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Two Reliability Acceptance Sampling Plans for Items Subject to Wiener Process of Degradation

Ji Hwan Cha^{*1}

Department of Statistics, Ewha Womans University
Seoul, 120-750, Rep. of Korea
e-mail: jhcha@ewha.ac.kr

Sophie Mercier

Laboratory of Mathematics and their Applications of Pau (LMAP)
University of Pau and Pays of Adour / IPRA / CNRS / E2S UPPA
64000 Pau, France
e-mail: sophie.mercier@univ-pau.fr

Abstract

Traditionally, in reliability acceptance sampling plans, the decision to accept or reject a lot is made by performing the life tests of items. However, when the item's deterioration is described by a degradation process, it can be made based on the observed deterioration levels of the items obtained from degradation tests. In this paper, two acceptance sampling plans are developed, based on the observation of the deterioration of the items, accumulated on a given period of time. To model the degradation of the items over time, the Wiener process with positive drift is employed. Algorithms to find the parameters of the proposed sampling plans are suggested. Conditionally on the acceptance in the test, the developed sampling plans are shown to improve the reliability performance of the items in the sense that the lifetimes of the items after the reliability sampling test are stochastically larger than those before the test. Also, we compare the two sampling plans both from a technical and economical points of view.

Keywords: Quality management; variables sampling plan; degradation test; Wiener process; stochastic ordering

¹Corresponding author

NOTATIONS

Θ	the frailty r.v. which represents the reliability characteristic of the population
T_θ	the lifetime of the item with reliability characteristic θ
$f_{T_\theta}(t)$	the pdf (probability density function) of T_θ
$\{W_\theta(t), t \geq 0\}$	the conditional degradation process given $\Theta = \theta$
$\{W(t), t \geq 0\}$	the unconditional degradation process
t_0	the specified testing duration
$\{W_1, \dots, W_n\}$	the observed degradation levels of the n tested items observed at time t_0
κ	the threshold level of degradation after which the failure of the item occurs
α	the producer's risk
β	the consumer's risk
(n, c, ξ)	the parameters of Sampling Plan I
$L_{n,\xi}^{(c)}(\theta)$	the lot acceptance probability of Sampling Plan I at θ
(n, ξ)	the parameters of Sampling Plan II
$L_{n,\xi}(\theta)$	the lot acceptance probability of Sampling Plan II at θ

1 Introduction

In an acceptance sampling plan, the decision on whether to accept or reject a lot of items is based on the observation of a random sample drawn from the lot. One can refer to Stephens (2001) and Montgomery (2012) for general introductions to acceptance sampling plans. In the specific case where the ‘lifetime’ of a product is the main characteristic of interest, sampling plans designed for testing the acceptability of a product are called life test reliability sampling plans. Based on the observation of the lifetimes of the items on tests or on the number of failures observed during a pre-specified testing time, a decision to reject or accept the corresponding lot is reached, taking into account both the producer's and consumer's risks. See, for example, Epstein (1954), Epstein and Sobel (1953, 1955), Blugren and Hewette (1973), Fairbanks (1988), Fertig and Mann (1980) and Schneider (1989) for different acceptance reliability sampling plans for exponential or Weibull distributions. Later on, more sophisticated reliability sampling plans have been developed. See, for instance, Balakrishnan et al. (1993, 2007), Aslam and Jun (2009) and Aslam et al. (2011a, 2011b, 2013a, 2013b, 2013c), Chen et al. (2004), Tsai and Wanbo (2009), Fernández et al. (2011), Wu et al. (2018). Recently, Cha (2016) has addressed the effect of acceptance sampling tests on the reliability performance of the items accepted in the test. Cha (2015) and Lee and Cha (2017) have also studied reliability sampling plans for repairable items.

However, frequently in practice, items are subject to degradation phenomena. See Cha and Badía (2020a, 2020b) for detailed examples of the light intensity of an optical item and the electric voltage or current of some electrical items. Pan et al. (2016, 2017) have studied reliability estimation approach based on a Wiener degradation process and an inverse Gaussian degradation process with random effect, respectively. See also Van Noortwijk (2009) for a review of the various applications of gamma degradation process to various reliability maintenance modeling. In such a case when items are subject to degradation phenomena, instead of observing items' lifetimes (which gen-

erally takes very long time), the degradation levels of the items can be observed on degradation tests (which are performed during a relatively short time). Then, based on this information, the decision on whether to accept or reject the corresponding lot can be made. Recently, Cha and Badía (2020a,2020b) studied sampling plans for items whose degradation process follows the gamma process and the inverse Gaussian process, respectively. In practice, the Wiener process is also frequently applied to model degradation phenomena (see Section 2) and there have been a few works on sampling plan based on the Wiener process. In Tsai et al. (2015), assuming the Wiener process of degradation, an acceptance test is proposed which minimizes the asymptotic variance of the estimate of the optimal acceptance testing time under a cost constrain. As a related work, one can also refer to Jin and Matthews (2014), where, assuming the Wiener process of degradation, a degradation-based testing procedure is developed to test whether the reliability of the product satisfies the required index or not.

In this paper, differently from Tsai et al. (2015) and Jin and Matthews (2014), we develop acceptance sampling plans which balance the producer's and consumer's risks based on the information about the degradation levels of the items, assuming the Wiener process of degradation. Furthermore, in addition to the sampling plan suggested in Cha and Badía (2020a,2020b), another new type of sampling plan which is based on the number of items whose degradation levels exceed a pre-determined level is proposed. Then, these two sampling plans are compared both from a technical and economical points of view. Note that classical reliability sampling plans based on life testing require the observation of failures during the test. Then, due to high reliability performance and long lifetimes of modern items, such plans can be of no use in practice as they can lead to impractically long testing durations. An important merit of the deterioration-based sampling plans proposed in this paper is that they do not require to observe failures of items, so that they can be implemented in much shorter time periods than the classical ones based on life testing.

The structure of this paper is as follows. In Section 2, a new reliability sampling plan for items subject to degradation phenomena is developed. This sampling plan is based on the number of items whose degradation levels exceed a pre-determined level. The parameter values of the sampling plan are determined by taking into account both producer's and consumer's risks. In addition, a convenient and efficient procedure is suggested for their practical implementation. Furthermore, we analyze the effect of the sampling plan on the reliability performance of the items in the population which has passed the testing procedure. In Section 3, we consider another reliability sampling plan for items subject to degradation phenomena, which is based on the sum of wear amounts accumulated by all the items put into the test. Similar issues are discussed for this second type of sampling plan as for the first one. Furthermore, the two sampling plans proposed in this paper are compared. Finally, concluding remarks are given in Section 4.

