Fractional operators for the magnetic dynamic behavior of ferromagnetic specimens: An overview

Benjamin Ducharne, P. Tsafack, Y.A. Tena Deffo, B. Zhang, G. Sebald

Fractional operators for the magnetic dynamic behavior of ferromagnetic specimens: an overview.

B. Ducharne[1,2*], P. Tsafack[3], Y.A. Tene Deffo[3], B. Zhang[4], G. Sebald[2]

[1] Laboratoire de Génie Electrique et Ferroélectricité – INSA de Lyon, 69100, Villeurbanne, France.

[2] ELyTMaX UMI 3757, CNRS – Université de Lyon – Tohoku University, International Joint Unit, Tohoku University, 980-8577 Sendai, Japan.

[3] Faculty of Engineering and Technology, University of Buea, 99999 Buea, Cameroon.

[4] Green Manufacturing R&D Laboratory, School of Mechanical, Electrical and Information Engineering, Shandong University, 264209 Weihai, China.

*corresponding author: benjamin.ducharne@insa-lyon.fr

Abstract:

This paper reviews the use of the fractional derivative operators for the dynamic magnetization of ferromagnetic specimens. Magnetic behaviors in ferromagnetic specimens are strongly nonlinear and frequency dependent. Magnetism has an atomic origin but the magnetic behavior as observed at the human scale is highly affected by phenomena occurring at larger scales. Under the influence of an external magnetic field, the homogeneity of a ferromagnetic sample magnetization is linked to the excitation dynamics. Models and simulations in this domain are strongly needed, they provide theoretical explanations and allow to anticipate complex phenomena, difficult to observe in a practical way.

On the one hand, such multi-scale dynamical behaviors can hardly be taken into account with the usual mathematical operators, on the other hand, correct simulation results on large frequency bandwidths can be obtained using fractional derivative operators. The use of fractional derivatives can be envisaged through different approaches: Lump models based on time fractional differential equations is one option, fractional anomalous diffusion equations is another. In this manuscript, these two methods are detailed and compared. Theoretical results are compared to experimental ones, and conclusions and perspectives are drawn such as possible improvements.

Keywords: Magnetic hysteresis, frequency dependence, fractional derivative, anomalous diffusion.

1 - Introduction:

Ferromagnetic behaviors and properties have been studied since more than a century but ferromagnetism is extremely complex [1]-[5]. The majority of electromagnetic devices rely on magnetic conversions, in this domain progresses are continuous. Such understanding is facilitated by numerical simulations based on theoretical concepts [6][7].

The influence of a magnetic excitation dynamics on the magnetization behaviors manifests itself though the frequency dependence of the hysteresis cycle [8]-[10].

The hysteresis cycle is the standard magnetic signature, it displays the evolution of the magnetic state *M* as a function of the magnetic excitation *H*, assuming both these vector quantities in a collinear situation [11].

Below a given frequency known as the quasi-static threshold, the frequency dependence of the hysteresis cycle is insignificant and the hysteresis shape remains unchanged. Beyond the quasi-static threshold under maximal *H* imposed conditions, the hysteresis area (also known as the hysteresis losses) grows first up to a maximum value then decreases continuously until a full disappearance under extreme frequency levels when a shield of macroscopic eddy currents prevents magnetic penetration in the tested specimen. On the one hand, the sequence of this two phases is always true, on the other hand due to the complexity of hysteresis and ferromagnetism, this trajectory is always a little different from one sample to another [12].

Under the quasi-static state, it is common to separate the magnetization into a reversible and an irreversible contribution. When the reversible contribution manifests itself through the anhysteretic curve, the irreversible one is linked to the hysteresis area, i.e. the irreversible domain wall motions and the irreversible magnetization rotation. In [13], author called these contributions reversible and irreversible work.

The frequency dependence of hysteresis finds its origin in two phenomena:

_ the effect of a dynamic excitation on the domain wall motions. Beyond the quasi-static threshold, the varying magnetic excitation induces a ripple effect on the domain wall motions which become larger and faster, generating a surplus of microscopic eddy currents and their corresponding Joules losses. According to the Bertotti's separation losses theory, this surplus can be assimilated to the "Excess losses".

