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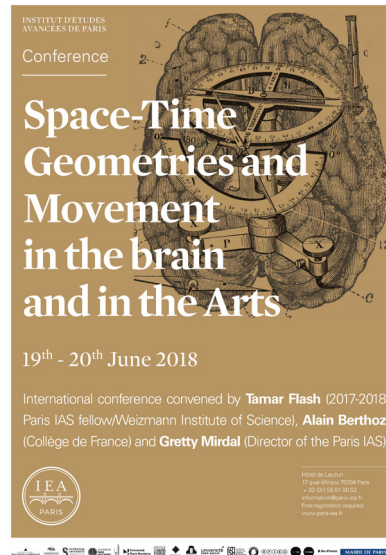
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From sketches to morphing: New geometric views on the epistemological role of drawing

Renaud Chabrier

(Preprint version, nov 2020)

Abstract

Geometry, in the modern sense of the term, is based on the study of transformations. The diversity of those transformations has recently been used to understand the diversity of spatio-temporal representations in the brain. I argue that this new association between geometry and neuroscience can also change the way we consider drawing. The role of transformations is thus evaluated here in three differing examples, all involving the drawing of living bodies: drawing from the nude, drawing a sequence of movements, and life sketching. This analysis indicates that “strokes”, rather than “lines”, play a fundamental role when drawing living beings. On this basis then, it is possible to put aside the classical approach to dealing with spatial representations through “central” or “linear” perspective, and to highlight alternative principles of spatial composition such as drawing “without a point of view”. This attempt to connect drawing with modern geometry and neuroscience leads to a re-evaluation of the epistemological importance of digital animation tools, according to their geometrical premises. I will discuss in more detail the role of morphing-based animation, which is based primarily on texture transformations rather than virtual cameras. The development of such a complete approach to drawing, including all its static, moving and transformational aspects, will emerge as a vital step in addressing the new challenges raised by the representation of life in the twenty-first century.

Introduction

Geometry maintained an important relationship with drawing for more than two millennia. Then, over the course of the last two centuries, the connection was gradually severed: geometry with all its developments in physics became seemingly too abstract to be studied with the aid of drawing. Indeed, the strange and fascinating aspects of space-time at the scale of atoms or galaxies had little to do with the experience of life at human scale, where drawing practice is deeply rooted.

While these new scientific fields were being explored, major discoveries and innovations were altering the practice of drawing in the Western world: modern art questioned the use of central (i.e. linear) perspective, animated cartoons explicitly introduced movement into images, and new methods introduced by both biomechanics and modern dance modified the way artists were looking at bodies. The discovery of techniques distant in time such as rock paintings, as well as distant in space such as Chinese painting or Japanese prints, also highlighted a certain universality of drawing.

In these circumstances, it may seem logical that a new, more global understanding of the practice of drawing should emerge, in both an international and holistic sense, but this has not been the case. As the philosopher Philippe Grossos (2017) indicates, the interest in symbols has led, over a long period, to neglect of the qualities of movement and presence that are fundamental in rock

art. Similarly, the interest in central perspective as a symbol of power has greatly overshadowed the analysis of living bodies represented in the same scenes (Chabrier, 2016). Indeed, the majority of reference studies such as those of Leroi-Gourhan (1965) on cave art archaeology or Panofsky (1927) on the aesthetic philosophy of renaissance painting prefer to distinguish separate symbolic systems, rather than appreciating the quality of spatio-temporal representations of living bodies in various context.

However, in recent decades, both cognitive science and neuroscience began to explore how the brain deals with space and time. An important idea was introduced: we animals and humans use not only one single universal geometry, but a set of different geometries depending on the task (Bennequin et al. 2009). In this chapter, we will see that this idea opens up the possibility of reconnecting modern geometry with drawing, especially in the case of the representation of living bodies. In the first part of this text, I will present very simply the fundamental role of transformations in the relatively recent re-definition of geometry, then I will discuss the application of this approach in neuroscience. I will propose using drawing both as a means of accessing the different geometries used by the brain and as an effective training tool to make good use of them.

In the second part, we will go on to examine the practice of drawing living bodies by looking for what transformations they imply. We will examine three situations respectively: drawing from the nude, sequence drawing, and life sketching. Each time, I will introduce historical elements to show how these drawing practices have evolved since the late nineteenth century. This analysis in terms of transformations will highlight the fundamental role played by the spatiality of the stroke. I will propose viewing the stroke as an exemplary case of “simplicity”, a tool for managing in a simple way a complex problem such as the representation of living beings. Using the word “stroke”, instead of “line”, will be a key aspect of this text. Indeed, the word “line” has a precise mathematical meaning: a line has a null curvature, otherwise it is described as a “curve”. Furthermore, those concepts of line and curve create abstraction from materiality: they do not take into account what the lines and curves are made of. Unlike “line”, the word “stroke” is not used in mathematics: it relates to a combination of gestures and traces on paper, with limitless possibilities of both trajectories and material textures. Consequently, speaking of “strokes” instead of “lines” or “curves”, when a precise mathematical meaning is not what matters, prevents confusion between different levels of abstraction.