2 Degradation-Based Sampling Plan I

2.1 Degradation Process and Sampling Plan

We now start our main discussion and first describe how to characterize the reliability level of an item. Frequently, there is variability in the reliability level of an item and some possible causes for it are the quality of resources and components supplied for the production, human errors, uncontrollable factors affecting the quality of the item, and so on. For instance, poor quality of resources and components may result in a low reliability level of produced items, and vice versa.

In this paper, the reliability level of an item is described through an Euclidian parameter θ that we call reliability characteristic. The possible variability in this reliability characteristic is translated through the use of a r.v. Θ , and we write $\Theta = \theta$ to indicate that the reliability characteristic is θ .

The degradation process of an items is denoted by $\{W(t), t \geq 0\}$, where $W(t)$ is the total amount of accumulated degradation until time t . As mentioned earlier, this degradation process depends on the reliability characteristic θ and $\{W_\theta(t), t \geq 0\}$ denotes the degradation process of an item given that $\Theta = \theta$. That is, $\{W_\theta(t), t \geq 0\}$ is the corresponding conditional degradation process given $\Theta = \theta$ and $W_\theta(t)$ is the total amount of accumulated degradation until time t given $\Theta = \theta$, i.e., $W_\theta(t) = (W(t)|\Theta = \theta)$. We assume that the failure of an item occurs if its accumulated degradation exceeds the predetermined failure threshold κ . In practice, items whose accumulated degradation exceeds this level, can still operate. However, in this case, the item is not usable any more and is defined to be in the failed state.

There is a huge literature devoted to the stochastic modeling of accumulated degradation, among which the most common models are gamma process (Van Noortwijk (2009)), inverse Gaussian process (Wang and Xu (2010)) and Wiener process (Doksum and Hoyland (1992)). As noted by Hu et al. (2015), the Wiener process model has been used for many applications in a variety of studies. For instance, it is used by Whitmore (1995) to analyse the degradation (gain) of a transistor, by Wang (2010) to model the deterioration of bridge beams due to chloride ion ingression, by Si et al. (2013) for the analysis of gyroscopic drift in an inertial navigation system, by Lu and Meeker (1993) to analyse the data from fatigue crack growth subject to loading cycles, to chose only a few among many other papers. We refer to Zhang et al. (2018) or Hu et al. (2015) for more examples and references. We also refer to Kahle et al. (2016) for some review on its use as a deterioration process and statistical inference. Based on the previous literature, Wiener process is an adequate and popular model for modeling deterioration. We here assume that the conditional degradation process $\{W_\theta(t), t \geq 0\}$ (given $\Theta = \theta$) follows a Wiener process with positive drift.

Specifically, in this paper, $W_\theta(t)$ follows the following Wiener process:

$$W_\theta(t) = \mu(\theta)t + \sigma B_t,$$

where $(B_t)_{t \geq 0}$ is a Brownian motion and $\mu(\theta)$ is strictly increasing in θ . Thus, $W_\theta(t) \sim \mathcal{N}(\mu(\theta)t, \sigma^2 t)$ and then, for example, $E[W_\theta(t)] = \mu(\theta)t$ is also monotonically increasing in θ , for all $t > 0$. Thus, it implies that the higher θ is, the higher the deterioration level

is. The effect of θ on the lifetime of an item can be made more precise using stochastic ordering. For the definitions of the classical stochastic orders used in this paper (usual stochastic order: \prec_{st} , hazard rate order: \prec_{hr} , reversed hazard rate order: \prec_{rh} , likelihood ratio order: \prec_{lr}), we refer to Müller and Stoyan (2002) and to Shaked and Shanthikumar (2007).

Let T_θ stand for the lifetime of an item with reliability characteristic θ , which is defined as the first-passage time of $(W_\theta(t))_{t \geq 0}$ over level $\kappa > 0$, namely:

$$T_\theta = \inf (t \geq 0 : W_\theta(t) \geq \kappa).$$

It is known that T_θ is inverse Gaussian distributed with pdf

$$f_{T_\theta}(t) = \frac{\kappa}{\sqrt{2\pi\sigma^2 t^3}} \exp\left(-\frac{(\kappa - \mu(\theta)t)^2}{2\sigma^2 t}\right), \forall t \geq 0$$

(see, e.g., Folks and Chhikara (1978)).

The following result explains that the lifetime of an item is decreasing in θ in a strong stochastic sense.

Proposition 1 T_θ decreases with respect to θ in the likelihood ratio ordering, i.e., $T_{\theta_2} \prec_{lr} T_{\theta_1}$, for all $\theta_1 < \theta_2$.

Proof. For $\theta_1 < \theta_2$, we have

$$\begin{aligned} & \frac{f_{T_{\theta_1}}(t)}{f_{T_{\theta_2}}(t)} \\ &= \exp\left(\frac{-(\kappa - \mu(\theta_1)t)^2 + (\kappa - \mu(\theta_2)t)^2}{2\sigma^2 t}\right) \\ &= \exp\left(\frac{\mu(\theta_2) - \mu(\theta_1)}{2\sigma^2} (t(\mu(\theta_1) + \mu(\theta_2)) - 2\kappa)\right). \end{aligned}$$

As $\mu(\theta_1) < \mu(\theta_2)$, the previous ratio increases with respect to t . This implies that $T_{\theta_2} \prec_{lr} T_{\theta_1}$ and the result. ■

As the likelihood ratio ordering implies the hazard rate and usual stochastic ordering, Proposition 1 also implies the ordering of T_{θ_1} and T_{θ_2} in terms of the corresponding hazard rate and survival function (see Shaked and Shanthikumar (2007)). Hence, the highest the reliability characteristic θ is, the lowest the reliability function $R_\theta(t) = \mathbb{P}(T_\theta > t)$ is (for all $t \geq 0$) and the lowest the mean time to failure $\mathbb{E}[T_\theta]$ is.

As explained in the beginning of this section, the reliability characteristic of an item depends on several features such as the quality of resources and components used in the production process, among others. Typically, such features are common to all the units produced in a same lot. Hence, although the reliability characteristic of produced items is variable, we here assume that all the items belonging to a same lot are produced by a sufficiently stable manufacturing process so that they share the same reliability

characteristic θ , and hence, the same degradation process $\{W_\theta(t), t \geq 0\}$. Also, as usual, the items are assumed to be conditionally identical and independent in a lot, given that their common reliability characteristic is θ (namely given $\Theta = \theta$).

