



HAL
open science

Theoretical and experimental study of micropolar elastic materials using acoustic waves in air

Erick Ogam, Zine El Abiddine Fella, Mohamed Fella, Claude Depollier

► To cite this version:

Erick Ogam, Zine El Abiddine Fella, Mohamed Fella, Claude Depollier. Theoretical and experimental study of micropolar elastic materials using acoustic waves in air. *Journal of Sound and Vibration*, 2021, 510, pp.116298. 10.1016/j.jsv.2021.116298 . hal-03266564

HAL Id: hal-03266564

<https://hal.science/hal-03266564>

Submitted on 11 Oct 2021

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

1 Theoretical and Experimental study of micropolar elastic
2 materials using acoustic waves in air

3 Erick Ogam^{a,*}, Zine El Abbidine Fellah^a, Mohamed Fellah^b, Claude Depollier^c

4 ^a*Laboratoire de Mécanique et d'Acoustique LMA - UMR 7031 AMU - CNRS - Centrale Marseille*

5 ^b*Laboratoire de physique théorique, Faculté de physique, Université des Sciences et de la Technologie*
6 *Houari Boumediene, BP 32 El Alia 16111 Bab Ezzouar Alger, Algeria.*

7 ^c*Laboratoire d'Acoustique de l'Université du Mans - UMR CNRS 6613 (LAUM), Avenue Olivier*
8 *Messiaen 72085 - Le Mans cedex 9 France.*

9 **Abstract**

This study focuses on the development of a theoretical and experimental acoustic method and setup for studying the micropolar behavior of materials. Two types of low density closed-cell foams made from polystyrene (expanded and extruded) materials that were previously reported as materials exhibiting the non-classical elastodynamic behavior were investigated in order to reveal the particular compoment using acoustic waves. Two different types of transducers were employed to provide a large frequency band to study the influence of the Cosserat parameters. The frequency band from 1-40 kHz was covered using a twitter loudspeaker whereas that from 30-120 kHz, by an air coupled ultrasonic transducer. The materials' micropolar elastic behavior were revealed through the comparison between the theoretical and experimental transmission coefficients. At the cut-off frequency where the theoretical phase velocity of the transverse acoustic mode was established to be infinite, the acoustic wave transmission by the panels was found to be very small. This makes the Cosserat materials appealing to harmonic noise reduction.

10 *Keywords:* Micropolar elasticity, Acoustic transmission coefficient, Air-filled closed-cell
11 foams, cut-off frequency, Recovery of Cosserat parameters

*Principal corresponding author
Email address: ogam@lma.cnrs-mrs.fr (Erick Ogam)

12 1. Introduction

13 The Cosserat brothers, Eugène and François, published a monograph [1] in 1909 in
14 which they presented a new variant of Continuum Mechanics. This detailed treatise
15 covered the idea of taking into account additionally the couple stresses, the rotational
16 degrees of freedom (micro-rotations) and their gradients of the material particles as
17 independent variables. The existence of the couple stresses was originally postulated by
18 Voigt in 1887 [2].

19 The couple stress theory using macrorotation as the true kinematical rotation for elas-
20 tic bodies in which the gradient of the rotation vector is used as a curvature tensor was
21 developed much later by Toupin [3], Mindlin and Tiersten [4], Koiter [5]. These formula-
22 tions use the four foundational continuum mechanical quantities (i.e., force, displacement,
23 couple, rotation) and are the fundamental pillars in the development of size-dependent
24 continuum mechanics. Heterogeneous materials exhibit intrinsic material length scales
25 (like in cellular structures, grains, particles and fibers). However, these formulations pose
26 some challenges like the indeterminacy of the spherical part of the couple-stress tensor
27 and the appearance of the body couple in the constitutive relation for the force-stress
28 tensor [4, 6]. Hadjefandiari and Dargush [7] resolved the inconsistencies by revealing
29 the subtle skew-symmetric character of the couple-stress tensor and showed that it was
30 a true vector. The constrained Cosserat theory is a particular case of the couple stress
31 theory in which kinematic constraints have been introduced in order to eliminate some
32 of the degrees of freedom [8] in order to obtain a simpler system of equations.

33 The extension of the linear Cosserat theory to include body microinertia effects is
34 attributed to Eringen[9]. He is also the one who renamed it as the micropolar theory
35 of elasticity [10]. A complete variant of this theory can be found in Kafadar and Erin-
36 gen [11]. Micropolar continuum mechanics [12] therefore incorporates a local rotation of
37 points (micropolar continua in the sense of Eringen) including the translation assumed
38 in classical elasticity; and a couple stress as well as the force stress.

39 Six material parameters are needed in the linear micropolar elasticity of isotropic

40 solids while only two Lamé moduli are necessary in the classical elasticity. A review and
41 survey of the literature on the theory of Cosserat continuum can be found in [13, 14].
42 This theory has been extended further to capture more complex micropolar material
43 behavior [15–17].

44 The majority of studies pertaining to the micropolar elastodynamics of materials
45 have been mainly theoretical. The most popular media geometry studied was that of an
46 infinite half-plane corresponding to the ground in geology and seismic studies. Many of
47 the problems reported were those concerning reflection and refraction of plane waves at
48 plane interfaces in which seismic waves generated within a semi-infinite half-plane mi-
49 cropolar solid media interact with, for example, a viscous liquid layer [18, 19]. The few
50 experimental measurements reported were mainly undertaken using mechanical (static,
51 quasi-static) means [20–23]. All six elastic constant parameters (like the apparent shear
52 modulus) have been found from torque and twist measurements in a pure torsion exper-
53 iment using a circular cylinder and the cylindrical bending of a rectangular plate [24].

54 In this study, a theoretical micropolar elastic model was developed and employed to
55 validate a new experimental method to study acoustic wave transmission through air-
56 filled closed-cell polystyrene panels considered to obey Cosserat elasticity (micropolar
57 continuum theory) law. The frequency domain ranges from the audible to the ultrasonic
58 regimes but < 120 kHz. The foams studied were two-phase composite materials in which
59 one phase was solid and the other, fluid (air). In this case, the size scale is large, the
60 material may no longer be assumed to be continuous [25, 26]. A table in reference [14]
61 classified the closed-cell polystyrene foams in the family of Cosserat elasticity materials.
62 The micropolar continuum theory model considered herein was that of the generalized
63 plane strain conditions assuming that the strain distributions do not depend on the third
64 coordinate geometry of the acoustic wave transmission through panels experiment. The
65 other reason motivating this study is that elastic data for closed-cell materials like the
66 EPS and XPS foams are hard to come by in the literature and the only existing methods
67 used by manufacturers are based on the static destructive mechanical methods declined

68 as standard test methods [27, 28].

69 2. Theoretical and experimental methods

70 2.1. Theoretical method for wave propagation in a Cosserat continuum

71 The loadings on a micropolar continuum of density ρ and rotational inertia J generate
 72 deformation of the body described by the displacement vector \mathbf{u} and the rotation vector
 73 $\boldsymbol{\Phi}$.

74 The equations of motion in the micropolar elastic continuum are [20, 29]

$$(\lambda + 2\mu + \mathcal{K}) \nabla(\nabla \bullet \mathbf{u}) - (\mu + \mathcal{K}) \nabla \times (\nabla \times \mathbf{u}) + \mathcal{K} \nabla \times \boldsymbol{\Phi} - \rho \frac{d^2}{dt^2} \mathbf{u} = 0,$$

$$(\alpha + \beta + \gamma) \nabla(\nabla \bullet \boldsymbol{\Phi}) - \gamma \nabla \times \nabla \times \boldsymbol{\Phi} + \mathcal{K} \nabla \times \mathbf{u} - 2\mathcal{K} \boldsymbol{\Phi} - \rho J \frac{d^2}{dt^2} \boldsymbol{\Phi} = 0, \quad (1)$$

75 where λ , μ are the Lamé constants, α , β , \mathcal{K} and γ are the new elastic constants
 76 usually referred to as the micropolar or Cosserat elastic constants.

77 In order to uncouple the equations of motion in Eqn. (1), the Helmholtz decomposi-
 78 tion is employed i.e., decompose the vectors \mathbf{u} and $\boldsymbol{\Phi}$ into their potential and solenoidal
 79 parts

$$\begin{aligned} \mathbf{u} &= \nabla q + \nabla \times \boldsymbol{\Pi}, \quad \nabla \bullet \boldsymbol{\Pi} = 0, \\ \boldsymbol{\Phi} &= \nabla \xi + \nabla \times \boldsymbol{\Theta}, \quad \nabla \bullet \boldsymbol{\Theta} = 0, \end{aligned} \quad (2)$$

80 The following uncoupled wave equations for the potentials q and ξ are obtained

$$\begin{aligned} c_1^2 \nabla^2 q - \frac{\partial^2}{\partial t^2} q &= 0, \\ c_3^2 \nabla^2 \xi - \omega_0^2 \xi - \frac{\partial^2}{\partial t^2} \xi &= 0, \end{aligned} \quad (3)$$

81 where $c_1^2 = \frac{\lambda+2\mu+\mathcal{K}}{\rho} = \frac{M+\mathcal{K}}{\rho}$, $c_3^2 = \frac{\alpha+\beta+\gamma}{\rho J}$, $\omega_0^2 = \frac{2\mathcal{K}}{\rho J}$.

82 The coupled system of wave propagation equations for the determination of the vector
83 potential for $\mathbf{\Pi}$ and $\mathbf{\Theta}$

$$\begin{aligned} c_2^2 \nabla^2 \mathbf{\Pi} + \frac{J\omega_0^2}{2} \nabla \times \mathbf{\Theta} - \frac{\partial^2}{\partial t^2} \mathbf{\Pi} &= 0, \\ c_4^2 \nabla^2 \mathbf{\Theta} - \omega_0^2 \mathbf{\Theta} + \frac{\omega_0^2}{2} \nabla \times \mathbf{\Pi} - \frac{\partial^2}{\partial t^2} \mathbf{\Theta} &= 0. \end{aligned} \quad (4)$$

84 where $c_2^2 = \frac{\mu+\mathcal{K}}{\rho}$, $c_4^2 = \frac{\gamma}{\rho J}$,

85 These equations govern the propagation of waves travelling in a Cosserat continuum.
86 The details of deriving the dispersion relations are found in the Appendix A or in
87 [30].

