Similarly, Firm B charges personalized prices to consumers on $\left[\tilde{x}_{1}, 1\right]$, and a homogeneous price on consumers located on $\left[0, \tilde{x}_{1}\right]$. Hence the location of the indifferent consumer in period 1 determines at the same time the ability of a firm and of its competitor to charge personalized prices to their past customers in period 2 .

Proposition 4 compares market equilibrium with and without strategic targeting, which allows to analyze the welfare implications of the adoption of strategic targeting by firms.

## Proposition 4

Compared with competition when firms target all their past customers, strategic targeting yields higher industry profits, lower consumer surplus, and a lower intensity of competition.

Proof: see Appendix A.4.
The proof proceeds in two steps, which compare welfare with and without strategic targeting in both competition periods. In period 2 , when firms target all consumers on whom they have collected information, they price aggressively new consumers that they have not served in the information acquisition period. Choe et al. (2018) show that a firm even charges a zero price to new consumers, which reduces the ability of its competitor to extract surplus from targeted consumers.

Using strategic targeting, a firm can profitably deviate from a situation where it targets all past customers by keeping some past customers untargeted. By doing so, it gives incentives to its competitor to increase its price, which allows the firm to charge higher prices to the remaining targeted customers. The loss of profits resulting from charging a homogeneous price instead of targeted prices to a share of past customers is covered by a better extraction of surplus from the remaining targeted customers. This relaxes competition, reduces consumer surplus, and increases the profits of the firms in period 2. ${ }^{17}$

[^10]As strategic firms only target their close-by consumers in period 2 , they do not compete for information acquisition in period 1, and the market outcome is the standard Hotelling equilibrium without information. Competition in period 1 is much more intense when firms are not strategic in their use of data. Indeed, if the indifferent consumer is located at $\tilde{x}_{1}$ in period 1, Firm A (Firm B) will price discriminate consumers on $\left[0, \tilde{x}_{1}\right]\left(\left[\tilde{x}_{1}, 1\right]\right)$ in period 2 . Hence the ability to compete in period 2 is determined by the number of consumers served in period 1 by each firm, and on whom information is collected. As Choe et al. (2018) have shown, intense competition takes place in period 1, under which one of the firms undercuts prices to constrain its competitor in the number of consumers on whom it collects data. The resulting market outcome has lower industry profits and higher consumer surplus than in the standard Hotelling competition framework without information, and therefore than in period 1 of our model with strategic consumer targeting.

### 4.2 First-party vs Third-party Data

This section compares profits and surplus in the targeting period with those in Bounie et al. (2021), where firms acquire information from a data broker. ${ }^{18}$ This allows us to assess the impact on market competition of a change from third to first-party data.

The central difference introduced by a data broker is its ability to internalize the competitive effect of information, by selling information to only one firm and keeping its competitor uninformed. In equilibrium, Bounie et al. (2021) show that Firm A (without loss of generality) acquires data on consumers located in $\left[0, \frac{3}{7}\right]$, and charges a homogeneous price to consumers in $\left[\frac{3}{7}, 1\right]$. The data broker does not sell information to Firm B, who charges a homogeneous price on the whole unit line. Proposition 5 compares profits and surplus with first and third-party data.

## Proposition 5

[^11]First-party data yield lower industry profits, higher consumer surplus, and a higher intensity of competition than third-party data acquired from a data broker. Proof: see Appendix A.5.

A data broker internalizes the business-stealing effect of information by keeping one of the firms uninformed. When firms choose their own targeting strategies using first-party data, they do not internalize the negative externality of targeting on their competitor, and more consumers are identified than with a data broker, which intensifies competition, increases consumer surplus, and reduces the profits of the industry.

## 5 Forward-looking Consumers

Consumers may anticipate in period 1 that firms collect their information to charge targeted prices in period 2 (Li and Jain, 2016). This ability of consumers to anticipate BBPD and purchase products accordingly is a classical consideration of the literature that usually finds a reduction of competition in the information acquisition period as demand becomes less price-sensitive (Fudenberg and Tirole, 2000).

This section considers consumers located in $\left[0, \frac{1}{3}\right]$ who anticipate that they will be charged a targeted price in period 2. It analyzes whether these consumers have interest to purchase from Firm B in period 1 in order to remain hidden from Firm A and pay a homogeneous price in period 2. When choosing which product to purchase in the first period, consumers maximize the sum of utilities over both periods by discounting period 2 with a factor $\delta_{c}$.

## Proposition 6

Strategic consumers purchase from the same firm in both competition periods and do not engage in hiding strategies.

Proof: see Appendix A.6.
Proposition 6 states that consumers do not have interest to hide from firms, and the equilibrium is identical to the baseline framework with myopic consumers.

This result contributes to previous literature that has shown that consumers have interest to change their consumption behavior when they are relatively indifferent between the products of both firms (Fudenberg and Tirole, 2000).

When firms adopt strategic targeting, only consumers with high valuations for a firm's product are targeted in the second period and may have interest to purchase their least preferred product in the first period. Purchasing Firm B's product in period 1 would induce an important opportunity cost for a consumer close to Firm A, which is not recovered in period 2, and consumers purchase their preferred product in both periods. Hence, consumers have no interest to hide when firms use strategic targeting.

## 6 Conclusion

This model of behavior-based price discrimination with strategic customer targeting has two important implications for firms and regulators. First, the analysis shows that the adoption of strategic targeting based on first-party data increases market competition compared with information acquisition from data brokers. Market competition and consumer surplus are higher when firms design their own data collection and targeting strategies than when they purchase information from third parties. Moreover, firms compete on a level playing field when they collect first-party data, while data brokers have incentives to engage in exclusionary practices and sell information to one firm only (Montes et al., 2019; Bounie et al., 2021). It is therefore important for competition authorities to assess whether the efficiency gains from having third-party data brokers specialized in data collection are not outweighed by a reduction in the intensity of competition and higher prices for consumers.

Secondly, with the introduction of new targeting strategies, competition authorities may have to reconsider the benefits of behavior-based price discrimination for consumer surplus. As firms become strategic in their uses of data, they can use information design to soften market competition. This result complements a recent report of the FTC calling for a better understanding of the implications of

Big-data analytics for markets and consumers. ${ }^{19}$
This simple two-period competition framework could be extended to account for positive data collection costs. Collecting, treating, and storing data is indeed costly, and may reduce the profitability of consumer targeting for firms. In particular, asymmetric data collection costs can provide a firm with a significant competitive advantage, and could restore asymmetric competition.

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## A Appendix

## A. 1 Proof of Proposition 1

I characterize the optimal targeting strategy of Firm A. For any given partition used by Firm B, I show that the optimal partition for Firm A is composed of one type (a) segment closest to its location where all consumers are charged personalized prices, and one type (b) segment on the rest of the line where consumers are charged a uniform price. The proof of the optimal partition for Firm B follows the same reasoning.