The third part will extend this reflection by examining the way in which several bodies, represented by means of the spatiality of the stroke, can be composed on the same sheet. I will highlight the existence of a geometry “without a point of view” whose use, highly developed in both the arts and the sciences, has been largely overshadowed by the attention paid to central perspective. I will present the technique of morphing-based animation, which can transform, in multiple ways, drawings created using a geometry without a point of view. Morphing will consequently be shown to be a natural extension of drawing, just as 3D computer graphics are a natural extension of both central perspective and photography. From sketching to morphing, this analysis in terms of transformations will emerge as a new overall framework for understanding the epistemological role of drawing. With the help of a philosophical reflection on the respective resources of Eastern and Western art, I will suggest the possibility of new collaborations between production, education and research.

Part 1: Multiple geometries in the drawer's brain

Geometry as a study of transformations

In the culture of non-mathematicians, the word “geometry” is generally used when speaking of distances and angles between various objects, or between different parts of the same object. The basic elements of geometry taught in school usually involve very static objects in two dimensions such as circles, lines, triangles or more complex polyhedrons. Consequently, the collective and popular understanding of the word “geometry” is still very close to the one expressed by Plato a long time ago: that this science is dedicated to the study of ideal, perfect and unchangeable forms.

In the mathematics community, however, the word “geometry” has taken on a very different meaning over the past 150 years. In 1872, Klein suggested replacing this study of static forms, inherited from Euclid, with another far more general discipline: the study of transformations allowing the passing from one “mathematical object” to the other. Some of those transformations are well known (displacements, also called translations, rotations...) while others are less familiar (affine transformations, perspective transformations...). The important concept is that each type of transformation allows us to associate or to differentiate among the objects that are studied: for example, two triangles presented one next to the other will be said to be equivalent for translation if a simple displacement makes the left one exactly cover the right one (or the opposite). With more complex transformations (rotation, change of scale, shearing...), mathematicians are able to create various classes of equivalence between the objects they are interested in, some of them being more intuitive than others. According to Klein's approach, then, a “geometry” is defined by what one can do with a certain kind of transformation; thus, there is not one, but many geometries.

Clearly, a deep epistemological gap lies between those two notions of geometry: one is “unique” and is based on unchangeable forms, the other is “multiple” and seeks possibilities for transformation. In the twentieth century, replacing the existing geometry with a more “modern” one in basic education was a fundamental challenge. However, despite the efforts of teachers to bring about this change (especially in France during the second half of the century), one must admit that the modern meaning of the word “geometry” was never adopted by the general public. Meanwhile, during the same period, the basic teaching of classical geometry progressively weakened, even in scientific and technical programs. One can remark that this double failure coincides with the neglect or even the rejection of the use of drawing in the teaching of mathematics, well before the more recent and radical change in representation practice due to digital tools. This separation between drawing and mathematics is partly due to the fact that drawing has long been seen as a “static” and “realist” representation tool in Western society. Thus, it seemed fundamentally incompatible with the development of a new concept of abstract transformation. Nonetheless, we can observe that a number of mathematicians make effective use of handmade drawings when explaining their research to a wider audience, such as Patrick Popescu-Pampu when he addresses the relationship between topology and geometry (2018; see also **Online resource [b]**).

The brain can use multiple types of transformations

Whatever the discussion of the teaching of “geometries” in education may be, we can observe that a perfectly normal brain without special mathematical training uses at least three geometries. They