_ the macroscopic eddy currents due to the magnetic excitation variations through the tested specimen [14]. This contribution is theoretically expressed through the magnetic field diffusion derived from the Maxwell equations:

$$\nabla^2 H = \sigma . \frac{dB}{dt} \tag{1}$$

B is the magnetic induction field and *σ* the electrical conductivity. This frequency dependent contribution is not limited to ferromagnetic materials and can be generalized to all electrical conductive materials.

The simultaneous simulation of these two contributions as observed experimentally for ferromagnetic materials is not straight forward. The kinetic of the magnetic domains interferes with the diffusion process forcing Eq. 1 resolution to divergent results, far from the experimental observations.

This distorted magnetic diffusion exhibits unusual answer and can be classify as anomalous. In [15][16], authors proposed to use alternative mathematical operators found in the framework of the fractional derivatives and adapt Eq. 1 to the simulation of such unfamiliar diffusion processes. By adjusting the fractional order, this alternative simulation method allows to match precisely the experimental observations.

But this is not the only way to simulate hysteresis precisely on a large frequency bandwidth, another solution consists on a phenomenological approach, based a lump model: a quasi-static hysteresis model extended to the frequency dependence through the adjunction of a fractional viscous-type dynamic effect. In [17][18], this method have been tested with success for the well-known Preisach and Jiles-Atherton (J-A) quasi-static hysteresis models.

In both methods fractional derivative operators bring flexibility in the simulation process. The fractional orders act as additional degrees of freedom which once adjusted provide simulation results of improved accuracy.

In this manuscript, we will review sequentially both methods, providing physical insights and additional perspectives. Descriptions and details of both the simulation methods will be described successively in the second section, with comparisons to experimental measurements in the third one, followed by conclusions and perspectives.

2 – Simulation methods

The magnetic core loss prediction is of large importance, exhaustive solutions are still absent but results obtained using time fractional derivative operators look promising [15]-[18].

2-1) Fractional anomalous diffusion equation

The concept of anomalous diffusion focuses on specific random processes, which have non-local properties of the corresponding dynamics or have a memory wait-time [19]. For the magnetic field diffusion in ferromagnetic specimens, we are facing the second case: the physical properties behind the magnetic dynamic are spatially homogeneous but the interactions between the microscopic flux variations due to the domain wall kinetic and the macroscopic one due to the magnetic stimulation generate additional lag which can be

interpreted as a long-time heavy tail decay. Therefore the adapted diffusion equation is obtained by replacing in Eq. 1 the first order time derivative with a fractional one:

$$\nabla^2 H = \sigma . \frac{d^\alpha B}{dt^\alpha} \tag{2}$$

The additional lag limits the diffusion process which becomes sub-diffusive, α is limited to the $0 < \alpha < 1$ interval. Illustration of the resolution of Eq. 2 for different experimental situations: Cross-section of a ferromagnetic toroidal core (left), lamination (center), can be found in [15][16]. For all these situations, a space discretization of Eq. 2 left term is done and a time fractional differential equation is solved for each node of the mesh.

Fractional calculus is defined as the branch of mathematical analysis dedicated to study the possibilities to extend derivatives or integrals to noninteger (real or complex) orders. Fractional derivatives and fractional integrals are hardly distinguishable since unlike classical Newtonian derivatives, a fractional derivative is defined via a fractional integral. Several definitions can be found in the litterature to describe fractional derivatives. Liouville [19]-[21] (Eq. 5), Grünwald-Letnikov [22][23], Caputo [24], are some of them, but the Riemann-Liouville [25] is the classical one:

$$\frac{d^\alpha f(t)}{dt^\alpha} = D^\alpha \left[f(t) \right] = \frac{1}{\Gamma(1-\alpha)} \frac{d}{dt} \int_{-\infty}^{t} (t-\tau)^{-\alpha} f(\tau) d\tau, \qquad -\infty < t < +\infty \tag{3}$$