88 Finally, there are four different waves propagating with four different phase velocities

$$v_1^2 = c_1^2 = \frac{\lambda + 2\mu + \mathcal{K}}{\rho}, \quad v_2^2 = c_3^2 + \frac{\omega_0^2}{k^2}, \quad v_{3,4}^2 = \frac{\omega^2}{k_{3,4}^2}. \quad (5)$$

89 Therefore, in a micropolar elastic medium of infinite extent propagates four types of
90 modes, 1) A longitudinal displacement wave mode propagating with a phase velocity,
91 v_1 also called Longitudinal Acoustic mode (LA-mode), 2) A longitudinal microrotation
92 wave (L0-mode) propagates with the velocity v_2 , 3) Transverse acoustic mode (TA-mode),
93 propagating with phase velocity v_3 , 4) Transverse Optic mode (TO-mode) propagating
94 with phase velocity v_4 .

95 It can be shown that the velocity of the longitudinal microrotation wave (v_2) is
96 dispersive (depends on frequency) when $\omega > \sqrt{2}\omega_0$

$$v_2^2 = \frac{\alpha + \beta + \gamma}{J\rho(1 - \frac{2\omega_0^2}{\omega^2})}. \quad (6)$$

97 However, when $\omega = \omega_c = \sqrt{2}\omega_0$, v_2 is infinite and the wave number equals zero.

98 When $\omega < \omega_c$, v_2 is purely imaginary. The characteristic frequency ω_c acts as a

99 cut-off frequency below which the wave number vanishes.

100 The TO velocity $v_3 = \infty$ at $\omega = \omega_c$. The sketches of the phase velocities v_3 and v_4
101 are given in reference [29]. It was shown that at $\omega = \omega_c$, $v_3 = \infty$. For $\omega < \omega_c^-$, $v_3 = -\infty$
102 and at $\omega > \omega_c^+$, $v_3 = +\infty$. The transverse optic mode wave velocity v_4 is finite over the
103 whole frequency range.

104 The experimental analysis of the phenomenon occurring around the cut-off frequency
105 of v_3 was therefore of significant interest in this study.

106 2.2. Plane strain problem geometry and boundary equations

107 The direct problem of acoustic wave transmission by a micropolar elastic panel was
108 simplified into a plane strain one. The objective consisted in determining the transmission
109 coefficient in a two dimensional setup in which a known plane wave impinged on the
110 boundary of an infinite micropolar elastic panel of finite thickness. In this configuration,
111 the panel is bounded on both sides by a fluid (air) of semi-infinite extent (Fig. 1).

112 A simple two dimensional problem where displacement \mathbf{u} and rotation $\mathbf{\Phi}$ fields depend
113 only on two space variables u_y, u_z and Φ_x and the time t was studied (Fig. 1). In this
114 case the field equations can be decomposed into an independent set (plane strain) with

$$\mathbf{u} = (0, u_y, u_z), \quad \mathbf{\Phi} = (\Phi_x, 0, 0), \quad \partial/\partial x = 0.$$

115 An incident acoustic plane wave propagating in the fluid Ω_0 impinges on the panel
116 at an incident angle θ and is reflected at the $z = 0$ plane at the boundary Γ_1 . Mode
117 transformation takes place i.e, from longitudinal wave in the fluid into the four micropolar
118 elastic wave modes (LA, TA, LO and TO wave modes) in the panel. Multiple internal
119 reflections of one of the modes within the layer is also represented in Fig. (1) for only a
120 single mode for clarity.

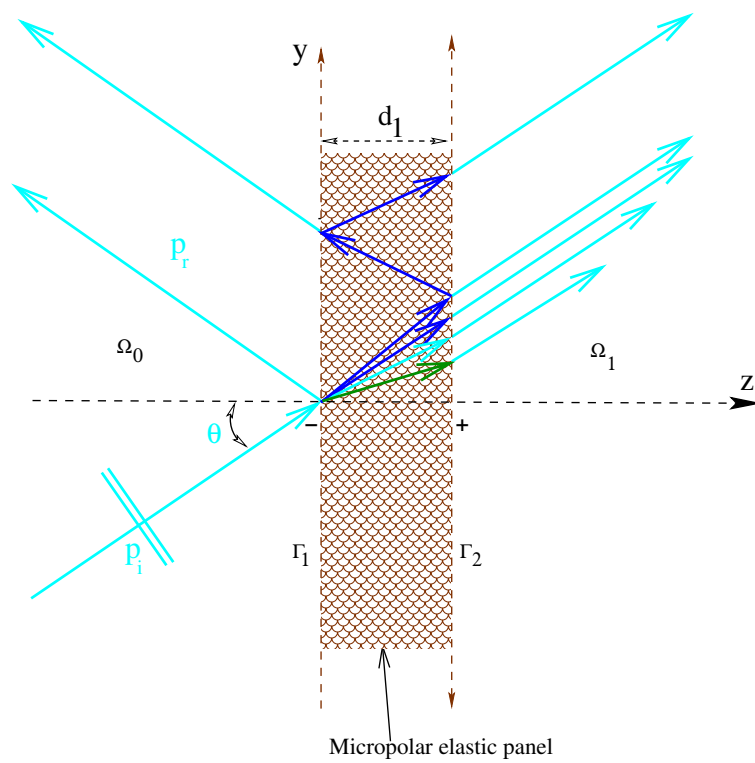


Figure 1: Geometry of the acoustic wave transmission problem. The panel is supposed to extend to infinity in the positive y - directions.

121 *2.2.1. The boundary conditions*

122 The boundary surface Γ_1 and Γ_2 are free from traction and couples, the equations for
 123 the boundary conditions for transmission/reflection through an infinite extent micropolar
 124 plate immersed in air. The stress tensors σ_{zz} , σ_{yz} and the couple stress tensor m_{yz} are
 125 employed

$$\begin{aligned}
 \sigma_{zz}(y, z) &= \lambda \left(\frac{\partial^2}{\partial y^2} q(y, z) + \frac{\partial^2}{\partial z^2} q(y, z) \right) + (2\mu + \mathcal{K}) \left(\frac{\partial^2}{\partial z^2} q(y, z) - \frac{\partial^2}{\partial z \partial y} \Pi_x(y, z) \right), \\
 \sigma_{zy}(y, z) &= (2\mu + \mathcal{K}) \frac{\partial^2}{\partial z \partial y} q(y, z) + (\mu + \mathcal{K}) \frac{\partial^2}{\partial z^2} \Pi_x(y, z) - \mu \frac{\partial^2}{\partial y^2} \Pi_x(y, z) + \\
 &\quad \mathcal{K} \left(\frac{\partial}{\partial y} \Theta_z(y, z) - \frac{\partial}{\partial z} \Theta_y(y, z) \right), \\
 m_{zx}(y, z) &= \gamma \left(\frac{\partial^2}{\partial z \partial y} \Theta_z(y, z) - \frac{\partial^2}{\partial z^2} \Theta_y(y, z) \right). \tag{7}
 \end{aligned}$$

126 The continuity of the acoustic particle velocity at the boundaries of the panel Γ_1 , and
 127 Γ_2 , are also sought. The velocity in the panel is given by

$$v_z(y, z) = i\omega \frac{\partial}{\partial z} q(y, z) - i\omega \frac{\partial}{\partial y} \Pi_x(y, z) \quad z \geq \Gamma_1, \cup z \leq \Gamma_2,$$

128 The 8 boundary conditions' equations are set using the following stress, torque and
 129 velocity equations

$$\begin{aligned}
 \sigma_{zz}(y, 0) &= -p_I, \quad z \in \Gamma_1, \quad \sigma_{zz}(y, d_1) = -p_T \quad z \in \Gamma_2, \\
 \sigma_{zy}(y, 0) &= 0, \quad z \in \Gamma_1, \quad \sigma_{zy}(y, d_1) = 0 \quad z \in \Gamma_2, \\
 m_{yz}(y, z) &= 0, \quad z \in \Gamma_1 \cup \Gamma_2, \\
 v_z^{\Gamma_1+}(y, 0) &= v_z^{\Gamma_1-}(y, 0), \quad v_z^{\Gamma_2+}(y, d_1) = v_z^{\Gamma_2-}(y, d_1). \tag{8}
 \end{aligned}$$

130 The acoustic pressure in the fluid medium at $z \leq \Gamma_1$ (p_I) and at $z \geq \Gamma_2$, (p_T) are

131 given by

$$\begin{aligned}
p_I(y, z) &= P_I \left(e^{\kappa_z^f z} + R e^{-\kappa_z^f z} \right) e^{\kappa_y^f y}, z \leq \Gamma_1, \\
p_T(y, z) &= T P_I e^{(\kappa_{z1} + \kappa_{z3} + \kappa_{z4})d_1 + \kappa_z^f (z-d_1)} e^{\kappa_y^f y}, z \geq \Gamma_2,
\end{aligned} \tag{9}$$

132 where R and T are the reflection and transmission coefficients respectively, $\kappa_z^f =$
133 $\frac{i\omega}{c_f} \cos(\theta_I)$ (θ_I is the oblique angle of incidence), $\kappa_y^f = \frac{i\omega}{c_f} \sin(\theta_I)$, c_f is the acoustic wave
134 velocity in the fluid. The wave velocities are computed using the expressions of the fluid
135 pressure above and the linear Euler equation, $v_z(y, z) = \frac{-1}{i\omega \rho_f} \frac{\partial}{\partial z} p(y, z)$. The complex
136 wave numbers $\kappa_{zn} = ik_n \cos(\theta_n)$ ($n=1 \dots 4$) for the waves propagating in the micropolar
137 elastic panel.