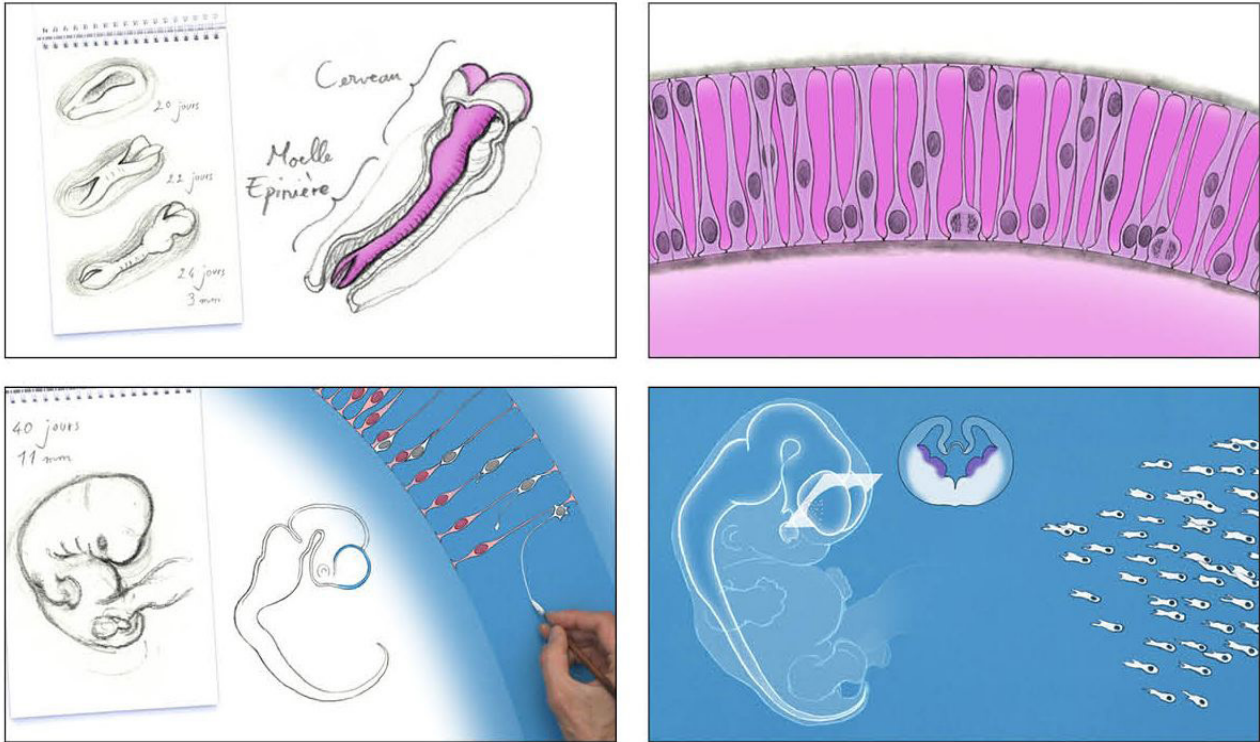


Figure 1: Showing transformations of the brain with transformations of drawings.

Images from “Birth of a brain”, directed and drawn by R. Chabrier for the “Brain” exhibition at Cité des Sciences et de l’Industrie, 2014.

are based respectively on simple displacement (translation), scale change, and rotation. These three “basic” transformations allow us to judge that several forms are not only alike, but alike in different ways and at different levels. Today, such transformations are widely used in modern animation and motion design, since animation software makes them very accessible. Before the widespread use of digital tools, however, artists like Norman McLaren showed the expressive richness of animations rigorously limited to a few transformations of this kind in short movies like “Vertical Lines” (1960). Similarly, psychologists have used very basic animations, such as moving triangles, in order to study the tendency for humans to assign a personality to moving objects (Heider, Simmel, 1944; White et al., 2011). Clearly, the conceptual facility of displacement, rotation and (small) scale change has allowed a natural connection between animation and cognitive psychology, both before and after the introduction of digital technologies: animated stimuli played an important part in the study of how the brain deals with transformations.

Owing to a long philosophical tradition going back to Aristotle, movement is generally considered to be a secondary attribute of form: you create a form, and then you can move it. However, the practice of animation enables a conceptual reversal, very similar to the one that we just discussed in mathematics. Indeed, in an animated sequence, it often appears that forms exist for the viewer mainly through their movement. Another way to describe this is to say that the forms presented to the audience remain invisible (or rather, neglected) if they are not carried by a work of transformation, which reveals them and orients the way they are perceived.

Transformation are present everywhere in animation. “Simple” parameters of position, size and rotation can be considered as fundamental transformations, especially in the context of documentary films. They are intended to be used with subtlety, in order to induce various evolutions in the imagination of the audience: indeed, a simple zoom on an image, centered on the right place with the right dynamics, can alter the intelligibility of a sequence. The short film “Birth of a Brain” (**Figure 1** and **Movie [1]**) is a good example. It aims to make brain development accessible to a wide audience by showing the richness of the associated spatiotemporal phenomena. A wide range of transformations are used in this film, including morphing, which we will discuss later. Many of them are based on the basic operations that we have just described.

While those very simple (but very efficient) examples do not challenge the geometric world we are familiar with, it appears that other more unexpected geometries also coexist in the brain. This idea has been developed from studies of “simple” curved movements in tracing tasks, and studies of trajectories in locomotion tasks. Bennequin, Fuchs, Berthoz and Flash (2009) found that such movements could be transformed one into the other according to a number of specific geometric rules. Bennequin and Berthoz (2017) also proposed extending this approach to more complex movements like prehension, with more advanced mathematical tools. Henceforth, the “objects” that those researchers are interested in are no longer static forms, possibly animated, like squares or triangles. Those objects are in fact defined in time and space: they can be in motion, and the geometries concerned can even be used to represent forces.