Eq. 3 can be understood in terms of convolution between *f*(t) and $t^\alpha/\Gamma(1-\alpha)$, where Γ(α) is the gamma function. In the Fourier domain, where every signal is harmonic and where a first order derivative is a jω multiplication, fractional derivatives become $(j\omega)^\alpha$ multiplications [26]-[28]. For a numerical resolution, Eq. 3 is expressed in its discrete form which lead to the direct Grünwald-Letnikov derivative definition:

$$\frac{d^{\alpha} f(t)}{dt^{\alpha}} = D^{\alpha} f(t) = \lim_{h \to 0} \frac{1}{h^{\alpha}} \sum_{k=0}^{\infty} (-1)^{k} \binom{\alpha}{k} f(t - kh)$$
$$\binom{\alpha}{k} = \frac{\Gamma(\alpha + 1)}{\Gamma(k+1)\Gamma(\alpha + 1 - k)} \tag{4}$$

The α derivative of a function *f*(t) at a given time t is a local time property only when α is an integer; this is not the case for non-integer power derivatives. In other words, it is not correct to say that the fractional derivative at t of a function *f*(t) depends only on values of f very near t, in the way that integer-power derivatives certainly do. Eq. 4 denotes weight coefficients which are expressed by:

$$\omega_{\alpha,k} = (-1)^{k} \binom{\alpha}{k} \tag{5}$$

As illustrated in Fig.1a-e below, these weight coefficients depend strongly on the fractional order. For sub-diffusive time fractional derivative equation ($0 < \alpha < 1$), the difference between the weight of the first coefficients with the remaining ones increased as α is getting closer to 0 meaning a larger influence of the past in the calculus of a given t time derivative.

Fig. 1 a-e, Weight coefficients. Fig. 1 f, Hysteresis area vs frequency for different fractional orders.

Eq. 6 below gives the expression of the fractional derivative diffusion equation in its Grünwald-Letnikov expression:

$$\nabla^{2} H = \sigma . \lim_{h \to 0} \frac{1}{h^{\alpha}} \sum_{k=0}^{\infty} (-1)^{k} \binom{\alpha}{k} B(t - kh)$$
$$\binom{\alpha}{k} = \frac{\Gamma(\alpha + 1)}{\Gamma(k+1)\Gamma(\alpha + 1 - k)} \tag{6}$$

The magnetic state of a ferromagnetic specimen and its variations are strongly dependent on its history. Using fractional derivative operators in the numerical method is coherent as it increases the influence of the past in the simulation of the magnetic behavior at a given time t.

In Fig. 1f is depicted an illustration of the fractional order influence on the balance of the frequency effect. The evolution of the hysteresis area is plotted as a function of the frequency for different values of fractional derivative order.

A decrease of α can be interpreted as a stretch of the hysteresis area <A> vs frequency *f* curve in the x direction but not only, it also affects the average permeability which becomes lower and reduces the losses maximum values.

2-2) Lump model combined to a fractional viscous-type dynamic effect

A lump-element model simplifies the description of a spatially distributed physical systems into a topology consisting of discrete entities. In our case, the whole ferromagnetic specimen is considered as a magnetic single lump-element, its inputs are external stimuli which can be of magnetic, mechanic or thermal nature and its output is the element magnetic state. Although the hysteresis cycle is a local information depending of space because of the eddy currents, a lump model (from spatial distribution to discrete entities) is still able to simulate the hysteresis cycle as measured experimentally (i.e. spatially averaged of *B* as a function of the surface tangent H). In a one hand, the advantages of using a lump model are clear: it limits the simulation approach to the replica of the measured quantities and it excludes any inconvenient due to the conformity of the magnetic state distribution. On the other hand, the physical interpretation is limited since the simulation results are limited to macroscopic average quantities and prohibit access to local information.