138 The incident and reflected waves are indicated by superscripts I and R propagate in
139 the slab. The potentials for the micropolar elastic medium are given by

$$q = a_1^I \exp [ik_1 (\sin \theta_1^I y + \cos \theta_1^I z) - i\omega_1^I t] + a_1^R \exp [ik_1 (\sin \theta_1^R y - \cos \theta_1^R z) - i\omega_1^R t], \tag{10}$$

$$\begin{aligned}
\Pi_x &= A_{3x}^I \exp [ik_3 (\sin \theta_3^I y + \cos \theta_3^I z) - i\omega_3^I t] + A_{3x}^R \exp [ik_3 (\sin \theta_3^R y - \cos \theta_3^R z) - i\omega_3^R t] + \\
&A_{4x}^I \exp [ik_4 (\sin \theta_4^I y + \cos \theta_4^I z) - i\omega_4^I t] + A_{4x}^R \exp [ik_4 (\sin \theta_4^R y - \cos \theta_4^R z) - i\omega_4^R t],
\end{aligned} \tag{11}$$

Since the transverse TA and TO waves are coupled we can write

$$\begin{aligned}
\Theta_y &= \eta_{3y} \{ A_{3x}^I \exp [ik_3 (\sin \theta_3^I y + \cos \theta_3^I z) - i\omega_3^I t] + A_{3x}^R \exp [ik_3 (\sin \theta_3^R y - \cos \theta_3^R z) - i\omega_3^R t] \} + \\
&\eta_{4y} \{ A_{4x}^I \exp [ik_4 (\sin \theta_4^I y + \cos \theta_4^I z) - i\omega_4^I t] + A_{4x}^R \exp [ik_4 (\sin \theta_4^R y - \cos \theta_4^R z) - i\omega_4^R t] \}
\end{aligned} \tag{12}$$

$$\Theta_z = \eta_{3z} \{ A_{3x}^I \exp [ik_3 (\sin \theta_3^I y + \cos \theta_3^I z) - i\omega_3^I t] + A_{3x}^R \exp [ik_3 (\sin \theta_3^R y - \cos \theta_3^R z) - i\omega_3^R t] \} + \eta_{4z} \{ A_{4x}^I \exp [ik_4 (\sin \theta_4^I y + \cos \theta_4^I z) - i\omega_4^I t] + A_{4x}^R \exp [ik_4 (\sin \theta_4^R y - \cos \theta_4^R z) - i\omega_4^R t] \}, \quad (13)$$

140 where the coupling coefficients $\eta_{3,4}$ were derived from Eqn. (A.3) using the amplitude
141 ratios

$$\bar{\Theta}_y = \frac{-i\omega_0^2}{2(\omega^2 - \omega_0^2 - c_4^2 k^2)} k_z \bar{\Pi}_x, \quad \bar{\Theta}_z = \frac{i\omega_0^2}{2(\omega^2 - \omega_0^2 - c_4^2 k^2)} k_y \bar{\Pi}_x, \quad (14)$$

142 and finally the amplitude ratios

$$\eta_{3,4y} = -\frac{i\omega_0^2}{2k_{3,4}(v_{3,4}^2 - \frac{\omega_0^2}{k_{3,4}^2} - c_4^2)} \cos \theta_{3,4}, \quad \eta_{3,4z} = \frac{i\omega_0^2}{2k_{3,4}(v_{3,4}^2 - \frac{\omega_0^2}{k_{3,4}^2} - c_4^2)} \sin \theta_{3,4}, \quad (15)$$

143 The complex wave-numbers $\kappa_f = i\vartheta_f$, $\vartheta_f = k_f \sin(\theta_f)$ (k_f is the compressional wave
144 number in the fluid). Simplifications can be done on the potentials by writing $\theta_n^I =$
145 θ_n^R ($n = 1 \dots 4$). The complex wave numbers $\kappa_{zn} = ik_n \cos(\theta_n)$ for the four waves
146 propagating in the micropolar elastic panel are related to the incident wave number in
147 the fluid through Snell-Descartes law of refraction and therefore can be written

$$\begin{aligned} \kappa_{z1} &= i\sqrt{k_1^2 - \vartheta_f^2}, & \kappa_{z2} &= i\sqrt{k_2^2 - \vartheta_f^2} \\ \kappa_{z3} &= i\sqrt{k_3^2 - \vartheta_f^2}, & \kappa_{z4} &= i\sqrt{k_4^2 - \vartheta_f^2} \end{aligned} \quad (16)$$

148 The hysteretic model of inherent damping of the panels were taken into account by
149 introducing a scalar-valued loss factor $\chi(\omega)$ into Young's modulus [31], such that $E_d(\omega) =$
150 $E_r(\omega) + iE_i(\omega) = E(\omega)(1 + i\chi(\omega))$ ($E_r(\omega)$ is called the storage modulus and $E_i(\omega)$ the
151 loss modulus of the panel, $M(\omega) = \frac{E(\omega)(1-\nu)}{(1+\nu)(1-2\nu)}$ where ν is the Poisson ratio). $E(\omega)$
152 is approximately constant in the chosen bandwidth (there is a basic lag between the
153 stress and the strain and therefore the area of the hysteresis cycle does not depend on
154 the frequency of the harmonic motion). The relationships between the different elastic

155 moduli can be found in reference [32].

156 A matrix equation is deduced from the 8 boundary conditions' equations

$$\begin{bmatrix} s\kappa_{z1} & -sk_{z1} & \cdots & & & -\frac{P_i \cos(\theta_I)}{c_f \rho_f} & 0 \\ & & & & & & \\ & & & & & & \\ & & & & & & \\ & & & \vdots & & & \\ & & & & & & \\ & & & & & & \\ & & & & & & \\ 0 & 0 & \gamma(\eta_{3z}\kappa_f\kappa_{z3} - \eta_{3y}\kappa_{z3}^2) & \cdots & & 0 & 0 \end{bmatrix} \times \begin{bmatrix} a_{1I} \\ a_{1R} \\ A_{3xI} \\ A_{3xR} \\ A_{4xI} \\ A_{4xR} \\ R \\ T \end{bmatrix} = \begin{bmatrix} -\frac{P_i \cos(\theta_I)}{c_f \rho_f} \\ 0 \\ -P_i \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix},$$

157 where $s = i\omega$.

158 The matrix equation is written as a linear system in the form $\mathbf{Ax} = \mathbf{b}$ (\mathbf{A} is an 8×8
159 matrix) and the equation solved to obtain T or R .

160 2.3. Materials, experimental method and setup

161 Expanded polystyrene (EPS) and Extruded polystyrene (XPS) are air-filled (95-98%)
162 closed-cell rigid foams [33] panels of different thicknesses were employed. These are two
163 types of foams of the same material (polystyrene) but are manufactured differently. EPS
164 is made from beads that are molded or cut into various sizes and shapes while XPS is
165 made from extruded sheets [33].

166 The experimental setup is shown in Fig. 2. A transient pulse wave was generated
167 using a loudspeaker driven by a waveform function generator (Agilent 33250B, Loveland
168 Colorado, USA) through a power amplifier Bruel and Kjaer (B&K2706, Naerum Den-
169 mark). The excitation signal generated was a positive rectangular pulse. The Fourier
170 transform of the excitation pulse $g(t)$ with amplitude A and width P_{wdt} in seconds is

$$G(f) = A P_{wdt} \text{sinc}(\pi f P_{wdt}). \quad (17)$$

171 As P_{wdt} becomes very large, the magnitude spectrum approaches a Dirac delta function
172 located at the origin. As the height of the pulse becomes higher and its width smaller, it
173 approaches a Dirac delta function and the magnitude spectrum flattens out and becomes
174 a constant. This latter case results in a signal with large frequency band-width being
175 generated. Shorter pulse widths also avoid the risk of damaging the loud-speaker when
176 the amplitude is high. When the panel is thin, the first mode has a high frequency
177 value, therefore, P_{wdt} is smaller, and relatively wider for a thick panel. The pulse width
178 was chosen $5\ \mu\text{s} < P_{wdt} < 12\ \mu\text{s}$. Even though the generated pulse is rectangular form,
179 the captured pulse of the incident wave will look sinusoidal because the generator signal
180 output is convoluted with the response of the loud-speaker.

181 The chosen loudspeaker was a Visaton (GmbH, Haan, Germany) magnetostatic rib-
182 bon tweeter reference MHT 12 - 8 Ohm with an opening, 60 mm wide and \times 84 mm
183 high. It was selected among several others because it had a low distortion factor, good
184 power-handling capabilities (100 Watts), highly linear impedance and amplitude fre-
185 quency response and a broad frequency response range (1 kHz - 50kHz). This is the
186 first time such a tweeter speaker is being employed in this frequency range. This wide
187 range is an advantage over the non-contact air-coupled ultrasonic transducers that are
188 band limited around their central working frequency e.g the Ultrason NCG50-S50 (State
189 College, PA USA) is a 50kHz center frequency transducer whose band-width is \approx 20 kHz.

190 The transmitted acoustic wave was captured using a small (1/8 inch) pressure-field
191 condenser microphone (Type B&K4138 , Naerum Denmark) connected to a frequency
192 analyzer (B&K2120, Naerum Denmark). The very low noise and high amplification an-
193 alyzer was used in the measuring microphone amplifier mode (2 Hz - 200kHz). The
194 signals from the microphone amplifier were then digitized using a Tektronix oscilloscope
195 model TDS3014B (Beaverton, OR USA). The transmission coefficient was obtained by
196 calculating the transfer function estimate between the captured incident acoustic pres-
197 sure(obtained without the panel in place). The transmitted acoustic pressure was ac-
198 quired when the panel was placed perpendicularly between the acoustic source and the

199 sensor (microphone).

200 The transmission coefficient was computed as a transfer function of a linear time-
 201 invariant (LTI) system giving the ratio between the incident pressure and the transmitted
 202 acoustic pressure. The transmission coefficient $T(f)$ was obtained from the quotient of
 203 the cross spectral power density S_{it} (between the incident, $p_i(t)$ and the transmitted,
 204 $p_t(t)$) acoustic pressure) and the auto-spectral power density S_{ii} ($p_i(t)$),

$$\tilde{T}(f) = \frac{\tilde{S}_{it}(f)}{\tilde{S}_{ii}(f)}. \quad (18)$$

205 where the tilde (\sim) indicates that the function is an estimate only. The quotient $\tilde{T}(f)$
 206 was calculated in Matlab® employing the function *tfestimate* [34] using the acquired
 207 signals. In order to avoid spectral leakage and attain accurate spectral measurements,
 208 the temporal signals were first windowed [35].