We are now interested in designing a sampling plan in order to ensure that the mean time to failure of an item is greater than a certain level. Based on the previously observed monotonicity of $\mathbb{E}[T_\theta]$ with respect to θ , the point is to propose a plan which ensures that the reliability characteristic θ is smaller than a certain level. In such a plan, the probability of rejecting a good lot is called the producer's risk, whereas the probability of accepting a bad lot is known as the consumer's risk. Let us introduce two bounds $\theta_i, i = 1, 2$, where $\theta_1 > \theta_2$, so that θ_1 and θ_2 stand for high and low quality bounds, respectively. The consumer demands that the lot acceptance probability should be smaller than the specified consumer's risk β when testing a lot with a low quality level $\theta \leq \theta_2$, whereas the producer requires that the lot rejection probability should be smaller than the specified producer's risk α , in case of a lot with a high quality level $\theta \geq \theta_1$ (where $\alpha, \beta \in (0, 1)$). Based again on the monotonicity of $\mathbb{E}[T_\theta]$ with respect to θ , it is equivalent to design the plan such that the lot acceptance probability is β when testing a lot with quality level θ_2 (consumer's risk) and that the lot rejection probability is α , in case of a lot with quality level θ_1 (producer's risk).

From the lot to be tested, n items are randomly selected and tested for a time interval t_0 , where t_0 is a specified testing duration. Let W_1, W_2, \dots, W_n stand for the degradation levels of these n items observed at time t_0 . Remembering that a larger value of θ (lower quality level) results in a larger value of $W_\theta(t)$, a possible acceptance test is as follows.

Rejection Rule of Sampling Plan I: If $\sum_{i=1}^n I(W_i > \xi) > c$, then the lot is rejected; otherwise the lot is accepted, where $\xi > 0$ and $c \in \{0, 1, 2, \dots, n-1\}$ are pre-determined parameters and $I(\cdot)$ is the indicator function, that is, $I(A) = 1$ if condition A is satisfied; $I(A) = 0$, otherwise.

Thus, by Plan I, the lot is rejected if there are strictly more than c items whose degradation level is larger than ξ and the proposed sampling plan is characterized by three parameters (n, c, ξ) . For $n \geq 1, c \in \{0, \dots, n-1\}$ and $\xi > 0$, the acceptance probability of a lot as a function of θ is given by

$$\begin{aligned} L_{n,\xi}^{(c)}(\theta) &= \mathbb{P}\left(\sum_{i=1}^n I(W_i > \xi) \leq c\right) \\ &= \sum_{i=0}^c \binom{n}{i} (\bar{F}_\theta(\xi))^i (F_\theta(\xi))^{n-i}, \end{aligned}$$

where $\bar{F}_\theta(\xi) = \mathbb{P}(W_\theta(t_0) > \xi)$ and $F_\theta(\xi) = 1 - \bar{F}_\theta(\xi)$.

It is now necessary to determine the parameters (n, c, ξ) so that the consumer's risk and the producer's risk are balanced as follows:

$$L_{n,\xi}^{(c)}(\theta_2) = \beta \text{ and } 1 - L_{n,\xi}^{(c)}(\theta_1) = \alpha. \quad (1)$$

The following result allows us to understand the influence of each parameter on the

acceptance probability $L_{n,\xi}^{(c)}(\theta)$ and to construct an efficient procedure in determining parameters (approximately) satisfying the two equations in (1).

Proposition 2 *With all parameters fixed except from one, $L_{n,\xi}^{(c)}(\theta)$ increases with respect to ξ and c , and decreases with respect to n and θ .*

Proof. Increasingness of $L_{n,\xi}^{(c)}(\theta)$ with respect to c is clear. Also, based on Cha (2016), a binomial random variable (r.v.) $\mathcal{B}(n, p)$ is known to increase in the likelihood ratio ordering with respect to n , and hence also in the usual stochastic ordering. Then $L_{n,\xi}^{(c)}(\theta)$ decreases with respect to n . Based on Klenke and Mattner (2010), $\mathcal{B}(n, p)$ increases in the usual stochastic ordering when p increases. Also, based on the fact that $W_\theta(t_0)$ increases with respect to θ in the likelihood ratio sense (see Müller and Stoyan (2002), page 62), $W_\theta(t_0)$ also increases in the usual stochastic order sense, so that $\bar{F}_\theta(\xi)$ increases with respect to θ and $L_{n,\xi}^{(c)}(\theta)$ decreases with respect to θ . Increasingness of $L_{n,\xi}^{(c)}(\theta)$ with respect to ξ is proved in a similar way. ■

Suppose that we search for (n, c) satisfying (1) for a fixed ξ . Considering successive $n = 1, 2, \dots$, we first look for c such that

$$L_{n,\xi}^{(c)}(\theta_1) = 1 - \alpha.$$

Denote this value of c by $c(n)$. If this $c(n)$ does not satisfy

$$L_{n,\xi}^{(c)}(\theta_2) = \beta,$$

then we fix the parameter of the number of tested items as $n + 1$ and look for the corresponding $c(n + 1)$, and so on. Due to Proposition 2, we already know that $c(n) \leq c(n + 1)$, $n = 1, 2, \dots$. Thus, $c(n)$ is a lower bound for $c(n + 1)$, which limits the range for $c(n + 1)$ to $\{c(n), \dots, n\}$.

In some cases, there cannot be exact values of (n, c, ξ) satisfying the two equations in (1). Thus, we take (n, c, ξ) which achieves the nearest acceptance probabilities:

$$L_{n,\xi}^{(c)}(\theta_2) \approx \beta \tag{2}$$

and

$$1 - L_{n,\xi}^{(c)}(\theta_1) \approx \alpha$$

or equivalently

$$L_{n,\xi}^{(c)}(\theta_1) \approx 1 - \alpha. \tag{3}$$

To find the parameters satisfying (2) and (3), the following algorithm is used, where n_{\max} is the maximum allowable sample size and $\varepsilon > 0$ is a precision threshold, which should be fixed in advance.

Algorithm 1

1. Fix ξ ;
2. Initialization for $n = 1$:
 - (a) Set $c(1) = 0$;
 - (b) If $\left|L_{1,\xi}^{(0)}(\theta_1) - 1 + \alpha\right| \leq \varepsilon$ and $\left|L_{1,\xi}^{(0)}(\theta_2) - \beta\right| \leq \varepsilon$, store the value $(n = 1, c(1) = 0)$;
3. For each $2 \leq n \leq n_{\max}$:
 - (a) Compute $L_{n,\xi}^{(c)}(\theta_1)$ for each $c \in \{c(n-1), \dots, n-1\}$;
 - (b) Find $c(n)$ which minimizes $\left|L_{n,\xi}^{(c)}(\theta_1) - 1 + \alpha\right|$ as a function of c ;
 - (c) If $\left|L_{n,\xi}^{(c(n))}(\theta_1) - 1 + \alpha\right| \leq \varepsilon$ and $\left|L_{n,\xi}^{(c(n))}(\theta_2) - \beta\right| \leq \varepsilon$, then store the value of $(n, c(n))$;
4. Among all $(n, c(n))$ stored in the previous stages, find the smallest n .