Firm A can choose any partition $X_{A} \in \Psi_{A}\left(\tilde{x}_{1}\right)$ for a given $\tilde{x}_{1}$. There are three types of segments to consider:

- Segments A, where Firm A is in constrained monopoly;
- Segments B, where Firms 1 and 2 compete.
- Segments C, where Firms 1 makes zero profit.

All segments in $X_{A}^{a}$ are necessarily of type A , while segments in $X_{A}^{b}$ may be of type A, B, and C.

To find the partition that maximizes the profits of Firm A, the proof proceeds in three steps. Step 1 analyzes type A segments, and shows that it is optimal for Firm A to first-degree price discriminate all consumers in these segments. Step 2
shows that all segments of type A are located closest to Firm A. Step 3 analyzes segments of type B and shows that it is always more profitable to target a union of such segments. Therefore, there is only one segment of type B, located furthest away from Firm A, and of size $1-x_{A}$. Finally, segments of type C can be discarded because information on consumers on these segments does not increase profits.

Step 1: I analyze segments of type $A$ where Firm $A$ is in constrained monopoly, and show that targeting all consumers is optimal.

Consider any segment $I=[i, i+l]$ of type A with $l, i$ verifying $i+l \leq 1$, such that Firm A is in monopoly on this segment, constrained by Firm B charging price $p_{B}$. I compare profits with first and third-degree price discrimination and I show that the former is more profitable for Firm A. I write $\pi_{A}^{\text {third }}$ and $\pi_{A}^{\text {first }}$ the profits of Firm A on $I$ with third-degree and first-degree price discrimination.

To prove this claim, I establish that $\pi_{A}^{f i r s t}$ is greater than $\pi_{A}^{t h i r d}$. First, profits with first-degree price discrimination is: $\pi_{A}^{f i r s t}=\int_{i}^{i+l} p_{A}(x) d x$. The demand is $l$ as Firm A gets all consumers by assumption.

$$
V-t x-p_{A}(x)=V-t(1-x)-p_{B} \Longrightarrow p_{A}(x)=t-2 t x+p_{B}
$$

Note that price $p_{B}$ is only affected by strategic interactions on the segments where firms compete, and therefore does not depend on the pricing strategy of Firm A on type A segments. I write the profit function for any $p_{2}$, replacing $p_{A}$ :

$$
\pi_{A}^{t h i r d}=l\left(t+p_{B}-2(l+i) t\right)
$$

Secondly, using a similar argument, I show that the profit with first-degree price discrimination is:

$$
\pi_{A}^{f i r s t}=\int_{i}^{i+l}\left(t-2 t x+p_{B}\right) d x
$$

Comparing $\pi_{A}^{\text {third }}$ and $\pi_{A}^{\text {first }}$ shows that the profit of Firm A using the firstdegree price discrimination is higher than under third-degree price discrimination, which establishes the claim. Therefore, in equilibrium, there is no type A segment in the set $X_{\theta}^{b}$.

Step 2: I show that all segments of type A are closest to Firm A (located at 0 on the unit line by convention).

Going from left to right on the Hotelling line, look for the first time where a type B interval, $J=[i ; i+l]$ of length $l$, is followed by an interval $I=[i+l, i+l+\epsilon]$ of type A.

A simple comparison allows to show that a reordering of the overall interval $J \cup I=[i, i+l+\epsilon]$ in two intervals $I^{\prime}=[i ; i+\epsilon]$ and $J^{\prime}=[i+\epsilon, i+l+\epsilon]$ increases
the profit of Firm A. Indeed, after the re-ordering, full surplus is extracted from consumers in $I^{\prime}$, who have the highest valuation on this interval. All consumers located on $J^{\prime}$ are charged a homogeneous price, which softens the competitive pressure on Firm B compared with $J \cup I$ and increases the competitive price charged by Firm B. By iteration, I conclude that type A segments are always at the left of type B segments.

Step 3: I now analyze segments of type $B$ where firms compete. For Firm A, starting from any partition with at least two segments of type $B, I$ show that a coarser partition always increases the profits of Firm A.

The two previous steps have shown that an optimal partition must be composed for each firm of one type A segment closest to its location and potentially several type B segments on the rest of the line, as illustrated in Figure 3.


Figure 3: Structures of the optimal partitions for each firm after applying steps 1 and 2.

The first two lines represent respectively the partitions used by Firm A and by Firm B. The thick black lines correspond to the consumers who are charged a personalized price by the closest firm. On each remaining segment of the first line, Firm A charges a homogeneous price: $p_{A 12}$ on the first segment starting from the left, $p_{A 22}$ on the second segment, and so on. Similarly on the second line, Firm B
charges homogeneous price $p_{B 12}$ on the first segment after the thick line, starting from the right, price $p_{B 22}$ on the second segment and so on.

The third line represents the resulting consumer demands, with in blue the demand of Firm A and in red the demand of Firm B. In each segment where a homogeneous price is charged by both firms, both firms reach a positive consumer demand, or else, one of the firms would charge personalized prices following step 1.

We want to show that the optimal partition has only one type B segment where firms charge a homogeneous price (contrary to four segments for Firm A and three segments for Firm B in the above example).

To do so, we show that if Firm A has a partition $X_{1}$ with at least two segments where it competes with Firm B, a coarser partition $X_{1}^{\prime}$ where the two adjacent segments located closest to Firm A are merged yields higher profits.

I compute the profits of Firm A on all the segments where firms compete. There are three types of segments to consider:

1. segments of type A that with partition $X_{1}$ that remain of type A with partition $X_{1}^{\prime}$.
2. segments of type B with partition $X_{1}$ that become of type A with partition $X_{1}^{\prime}$.
3. segments of type B with partition $X_{1}$ that remain of type B with partition $X_{1}^{\prime}$.

Note that there exists $\tilde{x}_{B}$ such that Firm B charges a homogeneous price on segment $\left[0, \tilde{x}_{B}\right]$ with $\tilde{x}_{B}>\tilde{x}_{1}$. Throughout the resolution we denote by $\hat{p}_{B}$ this price under partition $X_{1}$ and $\hat{p}_{B}^{\prime}$ with partition $X_{1}^{\prime}$. Similarly, there exists $\tilde{x}_{A}$ such that Firm A charges a homogeneous price on segment $\left[\tilde{x}_{A}, 1\right]$ with $\tilde{x}_{A}<\tilde{x}_{1}$. Throughout the resolution we denote by $\hat{p}_{A}$ this price under partition $X_{1}$ and $\hat{p}_{A}^{\prime}$ with partition $X_{1}^{\prime}$. Moreover, for simplicity we denote by $\hat{p}_{A i}$ and $\hat{p}_{B i}$ the homogeneous prices charged by Firm A and Firm B on their type B segments.

