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Behavior-Based Price Discrimination with endogenous data collection and strategic customer targeting*

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Abstract

This article analyzes behavior-based price discrimination in a two-period competition framework where firms endogenously collect consumer data and strategically target past customers. When firms strategically target customers: (i) they price-discriminate high valuation customers; (ii) they charge a homogeneous price to low valuation customers, even when they have precise information on them. Strategic targeting questions the main classical results of the literature: in a symmetric equilibrium firms do not compete for customer information acquisition and there is no consumer poaching. Sufficiently asymmetric data collection costs can restore previous results of the literature, and we discuss their implications for firms' data strategies and competition in digital markets.

Keywords: Behavior-based price discrimination; Strategic Targeting; Data collection

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1 Introduction

With the advance of information technology, companies now collect, store, and treat large amounts of customer data that they use to charge targeted prices to their past customers. This practice of behavior-based price discrimination (BBPD) has become increasingly common on the Internet ([Gorodnichenko et al., 2018](#)), and a firm such as Amazon can collect customer data such as search behavior, GPS localization, and any personal information to feed machine-learning algorithms to personalize ads, products, and prices to the needs of its customers ([Shiller et al., 2013](#)). More data collected allows firms to price-discriminate customers with an increasing precision ([Choe et al., 2018](#)), which can lower consumer surplus ([Aryal et al., 2021](#)). Recent studies document practices of BBPD in various industries such as newspapers ([Asplund et al., 2008](#)), credit markets ([Ioannidou and Ongena, 2010](#)) and mortgage markets ([Thiel, 2019](#)) among many others.

While behavior-based price discrimination has been an important topic in recent economic literature ([Fudenberg and Tirole, 2000](#); [Acquisti and Varian, 2005](#); [Fudenberg and Villas-Boas, 2006](#)), authors usually assume that firms are not strategic in their use of data to target customers. Firms use all customer information available and price discriminate as many consumers as possible. This assumption has recently been questioned by the literature on customer information design. As [Bounie et al. \(2021\)](#) have shown, strategic firms optimally price-discriminate only consumers with the highest valuation for their product. They charge a homogeneous price to other consumers with a lower valuation, even when they have precise information on these consumers, in order to soften the competitive effect of information.

This article embeds the static framework of [Bounie et al. \(2021\)](#) in a two-period model of behavior-based price discrimination. In the first period, firms have no information, and they can collect data on their customers. Information is modeled as a partition of the consumer demand into segments of variable length. Collecting more information reduces the length of the segments and allows firms to better identify consumers. Varying the size of the segments can be interpreted as changing the precision of information, which will be determined by the data

collection strategy of each firm. In the second period, firms strategically target some of their past customers, and they charge a homogeneous price on the rest of the consumer demand.

This framework with strategic data collection and strategic targeting allows us to derive four main contributions. First, we show that consumer poaching does not occur in a symmetric equilibrium with strategic targeting. Instead, by leaving a share of consumers unidentified in the middle of the line, Firms can soften the competitive effect of customer targeting. This result contrasts with previous literature that has overestimated the competitive effect of BBPD on competition by assuming that firms price discriminate all past customers (Choe et al., 2018; Choe and Matsushima, 2021).

Secondly, we show that in a symmetric equilibrium, there is no impact of BBPD on competition in period 1. Contrary to classical results of the literature (Villas-Boas, 1999; Fudenberg and Tirole, 2000), firms do not fight to acquire customer information in period 1 and they compete à la Hotelling. As consumers in the middle of the line are not targeted in period 2, firms have no interest to fight to acquire information on these consumers. Thus previous literature has also overestimated the competitive effect of BBPD at the customer information acquisition stage (Villas-Boas, 1999; Fudenberg and Tirole, 2000). The implications of these results are two folds. On the one hand, the profitability of BBPD for firms has been underestimated by previous studies. A firm that has developed strategic customer targeting will make significantly higher profits than a non-strategic firm, both when competing for consumer information acquisition and when competing with targeted pricing. On the other hand, BBPD with strategic targeting reduces market competition compared with previous findings, and competition authorities should reconsider the benefits of BBPD for consumer surplus.

Thirdly, a firm can strategically limit data collection by its competitor by undercutting prices in the data collection stage. A firm that undercuts prices reduces the demand of its competitor and can limit its ability to target consumers. We show that a necessary condition for a firm to be able to constraint its competitor on data collection is to incur a much lower data collection cost. This result

has important managerial implications: a firm should invest in data collection capacities not to be left behind in market competition with data. Competition authorities should investigate whether markets present asymmetries in data collection capacities in order to identify whether such constraining strategies take place.

Fourthly, we show that in a symmetric equilibrium when consumers are forward-looking, they do not hide in period 1 by purchasing their less preferred product. As targeted consumers are those with the highest valuation for a firm's product, buying from the other firm would lower drastically their utility. The benefit from paying a homogeneous price in period 2 does not cover this loss, and in equilibrium, they prefer not to hide. This result contrasts with previous literature that shows that consumers change their purchasing behavior when they anticipate higher prices due to BBPD (Fudenberg and Tirole, 2000; Chen et al., 2020). In these models, consumers located in the middle of the line hide in period 1: they pay personalized prices in period 2 as firms target all past customers.

This article contributes to the literature on behavior-based price discrimination and its impact on market competition and consumer surplus. A general result of this literature is that BBPD can lower firms' profits. Villas-Boas (1999) and Fudenberg and Tirole (2000) show that firms that can distinguish their rival customers will engage in consumer poaching intensifying market competition.¹ Villas-Boas (2004) show that BBPD lowers the profits of a monopolist when forward-looking consumers anticipate that they will pay a higher price once they are identified. Acquisti and Varian (2005) consider a seller's ability to commit to a pricing policy and find that conditioning prices on purchase histories is generally unprofitable.²

Finally, a recent stream of the literature analyzes asymmetry between firms (Gehrig et al., 2011). Carroni (2016) shows that when firms are asymmetric, they engage into anti-competitive market sharing agreements and BBPD harms

¹Additionally, Fudenberg and Tirole (2000) show that when customers can anticipate BBPD, they are less price-sensitive in the first period of the game which softens competition.

²Pazgal and Soberman (2008) explore the possibility of adding value to past customers. Although firms can lock in their customers in this way, they compete more aggressively for customers in period 1.

consumers. [Choe et al. \(2018\)](#) and [Choe and Matsushima \(2021\)](#) consider the location strategies of firms on a Hotelling line, and how optimal locations vary with the timing of the game. In particular, [Choe et al. \(2018\)](#) show that BBPD with perfect consumer recognition necessarily leads to an asymmetric equilibrium, even when firms are initially symmetric. We will show how this result depends on the assumption that firms target all their customers, and that when allowing for strategic targeting, the equilibrium is symmetric.

The remainder of this article is organized as follows. In [Section 2](#) we describe the model, and we solve the benchmark case with perfect information in [Section 3](#). We analyze the targeting strategies of firms that have information on past customers in period 2 in [Section 4.1](#). We then analyze competition in period 1 in [Section 4.2](#) and the data collection strategy of firms in [Section 5](#). We discuss how forward-looking consumers impact market equilibrium in [Section 6](#). [Section 7](#) concludes.

2 Description of the model

Two horizontally differentiated firms Firm 1 and Firm 2 compete in a product market. We consider two competition periods $s = 1, 2$, in which firms sell non-durable goods. Both firms incur the same marginal cost of production, which is normalized to zero, and in each period consumers have unit demands.

2.1 Consumers

Consumers are uniformly distributed on a unit line $[0, 1]$, and at each period s they can buy one product at a price p_{1s} from Firm 1 located at 0, or p_{2s} from Firm 2 located at 1.³ Since firms will be able to price discriminate when they have information, different consumers may pay different prices. A consumer located at $x \in [0, 1]$ derives a utility V from purchasing the product. He incurs a transportation cost $t > 0$ so that buying from Firm 1 (resp. from Firm 2), has a total cost tx (resp. $t(1-x)$). At each period, consumers purchase the product for which they have the highest utility.

³We assume that the market is covered. This assumption is common in the literature. See for instance [Bounie et al. \(2021\)](#) or [Montes et al. \(2018\)](#).

In period $s = 1, 2$, consumer located at x has a utility function defined by:

$$u_s(x) = \begin{cases} V - p_{1s} - tx, & \text{if he buys from Firm 1,} \\ V - p_{2s} - t(1 - x), & \text{if he buys from Firm 2.} \end{cases} \quad (1)$$

In the main analysis, consumers are myopic and maximize their utility at each consumption period.⁴ We will analyze in Section 6 forward-looking consumers who anticipate in the first period the prices that they will pay in the second period. We will show that in this model where firms strategically target consumers in period 2, forward-looking consumers do not hide in a symmetric equilibrium, and there is no difference with myopic consumers.

2.2 Firms

Firms collect information on their customers in period 1, which they use to price discriminate past customers in period 2. We characterize in this section the data collection and selling strategy of each firm.

2.2.1 Collecting data

We characterize the data collection strategy of Firm 1 and Firm 2 in period 1. Each firm collects data on its customers. Let's denote by \tilde{x} the consumer indifferent between buying from Firm 1 and Firm 2 in the first period, such that Firm 1 serves consumers on $[0, \tilde{x}]$ and Firm 2 serves consumers on $[\tilde{x}, 1]$. Data allows firms to partition consumer demand into segments of size $\frac{1}{k}$. We denote by $k_1\tilde{x}$ and $k_2(1 - \tilde{x})$ the number of consumer segments collected respectively by Firm 1 and Firm 2 on their customers (we drop subscripts for the remaining of this section). Modeling information through a partition of the consumer demand was first introduced by [Liu and Serfes \(2004\)](#). They consider firms that can purchase a partition of the whole unit line into k segments with an exogenous value of k . We extend their framework to account for the data collection strategies of each

⁴This assumption is standard in a stream of the literature focusing on information acquisition by the firms and their pricing strategies ([Caillaud and De Nijs, 2014](#); [Esteves and Vasconcelos, 2015](#)).

firm. In our model, firms collect information on their customers only, and not on the whole unit line. Moreover, each firm endogenously chooses the number of consumer segments k that it collects.

The number of consumer segments k corresponds to the precision of the information, and a firm that has information can third-degree price-discriminate consumers by charging different prices on different segments. This approach allows us to analyze varying levels of information precision, and to characterize the data collection strategies of competing firms. In the limit case where $k \rightarrow \infty$, information is perfect and firms can first-degree price-discriminate their past customers. First-degree price discrimination has been extensively studied in the literature on BBPD (Choe et al., 2018; Chen et al., 2020; Choe and Matsushima, 2021), and we will show how our model with endogenous data collection and strategic targeting complements previous studies.

We illustrate in Figure 1 the partitions collected by Firm 1 when $k_1 = 4$ and by Firm 2 when $k_2 = 8$. Information partitions are represented on distinct lines for clarity.

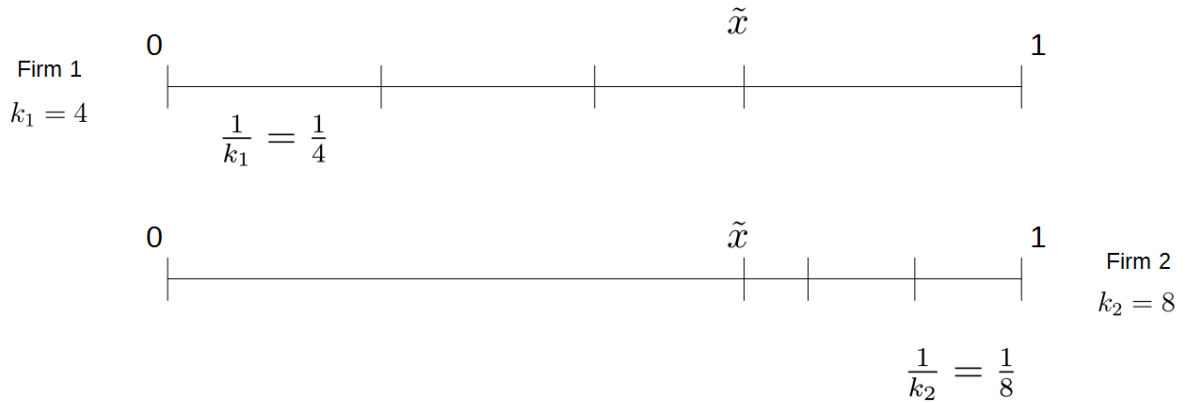


Figure 1: Data collection by Firm 1 and Firm 2: $k_1 = 4$ and $k_2 = 8$.