A lump approach is proposed by Zirka and al. in [29], to consider hysteresis in a ferromagnetic specimen:

$$\begin{cases} \dfrac{dB}{dt} = r(B)(H - H_0) \\ r(B) = R_m\left(1 - \dfrac{B^2}{B_s^2}\right) \end{cases} \quad (7)$$

r(B) is introduced as a dynamic magnetic resistivity which can supposed to be constant or *B* dependent, H_0 is a threshold field. But this approach gives limited accuracy in a restrained frequency bandwidth, then to improve it, authors proposed the following extension:

$$\frac{dB(t)}{dt} = r(B)\left[H(t) - H_{st}(B)\right]^v \quad (8)$$

Where $H_{st}(B)$ is a quasi-static contribution calculated from a quasi-static hysteresis model (Preisach, Jiles-Atherton …) and *v* a real number which is supposed to be controlling the model dynamics (i.e. its temporal and frequency properties). In [16][17], authors use similar equation but the balance of the model dynamic is maintained through the use of a fractional derivative operator:

$$\rho.\frac{d^\alpha B(t)}{dt^\alpha} = H_{dyn}(t) - f_{static}^{-1}(B(t)) \quad (9)$$

Just like the anomalous diffusion equation described in the previous section, adjusting the fractional order gives latitude in the simulation method and allows to match precisely with the experimental results on large frequency bandwidths.

3 – Experimental validation

Different experimental measuring set-up have been developed to confront the simulation results to experimental ones. Whatever the shape of the specimen tested, the basic experimental principles remain always the same:

_ a large turns number coil supplied with a current controlled power amplifier ensures the magnetic excitation (a magnetic yoke can also be associated to drive the magnetic excitation up to the tested area if necessary).

_ a noise shielded radiometric linear Hall probe positioned ideally as close as possible to the magnetic sample measured the surface excitation magnetic field H_{surf}.

_ a wounded search coil ensures the magnetic flux variations monitoring and the specimen local magnetic state evaluation.

On the left hand side of Fig. 2 below is depicted the single sheet tester experimental device we have been using to characterized ferromagnetic laminations. On the right hand side is displayed a micro Helmholtz coil system designed specifically to characterize cylindrical, low diameter specimens.

Fig. 2 – Experimental setup for the magnetic characterization of ferromagnetic lamination (left) and cylindrical specimens (right).

In [17][18] experimental Hysteresis cycles measured on ferromagnetic laminations are compared with success to simulation results obtained with the fractional lump model. Similar comparisons also successful with results coming from the resolution of the fractional anomalous diffusion equation can be found in [16].

Fig. 3 below shows the comparison of the experimental hysteresis cycles obtained using the micro Helmholtz coil characterization device and numerical results coming from both the fractional lump model and the anomalous fractional diffusion equation. Low carbon steel samples are tested, their chemical composition is given in Tab. 1 below:

Tab. 1 – Chemical composition of the low carbon steel specimens tested.

Element	Fe	Mn	Cu	C	Cr	Ni	Al	Si	P	S	Ti
Composition (Wt%)	99.539	0.181	0.082	0.075	0.036	0.032	0.023	0.014	0.009	0.007	0.002

The fractional diffusion equation is solved using polar coordinates. The details of the numerical resolution will be the object of a future communication.

Fig. 3 – Comparison simulation/measurement, for both the fractional diffusion equation and the fractional lump model.

For both simulation method, the J-A model has been used as quasi-static contribution. The anhysteretic curve is simulated using Eq. 10:

$$M_{anh} = M_s \tan^{-1}\left(\frac{H_e}{a}\right) + bH_e \tag{10}$$

The simulation parameters (Fig. 3) have been set using the optimization methods described in details in [15][29]. A complete hysteresis cycle is obtained in less than 4 seconds for the fractional lump model (1 10^6 samples.period^{-1}), 14 seconds are necessary for the fractional diffusion equation, with a mesh consisting of 80 nodes. The accuracy of the fractional diffusion equation is higher, in particularly for the magnetic behavior close to the inflection point, just before saturation. It is worth mentioning here that in a recent article [30], Liu & al. obtained improved results with the lump method by adding a second term, the fractional contribution is restricted to the classic losses (first term) and the excess losses are taken into account according to the Bertotti separation losses theory (second term).