For each ξ , the above algorithm allows us to obtain the corresponding $(n(\xi), c(\xi))$ such that $\left|L_{n(\xi),\xi}^{(c(\xi)(n))}(\theta_1) - 1 + \alpha\right| \leq \varepsilon$ and $\left|L_{n(\xi),\xi}^{(c(\xi)(n))}(\theta_2) - \beta\right| \leq \varepsilon$, namely such that $L_{n(\xi),\xi}^{(c(\xi))}(\theta_1) \simeq 1 - \alpha$ and $L_{n(\xi),\xi}^{(c(\xi))}(\theta_2) \simeq \beta$. Next, we consider various ξ in $(0, \xi_0]$ (where ξ_0 is a maximal threshold fixed in advance) and for each $\xi \in (0, \xi_0]$, we search for the optimal ξ (ξ_{opt} , say) which minimizes $n(\xi)$. Then the obtained solution $(\xi_{opt}, n(\xi_{opt}), c(\xi_{opt}))$ satisfies (2) and (3). Also the solution minimizes the total number of items used in the test, and hence minimizes the corresponding cost.

Example 1 Let $\theta_1 = 1; \theta_2 = 1.5; t_0 = 1; \mu(\theta) = \theta; \sigma = 0.5; \alpha = 0.05; \beta = 0.1; n_{\max} = 30$ and $\xi \in [0.01, 2]$. Using Algorithm 1, the corresponding $n(\xi), c(\xi), R_1(\xi), R_2(\xi)$ are obtained and plotted with respect to ξ in Figure 1, where

$$R_1(\xi) = L_{n(\xi),\xi}^{(c(\xi))}(\theta_1) - 1 + \alpha,$$

$$R_2(\xi) = \beta - L_{n(\xi),\xi}^{(c(\xi))}(\theta_2).$$

For $\varepsilon = 0.01$ we obtain: $\xi_{opt} = 1.23, n(\xi_{opt}) = 12, c(\xi_{opt}) = 6, R_1(\xi_{opt}) = -6.4 \times 10^{-3}$ and $R_2(\xi_{opt}) = -9.5 \times 10^{-3}$. The value of ξ_{opt} is highlighted with a red dotted line in Figure 1. As can be seen in the figure, the procedure chooses ξ_{opt} which minimizes $n(\xi_{opt})$ under the condition that $R_1(\xi_{opt})$ and $R_2(\xi_{opt})$ are near zero.

Example 2 We keep all the previous parameters except for θ_2, σ and ε (and n_{\max}), for which several values are envisioned. The results are provided in Table 1 for $\sigma = 0.5$ (and $n_{\max} = 55$), in Table 2 for $\sigma = 0.75$ (and $n_{\max} = 100$) and in Table 3 for $\sigma = 0.25$ (and $n_{\max} = 20$), where, in each case, ε has been mostly adjusted to get the best precision and n_{\max} is chosen such that ξ_{opt} exists, which complies with the conditions given in Algorithm 1.

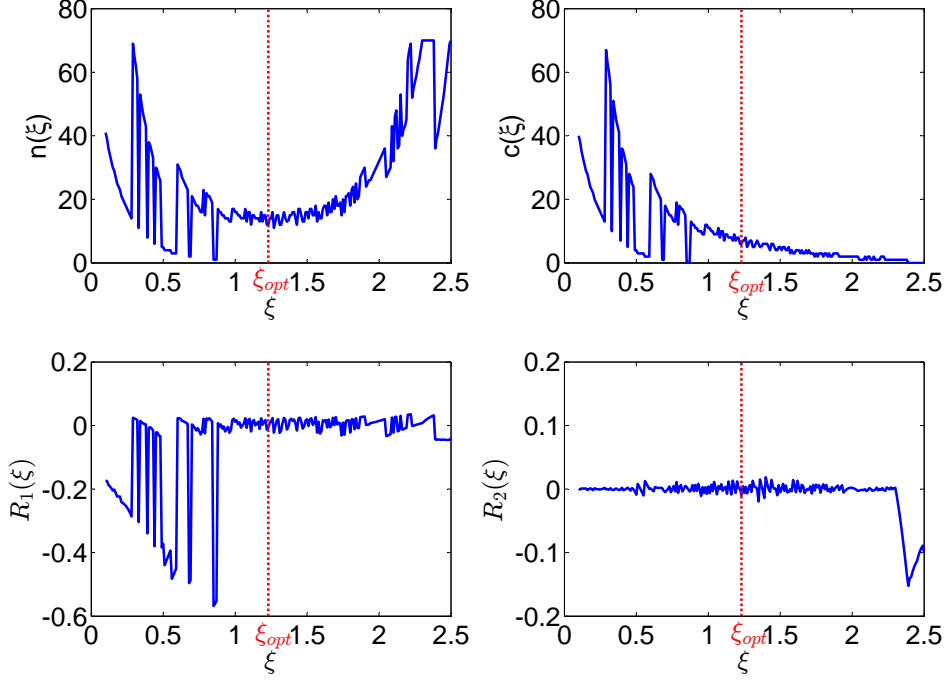


Figure 1: $n(\xi)$, $c(\xi)$, $R_1(\xi)$, $R_2(\xi)$ with respect to ξ together with ξ_{opt} , Example 1

θ_2	ε	ξ_{opt}	$n(\xi_{opt})$	$c(\xi_{opt})$	$R_1(\xi_{opt})$	$R_2(\xi_{opt})$
1.25	0.01	1.10	50	26	-9.6×10^{-3}	-1.4×10^{-3}
1.50	0.01	1.23	12	6	-6.4×10^{-3}	-9.5×10^{-3}
1.75	0.01	1.42	5	2	-8.3×10^{-3}	-8.5×10^{-3}
2	0.01	1.56	3	1	2.8×10^{-3}	5.9×10^{-3}
2.25	0.01	1.41	2	1	7.5×10^{-3}	9.2×10^{-3}
2.50	0.01	1.83	1	0	1.5×10^{-3}	9.9×10^{-3}
2.75	0.1	1.83	1	0	1.5×10^{-3}	6.7×10^{-2}
3	0.1	1.83	1	0	1.5×10^{-3}	9.0×10^{-2}
4	0.1	1.83	1	0	1.5×10^{-3}	1.0×10^{-1}

Table 1: ξ_{opt} , $n(\xi_{opt})$, $c(\xi_{opt})$, $R_1(\xi_{opt})$, $R_2(\xi_{opt})$ with respect to θ_2 , Example 2, case $\sigma = 0.5$ (and $n_{max} = 55$)