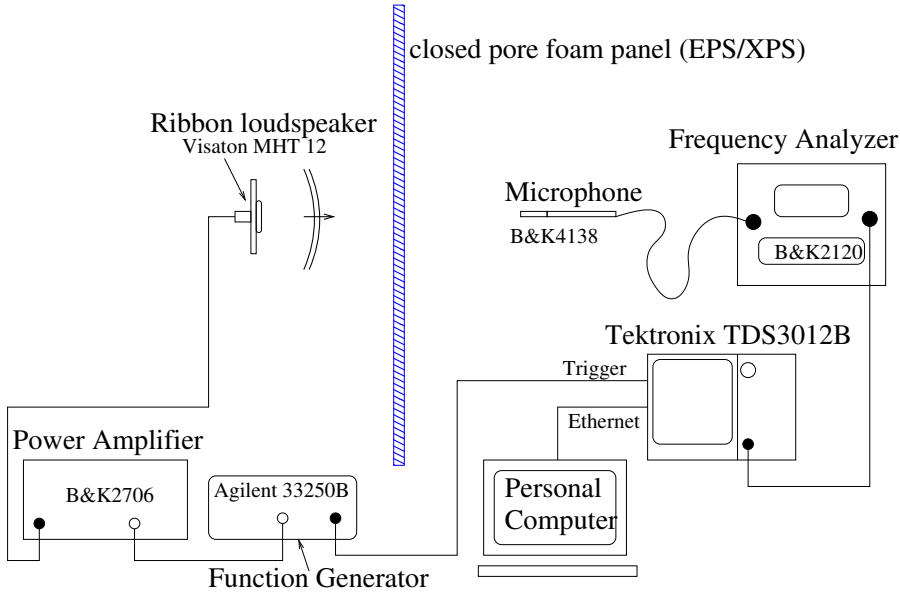


Figure 2: The experimental setup for the acoustic wave transmission by a panel.

209 *Using a 50 kHz ultrasonic transducer as acoustic source to extend the measurement band-*
210 *width*

211 The measurement frequency bandwidth beyond 40 kHz was extended by replacing
212 the loudspeaker by an air-coupled ultrasonic probe as an acoustic excitation source [36].
213 The exciter probe (Ultran NCG50-S50, State College, PA USA) had a center frequency
214 at 50 kHz and was connected to a Panametrics 5058-PR (Waltham USA) pulser. The
215 replacement of the acoustic exciter just modified the experimental setup slightly as the
216 acoustic receiver remained the same as in the loudspeaker source/microphone configura-
217 tion. If the ultrasonic detector had been used as receiver in place of the microphone, it
218 would have engendered undesirable reflections from its non-negligible larger surface area
219 rendering the modeling more complex. It was actually found to reflect the transmitted
220 waves backwards towards the panels due to its large surface area. The NCG50-S50 has a
221 large (50 mm \times 50 mm) active area. This is the reason why a tiny 1/8 inch in diameter
222 B&K 4138 microphone was preferred as receiver/detector.

223 **3. Results**

224 *3.1. XPS Panels*

225 The temporal signals representing the incident pressure (without the 4cm thick XPS
226 panel) and transmitted pressure (with the panel placed between the speaker and the
227 microphone) and their frequency spectrum are depicted in Figs. (3)a and b, respectively.
228 A rectangular window providing a better spectral accuracy was employed during the
229 signal processing.

230 From the zoomed view of the temporal signals (Fig. (3)a), it can be noticed that the
231 transmitted acoustic pressure wave is attenuated but arrives at the microphone position
232 earlier than the incident waves. This is because the acoustic waves travel faster in the
233 micropolar elastic solid than in air and are therefore accelerated in a small portion of the
234 solid panel layer.

235 The frequency spectrum depicted in Fig. (3)b shows that the spectrum of the trans-
 236 mitted pressure wave is cut-off at around 5-6 kHz (calculated ω_c is 5319 Hz such that
 237 the amplitude is diminished around this frequency value. The comparison between the
 238 theoretical micropolar elastic model and the experimentally determined transmission co-
 239 efficients (TC) are shown in Fig. (3)c. The theoretical cut-off frequency f_c is ≈ 5319
 240 Hz.

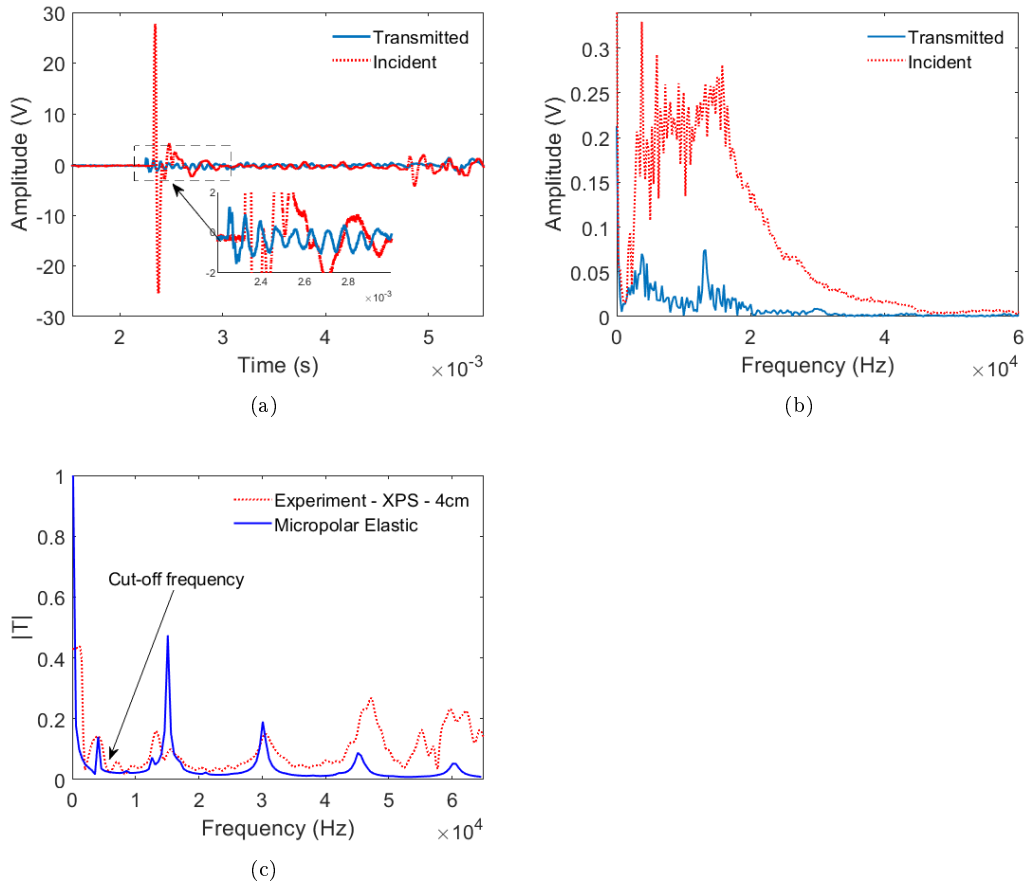


Figure 3: For the 4.0 cm thick XPS panel, (a) The temporal signals representing the incident and transmitted pressure. The transmitted wave is attenuated, albeit faster than the incident wave. (b) The frequency spectrum. Comparisons between the theoretical micropolar elastic and classical elastic models with experimental data (c) transmission coefficients. The computed cut-off frequency is at 5319 Hz.

241 The measurements results for the 2.0 cm thick XPS panel are shown in Fig. (4)a -

242 c. It can be noted that the frequency of the modes also depend on the thickness of the
 243 panel. The thicker the panel, the lower the frequency values of the modes.

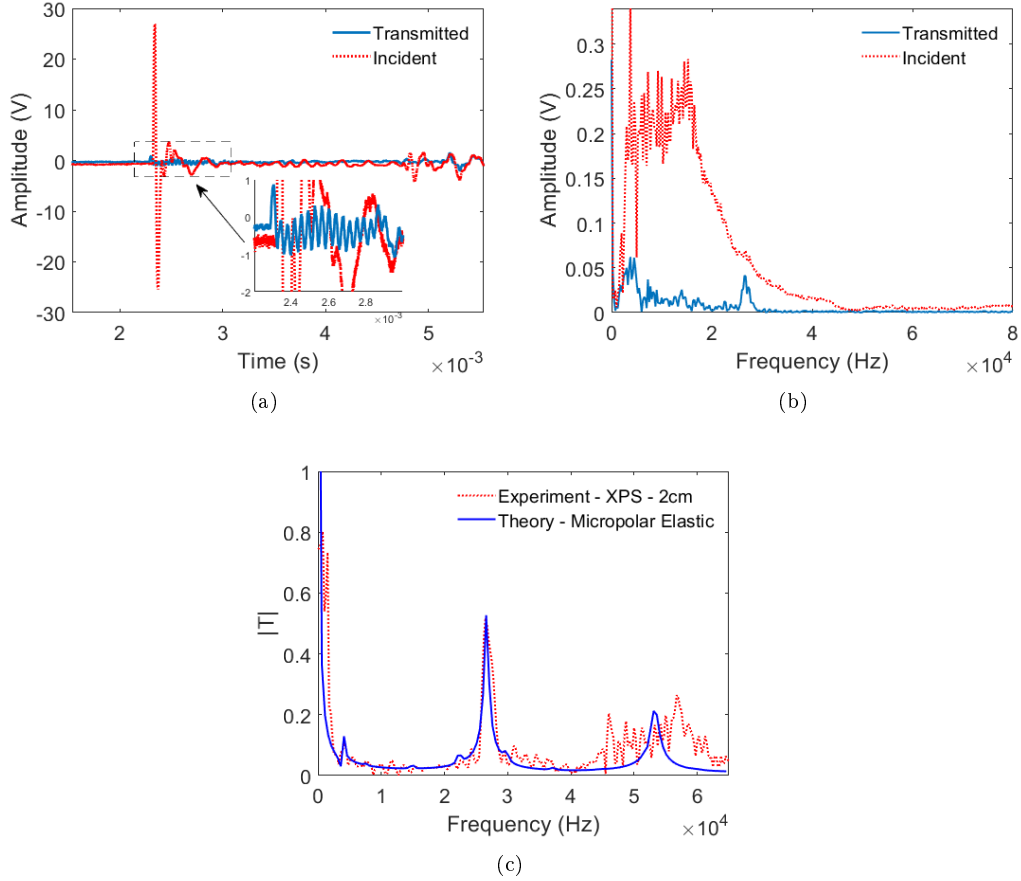


Figure 4: For the 2.0 cm thick XPS panel.(a) The temporal signals representing the incident and transmitted pressure (b) Their frequency spectrum. Comparisons between the theoretical and experimental, (c) the corresponding transmission coefficient.