The ratio $\frac{1}{k}$ can be interpreted as the precision of information collected by a firm. For instance, in Figure 1 $k_1 = 4$ and $\tilde{x} \in [\frac{1}{2}, \frac{3}{4}]$: Firm 1 can distinguish whether consumers belong to $[0, \frac{1}{4}]$, $[\frac{1}{4}, \frac{1}{2}]$, $[\frac{1}{2}, \tilde{x}]$, and to $[\tilde{x}, 1]$. For Firm 2, $k_2 = 8$ and $\tilde{x} \in [\frac{5}{8}, \frac{3}{4}]$: Firm 2 can distinguish whether consumers belong to $[0, \tilde{x}]$, $[\tilde{x}, \frac{3}{4}]$

or $[\frac{3}{4}, 1]$. At the other extreme, when k converges to infinity, an informed firm knows the exact location of each customer: Firm 1 has perfect information on $[0, \tilde{x}]$ and Firm 2 has perfect information on $[\tilde{x}, 1]$. Collecting more information by increasing the number of segments thus allows a firm to better identify consumers and to extract more surplus in period 2, and thus to increase its profits.

The cost of collecting data is equal to $c(k)$ for a unit mass of consumers, which satisfies the Inada conditions. The data collection cost is proportional to the number of consumers on whom data is collected. Thus Firm 1 collects data at cost $c(k_1)\tilde{x}$ and Firm 2 collects data at cost $c(k_2)(1 - \tilde{x})$. This cost encompasses various dimensions of the activity of firms that collect customer data, such as storing and handling data or any other infrastructure-related costs.⁵ In the analysis, each firm chooses how many consumer segments k_1 and k_2 it wants to collect and each firm knows the amount of data collected by its competitor. The data collection cost will depend on the share of consumers that each firm serves in period 1, \tilde{x} for Firm 1 and $1 - \tilde{x}$ for Firm 2.

2.2.2 Targeting consumers

We now describe the targeting strategy of firms in period 2. Firms have collected data on their customers in period 1, which they use for price-discrimination purposes in period 2.

Firms can use any combination of segments that they have collected to target consumers, contrary to previous literature where firms could use all information or no information at all (Liu and Serfes, 2004; Choe et al., 2018). This approach is proposed by Bounie et al. (2021), who study data brokers selling strategic information to competing firms, and we use it in this framework for the following reasons. Using all available information is not optimal for a firm, as there are two opposite effects of information on its profits. On the one hand, an informed firm can price discriminate consumers, thus increasing its profits through this rent extraction effect. On the other hand, information also increases competition in the market, which reduces the profits of both firms. An optimal partition maximizes

⁵Such cost structure is introduced by Bounie et al. (2020).

consumer surplus extraction while internalizing the competitive effect of information. A strategic firm can thus weaken or strengthen the intensity of competition on the market by determining the quantity of information that it uses to target consumers.

Bounie et al. (2021) have shown that an optimal information structure for a firm (say partition \mathcal{P}_1 for Firm 1) has the following features in this model. Partition \mathcal{P}_1 divides the unit line into two intervals: the first interval consists of j_1 segments (with j_1 an integer in $[0, k_1]$ and $\frac{j_1}{k_1} \leq \tilde{x}$) of size $\frac{1}{k_1}$ on $[0, \frac{j_1}{k_1}]$. We refer to this interval as the share of targeted consumers. Firm 1 does not target consumers in the second interval of size $1 - \frac{j_1}{k_1}$, and charges a uniform price on this second interval. We refer to this interval as the share of untargeted consumers. Similarly, Firm 2 will optimally target consumers belonging to the $j_2 \leq k_2$ segments located closest to its location (with $\frac{j_2}{k_2} \leq 1 - \tilde{x}$), the segments of consumers targeted by Firm 2 will belong to $[\frac{j_2}{k_2}, 1]$, and consumers on $[0, \frac{j_2}{k_2}]$ will be charged a homogeneous price by Firm 2. Consumers on $[\frac{j_1}{k_1}, \frac{j_2}{k_2}]$ are not targeted by firms in period 2, even though firms have collected data about them in period 1. Bounie et al. (2021) have shown that, by leaving a share of consumers untargeted by firms, these optimal partitions balance the rent extraction and the competition effects of information. While previous literature has assumed that firms price-discriminate all consumers that they have identified, we will show by relaxing this assumption that it is optimal for firms to keep a large share of consumers untargeted, even when firms have information about them.

Any optimal partition must be similar to partition \mathcal{P}_1 , and the optimization problem boils down to choosing a single value $j_1(k_1)$ for Firm 1, and $j_2(k_2)$ for Firm 2. We assume that firms choose j_1 and j_2 simultaneously and that each firm can observe the targeting strategy of its competitor. This assumption is standard in the literature where firms can observe prices set by their competitors (Acquisti and Varian, 2005; Fudenberg and Villas-Boas, 2006), and in equilibrium, j_1 will be chosen by Firm 1 as the best response to j_2 and reciprocally.

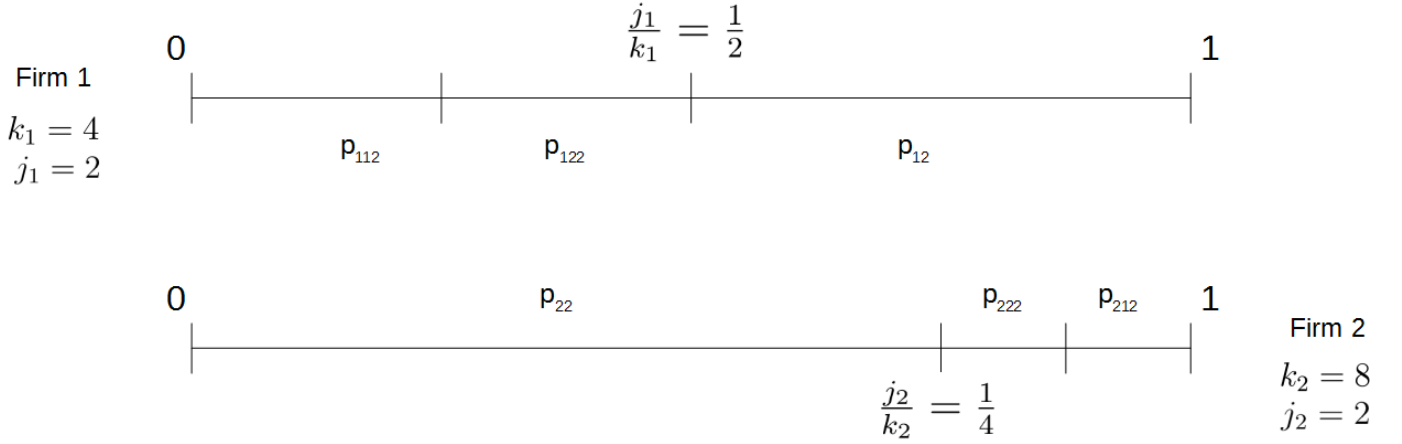


Figure 2: Targeting strategies of firms in period 2 with $k_1 = 4$, $j_1 = 2$, $k_2 = 8$, $j_2 = 2$.

Figure 2 illustrates the optimal partitions chosen by Firm 1 and Firm 2.⁶ Consumers on segment $[\frac{j_1}{k_1}, 1]$ and $[0, \frac{j_2}{k_2}]$ are charged a homogeneous price by Firm 1 and Firm 2 respectively, and we refer to these segments as untargeted consumers. In particular consumers on $[\frac{j_1}{k_1}, \frac{j_2}{k_2}]$ are untargeted by both firms.

The data collection stage can have an impact on the targeting strategy of firms. In period 1, Firm 1 collects information on $[0, \tilde{x}]$ and Firm 2 collects information on $[\tilde{x}, 1]$. Therefore, the targeting strategy of each firm must verify: $\frac{j_1}{k_1} \leq \tilde{x}$ and $\frac{j_2}{k_2} \leq 1 - \tilde{x}$. By assuming that firms target all past customers, previous literature focuses on the limit case of our model where $\frac{j_1}{k_1} = \tilde{x}$ and $\frac{j_2}{k_2} = 1 - \tilde{x}$. We will show that these equalities do not hold in a symmetric equilibrium. We will show in Section 4.2 that there exists asymmetric equilibrium in which a firm can undercut prices in period 1 to change the value of \tilde{x} and to limit data collection by its rival. By doing so, it can constraint the targeting strategy of its rival in period 2.

2.2.3 Profits

We describe the profits of the firms over both periods. At the beginning of period 1, firms maximize the sum of their profits on both periods by discounting period

⁶Firms compete with information on the same unit line. We represent the prices charged by each firm on separate lines for clarity purposes.

2 with factor δ .

The profits of each firm on period 1 are those in the standard Hotelling model without information. Without information on customers, firms only know that consumers are uniformly distributed on the unit line. Firm θ sets $p_{\theta 1}$ in equilibrium, and the resulting demand is $d_{\theta 1} = \frac{p_{-\theta 1} - p_{\theta 1}}{2t}$, where $d_{11} = \tilde{x}$ and $d_{21} = 1 - \tilde{x}$. The profits of Firm θ are $\pi_{\theta 1} = d_{\theta 1} p_{\theta 1}$.

In period 2, firms maximize their profits after having selected a partition j_{θ} of consumers to price discriminate. We denote by $d_{\theta i 2}$ the demand of Firm θ on the i th segment of consumer that it target. $d_{\theta i 2}$ depends on the size of the segment which is defined by the number of data collected k_{θ} : for Firm 1, $d_{1 i 2} = \frac{1}{k_1}$, and for Firm 2 $d_{2 i 2} = \frac{1}{k_2}$. An informed Firm θ maximizes the following profit function with respect to $p_{\theta 12}, \dots, p_{\theta n 2}$:

$$\begin{aligned}\pi_{12}(p_{1i2}, p_{12}) &= \sum_{i=1}^{j_1} d_{1i2} p_{1i2} + p_{12} d_{12}, \\ \pi_{22}(p_{2i2}, p_{22}) &= \sum_{i=1}^{j_2} d_{2i2} p_{2i2} + p_{22} d_{22}.\end{aligned}\tag{2}$$

In period 2, firms simultaneously set prices on each segment of the unit line where they have information. Firm θ sets prices in two stages. First, she sets prices on segments where she shares consumer demand with its competitors. Then, on segments where she is a monopolist, she sets a monopoly price. Each firm knows whether its competitor is informed, and the partition $j_{-\theta}$. Sequential pricing decision avoids the nonexistence of Nash equilibrium in pure strategies and is common in the literature supported by managerial practices. For instance, [Acquisti and Varian \(2005\)](#) use sequential pricing to analyze intertemporal price-discrimination with incomplete information on consumer demand. [Jentzsch et al. \(2013\)](#) and [Belleflamme et al. \(2020\)](#) also focus on sequential pricing where a higher personalized price is charged to identified consumers after a firm sets a uniform price. Sequential pricing is also common in business practices. Recently, Amazon has been accused to show higher prices for Amazon Prime subscribers, who pay an annual fee for unlimited shipping services, than for non-subscribers ([Lawsuit alleges](#)

Amazon charges Prime members for "free" shipping, Consumer affairs, August 29, 2017). Thus Amazon first sets a uniform price and then increases prices for high-value consumers who are better identified when they join the Prime program.