From Tables 1, 2 and 3, we find that $n(\xi_{opt})$ decreases as θ_2 increases (except for the last line of Table 2), namely, when there is a larger difference between θ_1 and θ_2 . Indeed, it seems natural that less units are required to make a distinction between the two levels when they are well separate. Furthermore, we observe that $c(\xi_{opt})$ decreases when θ_2 increases (except for the last line of Table 2). Also, from $\theta_2 \simeq 2.50$, the values of ξ_{opt} , $n(\xi_{opt})$, $c(\xi_{opt})$ and $R_1(\xi_{opt})$ become stable. However, $R_2(\xi_{opt})$ increases and, for large values of θ_2 , it becomes more difficult to comply with the constraint $R_2(\xi_{opt}) \simeq 0$. For a better understanding of the behavior of the procedure for large θ_2 , we refer to Figure

θ_2	ε	ξ_{opt}	$n(\xi_{opt})$	$c(\xi_{opt})$	$R_1(\xi_{opt})$	$R_2(\xi_{opt})$
1.25	0.05	1.09	99	52	-9.6×10^{-3}	-3.7×10^{-2}
1.50	0.05	1.13	27	15	-1.8×10^{-2}	-5.0×10^{-4}
1.75	0.05	1.40	11	5	-2.5×10^{-2}	-3.6×10^{-3}
2	0.05	1.36	6	3	-3.4×10^{-2}	5.1×10^{-3}
2.25	0.05	1.67	3	1	-4.1×10^{-2}	-2.4×10^{-2}
2.50	0.1	1.77	3	1	-1.3×10^{-2}	2.7×10^{-2}
2.75	0.1	1.78	1	0	-9.9×10^{-2}	2.1×10^{-3}
3	0.1	1.89	1	0	-6.8×10^{-2}	3.1×10^{-2}
4	0.1	1.67	2	1	1.6×10^{-2}	9.8×10^{-2}

Table 2: ξ_{opt} , $n(\xi_{opt})$, $c(\xi_{opt})$, $R_1(\xi_{opt})$, $R_2(\xi_{opt})$ with respect to θ_2 , Example 2, case $\sigma = 0.75$ (and $n_{max} = 100$)

θ_2	ε	ξ_{opt}	$n(\xi_{opt})$	$c(\xi_{opt})$	$R_1(\xi_{opt})$	$R_2(\xi_{opt})$
1.25	0.05	1.11	12	6	-1.3×10^{-2}	6.0×10^{-4}
1.35	0.05	1.19	2	1	-2.8×10^{-2}	-1.6×10^{-2}
1.45	0.05	1.23	1	0	-3.5×10^{-2}	5.9×10^{-3}
1.75	0.05	1.38	1	0	-1.4×10^{-2}	3.1×10^{-2}
2.00	0.1	1.59	1	0	4.1×10^{-2}	4.9×10^{-2}
2.25	0.1	1.42	1	0	3.5×10^{-2}	9.9×10^{-2}
2.5	0.1	1.42	1	0	3.5×10^{-2}	1.0×10^{-1}
3	0.1	1.42	1	0	3.5×10^{-2}	1.0×10^{-1}

Table 3: ξ_{opt} , $n(\xi_{opt})$, $c(\xi_{opt})$, $R_1(\xi_{opt})$, $R_2(\xi_{opt})$ with respect to θ_2 , Example 2, case $\sigma = 0.25$ (and $n_{max} = 20$)

2, which corresponds to $\theta_2 = 4$. We can see in that figure that, indeed, when θ_2 is large in front of θ_1 , it becomes difficult to comply with the two constraints. However, in that case, we have $L_{n(\xi),\xi}^{(c(\xi))}(\theta_1) \approx 1 - \alpha$ and $L_{n(\xi),\xi}^{(c(\xi))}(\theta_2) < \beta$ and the aim of the sampling plan is still fulfilled, with an even lower consumer's risk than required.

2.2 Analysis of Reliability Characteristic of an Accepted Lot

In this section, we investigate the effect of the sampling plan developed in the previous subsection on the reliability characteristic of an item belonging to a lot which has been accepted by the testing procedure. Before discussing the lifetime of the item, we first study its amount of degradation in an arbitrary time interval $W(t_2) - W(t_1)$, $t_1 < t_2$. As stated before, the reliability characteristic of the population is variable due to many different reasons and the variability is represented by the r.v. Θ . We set f_Θ to be its pdf. Before the testing procedure, the distribution of the increment $W(t_2) - W(t_1)$ is a mixed distribution with survival function

$$\bar{F}_{t_1,t_2}(x) = \int_{\mathbb{R}_+} \bar{F}_{t_1,t_2,\theta}(x) f_\Theta(\theta) d\theta, \forall x \geq 0$$

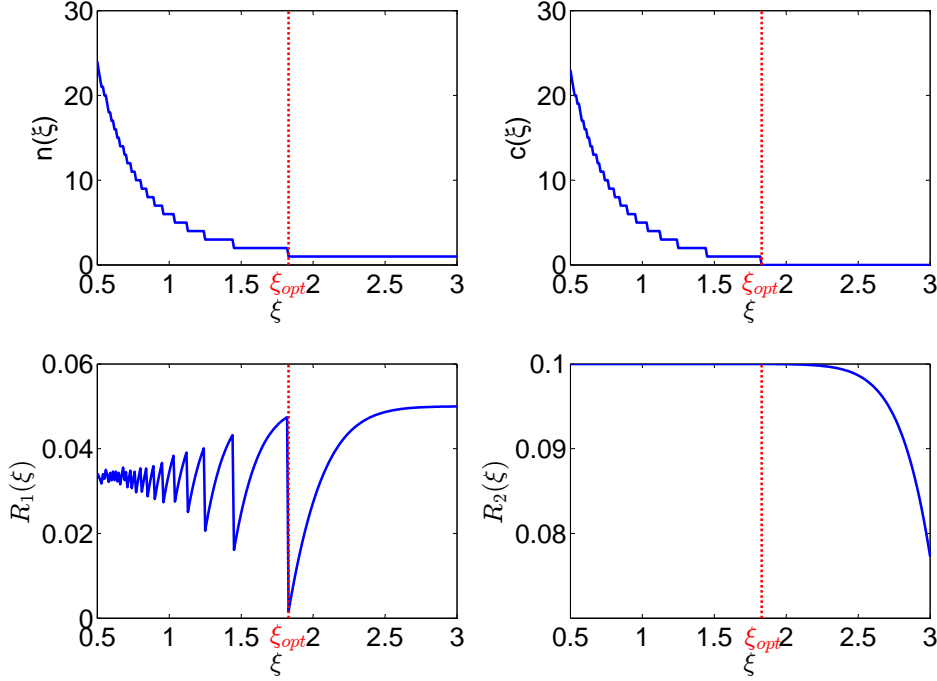


Figure 2: $n(\xi), c(\xi), R_1(\xi), R_2(\xi)$ with respect to ξ together with ξ_{opt} , case $\theta_2 = 4$, Example 2

and pdf:

$$f_{t_1, t_2}(x) = \int_{\mathbb{R}_+} f_{t_1, t_2, \theta}(x) f_{\Theta}(\theta) d\theta, \forall x \geq 0,$$

where $\bar{F}_{t_1, t_2, \theta}$ and $f_{t_1, t_2, \theta}$ are the survival function and the pdf of $W_{\theta}(t_2) - W_{\theta}(t_1)$, respectively, which is Gaussian distributed $\mathcal{N}(\mu(\theta)(t_2 - t_1), \sigma^2(t_2 - t_1))$.