Profits always increase on segments that are of type A with partitions $X_{1}$ and $X_{1}^{\prime}$. Indeed, I will show that $\hat{p}_{B}^{\prime}$ with partition $X_{1}^{\prime}$ is higher than $\hat{p}_{B}$ with partition $X_{1}$, and thus the profits of Firm A on type A segments increase.

It will also be useful to introduce the following notations. On interval $\left[0, \tilde{x}_{A}\right]$, there are $n$ segments where firms compete. Among them, there are $m$ segments which are type $B$ in partition $X_{1}$, but are no longer necessarily of type $B$ in partition $X_{1}$ (and are therefore of type A). There are $n+1-m$ segments of type B with partition $X_{1}$ that remain of type B with partition $X_{1}^{\prime}$. I compute prices and profits on these $n+1+m$ segments.

On interval $\left[\tilde{x}_{A}, \tilde{x}_{B}\right]$, withe partition $X_{1}\left(X_{1}^{\prime}\right)$ Firm A charges price $\hat{p}_{A}\left(\hat{p}_{A}^{\prime}\right)$ and Firm B charges price $\hat{p}_{B}\left(\hat{p}_{B}^{\prime}\right)$.

To compare the profits of the informed firm under both partitions, I first characterize type $B$ segments. A segment of type $B$ is non null, if the following
restrictions imposed by the structure of the model, are met: respectively positive demand and the existence of competition on segments of type B. In order to characterize type A and type B segments, it is useful to consider the following inequality:

$$
\begin{equation*}
\forall i, l \in[0,1] \text { s.t. } 0 \leq l \leq 1-i, i \leq \frac{\hat{p}_{B}+t}{2 t} \text { and } \frac{\hat{p}_{B}+t}{2 t}-l \leq i+l . \tag{5}
\end{equation*}
$$

In particular, I use the relation that Eq. 5 draws between price $\hat{p}_{B}$ and segments endpoints $i$ and $i+l$ to compare the profits of Firm A with partitions $X_{1}$ and $X_{1}^{\prime}$. Without loss of generality, I rewrite the notation of type A and B segments. The segment of type A is of size $\epsilon$ and is located at $u_{i}-\epsilon$, and segments of type B , are located at $s_{i}$ and are of size $l_{i} \cdot{ }^{20}$ On interval $\left[0, \tilde{x}_{B}\right]$, here are $n \in \mathbb{N}$ segments of type B , where prices are noted $\hat{p}_{A i}^{B}$. On interval $\left[\tilde{x}_{B}, 1\right]$, there are $n^{\prime}$ segments where firms compete with Firm A charging price $\hat{p}_{A}$ and Firm B charging prices $\hat{p}_{B}$ and $\hat{p}_{B i}^{B}$.

Our next point will be to show that profits on interval $\left[0, \tilde{x}_{B}\right]$ where Firm B charges a homogeneous price increase with partition $X_{1}^{\prime}$. Profits on interval $\left[\tilde{x}_{B}, 1\right]$ clearly increase, as we will show that $\hat{p}_{B}$ increases, relaxing the competitive pressure on Firm A in this interval.

I find the demand for Firm A on segments in $\left[0, \tilde{x}_{B}\right]$ using the location of the indifferent consumer:

$$
d_{A i}=x-s_{i}=\frac{\hat{p}_{B}-\hat{p}_{A i}^{B}+t}{2 t}-s_{i} .
$$

I can rewrite the profits of Firm A as the sum of three terms. The first term represents the profits on segments of type A . The second term represents the profits on segments of type B on interval $\left[0, \tilde{x}_{B}\right]$, the third term represents the profits on segments of type B on interval $\left[0, \tilde{x}_{B}\right]$.

$$
\begin{aligned}
\pi_{A}\left(X_{1}\right) & =\int_{0}^{s_{1}} \hat{p}_{A}^{A}(x) d x+\sum_{i=1}^{n} \hat{p}_{A i}^{B}\left[\frac{\hat{p}_{B}-\hat{p}_{A i}^{B}+t}{2 t}-s_{i}\right] \\
& +\hat{p}_{A}\left[\frac{\hat{p}_{A}-\hat{p}_{B}+t}{2 t}-\tilde{x}_{A}\right]+\sum_{i=1}^{n^{\prime}} \hat{p}_{A}\left[\frac{\hat{p}_{B i}^{B}-\hat{p}_{A}+t}{2 t}-s_{i}\right] .
\end{aligned}
$$

The price $\hat{p}_{B}$ is chosen by Firm B to maximize local profits generated on segments of type B only, where the demand for Firm B is:

$$
d_{B i}=s_{i}+l_{i}-x=\frac{\hat{p}_{A i}^{B}-\hat{p}_{B}-t}{2 t}+s_{i}+l_{i} .
$$

[^13]Firm B sets price $\hat{p}_{B}$ to maximize profits on interval $\left[0, \tilde{x}_{B}\right]$, which can be written as:

$$
\begin{equation*}
\pi_{B l}\left(X_{1}\right)=\sum_{i=1}^{n} \hat{p}_{B}\left[\frac{\hat{p}_{A i}^{B}-\hat{p}_{B}-t}{2 t}+s_{i}+l_{i}\right] . \tag{6}
\end{equation*}
$$

Firm A maximizes profits $\pi_{A}\left(X_{1}\right)$ with respect to $\hat{p}_{A}^{A}(x), \hat{p}_{A i}^{B}$ and $\hat{p}_{A}$, and Firm B maximizes $\pi_{B l}\left(X_{1}\right)$ with respect to $\hat{p}_{B}$, both profits are strictly concave.

Equilibrium prices are:

$$
\begin{aligned}
\hat{p}_{B} & =-\frac{t}{3}+\frac{4 t}{3(n+1)} \sum_{i=1}^{n+1}\left[\frac{s_{i}}{2}+l_{i}\right], \\
\hat{p}_{A i}^{B} & =\frac{\hat{p}_{B}+t}{2}-s_{i} t \\
& =\frac{t}{3}+\frac{2 t}{3(n+1)} \sum_{i=1}^{n+1}\left[\frac{s_{i}}{2}+l_{i}\right]-s_{i} t .
\end{aligned}
$$

Let $\hat{p}_{1 s}^{B}$ and $\hat{p}_{1 s+l}^{B}$ be the prices on the first two segments when the partition is $x_{1}$.

$$
\begin{gathered}
\hat{p}_{1 s}^{B}=\frac{\hat{p}_{B}+t}{2}-\frac{s t}{k}, \\
\hat{p}_{1 s+l}^{B}=\frac{\hat{p}_{B}+t}{2}-\frac{s+l}{k} t,
\end{gathered}
$$

$\hat{p}_{B}^{\prime}$ is the price set by Firm B with partition $X_{1}^{\prime}$, and $\hat{p}_{1 s}^{B^{\prime}}$ is the price set by Firm A on the last segment of partition $X_{1}^{\prime}$.