The rigorous expression of these last mathematical ideas is very abstract, and somewhat inaccessible even with a scientific background, such as the “geometries without points” of Elie Cartan (Ivey T, Landsberg, 2003). Moreover, those concepts are applied to neuroscientific observations, which can also be difficult to understand. In addition, animation techniques that use more complex trans-

formations than displacement, rotation and scale change require a distinct professional training, incompatible with a scientific career. Thus, the connection between animation and science has become quite strained. But if the brain indeed works this way at the functional level, we can infer several consequences.

Drawing as a way to access to the diversity of geometries

First, it should be possible to detect the influence of those multiple geometries in many human activities, and perhaps find examples that are easier to understand, to experiment with, or to sense, than their mathematical expression. Drawing makes a very good candidate for this investigation, because it can change both “three-dimensional reality” and “abstract thinking”, into fascinating fields for geometrical explorations. Escher’s etchings already provide well-known examples of this fact in imaginary worlds, but further on we will see that interesting geometrical challenges can be found in the reality in front of us, as soon as we pay attention to the spatiality of living beings.

Second, it may be necessary to assume the geometrical richness of human activities, in order to better evaluate the benefit that the brain derives from their practice. In particular, many manual activities related to “arts and crafts” like woodworking or modelling, are better appreciated today for their balancing function. In contrast with “pure” digital practices, they involve gesture, in which trajectories and forces are closely embedded. Adopting Daniel Bennequin’s proposition, we can consider those actions to be necessarily related to more complex geometries than the Euclidian one, integrating forces in particular.

Once again, drawing can play here an interesting role of mediation between the concrete and the abstract. Indeed, while aiming at the work of representation prior to production, fabrication or construction, the materiality of the paper and the tools (pen, charcoal, black stone, brush...) implies a play of forces between thrust, traction and friction, which must be partially integrated in planning a gesture.

While the proposition of Berthoz, Flash and Bennequin initiates a new way of considering spatiotemporal representations in the brain, it also encourages a change in the way we consider the activities that generate those representations, which can lead in turn to a fundamental reevaluation of manual craft. If we follow the logic of the authors, initially proposed by Klein, the key lies in the possibilities of transformation. Consequently, we will now take a look at drawing in this way, focusing, at first, on sketching living beings, then on composition, and finally on morphing-based animation, in order to understand which geometries arise from those practices.