244 3.2. EPS12 Panel

245 The results for the 2 cm thick EPS12 panels are shown in Fig. (5)a - c and the trans-
 246 mission coefficient for the 3 cm panel, in Fig. (5)d (the theoretical cut-off frequencies,
 247 $f_{c_{2cm}} = 2171$ Hz and $f_{c_{3cm}} = 3071$ Hz). The transmission coefficient dips at the cut-off
 248 frequency (velocity of the transverse acoustic mode is ∞). The frequency of the first
 249 peak representing the first mode in the EPS12 panel is lower than that of the XPS panel

250 of the same thickness (Fig. 4c and Fig. 5c) meaning that the values of its micropolar
 251 and elastic moduli are lower.

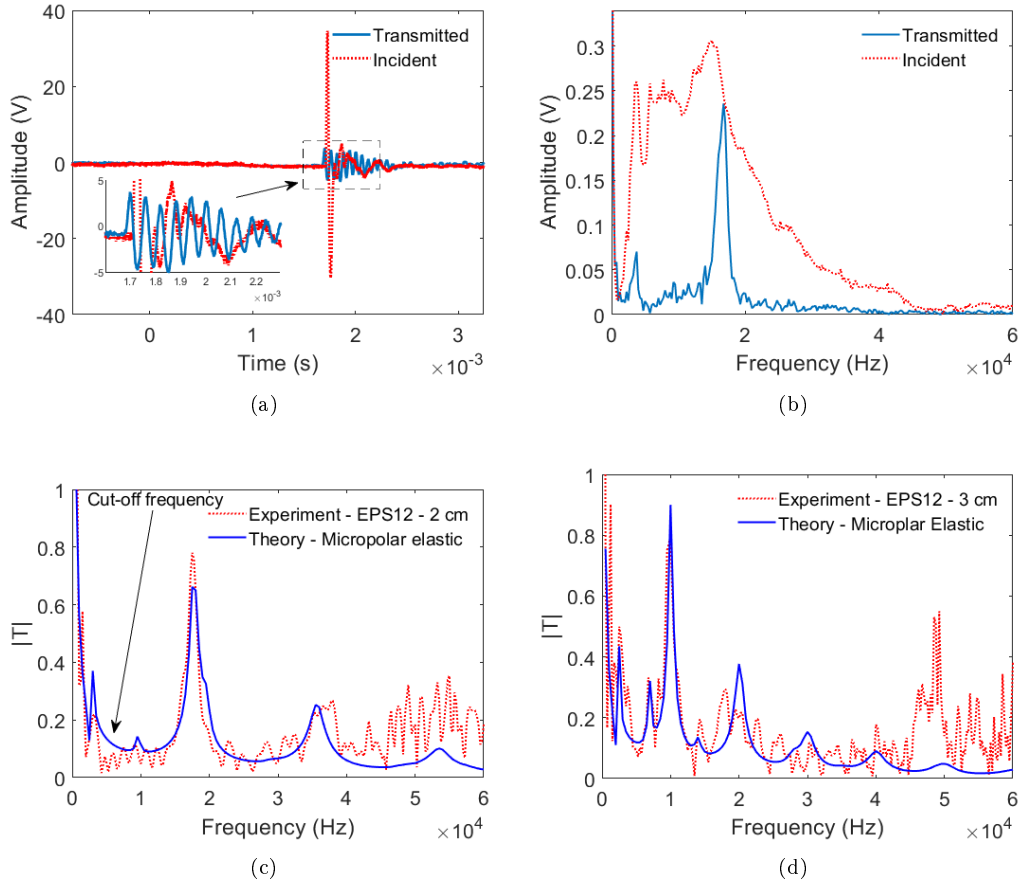


Figure 5: For the 2.0 cm thick EPS12 panel, (a) The temporal signals representing the incident and transmitted pressure (b) Their frequency spectrum. (c) Comparisons between the theoretical and experimental transmission coefficients. (d) The transmission coefficient for the 3.0 cm thick EPS12 panel

252 The test results for an EPS12 4.0 cm thick panel are shown in Fig. fig:A-4cm-thick
 253 xps panelc, (6). The acceleration (or early arrival) of the acoustic wave transmitted
 254 by the layer as compared to the incident acoustic pressure is more remarkable on the
 255 temporal signal of this panel, even without zooming.

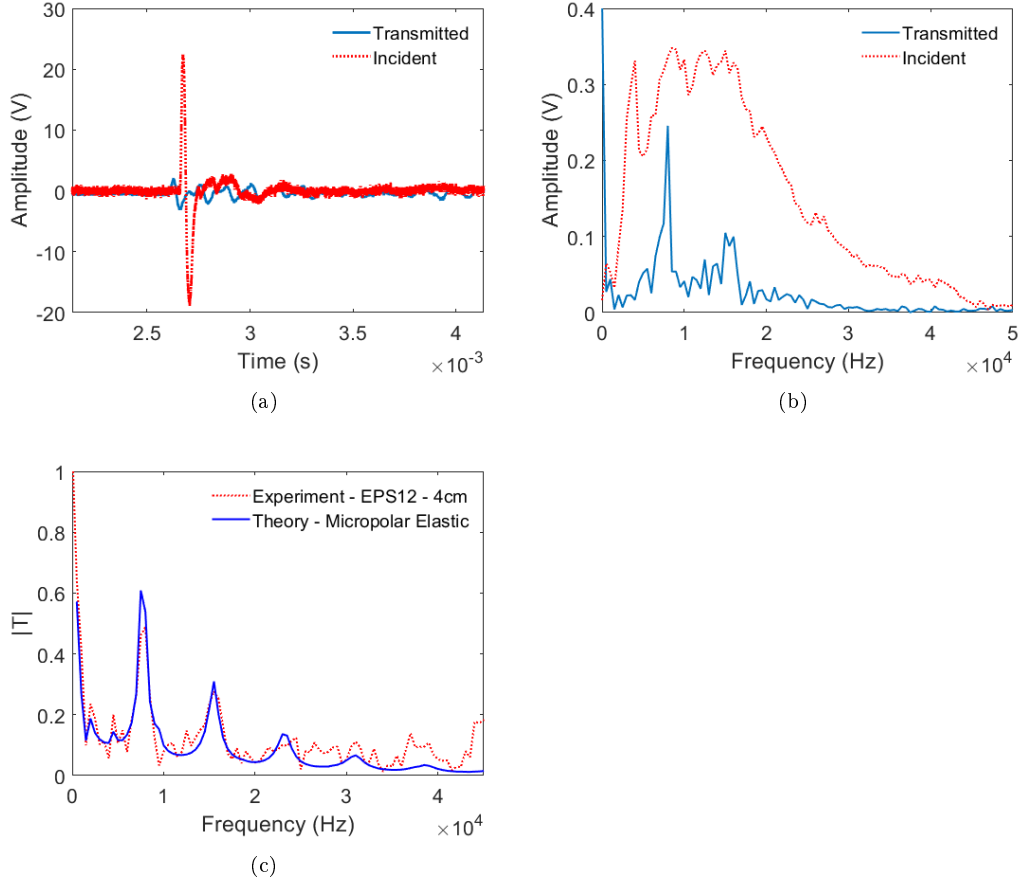


Figure 6: For the 4.0 cm thick EPS12 panel, (a) The temporal signals representing the incident and transmitted pressure (b) Their frequency spectrum. (c) Comparisons between the theoretical and experimental transmission coefficients.

256 *Extended bandwidth using a 50 kHz ultrasonic transducer as acoustic exciter source*

257 The frequency band covered by the ribbon twitter loudspeaker was from 2 - ≈ 35 kHz
 258 (see Figs. 3c, 4c, 5c and d, 6c). The peaks of the modes were not well marked beyond
 259 that bandwidth due to the low signal to noise ratio (SNR). This was due to the limited
 260 bandwidth of the loudspeaker. The SNR were lower for the thicker panels. In order to
 261 cover the higher frequencies following i.e, from ≈ 30 -90 kHz, an air coupled ultrasonic
 262 transducer was employed in place of the loudspeaker.

263 The incident temporal acoustic wave pressure signals and the comparison between the

264 captured experimental and the computed theoretical transmission coefficients are shown
 265 in Fig. (7). The spectra corresponding to the temporal signals are shown in Fig. (7)b.
 266 The transmission coefficient is shown in Fig. (7)(c). The acoustic modes up to 120 kHz
 267 are excited by this transducer and captured by the microphone. The transducer response
 268 covers the high frequency zones where the loudspeaker response is lower, however, the
 269 lower frequencies are not well reproduced with the former. The situation is improved
 270 when the loudspeaker is employed because of its superior frequency response in the lower
 271 frequency range.