Inequalities in Eq. 5 might not hold as price $\hat{p}_{B}$ varies depending on the partition acquired by Firm A. This implies that segments which are of type B with partition $X_{1}$ are then of type A with partition $X_{1}^{\prime}$. This is because the coarser the partition, the higher $\hat{p}_{B}$. I note $\tilde{s}_{i}$ the $m$ segments where it is the case. I then have:

$$
\begin{aligned}
\hat{p}_{B}^{\prime} & =\frac{4 t}{3(n-m)}\left[-\frac{n-m}{4}+\sum_{i=1}^{n}\left[\frac{s_{i}}{2}+l_{i}\right]-\sum_{i=1}^{m} \frac{\tilde{s}_{i}}{2}\right] \\
& =\frac{4 t}{3(n-m)}\left[-\frac{n+1}{4}+\sum_{i=1}^{n+1}\left[\frac{s_{i}}{2}+l_{i}\right]+\frac{m+1}{4}-\sum_{i=1}^{m} \frac{\tilde{s}_{i}}{2}-\frac{s+l}{2}\right] \\
& =\hat{p}_{B}+\frac{4 t}{3(n-m)}\left[\frac{3(m+1) \hat{p}_{B}}{4 t}+\frac{m+1}{4}-\sum_{i=1}^{m} \frac{\tilde{s}_{i}}{2}-\frac{s+l}{2}\right] \\
& \geq \hat{p}_{B}+\frac{4 t}{3(n-m)}\left[\frac{3}{4 t} \hat{p}_{B}+\frac{m \hat{p}_{B}}{2 t}+\frac{1}{4}-\frac{s+l}{2}\right], \\
\hat{p}_{1 s}^{B^{\prime}} & =\frac{\hat{p}_{B}+t}{2}-\frac{s t}{k},
\end{aligned}
$$

$$
\begin{aligned}
\pi_{A}\left(X_{1}\right) & =\sum_{i=1, s_{i} \neq \tilde{s}_{i}}^{n} p_{A i}\left[\frac{\hat{p}_{B}+t}{4 t}-\frac{s_{i}}{2}\right] \\
& +\sum_{i=1}^{m} \hat{p}_{A i}^{B}\left[\frac{\hat{p}_{B}+t}{4 t}-\frac{\tilde{s}_{i}}{2}\right]+\hat{p}_{1 s+l}^{B}\left[\frac{\hat{p}_{B}+t}{4 t}-\frac{s+l}{2}\right]+\sum_{i=1}^{n^{\prime}} \hat{p}_{A}\left[\frac{\hat{p}_{B i}^{B}-\hat{p}_{A}+t}{2 t}-s_{i}\right] \\
\pi_{A}\left(X_{1}^{\prime}\right) & =\sum_{i=1, s_{i} \neq \tilde{s}_{i}}^{n} \hat{p}_{A i}^{B^{\prime}}\left[\frac{\hat{p}_{B}^{\prime}+t}{4 t}-\frac{s_{i}}{2}\right]+\sum_{i=1}^{m} \frac{\tilde{l}_{i}}{k}\left[\hat{p}_{B}^{\prime}+t-2 t \frac{\tilde{s}_{i}+\tilde{l}_{i}}{k}\right]+\sum_{i=1}^{m^{\prime}} \hat{p}_{A}^{\prime}\left[\frac{\hat{p}_{B i}^{B^{\prime}}-\hat{p}_{A}^{\prime}+t}{2 t}-s_{i}\right] .
\end{aligned}
$$

I compare the profits of Firm A in both cases in order to show that $X_{1}^{\prime}$ induces higher profits. Clearly, because $\hat{p}_{B}^{\prime}>\hat{p}_{B}$, we have that:

$$
\hat{p}_{A}\left[\hat{p}_{B}-\frac{\hat{p}_{A}+t}{2 t}-\tilde{x}_{A}\right]+\sum_{i=1}^{n^{\prime}} \hat{p}_{A}\left[\hat{p}_{B i}^{B}-\frac{\hat{p}_{A}+t}{2 t}-s_{i}\right]>\hat{p}_{A}^{\prime}\left[\hat{p}_{B}^{\prime}-\frac{\hat{p}_{A}^{\prime}+t}{2 t}-\tilde{x}_{A}\right]+\sum_{i=1}^{m^{\prime}} \hat{p}_{A}^{\prime}\left[\frac{\hat{p}_{B i}^{B^{\prime}}-\hat{p}_{A}^{\prime}+t}{2 t}-s_{i}\right] .
$$

Hence, we focus on the rest of the expression in the remaining of the proof:

$$
\begin{aligned}
\Delta \pi_{A} & =\pi_{A}\left(X_{1}^{\prime}\right)-\pi_{A}\left(X_{1}\right) \\
& \left.\geq \sum_{i=1, s_{i} \neq \tilde{s}_{i}}^{n} \hat{p}_{A i}^{B^{\prime}} \frac{\hat{p}_{B}^{\prime}+t}{4 t}-\frac{s_{i}}{2}\right]-\sum_{i=1, s_{i} \neq \tilde{s}_{i}}^{n} \hat{p}_{A i}^{B}\left[\frac{\hat{p}_{B}+t}{4 t}-\frac{s_{i}}{2}\right] \\
& +\sum_{i=1}^{m} \frac{\tilde{l}_{i}}{k}\left[\hat{p}_{B}^{\prime}+t-2 t \frac{\tilde{s}_{i}+\tilde{l}_{i}}{k}\right]-\sum_{i=1}^{m} \hat{p}_{A i}^{B}\left[\frac{\hat{p}_{B}+t}{4 t}-\frac{\tilde{s}_{i}}{2}\right]-\hat{p}_{1 s+l}^{B}\left[\frac{\hat{p}_{B}+t}{4 t}-\frac{s+l}{2}\right] \\
& =\frac{t}{2} \sum_{i=1, s_{i} \neq \tilde{s}_{i}}^{n}\left[\frac{\hat{p}_{B}^{\prime}+t}{2 t}-s_{i}\right]^{2}-\frac{t}{2} \sum_{i=1, s_{i} \neq \tilde{s}_{i}}^{n}\left[\frac{\hat{p}_{B}+t}{2 t}-s_{i}\right]^{2} \\
& +\frac{t}{2} \sum_{i=1}^{m} \frac{\tilde{l}_{i}}{k}\left[2 \frac{\hat{p}_{B}^{\prime}+t}{t}-4 \frac{\tilde{s}_{i}+\tilde{l}_{i}}{k}\right]-\frac{t}{2} \sum_{i=1}^{m}\left[\frac{\hat{p}_{B}+t}{2 t}-\frac{\tilde{s}_{i}}{2}\right]^{2}-\frac{t}{2}\left[\frac{\hat{p}_{B}+t}{2 t}-\frac{s+l}{k}\right]^{2} .
\end{aligned}
$$