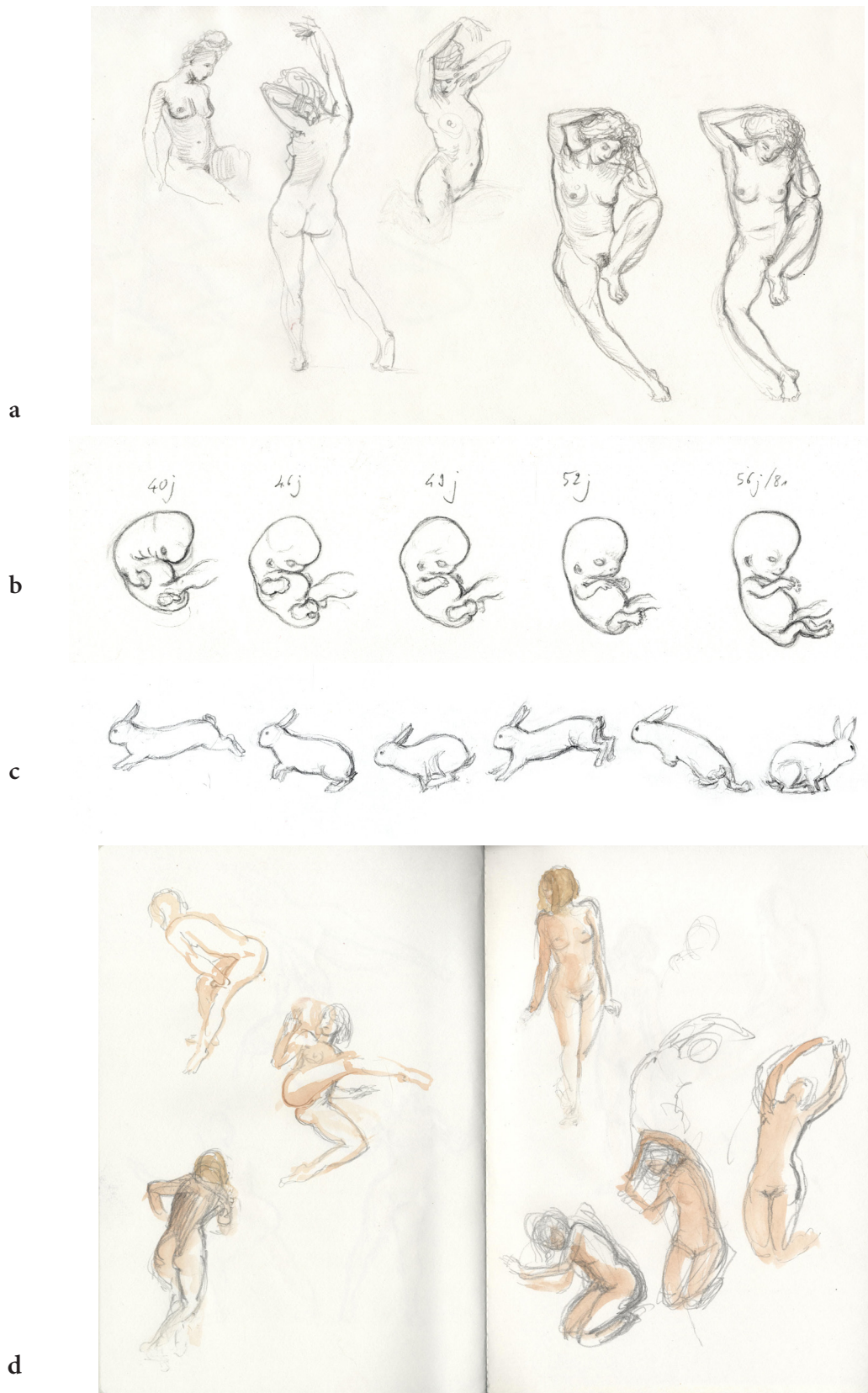


Figure 2: Various ways of drawing involve different attitudes towards transformations in general, and body movements in particular. Drawings by R. Chabrier.
a: Drawing from the nude asks the real model to suspend his/her transformations for a few minutes.
b, c: Sequence drawing develops the transformations of an imaginary body in an ordered, analytical way.
d: Life sketches let the model or subject free to transform itself, and use this information for free composition.

Part 2: Drawing living beings, from transformations to strokes

Fundamentally, drawing is a temporal process. It simply cannot be idealized as an instant event. This clearly distinguishes it from photography, which is generally based on extremely rapid shooting. As soon as one tries to draw living beings that can move and change, the time taken by the act of drawing opens the door to many transformations, as much in the subject drawn as in the draftsman himself. However, these transformations will appear very differently depending on the contexts in which the drawing is practiced. I will now review drawing from the nude, drawing in sequence, and life sketching. As we will see, those three approaches to representing life through drawing are related to different epistemological backgrounds, that may hide or highlight the way transformations are involved. Thus the main purpose of this section is to clarify this situation. It will become clear that the concept of “stroke” provides a more accurate basis than the usual concept of “line”, if we are interested in thinking about drawing in terms of transformations.

Suspending transformations: drawing from the nude

The nude illustrates well the temporality of drawing. A session usually lasts three hours, during which the model adopts poses of different durations (2, 5, 10, 20, 30 or 45 min ...). During each pose, the models focus on suspending the transformation of their body, in order to let themselves be drawn. But throughout these three hours, many other transformations occur in both the draftsmen and the models: the postures are relaxed or refined, the breathing is stabilized, time takes on a different density, the hand adapts to the attrition and sharpening of the tool, the drawing itself evolves. These are “invisible” transformations that are well known to trained draftsmen and models, but are at the same time difficult to describe and analyse.

Once the session is over, the spatial presence of the model, translated by the drawing, remains on the sheet of paper. This presence can also be interpreted in terms of transformations: the drawing suggests what would happen if one started to turn the drawn body around, or if this drawn body started to move. However, this way of seeing requires some training, and some sensitivity to the anticipatory processes involved in the brain. In the example that I give (**Figure 2a**), it can be seen that my drawing evolves as the session progresses. In addition, the first relatively short poses may suggest a movement of the model, whereas a longer pose, drawn from two different angles, gives an idea of the model’s spatiality through a turning movement of the viewer, if one passes the eye from one drawing to another.

In general, for cultural reasons that are deeply rooted in Western society, drawings produced from the nude are rarely interpreted in terms of transformation. Rather, they are considered as “still images” fixing the presence of the model on the paper. This stillness was indeed deliberately sought in academic drawing in the eighteenth and especially in the nineteenth century, treating living bodies as marble sculptures. As evoked by Pierre Bourdieu (2013) in his sociological study of the pioneers of modern art, the organization of life drawing workshops in France in the nineteenth century imposed a very strict hierarchy amongst the students, using geometrical constraints: on the one hand, they were asked to draw by rigorously obeying the laws of central perspective, and on the other, only the highest-ranked students were permitted to choose their “point of view”. From the 1860s, Manet and subsequent modern artists began to reject this form of drawing, since they considered it both fixed and constraining.