Demands in period 1 will impact the profits of firms in two ways. First, consumer data collection is costly and increases with demand. A firm with high data collection costs has interest to charge a high price in period 1 to serve few consumers. Secondly, a firm that has served few customers in period 1 can be limited on its targeting strategy j_θ in period 2. When choosing prices in period 1, firms take these two effects into account and maximize their aggregate profits over both periods with a discount factor. To emphasize the impact of prices in period 1 on market outcome in period 2, we write the location of the indifferent consumer $\tilde{x}(p_{11}, p_{21})$. Overall the objective functions of the firms at the beginning of the game are:

$$\text{For Firm 1: } \max_{p_{11}} \{ \pi_{11}(p_{11}, p_{21}) - c(k_1)\tilde{x}(p_{11}, p_{21}) + \delta\pi_{12}(p_{1i2}, p_{12}, \tilde{x}(p_{11}, p_{21})) \}$$

$$\text{For Firm 2: } \max_{p_{21}} \{ \pi_{21}(p_{21}, p_{11}) - c(k_2)(1 - \tilde{x}(p_{11}, p_{21})) + \delta\pi_{22}(p_{2i2}, p_{22}, \tilde{x}(p_{11}, p_{21})) \} \quad (3)$$

2.3 Timing

We summarize the timing of the game. In period 1, firms choose the number of data k_1 and k_2 that they will collect on their customers. k_1 and k_2 are known to both firms, and after having chosen these values, firms compete and collect data on their customer. In period 2, firms choose which partitions $j_1(k_1)$ and $j_2(k_2)$ they use to price discriminate consumers. Then firms set prices on the segment of untargeted consumers where they compete, and in the last stage, firms set prices on the monopolistic segments. The timing of the game is the following:

- Period 1:
 - Stage 1: Each firm chooses a number k_θ of consumer segments to collect at a unit cost $c(k_\theta)$.

- Stage 2: Firms compete by setting prices p_{11} and p_{21} , and collect k_θ segments of information on their customers.
- Period 2:
 - Stage 1: Each Firm θ simultaneously chooses partition $j_\theta(k_\theta)$ that it will use to price discriminate consumers.
 - Stage 2: Firms set prices p_{12} and p_{22} on the competitive segments of the unit line.
 - Stage 3: Firms set prices $p_{\theta i2}$ on consumers that they price discriminate.

3 Benchmark: perfect information on past customers

We begin our analysis by considering the benchmark case where data collection is costless and firms collect perfect information on their past customers. Analyzing perfect information allows us to focus on the impact of strategic targeting on market equilibrium in period 2, and how it impacts in turn competition in period 1. This framework has recently been used by [Choe et al. \(2018\)](#), who show that the equilibrium of this model is necessarily asymmetric, and poaching occurs in period 2. We will show that with strategic targeting in period 2, the equilibrium of the game is symmetric, and poaching never occurs.

3.1 Period 2: strategic targeting with perfect information

We analyze in this section the optimal targeting strategies of firms when they have perfect information on their past customers. With perfect information, firms first-degree price discriminate some of their customers, and the targeting strategy corresponds to the number of consumers that a firm targets. We first compute prices and demands depending on the targeting strategy of each firm. We then characterize the optimal targeting strategies of firms.

3.1.1 Prices and demands

We compute prices and demands in period 2 when firms have perfect information and target consumers strategically. With strategic targeting, Firm 1 chooses the value of x_1 , the last consumer that it price discriminates. Thus Firm 1 first-degree price discriminates consumers on $[0, x_1]$, and charges consumers on $[x_1, 1]$ a homogeneous price. Similarly, Firm 2 price discriminates consumers on $[x_2, 1]$, and charges consumers on $[0, x_2]$ a homogeneous price. The choices of x_1 and x_2 correspond to the targeting strategies of Firm 1 and Firm 2, and we will analyze their optimal values in the next section.

Lemma 1 gives the equilibrium prices that we will use to compute the profits of firms as well as consumer surplus.

Lemma 1

In period 2, the equilibrium prices with perfect information are the following:

- *Targeted consumers located on $[0, x_1]$ purchase the product of Firm 1, those on $[x_2, 1]$ purchase the product of Firm 2, and they pay the following prices:*

$$p_{12}(x) = 2t \left[1 - \frac{x_1}{3} - \frac{2x_2}{3} - x \right],$$

$$p_{22}(x) = 2t \left[1 - \frac{x_2}{3} - \frac{2x_1}{3} - (1 - x) \right].$$

- *On the segment where consumers are not targeted by Firm 1 and Firm 2 respectively:*

$$p_{12} = t \left[1 - \frac{4}{3}x_1 - \frac{2}{3}x_2 \right],$$

$$p_{22} = t \left[1 - \frac{4}{3}x_2 - \frac{2}{3}x_1 \right].$$

Proof: See Appendix A.1.

The targeting strategies have opposite effects on the profits of the firms. On the one hand, the higher the x_θ , the more consumers are targeted by Firm θ . Targeted consumers pay a higher price than untargeted consumers, and targeting more consumers increases the profits of a firm. On the other hand, increasing x_θ also increases competition between firms and lowers the price that they can charge to consumers. The optimal targeting strategies of firms balance these two effects of information on their profits.

Replacing prices in the expression of the profit functions, we write in Lemma 2 the profits of the firms with respect to x_1 and x_2 .

Lemma 2

In period 2, the profit of Firm θ with respect to x_θ and $x_{-\theta}$ is the following:

$$\pi_{\theta 2} = \frac{t}{2} - \frac{7}{9}x_\theta^2 t + \frac{2}{9}x_{-\theta}^2 t - \frac{4}{9}x_\theta x_{-\theta} t + \frac{2}{3}x_\theta t - \frac{2}{3}x_{-\theta} t.$$

It is clear that profits are strictly concave functions with respect to x_1 and x_2 , and therefore, they have a unique maximum that we characterize in the next section.

3.1.2 Strategic customer targeting

We characterize the optimal targeting strategies of firms in the benchmark case when they have perfect information on past customers. An important element of the analysis is the value of the indifferent consumer in period 1, \tilde{x} , and in period 2, \hat{x} . Indeed, it is common in the literature that BBPD result in poaching practices: some consumers purchase from one firm in period 1, and then from its competitor in period 2. Poaching is considered beneficial for consumers as it results from a more competitive market in period 2 thanks to BBPD.

Lemma 3 provides the equilibrium values of x_1 and x_2 , the equilibrium profits, as well as \hat{x} , the location of the indifferent consumer in period 2.

Lemma 3

In period 2, the benchmark equilibrium with strategic targeting is characterized by the following values:

$$x_1^* = x_2^* = \frac{1}{3}, \quad \hat{x} = \frac{1}{2},$$

$$\pi_{\theta 2}^* = \frac{7t}{18}.$$

Proof: see Appendix [A.2](#).

Each firm has the same optimal targeting strategy, in which they target only part of their past customers. Firms have information on consumers in $[\frac{1}{3}, \frac{2}{3}]$, but they charge them a homogeneous price in period 2. Moreover, the indifferent consumer is located in the middle of the line: $\hat{x} = \frac{1}{2}$. We will see in the next section that in a symmetric equilibrium, the consumer indifferent between both firms in period 1 is also located in the middle of the line, and that consumer poaching does not occur when firms target consumers strategically.

Each firm can target in period 2 customers that it has served in period 1. The location of the indifferent consumer in period 1, \tilde{x} , is therefore essential for the targeting strategy of firms in period 2. Indeed, if $\tilde{x} \in [\frac{1}{3}, \frac{2}{3}]$, both firms can target their optimal number of consumers in period 2. On the contrary, if $\tilde{x} \in [0, \frac{1}{3}]$ or if $\tilde{x} \in [\frac{2}{3}, 1]$, respectively Firm 1 and Firm 2 will not be able to target their optimal number of consumers, and will be constrained on their targeting strategy. Lemma 4 characterizes such constrained equilibrium in the special case where $\tilde{x} \in [0, \frac{1}{3}]$ and Firm 1 is constrained on its targeting strategy (the case where Firm 2 is constrained is identical).

Lemma 4

In period 2 when $\tilde{x} \in [0, \frac{1}{3}]$, the benchmark equilibrium with strategic targeting is characterized by the following values:

$$x_1^* = \tilde{x}, \quad x_2^* = \frac{3}{7} - \frac{2\tilde{x}}{7}, \quad \hat{x} = \frac{6\tilde{x} + 5}{14},$$

$$\pi_{12}^* = \frac{25t}{98} + \frac{30t}{49}\tilde{x} - \frac{31t}{49}\tilde{x}^2,$$

$$\pi_{22}^* = \frac{9t}{14} - \frac{6t}{7}\tilde{x} + \frac{2t}{7}\tilde{x}^2.$$

Proof: see Appendix A.3.

The profits of Firm 1, π_{12}^* , are lower than in the unconstrained equilibrium. They decrease when \tilde{x} decreases as the constraint becomes more binding. On the contrary, the profits of Firm 2, π_{22}^* , always increase when \tilde{x} decreases, and it is beneficial for Firm 2 to face a constrained competitor. When Firm 1 targets fewer consumers, competition is relaxed and Firm 2 makes higher profits.

We will see in the next section how a firm can constraint the targeting strategy of its competitor by undercutting prices in period 1, which limits the number of consumers that its serves and on whom it collects information.

3.2 Period 1: consumer data collection

We consider now competition in period 1 when firms collect data on their customers. We first characterize the symmetric equilibrium in period 1. We then analyze whether it is profitable for a firm to deviate from the symmetric equilibrium to constraint the targeting strategy of its competitor in period 2. We will show that deviation is never profitable and that the equilibrium is always symmetric.

In this benchmark case, we focus on perfect data collection, without collection cost. In the symmetric equilibrium, firms maximize profits in period 1 and market equilibrium is as in standard Hotelling competition without data collection.

Lemma 5

In period 1, the symmetric equilibrium has the following properties:

$$p_{\theta 1} = t, \quad \tilde{x} = \frac{1}{2}, \quad \pi_{\theta 1} = \frac{t}{2}.$$

The proof is available upon request.

In this symmetric equilibrium, firms serve the same consumers in period 1 and period 2 and: $\tilde{x} = \hat{x}$. Thus, contrary to previous literature (Fudenberg and Tirole, 2000; Choe et al., 2018) there is no consumer poaching even though firms engage in BBPD. Additionally, as firms target consumers strategically in period 2, they do not fight for consumer information acquisition in period 1. Therefore, competition and consumer surplus are much lower than in previous models of BBPD, and these results question the benefits of BBPD for consumers.

In the symmetric equilibrium, firms maximize their profits over both periods independently, and the sum of profits for Firm 1 and Firm 2 over both periods are: $\pi_\theta = \frac{t}{2} + \delta \frac{7t}{18}$.

We now consider whether it is profitable for a firm, say Firm 2 without loss of generality, to deviate from this symmetric equilibrium and undercut prices in period 1. By doing so, it can constraint the targeting strategy of Firm 1 and make higher profits in period 2.

In a constraining equilibrium, Firm 2 maximizes the sum of profits in period 1 and period 2 discounted by the factor δ . Proposition 1 shows that firms never deviate from the symmetric equilibrium in the benchmark case.

Proposition 1

It is never profitable for a firm to deviate from the symmetric equilibrium, and constraining strategies never occur in the benchmark case with strategic targeting.

Proof: see Appendix A.4.

Deviation from the symmetric equilibrium is never profitable for firms. Indeed, in order to constraint their competitor in period 2, they must undercut prices so that the indifferent consumer is very close to the competitor's location in period 1. For instance, if Firm 2 want to constraint Firm 1, it must be that $\tilde{x} < \frac{1}{3}$, and we can show that it requires that $p_{21} \leq \frac{t}{3}$. There is thus an important loss in period 1 for a firm that deviates from the symmetric equilibrium, and the benefits in period 2 do not cover this loss. Therefore, the conclusions of the symmetric

equilibrium hold: firms do not compete to obtain consumer information in period 1, and consumer poaching does not occur.

We generalize this benchmark in the remaining of the paper, in order to account for strategic data collection by firms. We will show that the structure of the data collection cost is essential to understand competition with BBPD. In particular, we will show that for a range of cost functions, constraining strategies are profitable for firms, and asymmetric equilibrium exist.

4 Model resolution

We generalize the benchmark model to account for the data collection strategies of the firms. In Section 4.1 we characterize the optimal targeting strategies of the firms when they have imperfect information and they third-degree price discriminate some of their past customers. In Section 4.2 we analyze competition in period 1 when firms collect data on their customers. Data collection is costly, and firms endogenously choose the number of segments that they collect on their customers in period 1.

4.1 Period 2: strategic targeting and competition

We characterize in this section the prices and profits of each firm when they target past customers with imperfect information in period 2. We then characterize the optimal targeting strategies of Firm 1 and Firm 2.