I consider the terms separately. First,

$$
\begin{aligned}
& \frac{t}{2} \sum_{i=1, s_{i} \neq \tilde{s}_{i}}^{n}\left[\frac{\hat{p}_{B}^{\prime}+t}{2 t}-s_{i}\right]^{2}-\frac{t}{2} \sum_{i=1, s_{i} \neq \tilde{s}_{i}}^{n}\left[\frac{\hat{p}_{B}+t}{2 t}-s_{i}\right]^{2} \\
= & \frac{t}{2} \sum_{i=1, s_{i} \neq \tilde{s}_{i}}^{n}\left[\left[\frac{2}{3(n-m)}\left[\frac{3}{4 t} \hat{p}_{B}+\frac{m \hat{p}_{B}}{2 t}+\frac{1}{4}-\frac{s+l}{2}\right]\right]^{2}\right. \\
& \left.+\left[\frac{\hat{p}_{B}+t}{2 t}-s_{i}\right]\left[\frac{4}{3(n-m)}\left[\frac{3}{4 t} \hat{p}_{B}+\frac{m \hat{p}_{B}}{2 t}+\frac{1}{4}-\frac{s+l}{2}\right]\right]\right] \\
\geq & \frac{t}{2}\left[\frac{\hat{p}_{B}+t}{2 t}-\frac{s+l}{k}\right] \frac{4}{3}\left[\frac{3}{4 t} \hat{p}_{B}+\frac{m \hat{p}_{B}}{2 t}+\frac{1}{4}-\frac{s+l}{2}\right] .
\end{aligned}
$$

Secondly, on segments of type B with partition $X_{1}$ that are of type A with partition $X_{1}^{\prime}$ :

$$
\frac{t}{2} \sum_{i=1}^{m} \frac{\tilde{l}_{i}}{k}\left[2 \frac{\hat{p}_{B}^{\prime}+t}{t}-4 \frac{\tilde{s}_{i}+\tilde{l}_{i}}{k}\right]-\frac{t}{2} \sum_{i=1}^{m}\left[\frac{\hat{p}_{B}+t}{2 t}-\frac{\tilde{s}_{i}}{2}\right]^{2} .
$$

On these $m$ segments, inequalities in Eq. 5 hold for price $\hat{p}_{B}^{\prime}$ but not for $\hat{p}_{B}$. Thus I can rank prices according to $\tilde{s}_{i}$ and $\tilde{l}_{i}$ :

$$
\frac{\tilde{s}_{i}+\tilde{l}_{i}}{k} \geq \frac{\hat{p}_{B}+t}{2 t}-\frac{\tilde{l}_{i}}{k} \quad \text { and } \quad \frac{\hat{p}_{B}^{\prime}+t}{2 t}-\frac{\tilde{l}_{i}}{k} \geq \frac{\tilde{s}_{i}+\tilde{l}_{i}}{k} .
$$

thus:

$$
2 \frac{\tilde{l}_{i}}{k} \geq \frac{\hat{p}_{B}+t}{2 t}-\tilde{s}_{i} \quad \text { and } \quad \frac{\hat{p}_{B}^{\prime}+t}{2 t}-2 \frac{\tilde{l}_{i}}{k} \geq \tilde{s}_{i} .
$$

By replacing $\tilde{s}_{i}$ by its upper bound value and then $\tilde{l}_{i}$ by its lower bound value I obtain:

$$
\frac{t}{2} \sum_{i=1}^{m} \frac{\tilde{l}_{i}}{k}\left[2 \frac{\hat{p}_{B}^{\prime}+t}{t}-4 \frac{\tilde{s}_{i}+\tilde{l}_{i}}{k}\right]-\frac{t}{2} \sum_{i=1}^{m}\left[\frac{\hat{p}_{B}+t}{2 t}-\frac{\tilde{s}_{i}}{2}\right]^{2} \geq 0 .
$$

Getting back to the profits difference, I obtain:

$$
\begin{align*}
\Delta \pi_{A} & \geq \frac{t}{2}\left[\frac{\hat{p}_{B}+t}{2 t}-\frac{s+l}{k}\right] \frac{4}{3}\left[\frac{3}{4 t} \hat{p}_{B}+\frac{m \hat{p}_{B}}{2 t}+\frac{1}{4}-\frac{s+l}{2}\right]-\frac{t}{2}\left[\frac{\hat{p}_{B}+t}{2 t}-\frac{s+l}{k}\right]^{2} \\
& \geq \frac{t}{2}\left[\frac{\hat{p}_{B}+t}{2 t}-\frac{s+l}{k}\right]\left[\frac{\hat{p}_{B}}{2 t}+\frac{s+l}{3 k}-\frac{1}{6}\right] . \tag{7}
\end{align*}
$$

The first bracket of Equation 7 is positive given Eq. 5. The second bracket is positive if $\frac{\hat{p}_{B}}{2 t}+\frac{s+l}{3 k} \geq \frac{1}{6}$. A sufficient condition for this result to hold is $\hat{p}_{B} \geq \frac{t}{3}$. I prove that this inequality is always satisfied by showing that the partition that contains all segments minimizes the price and profit of Firm B, and that in this case, $\hat{p}_{B} \geq \frac{t}{2} .{ }^{21}$ And as this price is greater than $\frac{1}{6}$, the second bracket of Equation 7 is positive. This proves that $\Delta \pi_{A} \geq 0$.

At this point of the step, we have shown that it is always optimal for each firm to use a partition with the following shape: each firm uses a partition with at most two segments of type B located furthest from its location. If both firms have only one such segment in the middle of the line, the proof is completed. If one firm has two segments of type B and the other firm has one segment of type $B$, the step above directly applies, and the transformation consisting in merging both type B segments is profit increasing for the firm.

If both firms have two type B segments, we must show that such partition denoted $X_{A}$ - always yields lower profits than the same partition but with the two

[^14]type b segments merged into one type B segment - denoted $X_{A}^{\prime}$ (we denote by $X_{B}$ the partition of Firm B). This is the last part of the proof, which we establish now. Such partition is depicted on Figure 4.




Firm B

Figure 4: Structures of the optimal partitions for each firm after applying steps 1, 2 , and the previous part of step 3 .