Today, therefore, for both cognitive and historical reasons, the existence of a strong geometric relationship between a drawing and a “still” model can easily lead to confusion. Firstly, it leads to the idea that the drawing “is” the thing it represents. This confusion related to identity is similar to what happens with photography, especially when it comes to very fine greyscale works with pencil. Secondly, the apparently direct link between the drawing and the model encourages the idea that the drawer, stroke by stroke or line by line, reproduces only “what he sees”. This interpretation does not sufficiently emphasize the process of transformation that must be gone through in order to find the coherence of a set of strokes, and it inhibits the imagination of possible transformations from the drawing as it is made. In fact, drawing from the nude can be connected to and practiced with a richer understanding of geometry, but in order to understand how, we might first consider other ways to draw life, further from the academic world.

Developing transformations: sequence drawing

Sequence drawing consists of drawing several steps in a process: most of the time, the steps are close together, in order to make the evolution more evident (especially if the sequence is not displayed as a movie, but printed on paper). Consequently, as opposed to drawing from the nude, sequence drawing is explicitly connected to transformations. For example, a sequence can show the development of the embryo, or a running rabbit (**Figure 2b** and **2c**). In the first case, modifications of the drawn body are particularly important. This corresponds to the meaning we usually give to the word “transformation”. However, we must remember once again that in mathematics this word also refers to modifications that do not affect the whole body shape and proportions; simple translations, changes of orientation and articulations of the limbs, that is to say everything we usually call “movement” or “motion”, can also be seen as different types of transformations.

These two drawing sequences are extracted respectively from a scientific film for the general public (“Birth of a brain”, Movie [1]), and from projections accompanying a musical comedy (“Snow White” directed by Guy Grimbert, Bobino Theater, Paris, 2013). Although they have been produced in order to be shown in totally different contexts, they have much in common. Critically, neither faithfully reproduces a “model” existing in reality; whether it is the embryo or the rabbit given as an example, they are created by drawing inspiration from several sources at once.

From an historical point of view, it is interesting to note that these two types of drawing “in sequence”, for either scientific or entertainment purposes, were initially developed during the same period: Haeckel popularized the depiction of embryonic development through his famous drawings in 1874, while Emile Reynaud met success with his first animated cartoons in 1877, thanks to his praxinoscope. However, despite the fact that scientific drawing and animated drawing generated a common and simultaneous interest in transformations, creating a relationship between the two disciplines was not straightforward at the time. Indeed, the idea of drawing the sequential development of a transformation was not received in the same way in the entertainment industry and in life sciences. Imagination was highly valued for entertainment. Conversely, Haeckel was criticized for introducing imaginary aspects into the sequential development of his drawings.

To understand this difference, it is helpful to go back to the historical and epistemological study of the notion of “Objectivity”, conducted by Lorraine Dalton and Peter Galison and published in the book of the same name (2007). In particular, the authors describe how contemporary scientists

reproached Haeckel for being overly guided by what he knew, or what he thought he knew, instead of recording correctly what he saw. In general, from the end of the nineteenth century, many scientists actively tried to reject any personal interpretation. Indeed, they were increasingly interested in phenomena that were not easily perceived by the human eye, and they realized that drawing from direct visual observation could transmit many interpretation errors, such as overlooking the diversity found in snowflakes or perceiving a non-existent symmetry in falling droplets. This led those scientists to reject drawing in favour of photography, in order to attain “mechanical objectivity”. Still images, in the literal photographic sense, became the main means of studying movement in science.

Hence, there was an early divergence between the art of animation and the sciences of movement, or transformations in general. Animation movie directors wanted their draftsmen to move everything that they could move, and let their imagination run free. Conversely, most scientists wanted to eliminate, as much as possible, human imagination from any visual information created. Animation continued to rely on manual craft to a large extent, while scientists began to value images obtained with automatic and mechanical recording tools. Ultimately, for many scientists in mathematics or physics (but also later in biology), the preference was to have no image at all, since images could interfere with essential concepts.