For $0 \leq t_1 < t_2$, let $\bar{F}_{t_1, t_2 | S}$ be the conditional survival function of $W(t_2) - W(t_1)$ for a randomly chosen item given that the test with the parameters (ξ, n, c) was successful (i.e., given that the lot is accepted), where $S = \{\sum_{i=1}^n I(W_i > \xi) > c\}$. Then we obtain the following result.

Proposition 3 For all $0 \leq t_1 < t_2$, we have:

$$\bar{F}_{t_1, t_2 | S}(x) = \frac{\int_{\mathbb{R}_+} \bar{F}_{t_1, t_2, \theta}(x) L_{n, \xi}^{(c)}(\theta) f_{\Theta}(\theta) d\theta}{\int_{\mathbb{R}_+} L_{n, \xi}^{(c)}(\theta) f_{\Theta}(\theta) d\theta}, \forall x \geq 0,$$

$$f_{t_1, t_2 | S}(x) = \frac{\int_{\mathbb{R}_+} f_{t_1, t_2, \theta}(x) L_{n, \xi}^{(c)}(\theta) f_{\Theta}(\theta) d\theta}{\int_{\mathbb{R}_+} L_{n, \xi}^{(c)}(\theta) f_{\Theta}(\theta) d\theta}, \forall x \geq 0.$$

Proof. Let $f_{\Theta|S}(\theta)$ be the conditional pdf of $\Theta|S$. We have:

$$\begin{aligned}\bar{F}_{t_1, t_2|S}(x) &= \int_{\mathbb{R}_+} \bar{F}_{t_1, t_2, \theta}(x) f_{\Theta|S}(\theta) d\theta, \\ f_{t_1, t_2|S}(x) &= \int_{\mathbb{R}_+} f_{t_1, t_2, \theta}(x) f_{\Theta|S}(\theta) d\theta,\end{aligned}$$

where

$$\begin{aligned}f_{\Theta|S}(\theta) &= f_{\Theta|\sum_{i=1}^n I(W_i > \xi) > c}(\theta) \\ &= \frac{\mathbb{P}(\sum_{i=1}^n I(W_i > \xi) > c | \Theta = \theta) f_{\Theta}(\theta)}{\mathbb{P}(\sum_{i=1}^n I(W_i > \xi) > c)} \\ &= \frac{L_{n, \xi}^{(c)}(\theta) f_{\Theta}(\theta)}{\int_{\mathbb{R}_+} L_{n, \xi}^{(c)}(\theta) f_{\Theta}(\theta) d\theta}\end{aligned}$$

which provides the two results. ■

The following result stochastically compares the r.v. Θ and $W(t_2) - W(t_1)$ before and after the testing procedure.

Proposition 4 *It holds that*

1. $[\Theta|S] \prec_{lr} \Theta$;
2. $[W(t_2) - W(t_1)|S] \prec_{lr} W(t_2) - W(t_1)$, for all $0 \leq t_1 < t_2$.

Proof. We have

$$\frac{f_{\Theta|S}(\theta)}{f_{\Theta}(\theta)} = C L_{n, \xi}^{(c)}(\theta),$$

(where C is a constant which does not depend on θ) and the ratio is decreasing with respect to θ (see Proposition 2). This provides the first point.

As for the second point, we have

$$\begin{aligned}f_{t_1, t_2}(x) &= \int_{\mathbb{R}_+} f_{t_1, t_2, \theta}(x) f_{\Theta}(\theta) d\theta, \\ f_{t_1, t_2|S}(x) &= \int_{\mathbb{R}_+} f_{t_1, t_2, \theta}(x) f_{\Theta|S}(\theta) d\theta.\end{aligned}$$

Also, we know from Müller and Stoyan (2002), page 62, that $W_{\theta}(t_2) - W_{\theta}(t_1)$ increases with respect to θ in the likelihood ratio order sense. As $[\Theta|S] \prec_{lr} \Theta$ from the first point, the second point now follows from Shaked and Shanthikumar (2007) (page 49, Theorem 1.C.17). ■

Obviously, before the testing procedure, the distribution of the first-passage time T of $W(t)$ over level $\kappa > 0$ of a randomly chosen item is a mixed distribution with pdf:

$$f_T(x) = \int_{\mathbb{R}_+} f_{T_{\theta}}(x) f_{\Theta}(\theta) d\theta, \forall x \geq 0.$$

Now, given that the test was successful (i.e., the lot is accepted), the conditional pdf of T given $S = \{\sum_{i=1}^n I(W_i > \xi) > c\}$ becomes

$$f_{T|S}(x) = \int_{\mathbb{R}_+} f_{T_\theta}(x) f_{\Theta|S}(\theta) d\theta, \forall x \geq 0.$$

In the following result, we compare the lifetimes before and after the testing procedure.

Proposition 5 *It holds that $T \prec_{lr} [T|S]$.*

Proof. Based on Proposition 1 and Shaked and Shanthikumar (2007) (page 46, Theorem 1.C.8), $-T_\theta$ increases with respect to θ in the likelihood ratio ordering. Using the first point of Proposition 4 and Shaked and Shanthikumar (2007) (page 49, Theorem 1.C.17), we derive that $[-T|S] \prec_{lr} -T$, which provides the result. ■

The above result entails for instance that $T \prec_{hr} [T|S]$, which means that the residual life of an item at any time t is stochastically larger if we know that the lot has been accepted:

$$[T - t|T > t] \prec_{sto} [T - t|S, T > t], \forall t \geq 0.$$

The above result also entails for instance that the mean residual life of an item at any time t is larger if we know that the lot has been accepted:

$$\begin{aligned} MRL_T(t) &\equiv \mathbb{E}(T - t|T > t) \\ &\leq \mathbb{E}(T - t|S, T > t) \\ &\equiv MRL_{T|S}(t) \end{aligned}$$

for all $t \geq 0$.