The profits of Firm A can be written:

$$
\begin{aligned}
\pi_{A}\left(X_{A}\right) & =\int_{0}^{s_{1}} \hat{p}_{A}^{A}(x) d x+\hat{p}_{A 1}\left[\frac{\hat{p}_{B 2}-\hat{p}_{A 1}+t}{2 t}-s_{1}\right] \\
& +\hat{p}_{A 2}\left[\frac{\hat{p}_{B 2}-\hat{p}_{A 2}+t}{2 t}-s_{2}\right]+\hat{p}_{A 2}\left[\frac{\hat{p}_{B 1}-\hat{p}_{A 2}+t}{2 t}-s_{3}\right] . \\
\pi_{B}\left(X_{B}\right) & =\int_{s_{4}}^{1} \hat{p}_{B}^{A}(x) d x+\hat{p}_{B 1}\left[s_{4}-\frac{\hat{p}_{B 1}-\hat{p}_{A 2}+t}{2 t}\right] \\
& +\hat{p}_{B 1}\left[s_{3}-\frac{\hat{p}_{B 1}-\hat{p}_{A 2}+t}{2 t}\right]+\hat{p}_{B 2}\left[s_{2}-\frac{\hat{p}_{B 1}-\hat{p}_{A 2}+t}{2 t}\right] .
\end{aligned}
$$

We can show that the profits of Firm A are lower than when merging the two type B segments and choosing an optimal $s_{1}^{*}$ such that all consumers are charged a targeted price on $\left[0, s_{1}^{*}\right]$ and one homogeneous price on $\left[s_{1}^{*}, 1\right]$. We need to consider two cases. Either Firm A makes positive profits on $\left[s_{3}, s_{4}\right]$ and makes profits as follows:

$$
\begin{aligned}
& \pi_{A}\left(X_{A}^{\prime}\right)=\int_{0}^{s_{1}^{*}} \hat{p}_{A}^{A}(x) d x+\hat{p}_{A 1}^{\prime}\left[\frac{\hat{p}_{B 2}-\hat{p}_{A 1}^{\prime}+t}{2 t}-s_{1}^{*}\right]+\hat{p}_{A 1}^{\prime}\left[\frac{\hat{p}_{B 1}-\hat{p}_{A 1}^{\prime}+t}{2 t}-s_{3}\right] . \\
& \pi_{B}\left(X_{B}^{\prime}\right)=\int_{s_{4}}^{1} \hat{p}_{B}^{A}(x) d x+\hat{p}_{B 1}\left[s_{4}-\frac{\hat{p}_{B 1}-\hat{p}_{A 1}^{\prime}+t}{2 t}\right]+\hat{p}_{B 2}\left[s_{3}-\frac{\hat{p}_{B 2}-\hat{p}_{A 1}^{\prime}+t}{2 t}\right] .
\end{aligned}
$$

Or Firm A does not make positive profits on $\left[s_{3}, s_{4}\right]$ and makes profits as follows:

$$
\begin{aligned}
\pi_{A}\left(X_{A}^{\prime}\right) & =\int_{0}^{s_{1}^{*}} \hat{p}_{A}^{A}(x) d x+\hat{p}_{A 1}^{\prime}\left[\frac{\hat{p}_{B 2}-\hat{p}_{A 1}^{\prime}+t}{2 t}-s_{1}^{*}\right] . \\
\pi_{A}\left(X_{B}\right) & =\int_{s_{4}}^{1} \hat{p}_{B}^{A}(x) d x+\hat{p}_{B 1}\left[s_{4}-\frac{\hat{p}_{B 1}-\hat{p}_{A 1}^{\prime}+t}{2 t}\right] \\
& +\hat{p}_{B 1}\left[s_{3}-\frac{\hat{p}_{B 1}-\hat{p}_{A 1}^{\prime}+t}{2 t}\right]+\hat{p}_{B 2}\left[s_{2}-\frac{\hat{p}_{B 2}-\hat{p}_{A 1}^{\prime}+t}{2 t}\right] .
\end{aligned}
$$

Having stated this last step of the problem, I keep its resolution available upon request to keep the proof as concise as possible. Overall, the last transformation increases the profits of Firm A.

## Conclusion

This result allows us to establish that it is always more profitable for Firm A to use a partition with one segment of type $B$ than to use a partition with several segments of type B.

These three steps prove that the optimal partition for each firm includes two intervals: Firm A first-degree price discriminates consumers on interval $\left[0, x_{A}\right]$, and charges a homogeneous price on the second interval located at $\left[x_{A}, 1\right]$. By symmetry, it is optimal for Firm B to target all consumers on interval $\left[1-x_{B}, 1\right]$ and to charge a homogeneous price to consumers on $\left[0,1-x_{B}\right]$.

## A. 2 Proof of Proposition 2

I characterize the optimal targeting strategies of the firms in period 2. I first compute prices and demands in period 2 when firms target consumers strategically. Firm A chooses the value of $x_{A}$ such that it price discriminates consumers on $\left[0, x_{A}\right]$, and charges consumers on $\left[x_{A}, 1\right]$ a homogeneous price. Similarly, Firm B price discriminates consumers on $\left[1-x_{B}, 1\right]$, and charges a homogeneous price to consumers on $\left[0,1-x_{B}\right]$. The choices of $x_{A}$ and $x_{B}$ correspond to the targeting strategies of Firm A and Firm B. I will provide prices and profits in period 2, and I will characterize $x_{A}$ and $x_{B}$.

## Prices and demand.

Firm A sets a price $p_{A 2}(x)$ for consumers located at $\left[0, x_{1}\right]$. Similarly, Firm B sets a price $p_{B 2}(x)$ for consumers located at $\left[1-x_{B}, 1\right]$. Firm $\theta$ then sets a unique price $p_{\theta 2}$ on the rest of the unit line. The price charged to targeted consumers by Firm A satisfies:

$$
\begin{aligned}
& V-t x-p_{A 2}(x)=V-t(1-x)-p_{B 2} \\
\Longrightarrow & x=\frac{p_{B 2}-p_{A 2}(x)+t}{2 t} \\
\Longrightarrow & p_{A 2}(x)=p_{B 2}+t-2 t x .
\end{aligned}
$$

Firm B charges homogeneous price $p_{B 2}$ on interval $\left[0,1-x_{B}\right]$, and charges targeted prices on $\left[1-x_{B}, 1\right]$ :

$$
p_{B 2}(x)=p_{A 2}+t-2 t x .
$$

Let denote $d_{A 2}$ the demand for Firm A (resp. $d_{B 2}$ the demand for Firm B) where firms compete. $d_{A 2}$ is determined by the indifferent consumer $\tilde{x}_{2}$ :
$V-t \tilde{x}_{2}-p_{A 2}=V-t\left(1-\tilde{x}_{2}\right)-p_{B 2} \Longrightarrow \tilde{x}_{2}=\frac{p_{B 2}-p_{A 2}+t}{2 t}$ and $d_{A 2}=\tilde{x}_{2}-x_{A}=$ $\frac{p_{B 2}-p_{A 2}+t}{2 t}-x_{A}\left(\right.$ resp. $\left.d_{B 2}=1-x_{B}-\frac{p_{B 2}-p_{A 2}+t}{2 t}\right)$.