4.1.1 Prices and demands

In period 2 each firm charges personalized prices on its targeted consumers, and a homogeneous price on the rest of the line. Lemma 6 gives the equilibrium prices that we will use to compute the profits of firms as well as consumer surplus.

Lemma 6

In period 2, the equilibrium prices are characterized by the following properties:

- For each segment $i = 1, \dots, j_1(k_1)$ and $i' = 1, \dots, j_2(k_2)$:

$$p_{1i2} = 2t \left[1 - \frac{j_1}{3k_1} - \frac{2j_2}{3k_2} - \frac{i}{k_1} \right],$$

$$p_{2i2} = 2t \left[1 - \frac{j_2}{3k_2} - \frac{2j_1}{3k_1} - \frac{i'}{k_2} \right].$$

- For the segment where consumers are not targeted by Firm 1 and Firm 2 respectively:

$$p_{12} = t \left[1 - \frac{4j_1}{3k_1} - \frac{2j_2}{3k_2} \right],$$

$$p_{22} = t \left[1 - \frac{4j_2}{3k_2} - \frac{2j_1}{3k_1} \right].$$

Proof: See Appendix A.5.

According to Lemma 6 the targeting strategy of each firm determines the intensity of competition. Homogeneous prices p_1 and p_2 and personalized prices $p_{\theta i2}$ decrease with j_1 and j_2 . This is the competitive effect of price discrimination that reduces the profits of firms. Moreover, personalized prices are higher than the homogeneous price: $p_{\theta i2} > p_{\theta 2}$. Targeting customers allows to extract more surplus and firms make higher profits through this rent extraction effect. We will show that the optimal targeting strategy balances these competitive and rent extraction effects of targeting.

We compute the resulting demands using the location of the consumer indifferent between buying from Firm 1 and Firm 2 in period 2, which is given by $\hat{x} = \frac{p_{22} - p_{12} + t}{2t} = \frac{1}{2} + \frac{j_1}{3k_1} - \frac{j_2}{3k_2}$. The demands of untargeted consumers who pay the homogeneous price are respectively:

$$d_{12} = \frac{1}{2} - \frac{2j_1}{3k_1} - \frac{j_2}{3k_2},$$

$$d_{22} = \frac{1}{2} - \frac{j_1}{3k_1} - \frac{2j_2}{3k_2}.$$

Demands decrease with $\frac{j_1}{k_1}$ and $\frac{j_2}{k_2}$. On the one hand, a firm that targets more consumers reduces the share of untargeted consumers. On the other hand, this share also decreases when the competitor increases $j_{-\theta}$ which intensifies the competitive pressure on the segment of untargeted consumers.

We replace prices and demand in the profit functions of firms in period 2 by their equilibrium values given in Lemma 6. The profits of each firm only depend on firms' targeting strategies j_1 and j_2 , and data collection k_θ .

Lemma 7

In period 2, the profit of Firm θ depending on firms' data collection and targeting strategies is:

$$\pi_{\theta 2} = \frac{t}{2} - \frac{7 j_\theta^2 t}{9 k_\theta^2} + \frac{2 j_{-\theta}^2 t}{9 k_{-\theta}^2} - \frac{4 j_\theta j_{-\theta} t}{9 k_\theta k_{-\theta}} + \frac{2 j_\theta t}{3 k_\theta} - \frac{2 j_{-\theta} t}{3 k_{-\theta}} - \frac{j_\theta t}{k_\theta^2}$$

Proof: see Appendix A.6.

The profit of Firm θ is a concave function with respect to j_θ , and thus has a unique maximum j_θ^* , which we characterize in the next section.

4.1.2 Strategic customer targeting

We analyze the targeting strategy of each firm in period 2, when firms have information on past customers. Firm 1 has collected $k_1 \tilde{x}$ consumer segments and chooses the numbers $j_1(k_1) \leq k_1 \tilde{x}$ of segments to whom it charges targeted prices p_{1i2} . Similarly, Firm 2 chooses the number $j_2(k_2) \leq k_2(1 - \tilde{x})$ of consumer segments to whom it charges targeted prices p_{2i2} . Each firm can observe the targeting strategy of its competitor, and j_1 and j_2 are chosen as simultaneous best response.⁷

There are two cases to consider depending on the number of consumers that firms have served and identified in period 1. In Section 4.1.2.1 we characterize the equilibrium in period 2 when firms are unconstrained on their targeting strategies. This allows us to provide the optimal numbers of consumers targeted by each firm.

⁷This assumption is standard in the literature on price discrimination (Bounie et al., 2021). Alternatively, we can assume that firms can instantaneously observe the homogeneous price of their competitor, as in standard Hotelling competition models (Thisse and Vives, 1988), and adjust their targeting and pricing strategies accordingly.

In Section 4.1.2.2 we characterize market equilibrium when a firm is constrained on its targeting strategy.

4.1.2.1 Equilibrium targeting strategy

We characterize in this section the optimal targeting strategy of each firm. We will show that firms do not target all past customers, and therefore, that they are not constrained in their targeting strategy in a symmetric equilibrium.

Proposition 2 provides the targeting strategies of Firm 1 and Firm 2, characterized by the locations of the last consumer targeted by Firm 1 $\frac{j_1(k_1)^*}{k_1}$ and by Firm 2 $\frac{j_2(k_2)^*}{k_2}$ in equilibrium.

Proposition 2

- *The optimal shares of consumers targeted respectively by Firm 1 and Firm 2 in period 2 are characterized by:*

$$\frac{j_1(k_1)^*}{k_1} = \frac{1}{3} + \frac{1}{5k_2} - \frac{7}{10k_1},$$

$$\frac{j_2(k_2)^*}{k_2} = \frac{1}{3} + \frac{1}{5k_1} - \frac{7}{10k_2}.$$

Proof: see Appendix A.7.

When $\tilde{x} \in [\frac{j_1(k_1)^*}{k_1}, \frac{j_1(k_2)^*}{k_2}]$ each firm can target its optimal share of past customers. Proposition 2 shows that firms won't target all available segments and will strategically withhold information on consumers. Keeping a share of consumers paying a homogeneous price allows firms to soften the competitive effect of price discrimination and to charge higher prices to target customers than in a situation where all customers are targeted.

Lemma 8 gives the unconstrained profits of Firm 1 and Firm 2 in period 2 with respect to the number of data k_1 and k_2 that they collect in period 1.

Lemma 8

Profits in period 2 are:

$$\pi_{12}(k_1, k_2, \tilde{x}) = \frac{7t}{18} - \frac{7t}{15k_1} + \frac{39t}{100k_1^2} + \frac{7t}{15k_2} - \frac{7t}{25k_1k_2} + \frac{7t}{50k_2^2},$$

$$\pi_{22}(k_1, k_2, \tilde{x}) = \frac{7t}{18} - \frac{7t}{15k_2} + \frac{39t}{100k_2^2} + \frac{7t}{15k_1} - \frac{7t}{25k_1k_2} + \frac{7t}{50k_1^2}.$$

Proof: see Appendix A.8.

From these expressions it is clear that the profit of a Firm θ increases with the number of consumer segments collected k_θ . This result contrasts with previous literature that finds that the profits of the firms decrease when information precision increases (Choe et al., 2018). In this model, finer segments allow a firm to better extract surplus from targeted consumers, which increases its profits. We will analyze in Section 5 how this relationship between data collection and profits impacts the data collection strategy of a firm in period 1.

We have focused in this section on the unconstrained targeting strategy of firms in period 2. We have seen that if the indifferent consumer in period 1 belongs to $[\frac{j_1(k_1)^*}{k_1}, \frac{j_1(k_2)^*}{k_2}]$, firms won't be constrained in their targeting strategy in period 2. In the next section, we analyze market equilibrium when one of the firms cannot target its optimal share of consumers in period 2. We will then analyze in Section 4.2 when it is profitable for a firm to deviate from a symmetric equilibrium in order to constraint the targeting strategy of its competitor.

4.1.2.2 Constraining targeting strategy

We now consider competition in period 2 when a firm constraints its competitor on its targeting strategy. Without loss of generality, we focus on the case where Firm 2 constraints Firm 1: $\tilde{x} < \frac{j_1^*}{k_1} = \frac{1}{3} + \frac{1}{5k_2} - \frac{7}{10k_1}$ and $\frac{j_1}{k_1} = \tilde{x}$. In this case, the equilibrium number of segments targeted by each firm is given in Lemma 9.⁸

Lemma 9

⁸Similarly, the optimal targeting strategies when Firm 2 is constrained by its share of identified consumers are: $\frac{j_2}{k_2} = 1 - \tilde{x}$, and $\frac{j_1}{k_1} = \frac{3}{7} + \frac{2\tilde{x}}{7k_1} - \frac{17}{14k_1}$.

In period 2, the optimal targeting strategies when Firm 1 is constrained on its share of identified consumers are:

$$\frac{j_1}{k_1} = \tilde{x},$$

$$\frac{j_2}{k_2} = \frac{3}{7} - \frac{2\tilde{x}}{7k_2} - \frac{9}{14k_2}.$$

Proof: see Appendix A.9.

Lemma 9 shows how Firm 2 changes its targeting strategy according to the constraint faced by Firm 1. When \tilde{x} decreases, the constraint on Firm 1 increases, and Firm 1 targets fewer consumers than in the unconstrained equilibrium. This relaxes the competitive pressure exerted by Firm 1 on the unit line in period 2. In turn, Firm 2 identifies more consumers when the constraint on Firm 1 increases: $\frac{j_2}{k_2}$ increases as \tilde{x} decreases. As competition becomes weaker, it is more profitable for Firm 2 to target consumers located further away from its location. When the constraint is maximal, $\tilde{x} = 0$ and Firm 2 price-discriminates consumers on $[\frac{3}{7} - \frac{9}{14k_2}, 1]$.

We write in Lemma 10 the profit of each firm when Firm 1 is constrained on j_1 by replacing j_1 and j_2 by their equilibrium values.

Lemma 10

The profits of firms in period 2 when Firm 1 is constrained on its targeting strategy are:

$$\pi_{12}^c(k_1, k_2, \tilde{x}) = \frac{25t}{98} + \frac{30t\tilde{x}}{49} - \frac{31t\tilde{x}^2}{49} - \frac{t\tilde{x}}{k_1} + \frac{18\tilde{x} + 15}{49k_2}t + \frac{9t}{98k_2^2},$$

$$\pi_{22}^c(k_1, k_2, \tilde{x}) = \frac{9t}{14} + \frac{2t\tilde{x}}{7}(\tilde{x} - 3) + \frac{2t}{7k_2}(\tilde{x} - 3) + \frac{9t}{28k_2^2}.$$

Proof: see Appendix A.10.

The profits of Firm 2 when facing Firm 1 constrained on targeting always increase when \tilde{x} decreases. There is thus an interest for a firm to face a constrained competitor. When Firm 1 cannot target its optimal number of consumers, competition

is less intense than in the unconstrained equilibrium and the profits of Firm 2 are higher.

On the contrary, the profits of Firm 1 decrease when \tilde{x} decreases following two effects. On the one hand, Firm 1 cannot target its optimal number of customers, and the stronger the constraint, the lower the profits of Firm 1. On the other hand, Firm 2 identifies more consumers when \tilde{x} decreases, which increases the competitive pressure on the market and reduces the profits of Firm 1.

We will analyze in the next section competition and data collection by firms in period 1, and how it impacts their targeting strategy in period 2. By undercutting its price in period 1, a firm can limit the ability of its competitor to target consumers in period 2 and make higher profits.