3 Degradation-Based Sampling Plan II

3.1 Sampling Plan

The second type of sampling plan based on the degradation tests is as follows. As in the previous section, n items are randomly chosen from the lot to be tested and tested for a time interval t_0 . From the test, the values of the degradation levels of these n items W_1, W_2, \dots, W_n are observed.

Rejection Rule of Sampling Plan II: If $\sum_{i=1}^n W_i > \xi$, then the lot is rejected; otherwise the lot is accepted, where $\xi > 0$ is a pre-determined parameter.

Then the proposed sampling plan is characterized by two parameters (n, ξ) . Note that

$\sum_{i=1}^n W_i \sim \mathcal{N}(n\mu(\theta)t_0, n\sigma^2t_0)$. Then, for $n \geq 1$ and $\xi > 0$, the acceptance probability of the lot as a function of θ is given by

$$L_{n,\xi}(\theta) = \mathbb{P}\left(\sum_{i=1}^n W_i \leq \xi\right) = F_{n,\theta}(\xi)$$

where $F_{n,\theta}$ is the Cdf (cumulative distribution function) $\mathcal{N}(n\mu(\theta)t_0, n\sigma^2t_0)$.

As before, it is now necessary to determine the parameters (n, ξ) so that the consumer's risk and the producer's risk are balanced as follows:

$$L_{n,\xi}(\theta_2) \approx \beta, \text{ and } L_{n,\xi}(\theta_1) \approx 1 - \alpha. \quad (4)$$

The following is the basic properties of the function $L_{n,\xi}(\theta)$.

Proposition 6 *With all parameters fixed except from one, $L_{n,\xi}(\theta)$ increases with respect to ξ , and decreases with respect to θ .*

Proof. The fact that $L_{n,\xi}(\theta)$ increases with respect to ξ is clear.

As $\sum_{i=1}^n W_i$ increases with respect to θ in the likelihood ratio order sense (see Müller and Stoyan (2002), page 62), $W_\theta(t_0)$ also increases in the usual stochastic order sense, so that $L_{n,\xi}(\theta)$ decreases with respect to θ . ■

Remark 1 *From the literature (see, e.g., Müller and Stoyan (2002), page 62), it is known that a Gaussian r.v. $\mathcal{N}(m, \sigma^2)$ increases when (m, σ) increases in the increasing convex ordering, but not in the (stronger) usual stochastic ordering. Then, there is no reason why $\sum_{i=1}^n W_i \sim \mathcal{N}(n\mu(\theta)t_0, n\sigma^2t_0)$ should be monotone with respect to n in the usual stochastic ordering and hence, there is no reason why $L_{n,\xi}(\theta)$ should be monotone with respect to n .*

Note that, using the Cdf of the normal distribution, for a fixed n , the parameter ξ satisfying $L_{n,\xi}(\theta_1) = 1 - \alpha$ can be easily obtained as $\xi = F_{n,\theta_1}^{-1}(1 - \alpha)$. As before, to find the parameters (n, ξ) , the following algorithm is used, where n_{\max} is the maximum allowable sample size and $\varepsilon > 0$ is the threshold for the precision, which should be fixed in advance.

Algorithm 2

1. For each $1 \leq n \leq n_{\max}$:
 - (a) Compute $\xi(n) = F_{n,\theta_1}^{-1}(1 - \alpha)$;
 - (b) Compute $L_{n,\xi(n)}(\theta_2)$;
2. Chose the smallest n such that $|L_{n,\xi(n)}(\theta_2) - \beta| \leq \varepsilon$.

The above algorithm allows us to obtain the corresponding optimal parameters $(n_{opt}, \xi(n_{opt}))$ such that n_{opt} is the minimum among all n 's satisfying (4). Note that this algorithm is much simpler than Algorithm 1 for Sampling Plan I.

Example 3 Let $\theta_1 = 1; t_0 = 1; \mu(\theta) = \theta; \alpha = 0.05; \beta = 0.1$. Considering several values for θ_2, σ and ε (and n_{\max}), the corresponding $(n_{opt}, \xi(n_{opt})), R_1(n_{opt}), R_2(n_{opt})$ are obtained using Algorithm 2. The results are provided in Table 4 for $\sigma = 0.5$ (and $n_{\max} = 50$), in Table 5 for $\sigma = 0.75$ (and $n_{\max} = 100$) and in Table 6 for $\sigma = 0.25$ (and $n_{\max} = 10$), where in each case, ε has been mostly adjusted to get the best precision and n_{\max} is chosen such that ξ_{opt} exists, which complies with the conditions given in Algorithm 2.

From Tables 4, 5 and 6, we can see that, in the same way as for the first acceptance sampling plan, the larger θ_2 is, the lower $n(\xi_{opt})$ is. Here again, a larger difference between θ_1 and θ_2 hence requires less units to make the distinction between the two levels. Also, for each value of σ , the values of $\xi(n_{opt})$ and n_{opt} become stable from a certain value $\bar{\theta}_2$ for θ_2 . Note that the larger σ is, the larger this threshold value $\bar{\theta}_2$ is, with $\bar{\theta}_2 \approx 1.75$ for $\sigma = 0.25$, $\bar{\theta}_2 \approx 2.5$ for $\sigma = 0.5$ and $\bar{\theta}_2 \approx 3$ for $\sigma = 0.75$. Such a remark is mostly valid for the first sampling plan as well, but the results are more stable and easy to interpret for the second one. Finally, here again, when θ_2 is large in front of θ_1 , we have $L_{n(\xi), \xi}^{(c(\xi))}(\theta_1) \approx 1 - \alpha$ and $L_{n(\xi), \xi}^{(c(\xi))}(\theta_2) < \beta$, so that the aim of the sampling plan is still fulfilled.

3.2 Analysis of Reliability Characteristic of an Accepted Lot

As before, we first study the amount of degradation in an arbitrary time interval $W(t_2) - W(t_1)$, $t_1 < t_2$. Let $S = \{\sum_{i=1}^n W_i \leq \xi\}$. We have a similar result as for Proposition 3, substituting $L_{n, \xi}^{(c(\xi))}$ by $L_{n, \xi}(\theta)$. The proof uses the same arguments and it is omitted.

Proposition 7 For all $0 \leq t_1 < t_2$, we have:

$$\bar{F}_{t_1, t_2 | S}(x) = \frac{\int_{\mathbb{R}_+} \bar{F}_{t_1, t_2, \theta}(x) L_{n, \xi}(\theta) f_{\Theta}(\theta) d\theta}{\int_{\mathbb{R}_+} L_{n, \xi}(\theta) f_{\Theta}(\theta) d\theta}, \forall x \geq 0,$$

$$f_{t_1, t_2 | S}(x) = \frac{\int_{\mathbb{R}_+} f_{t_1, t_2, \theta}(x) L_{n, \xi}(\theta) f_{\Theta}(\theta) d\theta}{\int_{\mathbb{R}_+} L_{n, \xi}(\theta) f_{\Theta}(\theta) d\theta}, \forall x \geq 0.$$

Regarding the stochastic comparison of Θ and $W(t_2) - W(t_1)$ before and after the testing procedure, similar arguments as for Proposition 4 can be used to obtain the following result.