## Profits of the firms.

The profits of the firms are:
$\pi_{A 2}=\int_{0}^{x_{A}} p_{A 2}(x) d x+d_{A 2} p_{A 2}=\int_{0}^{x_{A}}\left(p_{B 2}+t-2 t x\right) d x+\left(\frac{p_{B 2}-p_{A 2}+t}{2 t}-x_{A}\right) p_{A 2}$,
$\pi_{B 2}=\int_{1-x_{B}}^{1} p_{B 2}(x) d x+d_{B 2} p_{B 2}=\int_{1-x_{B}}^{1}\left(p_{A 2}+t-2 t x\right) d x+\left(\frac{p_{A 2}-p_{B 2}+t}{2 t}-x_{B}\right) p_{B 2}$.

## Prices and demands in equilibrium.

I now compute the optimal prices and demands, using first-order conditions on $\pi_{\theta}$ with respect to $p_{\theta}$. Prices in equilibrium are:

$$
\begin{aligned}
p_{A 2} & =t\left[1-\frac{2}{3} x_{B}-\frac{4}{3} x_{A}\right], \\
p_{B 2} & =t\left[1-\frac{2}{3} x_{A}-\frac{4}{3} x_{B}\right] .
\end{aligned}
$$

I rule out negative prices from the analysis: $p_{\theta 2}$ is taken equal to zero in case its above expression is negative.

Replacing these values in the above demands and prices gives:

$$
\begin{aligned}
& p_{A 2}(x)=2 t-\frac{4 t}{3} x_{B}-\frac{2 t}{3} x_{A}-2 t x, \\
& p_{B 2}(x)=2 t-\frac{4 t}{3} x_{A}-\frac{2 t}{3} x_{B}-2 t x
\end{aligned}
$$

Demands in equilibrium are as follows:

$$
\begin{aligned}
d_{A 2} & =\frac{1}{2}-\frac{2}{3} x_{A}-\frac{1}{3} x_{B}, \\
d_{B 2} & =\frac{4}{3} x_{B}-\frac{1}{2}-\frac{1}{3} x_{A} .
\end{aligned}
$$

## Profits in equilibrium.

I compute profits by replacing prices and demands by their equilibrium values:

$$
\pi_{\theta 2}=\frac{t}{2}-\frac{7}{9} x_{\theta}^{2} t+\frac{2}{9} x_{-\theta}^{2} t-\frac{4}{9} x_{\theta} x_{-\theta} t+\frac{2}{3} x_{\theta} t-\frac{2}{3} x_{-\theta} t .
$$

Profits are strictly concave functions with respect to $x_{A}$ and $x_{B}$, and they have a unique maximum.

## Optimal targeting strategies: unconstrained.

I derive the optimal targeting strategies $x_{A}^{*}$ and $x_{B}^{*}$ of Firm A and Firm B. The targeting strategies $x_{A}^{*}$ and $x_{B}^{*}$ are chosen as simultaneous best responses. I apply the first-order condition on $\pi_{A 2}$ with respect to $x_{A}$ and to $\pi_{B 2}$ with respect to $x_{B}$, and I find:

$$
x_{A}^{*}=x_{B}^{*}=\frac{1}{3} .
$$

As $p_{A 2}^{*}=p_{B 2}$, the indifferent consumer in period 2 is located at $\tilde{x}_{2}=\frac{1}{2}$.
By replacing $x_{A}^{*}$ and $x_{B}^{*}$ into $\pi_{A 2}$ and $\pi_{B 2}$ I obtain:

$$
\pi_{\theta 2}^{*}=\frac{7 t}{18} .
$$

## Optimal targeting strategies: constrained.

I derive the optimal targeting strategy $x_{B}^{*}$ of Firm B when Firm A is constrained and $x_{A}^{*}=\tilde{x}_{1}$.
$x_{B}^{*}$ is chosen as a best response to $x_{A}^{*}=\tilde{x}_{1}$. The profits of the firms are the following:

$$
\begin{aligned}
& \pi_{A 2}=\frac{t}{2}-\frac{7}{9} \tilde{x}_{1}^{2} t+\frac{2}{9} x_{B}^{2} t-\frac{4}{9} \tilde{x}_{1} x_{B} t+\frac{2}{3} \tilde{x}_{1} t-\frac{2}{3} x_{B} t . \\
& \pi_{B 2}=\frac{t}{2}-\frac{7}{9} x_{B}^{2} t+\frac{2}{9} \tilde{x}_{1}^{2} t-\frac{4}{9} x_{B} \tilde{x}_{1} t+\frac{2}{3} x_{B} t-\frac{2}{3} \tilde{x}_{1} t .
\end{aligned}
$$

I apply first-order conditions on $\pi_{B 2}$ with respect to $x_{B}$ :

$$
x_{B}^{*}=\frac{3}{7}-\frac{2 \tilde{x}_{1}}{7} .
$$

Replacing $x_{A}^{*}$ and $x_{B}^{*}$ into $p_{A 2}^{*}$ and $p_{B 2}^{*}$, I find that the indifferent consumer in period 2 is located at $\tilde{x}_{2}=\frac{6 \tilde{x}_{1}+5}{14}$.

By replacing $x_{A}^{*}$ and $x_{B}^{*}$ by their expressions into $\pi_{A 2}$ and $\pi_{B 2} \mathrm{I}$ obtain:

$$
\begin{gathered}
\pi_{A 2}^{*}=\frac{25 t}{98}+\frac{30 t}{49} \tilde{x}_{1}-\frac{31 t}{49} \tilde{x}_{1}^{2} \\
\pi_{B 2}^{*}=\frac{9 t}{14}-\frac{6 t}{7} \tilde{x}_{1}+\frac{2 t}{7} \tilde{x}_{1}^{2}
\end{gathered}
$$

## A. 3 Proof of Proposition 3

I show that the only equilibrium of the game is symmetric. In the previous proof, prices and targeting strategies in the symmetric case are computed as simultaneous best responses, deviation is not profitable and these strategies constitute an equilibrium.

I now show that the constrained, asymmetric strategy when Firm B undercuts prices is not sustainable, as a firm willing to constrain its competitor by undercutting prices always benefits from deviating to the symmetric equilibrium.

Profits in the symmetric equilibrium are equal to $\frac{7 t}{18}$ in period 2 and $\frac{t}{2}$ in period 1 and overall, the payoff of firms in the symmetric equilibrium is $\delta \frac{7 t}{18}+\frac{t}{2}$.

I provide an upper bound to the profits of a firm adopting a constraining strategy. The maximal profit in period 2 in the constrained case is reached when $\tilde{x}_{1}=0$ (when Firm A is constrained) and is equal to $\frac{9 t}{14}$.