4 – Conclusions

In this manuscript is reviewed the use of fractional derivative operators for taking into account the frequency dependent magnetization in ferromagnetic specimens. In the first method called fractional magnetic field diffusion equation, the time derivative term of the classic diffusion equation is replaced by a fractional one. The sub-diffusive resulting equation exhibits an additional lag in the diffusion process which can be adjusted by fixing the fractional order. A space discretized method is used for the resolution of this equation which gives access to local information. In the second method, a lump approach is proposed and the dynamic contribution is considered through the product of a constant ρ to the time fractional derivative of the induction field *B*. This method is faster in terms of simulation time, but for the cylindrical

shape low carbon steel specimens tested in this study the accuracy is lower. Future works and perspectives in this domain include to test the methods on other experimental situations (like a flat surface excited by a pancake coil) to extend the dynamic consideration to alternative excitation sources (mechanical, thermal …) and to progress again with the physical interpretation and meaning of the fractional derivative operators.

Data availability statement:

The data that support the findings of this study are available from the corresponding author upon reasonable request.

References:

[1] F. Bitter, “On inhomogeneities in the magnetization of ferromagnetic materials”, Phys. Rev., Vol. 38, 1903.

[2] P. Weiss, “L'hypothèse du champ moléculaire et la propriété ferromagnétique’’, J. de Phys., vol. 6, pp. 661-690, 1907.

[3] R.H. Pry, C.P. Bean, “Calculation of the energy loss in magnetic sheet materials using a domain model”, J. Appl. Phys., Vol. 29, n°532, 1958.

[4] G. Bertotti, “General properties of power losses in soft ferromagnetic materials”, IEEE Trans. Magn., vol. 24, iss. 1, pp. 621-630, 1988.

[5] R. M. Bozorth, “Ferromagnetism”, Wiley-IEEE Press, Michigan University, 1993.

[6] J. Gyselinck, P. Dular, C. Geuzaine, W. Legros, “Harmonic-balance finite-element modeling of electromagnetic devices: a novel approach”, IEEE Trans. Magn., vol. 38, iss. 2, pp. 521-524, 2002.

[7] S. Bouissou, F. Piriou, “Study of 3D formulations to model electromagnetic devices”, IEEE Trans. Magn., vol. 30, iss. 5, pp. 3228-3231, 1994.

[8] S.E. Zirka, Y.I.Moroz, P. Marketos, A.J. Moses, “Dynamic hysteresis modelling”, Phys. B: Cond. Mat., vol. 343, iss. 1-4, pp. 90-95, 2004.

[9] D.C. Jiles, “frequency dependence of hysteresis curves in conducting magnetic materials”, J. Appl. Phys., vol. 76, n° 5849, 1994.

[10] M. Petrun, S. Steentjes, “Iron-Loss and magnetization dynamics in non-oriented electrical steel: 1D excitations up to high frequencies”, IEEE Access, vol. 8, pp. 4568-4593, 2020.

[11] F. Fiorillo, “Measurement and characterization of magnetic materials”, Elsevier academic press, 2004.

[12] F. Broner, G.H. Goldsztein, S.H. Strogatz, “Dynamical hysteresis without static hysteresis: scaling laws and asymptotic expansions”, SIAM J. Appl. Math., vol. 57, iss. 4, pp. 1163-1187, 2006.

[13] H. Hauser, “Energetic model of ferromagnetic hysteresis: isotropic magnetization”, J. Appl. Phys., vol. 96, n° 5, pp. 2753-2767, 2004.

[14] M. A. Raulet, B. Ducharne, J.P. Masson, and G. Bayada, “The magnetic field diffusion equation including dynamic hysteresis: a linear formulation of the problem”, IEEE Transactions on Magnetics, Vol. 40, n° 2, pp. 872-875, 2004.