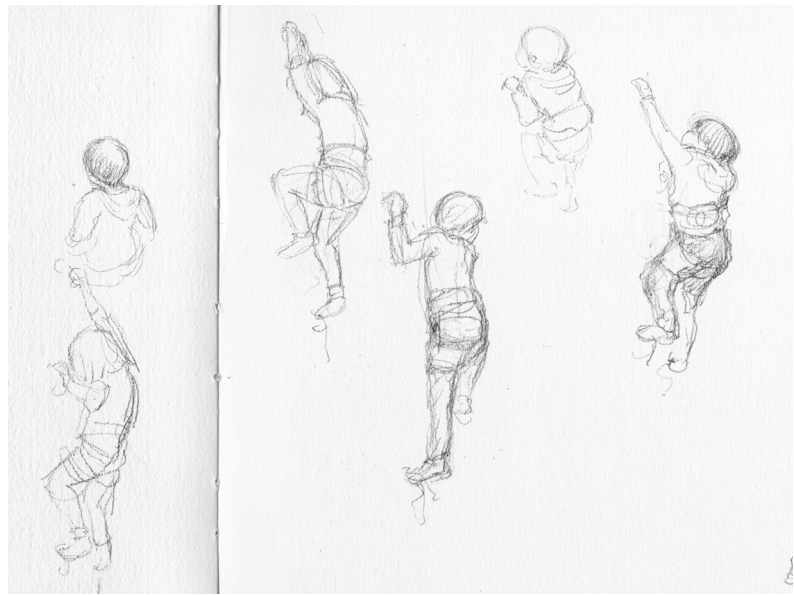
However, as distant as their practice could be during the course of the twentieth century, animation and science shared a common concern for an aspect of transformations that I have not addressed until this point: invariance. Invariants are aspects or properties of objects that remain unchanged by transformation. In mathematics, Klein’s new way of studying geometries was based on such invariants, which were considered as characteristics of the various kinds of transformations. In an animated fiction movie, invariance is also an important issue: some transformations might affect the very identity of a character, while others will allow you to believe in its integrity from one frame to the other. For example, if we consider again figures 2b and 2c, those transformations are intuitively related to the same embryo or the same rabbit.

Such an assumed invariance of a character, in a sequence of drawing that shows a movement broken down into a few steps, suggests that this transformation is in fact continuous. In mathematics, this would mean that analytical tools can be used. But in drawing, too, the realization of such sequences requires a strong “analytical” component: in order to ensure that the views are correctly linked to each other, the eye of the draftsman constantly compares, differentiates and regulates the steps. Very often, in order to support his strokes, the draftsman (or the animator) uses simple trajectories or volumes whose transformations are easier to control. This realization process, which can be very conceptual, brings us closer to the idea that the draftsman draws “what he knows” much more than “what he sees”.

In reviewing the cases of drawing from the nude and drawing movement sequences, my purpose was, first, to present two different approaches for drawing living bodies, and second, to show the reasons why one can so easily be reduced to “a pure perceptual product”, while the other can most often be interpreted as “a pure conceptual product”. As Seymour Simmons has shown in his historical study of drawing instructions (2011), both these ways of considering the practice of drawing have a clear philosophical background. They are closely associated, respectively, with the “realist” and “rationalist” schools of thought, which competed with each other during the nineteenth century and still remain influential; in British and American culture, for example, scientific drawing



a



b

Figure 3: Life sketches, in the case of animals or children, highlight the relationship between transformations and composition. Drawings by R. Chabrier.

a: Different drawings of the same flamingo at different times can also be interpreted as different flamingos within the space.

b: The composition of several individual sketches generates a spatial representation at a larger scale. A unique point of view cannot be identified in such a composition.

is still associated with the realism of Ruskin, with an emphasis on perception, while in France the importance of rationalism led to the early decline of drawing in many scientific fields, in favour of conceptual schematics.

According to Seymour Simons, both perceptual and conceptual approaches to drawing tend to neglect the haptic dimension of this practice, and more modern approaches to drawing can be related to “pragmatism”. Typically, in the drawing instructions provided by Kimon Nicolaides (1941), the gestures of the draftsman can be easily identified, and the strokes produce not only the shape of the bodies, but also aspects that are more difficult to see such as energy or action lines running inside the body. In the following section, I will present a slightly different approach to drawing, similar in the sense that it cannot be reduced to “what I see” or “what I know”, but different in the sense that it will not focus on gesture expressivity. Rather, in examining life sketches we will be largely concerned with the relation between strokes and transformations.

Getting inspiration from transformations: life sketches

Life sketching simply consists of drawing living beings that do not pose. It may be as well to sketch animals, as to work with a model in motion (**Figure 2d**), as Rodin experimented with from the 1880s. In such cases, all transformations are possible (let us repeat that we use “transformations” in the broad sense of the term, including very simple “movements”). This freedom, left to the subject, offers a great wealth of information on body structure, and it often reveals forms more clearly than the static observation of a motionless body. However, this information, by nature, is accessible only during the time of movement, or change. It has to be stabilized and completed with other information, in the brain. This situation forces the draftsman to acknowledge that the drawing action is located largely outside the strict limits of the visual world.