For Firm B to constrain Firm A in period 1, it must be the case that $\tilde{x}_{1} \leq \frac{1}{3}$. Let us consider the least constraining case where $\tilde{x}_{1}=\frac{1}{3}$, which leads to the highest profits of Firm B among the set of constraining strategies in period 1.

It is easy to show that to obtain $\tilde{x}_{1}=\frac{1}{3}$, Firm B must charge $p_{B 1}=\frac{t}{3}$ yielding profits in period 1 equal to $\frac{2 t}{9}$.

Thus the sum of profits over both periods in the constraining case is therefore lower than $\frac{2 t}{9}+\delta \frac{9 t}{14}<\frac{t}{2}+\delta \frac{7 t}{18}$, and profits are higher in the symmetric equilibrium.

Hence deviation is profitable, asymmetric pricing is not sustainable, and the only equilibrium of the game is symmetric.

## A. 4 Proof of Proposition 4

I show that the profits of the firms are greater, and consumer surplus lower in this model with strategic targeting than when firms target all consumers, as in the model of Choe et al. (2018). I cannot directly compare the values of profits and surplus between both models, as they consider a Hotelling competition framework with quadratic transportation costs while I consider linear transportation costs.

Hence I compare profits in both models with those in the standard Hotelling competition framework without information. In this article, profits in period 1 are those in the standard Hotelling model, while profits in period 1 in the model of Choe et al. (2018) are lower than in Hotelling.

Moreover, the profits of the firms are also lower in the targeting period as firms target more consumers in the model of Choe et al. (2018) than with strategic targeting. The resulting consumer surplus is lower with strategic targeting. Because profits are greater with than without strategic targeting in each competition period taken separately, the proof holds for any value of $\delta \in[0,1]$.

## A. 5 Proof of Proposition 5

Our model is directly comparable with market outcomes in Bounie et al. (2021), in which the data broker sells information to Firm A only, who can identify $x_{D B}=\frac{3}{7}$ consumers on $\left[0, \frac{3}{7}\right]$. Moreover, Firm B makes profits equal to $\frac{25 t}{98}$ and Firm A makes profits equal to $\frac{9 t}{14}$.

In this model with first-party data, profits in the second stage are equal to $\pi_{\theta 2}^{*}=\frac{7 t}{18}<\frac{9 t}{14}$.

Moreover, consumer surplus is greater with first-party data where both firms target consumers and compete on a level playing field, and the total share of targeted consumers is equal to $\frac{2}{3}>\frac{3}{7}$.

## A. 6 Proof of Proposition 6

I compare the utility of a consumer located at $x \in\left[0, \frac{1}{3}\right]$ when purchasing its preferred product with its utility from purchasing Firm B's product in period 1 and paying the homogeneous price in period 2 .

The utility when purchasing from Firm A is $u_{1}(x)=V-t x-t$ in period 1 and $u_{2}(x)=V-t x-\frac{4 t}{3}+2 t x$ in period 2 .

The utility when purchasing from Firm B in period 1 is $u_{1 B}(x)=V-t(2-x)$ and $u_{2 A}(x)=V-t x-\frac{t}{3}$ when paying the homogeneous price in period 2 .

For all $\delta_{c} \leq 1, u_{1}(x)+\delta_{c} u_{2}(x) \geq u_{1 B}(x)+\delta_{c} u_{2 A}(x)$ and consumers do no hide.


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[^1]:    ${ }^{1}$ ACNielsen Corporation History, Funding Universe, last accessed October 12, 2021.
    ${ }^{2}$ Kesko Annual Report 2015, last accessed October 12, 2021.

[^2]:    ${ }^{3}$ See also Chen and Pearcy (2010) who allow firms to reward loyal consumers, and Esteves et al. (2022) who consider general distributions of consumer preferences.

[^3]:    ${ }^{4}$ Facebook gave Lyft and others special access to user data; engadget, May 12th, 2018.

[^4]:    ${ }^{5}$ Other models also consider an infinite number of competition periods (Villas-Boas, 1999, 2004). The results of this article are not affected by these different timing structures, and we focus on a two-stage framework for simplicity.
    ${ }^{6}$ Uniform consumer distribution is a standard specification of the literature, whose limits have recently been discussed by Esteves et al. (2022).
    ${ }^{7}$ The market is assumed to be covered, which is a standard consideration of the literature. See for instance Thisse and Vives (1988), Liu and Serfes (2004), Stole (2007), Ulph and Vulkan (2000), Montes et al. (2019), and Bounie et al. (2021).

[^5]:    ${ }^{8}$ This assumption is standard in a stream of the literature focusing on information acquisition by the firms and their pricing strategies (Caillaud and De Nijs, 2014; Esteves and Vasconcelos, 2015).
    ${ }^{9}$ Choe et al. (2018) also adopt a model where data allows firms to perfectly learn the location of each customer that it serves. This specification is required for firms to have sophisticated targeting strategies, which is the focus of this article.
    ${ }^{10}$ These possible targeting strategies are in line with recent literature on information design (Bounie et al., 2021).

[^6]:    ${ }^{11}$ Note that the situation where firms target all past customers is a special case of this model where Firm A uses only one type (a) interval on $\left[0, \tilde{x}_{1}\right]$ and similarly for Firm B on $\left[\tilde{x}_{1}, 1\right]$.
    ${ }^{12}$ The equilibrium concept is discussed in detail in Section 2.4.
    ${ }^{13}$ Considering a discounted future for the firms is in line with the literature (Fudenberg and Villas-Boas, 2006).

[^7]:    ${ }^{14} \mathrm{An}$ alternative timing may consider simultaneous targeting and pricing decisions by each firm in period 2. This would not alter the qualitative insights of the analysis, which, as will be shown in Section 3.1.1, rely on the ability of strategic firms to design information partitions that soften the competitive effect of targeting.

[^8]:    ${ }^{15}$ The contribution of this article will be to show that, contrary to previous literature, when firms target consumers strategically in period 2, they do not fight for information acquisition in period 1. Hence firms set prices in period 1 only accounting for their present profits, and competition is identical to the standard Hotelling framework.

[^9]:    ${ }^{16}$ Proposition 1 also generalizes the results of Bounie et al. (2021) by characterizing the optimal targeting strategies of firms using first-party data.

[^10]:    ${ }^{17}$ This result contributes to the literature on BBPD that emphasizes the risk for consumers of an intense extraction of their surplus (Gehrig et al., 2011).

[^11]:    ${ }^{18}$ There is no equivalent of the information acquisition period (period 1 ) in the framework of Bounie et al. (2021), as firms can purchase data from a third-party, and do not need to serve consumers to learn their preferences.

[^12]:    ${ }^{19}$ Big Data - A Tool for Inclusion or Exclusion?, January 6, 2016.

[^13]:    ${ }^{20}$ With $u_{i}$ and $s_{i}$ lower than 1.

[^14]:    ${ }^{21}$ As shown in Liu and Serfes (2004).