[15] B. Ducharne, P. Tsafack, Y.A. Tene Deffo, B. Zhang, G. Sebald, “Anomalous fractional magnetic field diffusion through the cross-section of a massive toroidal ferromagnetic core”, Com. in Nonlin. Sci. and Num. Sim., vol. 92, 105450, 2020.

[16] B. Ducharne, Y.A. Tene Deffo, B. Zhang, G. Sebald, “Anomalous fractional diffusion equation for magnetic losses in a ferromagnetic lamination”, The Eur. Phys. J. Plus, 135:325, 2020.

[17] B. Zhang, B. Gupta, B. Ducharne, G. Sebald, T. Uchimoto, “Dynamic magnetic scalar hysteresis lump model, based on JilesAtherton quasi-static hysteresis model extended with dynamic fractional derivative contribution”, IEEE Trans. Magn., Iss. 99, pp. 1-5, 2018.

[18] B. Zhang, B. Gupta, B. Ducharne, G. Sebald, T. Uchimoto, “Preisach’s model extended with dynamic fractional derivation contribution”, IEEE Trans. Magn., Vol. 54 iss. 3, 2017.

[19] K. S. Miller and B. Ross, “An Introduction to the Fractional Calculus and Fractional Differential Equations”, John Wiley & Sons, New York, NY, USA, 1993.

[20] K. B. Oldham and J. Spanier, “The Fractional Calculus: Theory and Application of Differentiation and Integration to Arbitrary Order”, Academic Press, New York, NY, USA, 1974.

[21] J. T. Machado, V. Kiryakova, and F. Mainardi, “Recent history of fractional calculus”, Communications in Nonlinear Science and Numerical Simulation, vol. 16, n° 3, pp. 1140-1153, 2011.

[22] A. K. Grünwald, “Über “begrenzte” derivationen und deren anwendung”, Zeitschrift für Mathematik und Physik, vol. 12, pp. 441–480, 1867.

[23] A. V. Letnikov, “Theory of differentiation with an arbitrary index”, Sbornik: Mathematics, vol. 3, pp. 1–66, 1868 (Russian).

[24] M. Caputo, “Linear models of dissipation whose q is almost frequency independent-ii”, Geophysical Journal of the Royal Astronomical Society, vol. 13, no. 5, pp. 529–539, 1967.

[25] B. Riemann, “Versuch Einer Allgemeinen Auffassung der Integration und Differentiation”, Gesammelte Mathematische Werke und Wissenschaftlicher Nachlass, Teubner, Leipzig, 1876, Dover, New York, NY, USA, 1953.

[26] D. Guyomar, B. Ducharne, G. Sebald, “Time fractional derivatives for voltage creep in ferroelectric materials: theory and experiment”, Journal of Physics D: Applied Physics, Vol. 41, Iss. 12, 125410, 2008.

[27] D. Guyomar, B. Ducharne, G. Sebald, “Dynamical hysteresis model of ferroelectric ceramics under electric field using fractional derivatives”, Journal of Physics D: Applied Physics, Vol. 40, Iss. 19, pp. 6048-6054, 2007.

[28] B. Ducharne, B. Newell, G. Sebald, “A unique fractional derivative operator to simulate all dynamic piezo ceramic dielectric manifestations: from aging to frequency dependent hysteresis”, IEEE Transactions on Ultrasonic, Ferroelectricity and Frequency Control, vol. 67, iss. 1, pp. 197-206, 2019.

[29] S. Zhang, B. Ducharne, T. Uchimoto, A. Kita, “Simulation tool for the Eddy Current Magnetic Signature (EC-MS) non-destructive method, J. of Mag. and Mag. Mat., vol. 513, 167221, 2020.

[30] R. Liu, L. Li, “Analytical prediction model of energy losses in soft magnetic materials over broadband frequency range”, IEEE Trans. Pow. Elect., vol. 36, iss. 2, pp. 2009-2017, 2021.