4.2 Period 1: competition and consumer data collection

We analyze in this section competition in period 1. Firms compete in price and collect data on consumers who purchase their product. Therefore period 1 has an impact on competition in period 2, as firms will adapt their targeting strategy depending on the number of customers that they can target. When choosing prices p_{11} and p_{21} , Firm 1 and Firm 2 maximize profits while accounting for the data collection cost that increases with consumer demand, and for their targeting strategy in period 2.

$$\begin{aligned}
 \text{For Firm 1: } & \max_{p_{11}} \left\{ (p_{11} - c(k_1)) \left(\frac{p_{12} - p_{11} + t}{2t} \right) + \delta \pi_{12}(p_{1i2}, p_{12}, \tilde{x}(p_{11}, p_{21})) \right\} \\
 \text{For Firm 2: } & \max_{p_{21}} \left\{ (p_{21} - c(k_2)) \left(\frac{p_{11} - p_{12} + t}{2t} \right) + \delta \pi_{22}(p_{2i2}, p_{22}, \tilde{x}(p_{11}, p_{21})) \right\}
 \end{aligned} \tag{4}$$

We first analyze in Section 4.2.1 market equilibrium when \tilde{x} is not constraining the targeting strategy of firms in period 2. We show that strategic targeting in period 2 has no impact on competition in period 1. However, as data collection is costly, firms charge higher prices in period 1 than in the standard Hotelling framework

without data collection. We then analyze in Section 4.2.2 market equilibrium when a firm limits the data collection strategy of its competitor by undercutting prices in period 1. Such constraining strategy prevents the firm with the lowest consumer demand to target its optimal number of consumers in period 2.

4.2.1 Consumer data collection: no price undercutting

We characterize competition in period 1 when firms do not constraint each other on data collection. In this case, prices in period 1 are chosen in order to maximize the profits of the firms, which are given in Lemma 11.

Lemma 11

Consider \tilde{x} that does not constraint a firm's targeting strategy in period 2.

In period 1, prices p_{11} and p_{21} are chosen to maximize the following profits functions:

$$\pi_{11}(p_{11}) = (p_{11} - c(k_1)) \left(\frac{p_{12} - p_{11} + t}{2t} \right)$$

$$\pi_{21}(p_{21}) = (p_{21} - c(k_2)) \left(\frac{p_{11} - p_{12} + t}{2t} \right)$$

Profits are those in standard Hotelling competition with asymmetric costs that are proportional to consumer demand. First-order conditions on π_{11} and π_{21} give us prices and profits in equilibrium for period 1.⁹

Proposition 3

Prices and profits in equilibrium are as follows:

$$p_{\theta 1}^* = t + \frac{2c(k_\theta) + c(k_{-\theta})}{3}$$

$$\pi_{\theta 1}^* = \frac{t}{2} + \frac{c(k_{-\theta}) - c(k_\theta)}{3t} + \frac{(c(k_{-\theta}) - c(k_\theta))^2}{18t}$$

⁹The proof is available upon request.

The location of the indifferent consumer is characterized by $\tilde{x} = \frac{1}{2} + \frac{c(k_2) - c(k_1)}{6t}$, and we assume that data collection costs are close enough so that $\tilde{x} \in [\frac{j_1^*}{k_1}, \frac{j_2^*}{k_2}]$. In this case, \tilde{x} is not constraining the targeting strategies of firms in period 2, and Proposition 3 shows that data collection costs have a positive impact on prices in period 1. By serving more consumers firms incur a cost to collect data, which increases the equilibrium prices of their product. There is a positive externality of a firm's data collection cost on its competitor: as a higher cost increases the equilibrium price set by a firm, its competitor raises also its price in response.

The effects of data collection costs on a firm's profits depend on the cost of its competitor. Consider the case where Firm 1 has a lower collection cost than Firm 2, without loss of generality: $c(k_1) < c(k_2)$. $\pi_{11}^* = \frac{t}{2} + \frac{c(k_2) - c(k_1)}{3t} + \frac{(c(k_2) - c(k_1))^2}{18t} > \frac{t}{2}$, and the profit of Firm 1 is higher than in the standard Hotelling model without information. As Firm 2 faces a higher cost than Firm 1, it is willing to serve fewer consumers to lower its data collection cost. Thus Firm 2 lowers its demand leaving Firm 1 with a larger share of consumers to serve.

On the contrary, the profit of Firm 2 depends on the term $\frac{c(k_1) - c(k_2)}{3t} + \frac{(c(k_2) - c(k_1))^2}{18t}$. For $c(k_2) \in [c(k_1), c(k_1) + 3t]$, this term is negative and Firm 2 makes lower profits than in the Hotelling model without data collection. Other values of $c(k_2)$ are ruled out of the analysis as the indifferent consumer would not belong to $[0, 1]$.

Finally, when data collection is costless, the profits of firms are those in the standard Hotelling model: $\pi_{\theta 1} = \frac{t}{2}$. As firms target customers strategically, competition between firms in period 1 is not impacted by competition in period 2. Proposition 4 summarizes this discussion.

Proposition 4

When firms strategically target consumers in period 2, there is no competitive effect of customer identification in period 1 in an unconstrained equilibrium.

Proposition 4 is an important result of this article. Previous models of behavior-based price discrimination usually observe an increase in competition in the first period of the game where firms want to collect data on as many consumers as possible (Pazgal and Soberman, 2008; Zhang, 2011; Choe et al., 2018). We show that

when firms strategically target consumers in period 2, they target high valuation customers only, and they do not compete in period 1 for information acquisition. Moreover, in a symmetric equilibrium, firms do not poach consumers in period 2. Thus strategic customer targeting suppresses the competitive effect of customer information acquisition.

We analyze in the next section the conditions under which it is profitable for a firm to undercut prices in period 1 in order to lower the number of consumers served by its competitor and to constraint its targeting strategy in period 2.

4.2.2 Consumer data collection: price undercutting and constraining strategies

Firm 2 (without loss of generality) can undercut its price in period 1 to lower the demand of Firm 1 and limit its ability to target consumers in period 2. In this case, prices set by each firm in period 1 impact their profits in period 2, and the overall profits of firms are given in Lemma 12.

Lemma 12

In period 1, when $\tilde{x} \in [0, \frac{j_1(k_1)^}{k_1}]$, firms maximize the following profits functions, with respect to p_{11} and p_{21} :*

$$\pi_1(p_{11}) = (p_{11} - c(k_1)) \left(\frac{p_{12} - p_{11} + t}{2t} \right) + \delta \pi_{12}^c(\tilde{x}(p_{11}, p_{12})),$$

$$\pi_2(p_{21}) = (p_{21} - c(k_2)) \left(\frac{p_{11} - p_{12} + t}{2t} \right) + \delta \pi_{22}^c(\tilde{x}(p_{11}, p_{12})).$$

These objective functions are composed for each firm of the sum of profits in periods 1 and 2, which are both concave with a unique maximum. Thus the objective function of each firm is also concave with a unique maximum, and we can apply first-order conditions on π_1 and π_2 w.r.t. p_{11} and p_{21} respectively, which give us prices in equilibrium and the location of $\tilde{x}(p_{11}, p_{21})$. For simplicity, we provide equilibrium values for $\delta = 1$.

Proposition 5

Prices and profits in equilibrium when Firm 1 is constrained on j_1 are the following:

$$p_{11}^* = \frac{30t}{41} + \frac{21t}{41k_1} - \frac{2t}{41k_2} + \frac{20c(k_2) + 21c(k_1)}{41},$$

$$p_{21}^* = \frac{91t}{164} + \frac{35t}{164k_1} + \frac{6t}{41k_2} + \frac{129c(k_2) + 35c(k_1)}{164},$$

$$\tilde{x}^* = \frac{135}{328} + \frac{4}{41k_2} - \frac{49}{328k_1} + \frac{49(c(k_2) - c(k_1))}{328t}.$$

Proof: see Appendix A.11.

A necessary condition for the targeting strategy of Firm 1 to be constrained in period 2 is to have $\tilde{x}^* < \frac{j_1^*}{k_1} = \frac{1}{3} + \frac{1}{5k_2} - \frac{7}{10k_1}$. This inequality to hold if data collection costs are sufficiently asymmetric: $c(k_1) - c(k_2) > \frac{11t}{21} + \frac{129t}{35k_1} - \frac{24t}{35k_2}$. In this case, Proposition 5 shows that Firm 2 can constraint Firm 1 on its targeting in period 2 by undercutting prices and serving far away customers in period 1. In period 1, competition is higher, and prices are lower than in the unconstrained equilibrium. When Firm 1 cannot target its optimal number of customers in period 2, its profits are smaller than without the constraint, while the profits of Firm 2 are higher in period 2.

We now consider profits in periods 1 and 2 when Firm 2 constraints Firm 1, and we compare them with profits in the unconstrained equilibrium.

Proposition 6

Compared with the unconstrained equilibrium, when Firm 2 constrains the data collection strategy of Firm 1 in period 1:

- (a) The profits of both firms are lower in period 1:

$$\pi_{\theta 1}^c < \pi_{\theta 1}.$$

- (b) *The profits of Firm 2 are higher in period 2:*

$$\pi_{22}^c > \pi_{22}.$$

- (c) *The profits of Firm 1 are lower in period 2:*

$$\pi_{12}^c < \pi_{12}.$$

Proof: see Appendix [A.12](#).

A firm with a data collection cost that is sufficiently lower than its competitor is willing to undercut prices and make lower profits in period 1 to limit the targeting strategy of its competitor in period 2. The expected benefits in period 2 compensate the loss of profits in period 1 according to the discount factor: the smaller the δ , the lower the incentives of a firm to constraint its competitor.

Constraining strategies arise in equilibrium under sufficient asymmetry in data collection cost in period 1. A firm that pays low data collection costs will dominate markets and serve a large share of consumer demand in both periods. On the contrary, if costs are symmetric, constraining strategies will not take place and firms won't undercut prices in period 1. It is thus important to understand the structure of the data collection cost of each firm. In particular, [Carroni \(2016\)](#) shows that asymmetric firms engage into market sharing agreements.¹⁰ In this model, the cost is proportional to consumer demand, which impacts the firms' pricing strategies in period 1, and asymmetric firms compete more fiercely. Different cost structures will have significant impacts on data collection and pricing strategies. We analyze in the next section the incentives of firms to collect data in period 1.

5 Strategic data collection

We consider in this section the strategies of firms when investing in data collection. In the first stage of the game, each Firm θ can choose how many segments k_θ

¹⁰He considers firms selling products with asymmetric quality, which can be transposed to other types of asymmetry, for instance on data collection costs.

it collects on consumer demand. In Section 5.1 we consider the incentives of firms to collect data in the unconstrained equilibrium, when data collection costs are sufficiently symmetric: $c(k_1) - c(k_2) < \frac{11t}{21} + \frac{129t}{35k_1} - \frac{24t}{35k_2}$. In this case, a higher k_θ is more costly to collect in period 1 but increases profits in period 2. Indeed, more precise information allows firms to better extract consumer surplus in period 2 and yields higher profits. Then in Section 5.2 we analyze the case where data collection costs are asymmetric so that a firm can constraint its competitor targeting strategy.

5.1 Data collection in equilibrium

Consider first the case where data collection costs are similar for both firms, and $c(k_1) - c(k_2) < \frac{11t}{21} + \frac{129t}{35k_1} - \frac{24t}{35k_2}$. We have seen in the previous section that in this case, it is not profitable for a firm to constraint its competitor on the number of consumers that it identifies in period 1 and targets in period 2. We show in Proposition 7 that a symmetric equilibrium exists in data collection.

Proposition 7

There exists a symmetric equilibrium in which both firms collect the same amount of data on their customers:

$$k_1 = k_2.$$

The proof of this proposition results from the two opposite effects of data collection on the profit of a firm. On the one hand, Proposition 3 shows that it is more profitable for Firm 1 in period 1 to collect fewer consumer data than Firm 2, and to have $c(k_1) < c(k_2)$. In turn, Firm 2 will have interest to collect less data than Firm 1, and consumer data collection will decrease. On the other hand, collecting data allows firms to increase profits in period 2 through better consumer surplus extraction. This second effect drives up data collection. In the symmetric equilibrium of the game, firms collect $k_1 = k_2 = k$ data, such that $-\pi'_{\theta_1}(k) = \delta\pi_{\theta_2}(k)'$: the discounted marginal gain in period 2 from collecting more data and extracting more surplus is equal to the marginal loss in period 1 from

losing market shares and collect costly data. The optimal value of k thus balances the two effects of data collection on profits

5.2 Data collection in constraining equilibrium

We now analyze how the data collection strategies of firms can lead to an equilibrium where one firm constrains the other on its targeting strategy. We have shown in Section 4.2 that a firm can undercut prices in period 1 to limit data collection by its competitor and constraint targeting in period 2. A necessary condition for such constraining strategy to take place is for the constrained firms to pay a higher data collection cost than its competitor, such as if $k_1 > k_2$: $c(k_1) - c(k_2) > \frac{11t}{21} + \frac{129t}{35k_1} - \frac{24t}{35k_2}$. In this case, Firm 2 finds it profitable to constraint Firm 1 on its targeting strategy by limiting the number of consumers that it identifies in period 1.