Proposition 8 It holds that

1. $[\Theta | S] \prec_{lr} \Theta$;
2. $[W(t_2) - W(t_1) | S] \prec_{lr} W(t_2) - W(t_1)$, for all $0 \leq t_1 < t_2$.

Regarding the stochastic comparison of lifetimes, here again similar arguments as for Proposition 5 can be used to obtain the following result.

Proposition 9 It holds that $T \prec_{lr} [T | S]$.

As a conclusion, just as for Sampling Plan I, Sampling Plan II also allows us to know that the lifetime of an item belonging to an accepted lot is larger than if the lot had not been tested.

θ_2	ε	$\xi(n_{opt})$	n_{opt}	$R_1(n_{opt})$	$R_2(n_{opt})$
1.25	0.005	38.80	34	-4.16×10^{-17}	-2×10^{-3}
1.50	0.025	10.33	8	-4.16×10^{-17}	-1.8×10^{-2}
1.75	0.025	5.64	4	-4.16×10^{-17}	1.2×10^{-2}
2	0.025	3.16	2	-4.16×10^{-17}	-1.8×10^{-2}
2.25	0.075	3.16	2	-4.16×10^{-17}	7.1×10^{-2}
2.50	0.075	1.82	1	-4.16×10^{-17}	1.2×10^{-2}
2.75	0.075	1.82	1	-4.16×10^{-17}	6.8×10^{-2}
3	0.1	1.82	1	-4.16×10^{-17}	9.1×10^{-2}
4	0.1	1.82	1	-4.16×10^{-17}	1.0×10^{-1}

Table 4: $\xi(n_{opt})$, n_{opt} , $R_1(n_{opt})$, $R_2(n_{opt})$ with respect to θ_2 (and ε), Example 3, case $\sigma = 0.5$ (and $n_{max} = 50$)

θ_2	ε	$\xi(n_{opt})$	n_{opt}	$R_1(n_{opt})$	$R_2(n_{opt})$
1.25	0.005	86.75	76	-6.94×10^{-17}	-4×10^{-3}
1.50	0.005	24.38	19	-4.16×10^{-17}	-4×10^{-3}
1.75	0.025	11.49	8	-4.16×10^{-17}	-1.8×10^{-2}
2	0.025	7.76	5	-4.16×10^{-17}	9.3×10^{-3}
2.25	0.025	5.14	3	-4.16×10^{-17}	-7.1×10^{-3}
2.50	0.025	3.75	2	-4.16×10^{-17}	-1.8×10^{-2}
2.75	0.075	3.74	2	-4.16×10^{-17}	5.1×10^{-2}
3	0.075	2.23	1	-4.16×10^{-17}	-5.3×10^{-2}
4	0.1	2.23	1	-4.16×10^{-17}	9.1×10^{-2}

Table 5: $\xi(n_{opt})$, n_{opt} , $R_1(n_{opt})$, $R_2(n_{opt})$ with respect to θ_2 (and ε), Example 3, case $\sigma = 0.75$ (and $n_{max} = 100$)

θ_2	ε	$\xi(n_{opt})$	n_{opt}	$R_1(n_{opt})$	$R_2(n_{opt})$
1.25	0.025	9.16	8	-4.16×10^{-17}	-1.8×10^{-2}
1.35	0.025	4.82	4	6.94×10^{-17}	-2.4×10^{-2}
1.45	0.050	3.71	3	-4.16×10^{-17}	3.0×10^{-2}
1.75	0.050	1.41	1	-4.16×10^{-17}	1.2×10^{-2}
2	0.1	1.41	1	-4.16×10^{-17}	9.1×10^{-2}
2.25	0.1	1.41	1	-4.16×10^{-17}	1.0×10^{-1}
2.50	0.1	1.41	1	-4.16×10^{-17}	1.0×10^{-1}
3	0.1	1.41	1	-4.16×10^{-17}	1.0×10^{-1}

Table 6: $\xi(n_{opt})$, n_{opt} , $R_1(n_{opt})$, $R_2(n_{opt})$ with respect to θ_2 (and ε), Example 3, case $\sigma = 0.25$ (and $n_{max} = 10$)

3.3 Comparison of Sampling Plans I and II

In Sections 2 and 3, we have proposed two types of sampling plans. Sampling Plan I is based on the statistic $\sum_{i=1}^n I(W_i > \xi)$, whereas Sampling Plan II is based on $\sum_{i=1}^n W_i$. The optimal parameters for each sampling plan are given in Tables 1-3 and Tables 4-6, respectively. As there are negligible differences in R_1 and R_2 , we compare the total number of items used in the testing procedure. Especially when there is very small difference between θ_1 and θ_2 , the total number of items used in the testing procedure n of Sampling Plan II is much smaller than that of Sampling Plan I. Thus, from the experimental comparison, we can conclude that Sampling Plan II spends less items in the degradation test. Furthermore, Sampling Plan II has smaller number of parameters and Algorithm 2 is much simpler than Algorithm 1. In addition, in Sampling Plan II, the parameter ξ can be conveniently obtained by using the Cdf of the normal distribution.

On the other hand, in some cases, whether the amount of degradation of an item reaches a predetermined level ξ (i.e., $W_i > \xi$) or not can be easily checked and directly reported by a censor built in the item. In this case, in applying Sampling Plan I, there is no need to observe the degradation levels of the items on the test and the observation cost can be substantially saved. In this case, Sampling Plan I would be preferable from an “economical point of view”.

4 Concluding Remarks

In this paper, based on the observation of the deterioration of the items, two types of variables acceptance sampling plans have been developed. The first type of sampling plan (Plan I) is based on the statistic $\sum_{i=1}^n I(W_i > \xi)$ and the second type (Plan II) is based on $\sum_{i=1}^n W_i$. Considering both the producer’s risk and consumer’s risk, efficient algorithms to determine the parameters of the sampling plans have been suggested. Furthermore, the effect of the proposed sampling plans on the accepted lots has been analyzed. It has been shown that the reliability characteristic of the accepted items after the test improves compared with that of the items before the test in a stochastic sense. Finally, we have compared the two types of sampling plans from both the economical and technical perspectives.

In this paper, we have considered the Wiener process with positive drift as the degradation process of the items. If more appropriate, other types of degradation processes could be employed and similar sampling plans could be developed based on them.

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