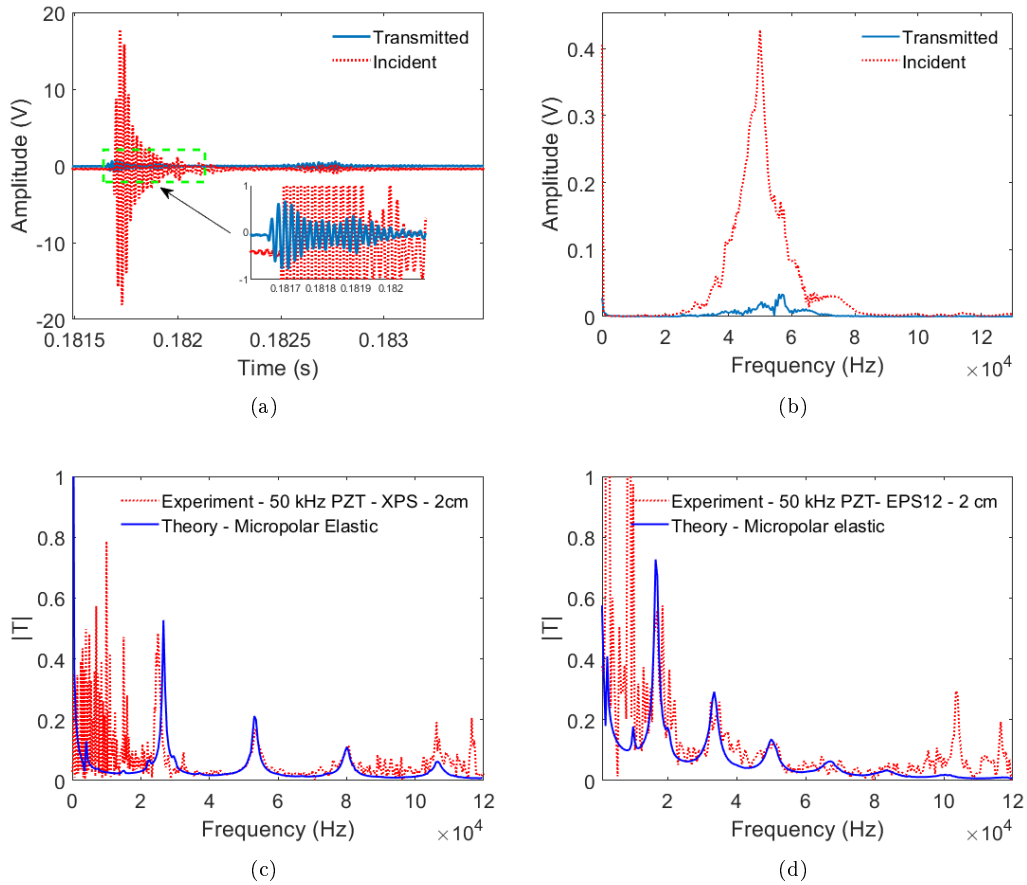


Figure 7: Using an Ultran air-coupled PZT transducer as acoustic source to probe a 2.0 cm XPS panel, (a) The incident and transmitted acoustic waves in the time domain, (b) Their spectrum. (c) Comparison between the theoretical and experimental transmission coefficients. (d) TC for the 2 cm thick EPS12 panel.

Table 1: Adjusted parameters employed in the theoretical model to fit the experimental curves. The microinertia is J .

Panel	d_1 (cm)	ρ (kg/m ³)	E (MPa)	Loss χ	ν	\mathcal{K} (MPa)	γ (MN)	J (N/m)
XPS	2	32	2.7	0.02	0.49	0.0510	1.68	2.35e-5
XPS	4	32	2.8	0.02	0.49	0.0510	0.57	2.35e-5
EPS12	2	12	1.4	0.06	0.45	0.070	0.86	3.13e-5
EPS12	3	12	1.5	0.06	0.43	0.070	1.2	3.13e-5
EPS12	4	12	1.2	0.075	0.45	0.037	0.87	3.9e-5

272 4. Discussion

273 4.1. Comparison between the Young's modulus recovered with those obtained using other 274 methods

275 A summary of the micropolar parameters for the problem are tabulated in Table (1).
276 The precise optimized values will be given in a future study involving the resolution
277 of a complete inverse problem for wave propagation in Cosserat elastic continuum. A
278 Manufacturer of EPS12 foams under the name of Geofam (ACH Foam Technologies®
279 (Denver, CO USA)), Geofams (AFM Corporation®, Lakeville, MN USA) indicates that
280 the Elastic modulus measured using the ASTM D6817 standard [27, 28] at a compression
281 of 1% is 1500 (kPa). Reference [37] also reports that the initial tangent modulus (deter-
282 mined from the linear portion of the stress-strain curve [37, 38]) for EPS12 is 1600 kPa.
283 These compare well with the value obtained from the experiment. The advantage of the
284 acoustic method is that it is non destructive and is lower compression rate as compared
285 to the Compression test methods where the materials are cut in cylindrical, rectangular
286 and cube form.

287 4.2. Phase wave velocities as a means of verification of some of the material parameters

288 The phase wave velocity of the P-wave modes (numbered n) propagating in the thick-
289 ness/layer of the panels are calculated from the following relationship

$$v_p^n \approx 2d_1 \frac{f_p^n}{n},$$

Table 2: Velocities calculated from the frequency of the first longitudinal mode (f_{L_1}) and the recovered micropolar parameter K and the P-wave modulus (longitudinal modulus) M . The cut-off frequency is f_c .

Panel	density (kg/m ³)	d_1 (cm)	velocity ($\sqrt{\frac{M+K}{\rho}}$) (m/s)	Velocity ($2f_{L_1}d_1$) (m/s)	f_c (Hz)
XPS	32	4	1229	1240	6000
XPS	32	2	1066	1064	6250
EPS	12	2	645	660	2171
EPS	12	3	602	600	3071
EPS	12	4	620	640	2242

where d_{th} is the panel thickness, f_p^n is the frequency of the n th mode determined from the transmission coefficient. Velocity can also be computed once the theoretical TC data is fitted to the experimental TC and found satisfactory. The following relationships engaging the mechanical moduli and density $v_p^1 = \sqrt{\frac{M+K}{\rho}}$ from the theoretical model is employed.

Knowledge of the cut-off frequency f_c and the phase velocity are important for determining the bounds of J . The expression for the phase velocity is rewritten as $(v_p^1)^2 = (M + 2\pi^2\rho J f_c^2)$. This shows that the velocity of the longitudinal modes also depend on the micropolar parameter J (the micro-inertia) among others. The measured and recovered phase velocities are in good agreement (difference < 4 percent) and are summarized in Table (2).

The influence of the Cosserat elastic constants and Poisson ratio on the transmission coefficient

The Sensitivity of the transmission coefficient vis-à-vis the micropolar parameters \mathcal{K} , J , γ and the Poisson ratio was studied by varying them while all the others remained fixed at their expected values. Their influence was analyzed by examining the changes occurring on the magnitude of the transmission coefficient as parameters were varied. It was observed that \mathcal{K} (the Cosserat couple modulus), influenced the magnitude of the transmission coefficient (Fig. (8)a and c) over the whole range of frequencies. The changes in magnitude of the TC with the variations of \mathcal{K} were found to be well marked. This was accompanied by a directly proportional increase/decrease of the resonance

311 frequency of the modes and as a consequence an advantage over the classical elastic model
 312 that has only two tuning parameters (E and ν). From this observation, the parameter \mathcal{K}
 313 can easily be recovered from experimental data since it has a significant influence on the
 314 TC. The rotational inertia J and the twist coefficient γ influence the amplitudes around
 315 the zoomed area of Figs. 8a and b. This is around the cut-off frequency area. Therefore
 316 these two parameters can be considered as low frequency parameters. Poisson ratio was
 317 also found to influence the TC (Figs. 8d)

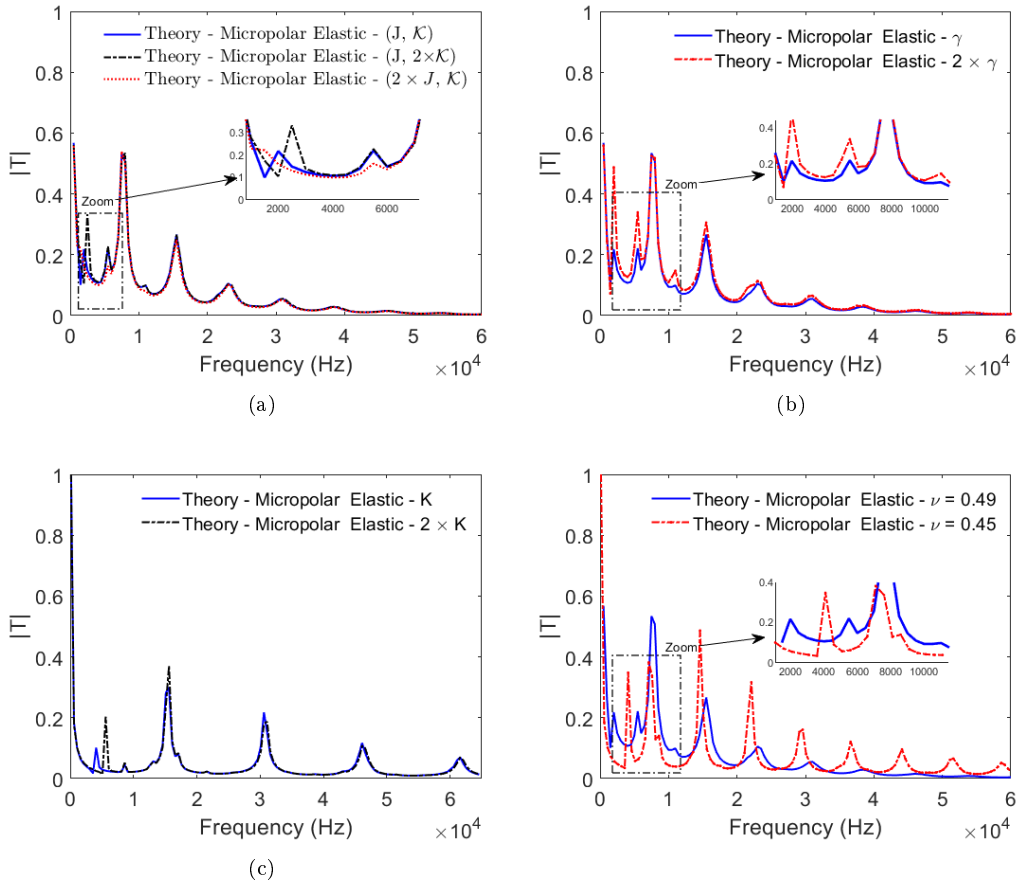


Figure 8: Sensitivity of the transmission coefficient to the variations of (a) The Cosserat couple modulus \mathcal{K} ($\mathcal{K} = 42$ kPa) and the rotational inertia $J = 4.7 \times 10^{-5}$ N/m and (b) The twist coefficient $\gamma = 5.6 \times 10^5$ N for an EPS12 4.0 cm thick panel. For the XPS 4.0 cm thick panel (c) Variation of \mathcal{K} ($\mathcal{K} = 51$ kPa). (d) Variation of Poisson ratio ν .