Proposition 8

There exists an asymmetric equilibrium in which one firm (say Firm 1) collects more consumer data than its competitor:

$$k_1 > k_2.$$

The proof of Proposition 8 results from an additional effect of data collection on the profits of the firms. Proposition 6 shows that a firm makes lower profits when it is constrained on its targeting strategy. There is thus a negative effect of data collection by Firm 1 in period 1 as it allows Firm 2 to exert a higher constraint on targeting in period 2. Therefore the ability of Firm 2 to constraint Firm 1 reduces the incentives of Firm 1 to collect data. Constraining strategies by Firm 2 also lower the value of k_2 : the lower k_2 , the higher the ability of Firm 2 to constraint Firm 1, and thus to increase its profits in period 2.

There is thus a trade-off for Firm 2 between collecting fewer data and constraining Firm 1 on targeting, or collecting more data and better targeting consumers. In our analysis, we have focused on firms with symmetric data collection costs: $c(k)$. However, companies can also choose to invest in data infrastructure in order

to lower this cost. A company that is more efficient to collect, store, and treat customer data, can constraint its competitor, and at the same time collect more data and extract more consumer surplus at the targeting stage. We leave the analysis of such a model with investments in data collection capacities for further research.

6 Forward-looking consumers

An important stream of the literature on behavior-based price discrimination has considered the purchasing strategies of forward-looking customers. For instance, [Fudenberg and Tirole \(2000\)](#) consider consumers who anticipate in period 1 that they will be targeted in period 2 by the firm from which they buy a product, and that they will pay a higher price for its product.

Consider a consumer that optimally purchases from Firm 1 in period 1. Does this consumer has interest to purchase from Firm 2 in period 1 in order to hide from Firm 1 and pay a lower price in period 2? Consumers who pay a higher price in period 2 are those targeted by a firm, and who belong to segments $i \leq j_1$ (which holds for j_k constrained and unconstrained). A consumer located at x loses utility $u(p_{21}, x) - u(p_{11}, x) = -t + 2tx - \frac{c(k_2) - c(k_1)}{3}$ from purchasing its less preferred product in period 1, and gains utility $u(p_{12}, x) - u(p_{1i2}, x) = 2t - 2tx + \frac{2tj_1(k_1)}{3k_1} - \frac{2j_2(k_2)t}{3k_2} - \frac{2it}{3k_1}$ from not being targeted in period 2.

We compare the loss of utility for a consumer who purchases its less preferred product in period 1 $u(p_{21}, x) - u(p_{11}, x)$ with its discounted utility gain from not being targeted in period 2 $\delta(u(p_{\theta 2}, x) - u(p_{\theta i 2}, x))$. Clearly when firms are symmetric - $j_1 = j_2$ and $c(k_1) = c(k_2)$, consumers have no interest to hide their true valuation, and there is no difference between the purchasing behavior of forward looking consumers and of myopic consumers. When $\delta < 1$ we can determine for each segment i the location x_i of the last consumer that has interest to hide:

$$x_i = \frac{2\delta - 1}{2\delta - 2} + \frac{c(k_2) - c(k_1)}{(6\delta - 6)t} + \frac{\delta}{(3\delta - 3)t} \left(\frac{j_1}{k_1} - \frac{j_2}{k_2} - \frac{i}{k_1} \right), \text{ under the condition that } x_i \in \left[\frac{i-1}{k_\theta}, \frac{i}{k_\theta} \right].$$

Proposition 9

- When Firm 1 and Firm 2 are symmetric and target consumers strategically, it is not profitable for forward-looking consumers to hide from firms, and they always purchase their preferred product.
- When firms are asymmetric in data collection, some customers hide from Firm θ with the lowest data collection in period 1 and are not targeted in period 2. These consumers are located on $[x_i, \frac{i}{k_\theta}]$, in the i th segments closest to Firm θ , with

$$x_i = \frac{2\delta - 1}{2\delta - 2} + \frac{c(k_2) - c(k_1)}{(6\delta - 6)t} + \frac{\delta}{(3\delta - 3)t} \left(\frac{j_1}{k_1} - \frac{j_2}{k_2} - \frac{i}{k_1} \right).$$

Proposition 9 is an important contribution of this article. When firms are perfectly symmetric, consumers have no interest to hide from the firm that serves their most preferred product. This contrasts with previous literature where consumer demand adapts to limit rent extraction by firms (Bonatti and Cisternas, 2020). For instance, Fudenberg and Tirole (2000) find that consumers are less price-sensitive in period 1 as they anticipate behavior-based price discrimination in period 2.¹¹ In this model with strategic targeting, only consumers located at the extremities of the unit line pay targeted prices, and consumers in the middle of the line pay a low homogeneous price. As targeted consumers are located close to a firm's location, it is costly for them to purchase their less preferred product located at the other side of the line, and this cost would not be covered by the gains from not being targeted and pay the homogeneous price in period 2. Thus previous literature overestimates the competitive effect of information in period 2, as well as the impacts of data collection on competition in period 1.

When data collection costs are sufficiently asymmetric, consumers hide from the firm with the highest data collection cost and the highest prices in period 1 by purchasing the product of its competitor. High valuation consumers who cannot be targeted by a firm in period 2 pay a homogeneous price that is lower than the targeted price. This benefits the firm with the lowest data collection cost, which can use its data collection strategy as a lever to serve more looking

¹¹Recent literature reconsiders this result, for instance by accounting for endogenous product design by firms (Zhang, 2011).

forward consumers in period 1 and to constraint its competitor targeting strategy in period 2. This additional effect drives down the number of data collected by each firm, which in turn increases consumer surplus.

7 Conclusion

This model of behavior-based price discrimination with strategic data collection and strategic customer targeting has three important implications for firms and regulators. A first implication concerns firms' managerial practices: a firm has to develop a data strategy for customer targeting. A firm that targets all available customer segments will achieve low profits. On the contrary, a firm that has developed sophisticated targeting strategies can make higher profits than its competitor and dominate digital markets. [DalleMule and Davenport \(2017\)](#) highlight the importance for firms to develop their data strategy that depends on the data capabilities of competitors among other factors.

Secondly, data collection costs are a central element of firms' pricing strategies and market equilibrium, and a firm has to develop an efficient data acquisition strategy: firms collecting all consumer data indifferently bear a data collection cost for non-profitable customers. Moreover, a firm with a high data collection cost will collect less data than an efficient firm and in turn, will have a lower ability to target customers. Conversely, a firm with an efficient data collection technology can dominate its competitor and prevent it from collecting data on valuable customers. Therefore, companies must invest in new efficient data collection capacities in order not to be left behind in market competition with customer data. It is also essential for regulators to understand whether markets present a strong asymmetry in data collection costs to understand if efficient firms constraint the strategies of their rivals and are engaged in abuses of dominant position. Recent advances in cloud computing have allowed costs of data storage and data processing to fall in the last decade ([Lambrecht and Tucker, 2015](#)).¹² However, it is clear that major tech companies such as Amazon have access to much better technological capacities

¹²See also [Can Cloud Storage Costs Fall to Zero?](#), Enterprise Storage Forum, last accessed 10/05/2021..

than their competitors, and can use them to collect and treat consumer data at a significantly lower cost, and in the end, dominate their competitors.

Thirdly, competition authorities should reconsider the benefits of behavior-based price discrimination for consumer surplus. In a symmetric equilibrium, we show that the competitive effects of BBPD have been overestimated in previous literature where firms are not strategic in customer targeting. It is thus essential to understand whether firms target consumers strategically to assess the impacts of BBPD on consumer welfare. Strategic targeting is supported by recent advances in algorithmic pricing, which have allowed firms to improve significantly their pricing strategies to the expense of consumers (Calvano et al., 2020).

This simple two-period competition framework could be extended to account for more dynamic data collection strategies. Recent studies show that data can lose its significance over time, and firms may need to collect a new batch of customer information at each competitive stage (Valavi et al., 2020). This new dimension would enrich the game by having firms develop simultaneously data collection and targeting strategies, which we believe offer an interesting path for further research.

References

- Alessandro Acquisti and Hal R Varian. Conditioning prices on purchase history. *Marketing Science*, 24(3):367–381, 2005.
- Gaurab Aryal, Charles Murry, and Jonathan W Williams. Price discrimination in international airline markets. *arXiv preprint arXiv:2102.05751*, 2021.
- Marcus Asplund, Rickard Eriksson, and Niklas Strand. Price discrimination in oligopoly: evidence from regional newspapers. *The Journal of Industrial Economics*, 56(2):333–346, 2008.
- Paul Belleflamme, Wing Man Wynne Lam, and Wouter Vergote. Competitive imperfect price discrimination and market power. *Marketing Science*, 39(5):996–1015, 2020.
- Alessandro Bonatti and Gonzalo Cisternas. Consumer scores and price discrimination. *The Review of Economic Studies*, 87(2):750–791, 2020.
- David Bounie, Antoine Dubus, and Patrick Waelbroeck. Market for information and selling mechanisms. 2020.
- David Bounie, Antoine Dubus, and Patrick Waelbroeck. Selling strategic information in digital competitive markets. *The RAND Journal of Economics*, 52(2):283–313,

2021. doi: <https://doi.org/10.1111/1756-2171.12369>. URL <https://onlinelibrary.wiley.com/doi/abs/10.1111/1756-2171.12369>.

- Bernard Caillaud and Romain De Nijs. Strategic loyalty reward in dynamic price discrimination. *Marketing Science*, 33(5):725–742, 2014.
- Emilio Calvano, Giacomo Calzolari, Vincenzo Denicolo, and Sergio Pastorello. Artificial intelligence, algorithmic pricing, and collusion. *American Economic Review*, 110(10):3267–97, 2020.
- Elias Carroni. Competitive customer poaching with asymmetric firms. *International Journal of Industrial Organization*, 48:173–206, 2016.
- Zhijun Chen, Chongwoo Choe, and Noriaki Matsushima. Competitive personalized pricing. *Management Science*, 66(9):4003–4023, 2020.
- Chongwoo Choe and Noriaki Matsushima. Behavior-based price discrimination and product choice. *Review of Industrial Organization*, 58(2):263–273, 2021.
- Chongwoo Choe, Stephen King, and Noriaki Matsushima. Pricing with cookies: Behavior-based price discrimination and spatial competition. *Management Science*, 64(12):5669–5687, 2018.
- Leandro DalleMule and Thomas H Davenport. What’s your data strategy. *Harvard Business Review*, 95(3):112–121, 2017.
- Rosa-Branca Esteves and Helder Vasconcelos. Price discrimination under customer recognition and mergers. *Journal of Economics & Management Strategy*, 24(3):523–549, 2015.
- Drew Fudenberg and Jean Tirole. Customer poaching and brand switching. *RAND Journal of Economics*, pages 634–657, 2000.
- Drew Fudenberg and J Miguel Villas-Boas. Behavior-based price discrimination and customer recognition. *Handbook on economics and information systems*, 1:377–436, 2006.
- Thomas Gehrig, Oz Shy, and Rune Stenbacka. History-based price discrimination and entry in markets with switching costs: a welfare analysis. *European Economic Review*, 55(5):732–739, 2011.
- Yuriy Gorodnichenko, Viacheslav Sheremirov, and Oleksandr Talavera. Price setting in online markets: Does it click? *Journal of the European Economic Association*, 16(6):1764–1811, 2018.
- Vasso Ioannidou and Steven Ongena. “time for a change”: loan conditions and bank behavior when firms switch banks. *The Journal of Finance*, 65(5):1847–1877, 2010.
- Nicola Jentzsch, Geza Sapi, and Irina Suleymanova. Targeted pricing and customer data sharing among rivals. *International Journal of Industrial Organization*, 31(2):131–144, 2013.

- Anja Lambrecht and Catherine E Tucker. Can big data protect a firm from competition? *Available at SSRN 2705530*, 2015.
- Qihong Liu and Konstantinos Serfes. Quality of information and oligopolistic price discrimination. *Journal of Economics & Management Strategy*, 13(4):671–702, 2004.
- Rodrigo Montes, Wilfried Sand-Zantman, and Tommaso Valletti. The value of personal information in online markets with endogenous privacy. *Management Science*, 2018.
- Amit Pazgal and David Soberman. Behavior-based discrimination: Is it a winning play, and if so, when? *Marketing Science*, 27(6):977–994, 2008.
- Benjamin Reed Shiller et al. *First degree price discrimination using big data*. Brandeis Univ., Department of Economics, 2013.
- Jurre H Thiel. Price discrimination, switching costs and welfare: Evidence from the dutch mortgage market. 2019.
- Jacques-Francois Thisse and Xavier Vives. On the strategic choice of spatial price policy. *The American Economic Review*, pages 122–137, 1988.
- Ehsan Valavi, Joel Hestness, Newsha Ardalani, and Marco Iansiti. Time and the value of data. *Harvard Business School Strategy Unit Working Paper*, 2020.
- J Miguel Villas-Boas. Dynamic competition with customer recognition. *The Rand Journal of Economics*, pages 604–631, 1999.
- J Miguel Villas-Boas. Price cycles in markets with customer recognition. *RAND Journal of Economics*, pages 486–501, 2004.
- Juanjuan Zhang. The perils of behavior-based personalization. *Marketing Science*, 30(1):170–186, 2011.

A Appendix

A.1 Proof of Lemma 1

We characterize the equilibrium prices in period 2 when firms targets respectively consumers on $[0, x_1]$ and on $[x_2, 1]$.

Prices and demand.

Firm 1 sets a price $p_{12}(x)$ for consumers located at $[0, x_1]$. Similarly, Firm 2 sets a price $p_{22}(x)$ for consumers located at $[x_2, 1]$. Firm θ then sets a unique price $p_{\theta 2}$ on the rest of the unit line. The price charged to targeted consumers by Firm 1 satisfies:

$$\begin{aligned}
V - tx - p_{12}(x) &= V - t(1 - x) - p_{22} \\
\implies x &= \frac{p_{22} - p_{12}(x) + t}{2t} \\
\implies p_{12}(x) &= p_{22} + t - 2tx.
\end{aligned}$$

p_{22} is the price set by Firm 2 on interval $[0, x_2]$ where it does not target consumers. Prices set by Firm 2 on segments in interval $[x_2, 1]$ are:

$$p_{22}(x) = p_{12} + t - 2tx.$$

Let denote d_{12} the demand for Firm 1 (resp. d_{22} the demand for Firm 2) where firms compete. d_{12} is determined by the indifferent consumer \hat{x} :

$$\begin{aligned}
V - t\hat{x} - p_1 &= V - t(1 - \hat{x}) - p_2 \implies \hat{x} = \frac{p_2 - p_1 + t}{2t} \text{ and } d_{12} = \hat{x} - x_1 = \frac{p_{22} - p_{12} + t}{2t} - x_1 \\
(\text{resp. } d_{22} &= 1 - x_2 - \frac{p_{22} - p_{12} + t}{2t}).
\end{aligned}$$

Profits of the firms.

The profits of the firms are:

$$\begin{aligned}
\pi_{12} &= \int_0^{x_1} p_{12}(x)dx + d_{12}p_{12} = \int_0^{x_1} (p_{22} + t - 2tx)dx + \left(\frac{p_{22} - p_{12} + t}{2t} - x_1\right)p_{12}, \\
\pi_{22} &= \sum_{i=1}^{j_2} d_{2i}p_{2i2} + d_{22}p_{22} = \int_{x_2}^1 (p_{12} + t - 2tx)dx + \left(\frac{p_{12} - p_{22} + t}{2t} - x_2\right)p_{22}.
\end{aligned}$$

Prices and demands in equilibrium.

We now compute the optimal prices and demands, using first-order conditions on π_θ with respect to p_θ . Prices in equilibrium are:

$$\begin{aligned}
p_{12} &= t\left[1 - \frac{2t}{3}x_2 - \frac{4t}{3}x_1\right], \\
p_{22} &= t\left[1 - \frac{2t}{3}x_1 - \frac{4t}{3}x_2\right].
\end{aligned}$$

Replacing these values in the above demands and prices gives:

$$\begin{aligned}
p_{12}(x) &= 2t - \frac{4t}{3}x_2 - \frac{2t}{3}x_1 - 2tx, \\
p_{22}(x) &= 2t - \frac{4t}{3}x_1 - \frac{2t}{3}x_2 - 2tx
\end{aligned}$$

and

$$\begin{aligned}
d_1 &= \frac{1}{2} - \frac{2}{3}x_1 - \frac{1}{3}x_2, \\
d_2 &= \frac{4}{3}x_2 - \frac{1}{2} - \frac{1}{3}x_1.
\end{aligned}$$

A.2 Proof of Lemma 3

We derive the optimal targeting strategies x_1^* and x_2^* of Firm 1 and Firm 2.

Each firm knows the strategy of its competitor, and x_1^* and x_2^* are chosen as simultaneous best response. The profits of the firms are the following:

$$\pi_{\theta 2} = \frac{t}{2} - \frac{7}{9}x_{\theta}^2t + \frac{2}{9}x_{-\theta}^2t - \frac{4}{9}x_{\theta}x_{-\theta}t + \frac{2}{3}x_{\theta}t - \frac{2}{3}x_{-\theta}t.$$

We apply first-order condition on π_{12} with respect to x_1 and to π_{22} with respect to x_2 , and we find:

$$x_1^* = x_2^* = \frac{1}{3}.$$

As $p_{12}^* = p_{22}$, it is straightforward that the indifferent consumer in period 2 is located at $\hat{x} = \frac{1}{2}$.

A.3 Proof of Lemma 4

We derive the optimal targeting strategy x_2^* of Firm 2 when Firm 1 is constrained and $x_1^* = \tilde{x}$.

x_2^* is chosen as a best response to $x_1^* = \tilde{x}$. The profits of the firms are the following:

$$\begin{aligned}\pi_{12} &= \frac{t}{2} - \frac{7}{9}\tilde{x}^2t + \frac{2}{9}x_2^2t - \frac{4}{9}\tilde{x}x_2t + \frac{2}{3}\tilde{x}t - \frac{2}{3}x_2t. \\ \pi_{22} &= \frac{t}{2} - \frac{7}{9}x_2^2t + \frac{2}{9}\tilde{x}^2t - \frac{4}{9}x_2\tilde{x}t + \frac{2}{3}x_2t - \frac{2}{3}\tilde{x}t.\end{aligned}$$

We apply first-order conditions on π_{22} with respect to x_2 and we find:

$$x_2^* = \frac{3}{7} - \frac{2\tilde{x}}{7}.$$

replacing the expression of x_1^* and x_2^* into p_{12}^* and p_{22}^* , we find that the indifferent consumer in period 2 is located at $\hat{x} = \frac{6\tilde{x}+5}{14}$.

The expressions of profits are found by replacing x_1^* and x_2^* by their expressions into π_{12} and π_{22} .

A.4 Proof of Proposition 1

We compare profits in the symmetric equilibrium and profits in the constrained equilibrium when firm 2 deviates. We show that the former is always higher than the former and that deviation is never profitable.

We focus on the case where $\delta = 1$, and deviation is the most profitable.

Profits in the symmetric equilibrium are equal to $\frac{7t}{18}$ in period 2 and $\frac{t}{2}$ in period 1 and overall to $\frac{7t}{18} + \frac{t}{2} = \frac{8t}{9}$.

The maximal profit in period 2 in the constrained equilibrium is reached when $\tilde{x} = 0$ (when Firm 1 is constrained) and is equal to $\frac{9t}{14}$.

For Firm 2 to constrain Firm 1 in period 1, it must be the case that $\tilde{x} \leq \frac{1}{3}$. Let us consider the least constraining case where $\tilde{x} = \frac{1}{3}$, which leads to the highest profits of Firm 2 among the set of constraining equilibrium in period 1.

Necessarily, it is easy to show that Firm 2 must charge $p_{21} = \frac{t}{3}$, yielding profits in period 1 equal to $\frac{t}{9}$.

Thus the sum of profits over both periods in the constraining equilibrium is equal to $\frac{95t}{126} < \frac{8t}{9}$, and deviation is never profitable.

A.5 Proof of Lemma 6

We characterize the equilibrium prices in period 2 when firms target j_1 and j_2 consumer segments respectively.

Prices and demand.

Firm $\theta = 1, 2$ sets a price $p_{\theta i2}$ for each segment of size $\frac{1}{k_\theta}$, and a unique price $p_{\theta 2}$ on the rest of the unit line. The demand for Firm θ on targeted segments is $d_{\theta i} = \frac{1}{k_\theta}$. The corresponding prices are computed using the indifferent consumer located on the right extremity of the segment, $\frac{i}{k_\theta}$. For Firm 1:

$$\begin{aligned} V - t \frac{i}{k_1} - p_{1i2} &= V - t \left(1 - \frac{i}{k_1}\right) - p_{22} \\ \implies \frac{i}{k_1} &= \frac{p_{22} - p_{1i2} + t}{2t} \\ \implies p_{1i2} &= p_{22} + t - 2t \frac{i}{k_1}. \end{aligned}$$

p_{22} is the price set by Firm 2 on interval $[0, \frac{j_2}{k_2}]$ where it does not target consumers. Prices set by Firm 2 on segments in interval $[\frac{j_2}{k_2}, 1]$ are:

$$p_{2i2} = p_{12} + t - 2t \frac{i}{k_2}.$$

Let denote d_{12} the demand for Firm 1 (resp. d_{22} the demand for Firm 2) where firms compete. d_{12} is determined by the indifferent consumer \hat{x} :

$$\begin{aligned} V - t\hat{x} - p_1 &= V - t(1 - \hat{x}) - p_2 \implies \hat{x} = \frac{p_2 - p_1 + t}{2t} \text{ and } d_{12} = \hat{x} - \frac{j_1}{k_1} = \frac{p_{22} - p_{12} + t}{2t} - \frac{j_1}{k_1} \\ \text{(resp. } d_{22} &= 1 - \frac{j_2}{k_2} - \frac{p_{22} - p_{12} + t}{2t}). \end{aligned}$$

Profits of the firms.

The profits of the firms are:

$$\begin{aligned}\pi_{12} &= \sum_{i=1}^{j_1} d_{1i} p_{1i2} + d_{12} p_{12} = \sum_{i=1}^{j_1} \frac{1}{k_1} (p_{22} + t - 2t \frac{i}{k_1}) + (\frac{p_{22} - p_{12} + t}{2t} - \frac{j_1}{k_1}) p_{12}, \\ \pi_{22} &= \sum_{i=1}^{j_2} d_{2i} p_{2i2} + d_{22} p_{22} = \sum_{i=1}^{j_2} \frac{1}{k_2} (p_{12} + t - 2t \frac{i}{k_2}) + (\frac{p_{12} - p_{22} + t}{2t} - \frac{j_2}{k_2}) p_{22}.\end{aligned}$$

Prices and demands in equilibrium.