318 It is worth noting that the characteristic length [39, 40] can be obtained from the
 319 micropolar twist (γ) and couple (\mathcal{K}) modulus. This length is equal to the spherical
 320 polystyrene bead diameter size that forms the EPS12 foam (their diameters are usually
 321 in the range 0.3mm - 4mm). It is an intrinsic dimension defining the heterogeneous
 322 structure of the material. The characteristic length in bending in micropolar theory was
 323 given by Yang et al [39], as $l_{b2} = [\gamma/(2(2\mu + \mathcal{K}))]^{1/2}$ in mm. In reference [41], \mathcal{K} in l_{b2}
 324 was neglected. The beads from the 4cm thick Eps12 foam had several diameters ranging
 325 from ≈ 0.3 mm to 4mm. The calculations with the recovered values herein gave $l_{b2} \approx 0.7$
 326 mm.

327 *4.3. Comparison between the theoretical micropolar elastic and classical elastic model*

328 The micropolar model reproduces most of the small but different peaks (frequency
 329 and amplitude) at the beginning of the transmission coefficient (around the cut-off fre-
 330 quency) and in the higher modes (Fig. 7). The transmission coefficients of the theoretical
 331 micropolar elastic (modeling of an XPS 4cm thick panel) and that of the classical elastic
 332 continuum model and experiment were compared. The classical model was implemented
 333 using the Transfer matrix method (TMM) in which the longitudinal and shear modes
 334 were taken into account [32]. Both formulations capture the resonance peaks in the
 335 experimental TC curve (Fig. 9a). The absence of J in the classical elastic model was
 336 simulated to examine the effect of its absence on the TC of the micropolar elastic model.
 337 It was given a very small value, $J = 1 \times 10^{-50} \text{ Nm}^{-1}$ (J cannot be zero, it is denominator
 338 in the expressions for ω_0 , c_3 and c_4). The connection between the two theories is more
 339 involved than merely making $J \rightarrow 0$ [14]. The resulting TC had a peak around the
 340 cut-off frequency missing (Fig. 9b).

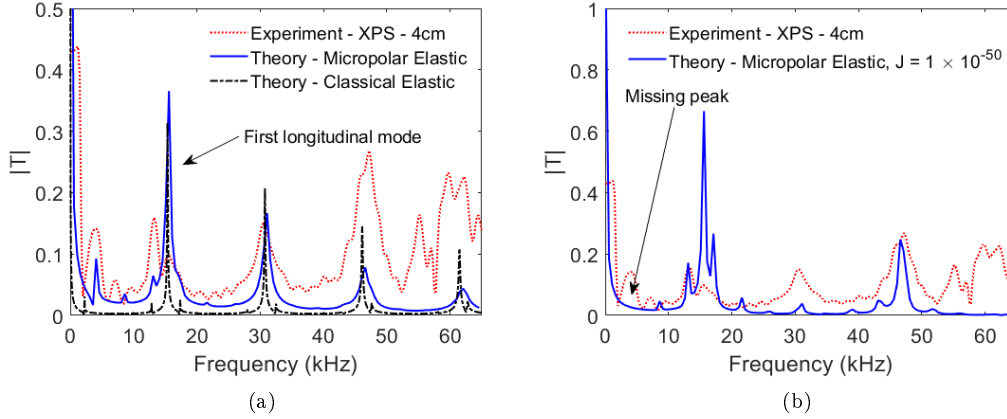


Figure 9: Comparison between the transmission coefficients for an XPS, 4.0 cm thick panel. (a) modeled by the micropolar elastic and the classical elastic formulations using the same E and ν . (b) The experimental and theoretical micropolar elastic model with a very small rotational inertia value, $J = 1.0 \times 10^{-50}$ N/m.

341 5. Conclusion

342 An experimental test rig was setup for the obtention of the transmission coefficient in
 343 *air* to recover the micropolar elastic properties of XPS and EPS panels in the audible and
 344 ultrasonic frequency ranges. Not only did the transmission coefficients agree with the
 345 theoretical ones but the elastic parameters also agreed well with those in the literature
 346 for EPS12 of the same density but obtained using a different measurement technique.

347 It was confirmed for the materials tested (two families of closed-cell polystyrene foams,
 348 XPS and EPS) that the Cosserat continuum model, which admits additional degrees of
 349 freedom associated with rotation of the microstructure, described best their behavior.
 350 It is a better alternative to the classical continuum model. The absence of transmission
 351 at the cut-off frequency and a better correspondence of the transmitted modes with the
 352 experiment, were found to be the main features that differentiates the micropolar elastic
 353 material behavior from the classical elastic one. These features were more marked in the
 354 transmission coefficient curves for the thicker EPS and XPS foams.

355 The Cosserat couple modulus, \mathcal{K} , the micro-inertia J and the twist modulus γ were
 356 found to influence the transmission coefficient of XPS and EPS thick plates in air. This

357 is an important finding because it implies that micropolar parameters: the micro-inertia
 358 J and \mathcal{K} can be recovered using the measurement method developed herein. Recall that,
 359 only M and ν can be recovered using the classical elastic continuum theory [32].

360 Finally, this new method appeals to the characterization of closed-cell porous panels
 361 and will provide elasticity data especially for the XPS foams that are hard to come by
 362 in the available literature.

363 Declaration of Competing Interest

364 The authors declare that they have no known competing financial interests or personal
 365 relationships that could have appeared to influence the work reported in this paper.

366 Appendix A. The dispersion relationships

367 Plane wave type of propagation is assumed for which the potentials take the following
 368 forms

$$(q, \xi, \mathbf{\Pi}, \mathbf{\Theta}) = (\bar{q}, \bar{\xi}, \bar{\mathbf{\Pi}}, \bar{\mathbf{\Theta}}) \exp [i(\mathbf{k} \bullet \mathbf{r} - \omega t)], \quad (\text{A.1})$$

369 where \mathbf{k} is the wave-number vector ($\mathbf{k} = k_x \hat{x} + k_y \hat{y} + k_z \hat{z}$), \mathbf{r} the position vector ($\mathbf{r} =$
 370 $x \hat{x} + y \hat{y} + z \hat{z}$), $\bar{q}, \bar{\xi}$ are complex amplitude constants, while $\bar{\mathbf{\Pi}}, \bar{\mathbf{\Theta}}$ are complex vectors.

371 By using the vector identities in Appendix B and also writing $\nabla \times \mathbf{\Pi} = i \mathbf{k} \times \mathbf{\Pi}$,
 372 Eqn. (4) can be written

$$(\omega^2 - k^2 c_2^2) \bar{\mathbf{\Pi}} + i \frac{J \omega_0^2}{2} \mathbf{k} \times \bar{\mathbf{\Theta}} = 0, \quad i \frac{\omega_0^2}{2} \mathbf{k} \times \bar{\mathbf{\Pi}} + (\omega^2 - \omega_0^2 - c_4^2 k^2) \bar{\mathbf{\Theta}} = 0. \quad (\text{A.2})$$

373 The dispersion relation is found through elimination of $\bar{\mathbf{\Pi}}$ and $\bar{\mathbf{\Theta}}$ from these equations

$$\bar{\mathbf{\Theta}} = \frac{-i \omega_0^2}{2(\omega^2 - \omega_0^2 - c_4^2 k^2)} \mathbf{k} \times \bar{\mathbf{\Pi}} \Rightarrow (\omega^2 - k^2 c_2^2) \bar{\mathbf{\Pi}} + \frac{J \omega_0^4}{4(\omega^2 - \omega_0^2 - c_4^2 k^2)} \mathbf{k} \times \mathbf{k} \times \bar{\mathbf{\Pi}} = 0 \quad (\text{A.3})$$

374 Developing the expression $\mathbf{k} \times \mathbf{k} \times \bar{\mathbf{\Pi}} = (\mathbf{k} \bullet \bar{\mathbf{\Pi}})\bar{\mathbf{\Pi}} - (\mathbf{k} \bullet \mathbf{k})\bar{\mathbf{\Pi}} = -k^2 \bar{\mathbf{\Pi}}$ (using the
 375 identity $\mathbf{A} \times (\mathbf{B} \times \mathbf{C}) = (\mathbf{A} \bullet \mathbf{C})\mathbf{B} - (\mathbf{A} \bullet \mathbf{B})\mathbf{C}$ and $\mathbf{k} \bullet \bar{\mathbf{\Pi}} = \mathbf{k} \bullet \bar{\mathbf{\Theta}} = 0$) then

$$k^4 + \left(\frac{\omega_0^2}{c_4^2} \left(1 - \frac{J \omega_0^2}{4 c_2^2} \right) - \frac{\omega^2}{c_2^2} \left(1 + \frac{c_2^2}{c_4^2} \right) \right) k^2 + \frac{\omega^2}{c_2^2 c_4^2} (\omega^2 - \omega_0^2) = 0.$$

376 which can also be written as a quartic equation of the type with solutions

$$k^4 - 2b k^2 + c = 0, \quad \Rightarrow \quad k_{3,4}^2 = -b \pm \sqrt{b^2 - 4c}, \quad (\text{A.4})$$

377 where $b = \frac{\omega^2}{2c_2^2} \left(1 + \frac{c_2^2}{c_4^2} \right) - \frac{\omega_0^2}{2c_4^2} \left(1 - \frac{J \omega_0^2}{4c_2^2} \right)$, $c = \frac{\omega^2}{c_2^2 c_4^2} (\omega^2 - \omega_0^2)$

378 The two wave numbers, k_3 and k_4 are solutions and correspond to a transverse acoustic
 379 mode (TA-mode) and transverse Optic mode (TO mode). The upper sign (+) in the
 380 equation of the coupled transverse displacement wave corresponds to k_3 ; the lower sign
 381 (-) corresponds to the wave number k_4 of the coupled transverse microrotational waves.