We now compute the optimal prices and demands, using first-order conditions on π_θ with respect to p_θ . Prices in equilibrium are:

$$\begin{aligned}p_{12} &= t[1 - \frac{2}{3} \frac{j_2}{k_2} - \frac{4}{3} \frac{j_1}{k_1}], \\ p_{22} &= t[1 - \frac{2}{3} \frac{j_1}{k_1} - \frac{4}{3} \frac{j_2}{k_2}].\end{aligned}$$

Replacing these values in the above demands and prices gives:

$$\begin{aligned}p_{1i2} &= 2t - \frac{4}{3} \frac{j_2 t}{k_2} - \frac{2}{3} \frac{j_1 t}{k_1} - 2 \frac{it}{k_1}, \\ p_{2i2} &= 2t - \frac{4}{3} \frac{j_1 t}{k_1} - \frac{2}{3} \frac{j_2 t}{k_2} - 2 \frac{it}{k_2}.\end{aligned}$$

and

$$\begin{aligned}d_1 &= \frac{1}{2} - \frac{2}{3} \frac{j_1}{k_1} - \frac{1}{3} \frac{j_2}{k_2}, \\ d_2 &= \frac{4}{3} \frac{j_2}{k_2} - \frac{1}{2} - \frac{1}{3} \frac{j_1}{k_1}.\end{aligned}$$

A.6 Proof of Lemma 7

We replace prices and demands in the expressions of profits of the firms in period 2 by their equilibrium values given in Lemma 6, and we write the profits of each firm as follows:

$$\begin{aligned}
\pi_{12} &= \sum_{i=1}^{j_1} \frac{1}{k_1} 2t \left[1 - \frac{j_1}{3k_1} - \frac{2j_2}{3k_2} - \frac{i}{k_1} \right] \\
&\quad + t \left[1 - \frac{4j_1}{3k_1} - \frac{2j_2}{3k_2} \right] \left(\frac{1}{2} - \frac{2j_1}{3k_1} - \frac{j_2}{3k_2} \right), \\
&= \frac{t}{2} - \frac{7j_1^2 t}{9k_1^2} + \frac{2j_2^2 t}{9k_2^2} - \frac{4j_1 j_2 t}{9k_1 k_2} + \frac{2j_1 t}{3k_1} - \frac{2j_2 t}{3k_2} - \frac{j_1 t}{k_1^2}
\end{aligned} \tag{5}$$

$$\begin{aligned}
\pi_{22} &= \sum_{i=1}^{j_2} \frac{1}{k_2} 2t \left[1 - \frac{j_2}{3k_2} - \frac{2j_1}{3k_1} - \frac{i'}{k_2} \right] \\
&\quad + t \left[1 - \frac{4j_2}{3k_2} - \frac{2j_1}{3k_1} \right] \left(\frac{1}{2} - \frac{j_1}{3k_1} - \frac{2j_2}{3k_2} \right) \\
&= \frac{t}{2} - \frac{7j_2^2 t}{9k_2^2} + \frac{2j_1^2 t}{9k_1^2} - \frac{4j_2 j_1 t}{9k_1 k_2} + \frac{2j_2 t}{3k_2} - \frac{2j_1 t}{3k_1} - \frac{j_2 t}{k_2^2}.
\end{aligned}$$

These expressions of profits in equilibrium only depend on firms' targeting strategies j_1 and j_2 , and on data collection k_θ . It is clear from these expressions that the profit of Firm θ is a concave function with respect to j_θ , and thus has a unique maximum j_θ^* , which we characterize in the next proof.

A.7 Proof of Proposition 2

We derive the optimal targeting strategies j_1^* and j_2^* of Firm 1 and Firm 2.

Each firm knows the choice of its competitor, and j_1^* and j_2^* are chosen as simultaneous best response. The profits of the firms are the following:

$$\begin{aligned}
\pi_{12} &= \sum_{i=1}^{j_1} \frac{1}{k_1} 2t \left[1 - \frac{j_1}{3k_1} - \frac{2j_2}{3k_2} - \frac{i}{k_1} \right] \\
&\quad + t \left[1 - \frac{4j_1}{3k_1} - \frac{2j_2}{3k_2} \right] \left(\frac{1}{2} - \frac{2j_1}{3k_1} - \frac{j_2}{3k_2} \right), \\
&= \frac{t}{2} - \frac{7j_1^2 t}{9k_1^2} + \frac{2j_2^2 t}{9k_2^2} - \frac{4j_1 j_2 t}{9k_1 k_2} + \frac{2j_1 t}{3k_1} - \frac{2j_2 t}{3k_2} - \frac{j_1 t}{k_1^2}
\end{aligned} \tag{6}$$

$$\begin{aligned}
\pi_{22} &= \sum_{i=1}^{j_2} \frac{1}{k_2} 2t \left[1 - \frac{j_2}{3k_2} - \frac{2j_1}{3k_1} - \frac{i'}{k_2} \right] \\
&\quad + t \left[1 - \frac{4j_2}{3k_2} - \frac{2j_1}{3k_1} \right] \left(\frac{1}{2} - \frac{j_1}{3k_1} - \frac{2j_2}{3k_2} \right) \\
&= \frac{t}{2} - \frac{7j_2^2 t}{9k_2^2} + \frac{2j_1^2 t}{9k_1^2} - \frac{4j_2 j_1 t}{9k_1 k_2} + \frac{2j_2 t}{3k_2} - \frac{2j_1 t}{3k_1} - \frac{j_2 t}{k_2^2}.
\end{aligned}$$

We apply first-order condition on π_{12} with respect to j_1 and to π_{22} with respect to j_2 , and we find:

$$\frac{j_1(k_1)^*}{k_1} = \frac{1}{3} + \frac{1}{5k_2} - \frac{7}{10k_1},$$

$$\frac{j_2(k_2)^*}{k_2} = \frac{1}{3} + \frac{1}{5k_1} - \frac{7}{10k_2}.$$

A.8 Proof of Lemma 8

We replace into the profit functions of each firm the equilibrium values of j_1^* and j_2^* provided in the proof above:

$$\pi_{12}(k_1, k_2, \tilde{x}) = \frac{7t}{18} - \frac{7t}{15k_1} + \frac{39t}{100k_1^2} + \frac{7t}{15k_2} - \frac{7t}{25k_1k_2} + \frac{7t}{50k_2^2}$$

$$\pi_{22}(k_1, k_2, \tilde{x}) = \frac{7t}{18} - \frac{7t}{15k_2} + \frac{39t}{100k_2^2} + \frac{7t}{15k_1} - \frac{7t}{25k_1k_2} + \frac{7t}{50k_1^2}$$

A.9 Proof of Lemma 9

We characterize the optimal targeting strategy of Firm 2 when Firm 1 is constrained on j_1^* . When $j_1^* > \tilde{x}$, the constraint is binding and Firm 1 targets as many customers as possible, that is, all customers on $[0, \tilde{x}]$.

The best response of Firm 2 is found by maximizing its profit function with $j_1 = \tilde{x}$:

$$\pi_{22} = \frac{t}{2} - \frac{7j_2^2t}{9k_2^2} + \frac{2\tilde{x}^2t}{9} - \frac{4j_2\tilde{x}t}{9k_2} + \frac{2j_2t}{3k_2} - \frac{2\tilde{x}t}{3} - \frac{j_2t}{k_2^2}.$$

Straightforward calculation give us the following equilibrium value of j_2 :

$$\frac{j_2}{k_2} = \frac{3}{7} - \frac{2\tilde{x}}{7k_2} + \frac{9}{14k_2}.$$

A.10 Proof of Lemma 10

We provide the profits of the firms in the equilibrium where Firm 1 is constrained on its targeting strategy, and

$$\frac{j_1}{k_1} = \tilde{x},$$

$$\frac{j_2}{k_2} = \frac{3}{7} - \frac{2\tilde{x}}{7k_2} + \frac{9}{14k_2}.$$

We replace these values in the profits of the firms:

$$\begin{aligned}\pi_{12} &= \frac{t}{2} - \frac{7j_1^2 t}{9k_1^2} + \frac{2j_2^2 t}{9k_2^2} - \frac{4j_1 j_2 t}{9k_1 k_2} + \frac{2j_1 t}{3k_1} - \frac{2j_2 t}{3k_2} - \frac{j_1 t}{k_1^2} \\ \pi_{22} &= \frac{t}{2} - \frac{7j_2^2 t}{9k_2^2} + \frac{2j_1^2 t}{9k_1^2} - \frac{4j_2 j_1 t}{9k_1 k_2} + \frac{2j_2 t}{3k_2} - \frac{2j_1 t}{3k_1} - \frac{j_2 t}{k_2^2}.\end{aligned}\tag{7}$$

And we obtain:

$$\begin{aligned}\pi_{12}^c(k_1, k_2, \tilde{x}) &= \frac{25t}{98} + \frac{30t\tilde{x}}{49} - \frac{31t\tilde{x}^2}{49} - \frac{t\tilde{x}}{k_1} + \frac{18\tilde{x} + 15}{49k_2}t + \frac{9t}{98k_2^2}, \\ \pi_{22}^c(k_1, k_2, \tilde{x}) &= \frac{9t}{14} + \frac{2t\tilde{x}}{7}(\tilde{x} - 3) + \frac{2t}{7k_2}(\tilde{x} - 3) + \frac{9t}{28k_2^2}.\end{aligned}$$

A.11 Proof of Proposition 5

We compute prices and demand in period 1 of the constrained equilibrium. At the beginning of the game, firms maximize the following objective functions:

$$\begin{aligned}\pi_1(p_{11}) &= (p_{11} - c(k_1)) \left(\frac{p_{12} - p_{11} + t}{2t} \right) + \delta \pi_{12}^c(\tilde{x}(p_{11}, p_{12})), \\ \pi_2(p_{21}) &= (p_{21} - c(k_2)) \left(\frac{p_{11} - p_{12} + t}{2t} \right) + \delta \pi_{22}^c(\tilde{x}(p_{11}, p_{12})).\end{aligned}$$

Where

$$\begin{aligned}\pi_{12}^c(k_1, k_2, \tilde{x}) &= \frac{25t}{98} + \frac{30t\tilde{x}}{49} - \frac{31t\tilde{x}^2}{49} - \frac{t\tilde{x}}{k_1} + \frac{18\tilde{x} + 15}{49k_2}t + \frac{9t}{98k_2^2}, \\ \pi_{22}^c(k_1, k_2, \tilde{x}) &= \frac{9t}{14} + \frac{2t\tilde{x}}{7}(\tilde{x} - 3) + \frac{2t}{7k_2}(\tilde{x} - 3) + \frac{9t}{28k_2^2}.\end{aligned}$$

We replace in $\pi_{12}^c(k_1, k_2, \tilde{x})$ and $\pi_{22}^c(k_1, k_2, \tilde{x})$ the value of \tilde{x} as a function of prices p_{11} and p_{21} : $\tilde{x} = \frac{p_{12} - p_{11} + t}{2t}$.

Finally we apply first order conditions with respect to p_{11} and p_{21} to the sum of profits and we obtain:

$$\begin{aligned}p_{11}^* &= \frac{30t}{41} + \frac{21t}{41k_1} - \frac{2t}{41k_2} + \frac{20c(k_2) + 21c(k_1)}{41}, \\ p_{21}^* &= \frac{91t}{164} + \frac{35t}{164k_1} + \frac{6t}{41k_2} + \frac{129c(k_2) + 35c(k_1)}{164}.\end{aligned}$$

We then derive the expression of \tilde{x}^* by replacing p_{11}^* and p_{21}^* in $\tilde{x} = \frac{p_{12} - p_{11} + t}{2t}$, and we obtain:

$$\tilde{x}^* = \frac{135}{328} + \frac{4}{41k_2} - \frac{49}{328k_1} + \frac{49(c(k_2) - c(k_1))}{328t}.$$

A.12 Proof of Proposition 6

The profits of both firms in period 1 are maximized in the unconstrained equilibrium. In the constraint equilibrium, prices p_{11} and p_{12} take into account the positive impacts of demand on profits in period 2. It is thus straightforward that profits are higher in the unconstrained equilibrium than in the constrained equilibrium.

By comparing profits, it is straightforward to show that profits in periods 2 in the constrained equilibrium are lower for the constrained firm, and higher for the constraining firm than in the unconstrained equilibrium.

$$\begin{aligned}\pi_{12}(k_1, k_2, \tilde{x}) &= \frac{7t}{18} - \frac{7t}{15k_1} + \frac{39t}{100k_1^2} + \frac{7t}{15k_2} - \frac{7t}{25k_1k_2} + \frac{7t}{50k_2^2}, \\ \pi_{22}(k_1, k_2, \tilde{x}) &= \frac{7t}{18} - \frac{7t}{15k_2} + \frac{39t}{100k_2^2} + \frac{7t}{15k_1} - \frac{7t}{25k_1k_2} + \frac{7t}{50k_1^2}, \\ \pi_{12}^c(k_1, k_2, \tilde{x}) &= \frac{25t}{98} + \frac{30t\tilde{x}}{49} - \frac{31t\tilde{x}^2}{49} - \frac{t\tilde{x}}{k_1} + \frac{18\tilde{x} + 15}{49k_2}t + \frac{9t}{98k_2^2}, \\ \pi_{22}^c(k_1, k_2, \tilde{x}) &= \frac{9t}{14} + \frac{2t\tilde{x}}{7}(\tilde{x} - 3) + \frac{2t}{7k_2}(\tilde{x} - 3) + \frac{9t}{28k_2^2}.\end{aligned}$$