382 Appendix B. Some vector calculus identities employed

383 The following vector calculus identities were employed

$$\nabla \bullet (\nabla \mathbf{q}) = \nabla^2 q, \quad \nabla \bullet (\nabla \times \mathbf{\Pi}) = 0, \quad (\text{B.1})$$

$$\nabla \times (\nabla q) = 0, \quad \nabla \times (\nabla \times \mathbf{\Pi}) = \nabla(\nabla \bullet \mathbf{\Pi}) - \nabla^2 \mathbf{\Pi} \quad (\text{B.2})$$

384 References

- 385 [1] E. Cosserat, F. Cosserat, Théorie des corps déformables, Librairie Scientifique, Hermann, Paris,
 386 1909.
- 387 [2] W. Voigt, Theoretische studien fiber die elastizitatsverhilttnisse der kristalle (theoretical studies on
 388 the elasticity relationships of crystals), Abh. Gesch. Wissenschaften (1887) 34.
- 389 [3] R. A. Toupin, Elastic materials with couple-stresses, Arch. Rational Mech. Ana. 11 (1962) 385–414.
- 390 [4] R. D. Mindlin, H. F. Tiersten, Effects of couple-stresses in linear elasticity, Arch. Rational Mech.
 391 Ana. 11 (1962) 415–488.

- 392 [5] W. T. Koiter, Couple-stresses in the theory of elasticity, in: Pt. I-II. Proc. Koninkl., Neterland
393 Akad., Wetensch B67, 1964, pp. 17–44.
- 394 [6] A. C. Eringen, Theory of micropolar elasticity, in: H. Liebowitz (Ed.), Fracture, Academic Press,
395 New York, 1968, pp. 621–729.
- 396 [7] A. R. Hadjesfandiari, G. F. Dargush, Couple stress theory for solids, *Int. J. Solids Struct.* 48 (18)
397 (2011) 2496–2510.
398 URL <https://www.sciencedirect.com/science/article/pii/S0020768311001727>
- 399 [8] M. Brand, M. Rubin, A constrained theory of a cosserat point for the numerical solution of dynamic
400 problems of non-linear elastic rods with rigid cross-sections, *Int. J. Nonlin. Mech.* 42 (2) (2007) 216–
401 232, special Issue in Honour of Dr Ronald S. Rivlin.
402 URL <https://www.sciencedirect.com/science/article/pii/S0020746206000849>
- 403 [9] A. C. Eringen, E. S. Suhubi, *Elastodynamics*, Vol. 2., Vol. Vol. 2, Academic Press, New York, 1975,
404 Ch. 5.
- 405 [10] A. C. Eringen, Linear theory of micropolar elasticity, *J. Math. Mech.* 15 (6) (1966) 909–923.
- 406 [11] C. Kafadar, A. Eringen, Micropolar media-i the classical theory, *Int. J. Eng. Sci.* 9 (3) (1971) 271
407 – 305.
408 URL <http://www.sciencedirect.com/science/article/pii/0020722571900401>
- 409 [12] R. Mindlin, Stress functions for a cosserat continuum, *Int. J. Solids Struct.* 1 (3) (1965) 265 – 271.
410 URL <http://www.sciencedirect.com/science/article/pii/0020768365900338>
- 411 [13] J. Altenbach, H. Altenbach, V. A. Eremeyev, On generalized Cosserat-type theories of plates and
412 shells: a short review and bibliography, *Arch. Appl. Mech.* 80 (1) (2010) 73–92.
413 URL <https://doi.org/10.1007/s00419-009-0365-3>
- 414 [14] S. Hassanpour, G. R. Hepler, Micropolar elasticity theory a survey of linear isotropic equations,
415 representative notations, and experimental investigations, *Math. Mech. Solids* 22 (2) (2017) 224–
416 242.
417 URL <https://doi.org/10.1177/1081286515581183>
- 418 [15] S. Duan, W. Wen, D. Fang, A predictive micropolar continuum model for a novel three-dimensional
419 chiral lattice with size effect and tension-twist coupling behavior, *J. Mech. Phys. Solids* 121 (2018)
420 23 – 46.
421 URL <http://www.sciencedirect.com/science/article/pii/S0022509618303016>
- 422 [16] A. Spadoni, M. Ruzzene, Elasto-static micropolar behavior of a chiral auxetic lattice, *J. Mech.*
423 *Phys. Solids* 60 (1) (2012) 156 – 171.
424 URL <http://www.sciencedirect.com/science/article/pii/S0022509611001864>
- 425 [17] R. S. Lakes, R. L. Benedict, Noncentrosymmetry in micropolar elasticity, *Int. J. Eng. Sci.* 20 (10)
426 (1982) 1161 – 1167.

- 427 URL <http://www.sciencedirect.com/science/article/pii/0020722582900969>
- 428 [18] R. Kumar, S. K. Tomar, Reflection and transmission of elastic waves at viscous liquid-micropolar
429 elastic solid interface, *Int. J. Math. Math. Sci.* 26 (11) (2001) 685–694.
- 430 URL <http://dx.doi.org/10.1155/S0161171201005415>
- 431 [19] D. Singh, S. Tomar, Longitudinal waves at a micropolar fluid/solid interface, *Int. J. Solids Struct.*
432 45 (1) (2008) 225 – 244.
- 433 URL <http://www.sciencedirect.com/science/article/pii/S0020768307003071>
- 434 [20] A. C. Eringen, *Microcontinuum Field Theories - I. Foundations and Solids*, 1st Edition, Classical
435 and Continuum Physics, Springer-Verlag New York, New York, USA, 1999.
- 436 [21] R. Lakes, W. J. Drugan, Bending of a cosserat elastic bar of square cross section: Theory and
437 experiment, *J. Appl. Mech.* 82 (9) (2015) 091002–091010.
- 438 URL <http://dx.doi.org/10.1115/1.4030626>
- 439 [22] H. Park, R. Lakes, Cosserat micromechanics of human bone: Strain redistribution by a hydration
440 sensitive constituent, *J. Biomech.* 19 (5) (1986) 385 – 397.
- 441 URL <http://www.sciencedirect.com/science/article/pii/0021929086900151>
- 442 [23] R. Lakes, Experimental microelasticity of two porous solids, *Int. J. Solids Struct.* 22 (1) (1986) 55
443 – 63.
- 444 URL <http://www.sciencedirect.com/science/article/pii/0020768386901034>
- 445 [24] R. D. Gauthier, W. E. Jahsman, A quest for micropolar elastic constants, *J. Appl. Mech.* 42 (2)
446 (1975) 369–374.
- 447 URL <http://dx.doi.org/10.1115/1.3423583>
- 448 [25] R. Lakes, Strongly Cosserat elastic lattice and foam materials for enhanced toughness, *Cell. Polym.*
449 12 (1993) 17–30.
- 450 [26] W. B. Anderson, , R. S. Lakes, Size effects due to Cosserat elasticity and surface damage in closed-
451 cell polymethacrylimide foam, *J. Mater. Sci.* 29 (24) (1994) 6413–6419.
- 452 URL <https://doi.org/10.1007/BF00353997>
- 453 [27] ASTM C165-07(2012), Standard Test Method for Measuring Compressive Properties of Thermal In-
454 sulations, American Society for Testing and Materials (ASTM) International, West Conshohocken,
455 PA (2012).
- 456 [28] ASTM D6817 D6817M 15, Standard Specification for Rigid Cellular Polystyrene Geofoam, Amer-
457 ican Society for Testing and Materials (ASTM) International, West Conshohocken, PA (2015).
- 458 [29] V. R. Parfitt, A. C. Eringen, Reflection of plane waves from the flat boundary of a micropolar
459 elastic half-space, *J. Acoust. Soc. Am.* 45 (5) (1969) 1258–1272.
- 460 URL <https://doi.org/10.1121/1.1911598>
- 461 [30] A. C. Eringen, *Microcontinuum Field Theories: I. Foundations and Solids*, Springer New York,

- 462 2012.
- 463 [31] J. M. Carcione, Wave Fields in Real Media, 1st Edition, Vol. 31 of Handbook of Geophysical
464 Exploration Seismic Exploration, Pergamon, Amsterdam, 2001.
- 465 [32] E. Ogam, Z. Fellah, G. Ogam, Identification of the mechanical moduli of closed-cell porous foams
466 using transmitted acoustic waves in air and the transfer matrix method, Compos. Struct. 135 (2016)
467 205 – 216.
- 468 [33] L. J. Gibson, M. F. Ashby, Cellular solids : Structure and properties, 2nd Edition, Cambridge Solid
469 State Science, Cambridge University Press, 1997.
- 470 [34] I. MathWorks, Signal Processing Toolbox for Use with MATLAB: User's Guide, version 7.2 (r2016a)
471 Edition, Computation, visualization, programming, The MathWorks, Natick, MA USA, 2016.
- 472 [35] A. Nuttall, Some windows with very good sidelobe behavior, IEEE Trans. Acoust., Speech, Signal
473 Process. 29 (1) (1981) 84–91.
- 474 [36] E. Ogam, Z. Fellah, N. Sebaa, J.-P. Groby, Non-ambiguous recovery of Biot poroelastic parameters
475 of cellular panels using ultrasonic waves, J. Sound Vib. 330 (6) (2011) 1074 – 1090.
476 URL <http://www.sciencedirect.com/science/article/pii/S0022460X10006371>
- 477 [37] Y. Beju, J. Mandal, Expanded polystyrene (eps) geofoam: Preliminary characteristic evaluation,
478 Procedia Eng. 189 (2017) 239 – 246, proceedings of the International Scientific Conference Trans-
479 portation Geotechnics and Geoecology (TGG-2017).
480 URL <http://www.sciencedirect.com/science/article/pii/S1877705817321598>
- 481 [38] ASTM D1621 - 10, Standard Test Method for Compressive Properties Of Rigid Cellular Plastics,
482 American Society for Testing and Materials (ASTM) International, West Conshohocken, PA (2010).
- 483 [39] J. Yang, R. S. Lakes, Experimental study of micropolar and couple stress elasticity in compact bone
484 in bending, Journal of Biomechanics 15 (2) (1982) 91–98.
485 URL <https://www.sciencedirect.com/science/article/pii/0021929082900409>
- 486 [40] F.-Y. Huang, B.-H. Yan, J.-L. Yan, D.-U. Yang, Bending analysis of micropolar elastic beam using
487 a 3-D finite element method, International Journal of Engineering Science 38 (3) (2000) 275–286.
488 URL <https://www.sciencedirect.com/science/article/pii/S0020722599000415>
- 489 [41] Z. Rueger, R. S. Lakes, Cosserat elasticity of negative poisson's ratio foam: experiment, Smart
490 Materials and Structures 25 (5) (2016) 054004.
491 URL <https://doi.org/10.1088/0964-1726/25/5/054004>