Suppose for example that I find myself in front of a group of flamingos in a zoo (**Figure 3a**). I start to draw a bird with its head down, but it then raises its neck. In order to go on, I have to use my short-term memory as well as my anatomical knowledge. More precisely, I have to fit this anatomical knowledge with what the flamingo does: this can be considered as a transformation. I also have an interest in taking into account the “embodiment”, which allows me to evaluate the animal’s balance by appealing to my own postural sensations. Once again, it is a kind of transformation from one body to the other. In parallel, I must keep in mind that the real bird, although it has moved, continues to be a reference with which my drawing can remain coherent in one way or another. Through all those transformations, a vast combination of information and representations (visual, haptic, proprioceptive ...) can be mobilized. At each stage of the drawing, all these processes lead to a simple stroke.

Thus the psychological foundation of the stroke represents a rich epistemological issue. In life sketching, the stroke is evidently not the result of a planned action with reference to a single fixed model, whether located “in front of the draftsman” or “in his mind”. On the contrary, transformations seem to be involved in the production of the stroke at all levels of abstraction that the brain allows. Consequently, the stroke can integrate and transmit a wide range of information. In this way, it can be considered a case of “simplicity”: a tool that makes it possible to handle great complexity in a simple way, without reducing this complexity to the point where oversimplification introduces new problems (Berthoz, 2009).

Indeed, a stroke is always simple, compared to the reality, and compared to the neural processes that motivate it. Simultaneously, the effective stroke in a sketch task is never so simple as to be reduced to a two-dimensional “graphic object”. In the case of life sketching, it is particularly inappropriate to ignore the spatiality of life: flat sketches lose the connection with reality, and very quickly become boring. Therefore, the stroke that synthesizes this wealth of information must also have a spatial value. This is obtained in many ways: not only by the path of the tool on the surface of the sheet of paper, but also, more precisely, by the variations of curvature, thickness and texture along this path. Those textures, which involve spatial frequencies, also reinforce the possibilities of connections between multiple strokes. Those relationships have a temporal dimension, since it takes a certain amount of time to appreciate each texture. All these factors depend on the force and friction applied in the contact between the tool and the sheet. They bring to the stroke additional dimensions allowing the observer’s brain to make a relatively clear qualitative distinction between a flat “graphic object”, which can be easily identified and isolated, and a “stroke”, which generates space and time, and which binds to other strokes in specific ways.

Intermediate conclusion: the geometries of strokes

I have highlighted the concept of “stroke” (in French, “le trait”) through the case of life sketches. However, one can observe that strokes are also involved in drawing from the nude or drawing in sequences. This community of means is evident when drawings are kept as sketches rather than being developed into “finished” images. In figures 2&3, I have purposefully chosen examples of this kind. If strokes are deeply related to transformations, as I have attempted to demonstrate, it follows that the geometries of strokes have a fundamental role in drawing.

However, this idea is quite novel in Western culture. A general preference for “finished” and “clean” images, rather than sketches, may explain why the concept of strokes was not greatly developed until recently. The variety of tools (charcoal, pen, pencil, black chalk, blood, wash, not to mention the different engraving techniques) was probably also confusing, since each technique has its specificities when it comes to generating space from a set of strokes, as evidenced by the drawing treatises of the nineteenth century (Fraipont, 1897). In any case, the standard concept associated with drawing has not been “stroke” but “line”. This vocabulary had huge consequences for the place of drawing in modern society, because it led to the development of a flawed relationship between drawing by hand and mathematics. Indeed, a “line” is by default considered a “straight line”. Of course, a line can be curved. But being curved, according to this logic, is only a secondary attribute.

Conversely, using this idea of “stroke”, one can develop an alternative concept where curvature is fundamental. This creates a natural connection with the immense developments achieved in mathematics since Klein. Hence, in her work on the analysis of human arm movement, Tamar Flash uses the moving frame, an analysis tool developed by Elie Cartan, in order to show that equiaffine geometry is more relevant than Euclidian geometry for understanding human tracing gestures (Flash & Handzel 2007). This sentence might of course mean nothing, without further explanation, for a non-mathematician reader, but Tamar Flash’s further conclusions make a great deal of sense to people who practice drawing; in the affine geometry that she associates with strokes, the most natural connection between a departure and an arrival is not a straight line. This is exactly

what happens when we draw living bodies (as opposed to drawing classical or modern architecture): if we have a clear feeling of what we draw, of how the pen starts and of how it ends, our hand simply doesn't take a straight path. Moreover, Tamar Flash defines an affine distance all along this path; she shows that the preferred curves, in human gesture, maximise this distance. Once again, this appears very consistent with the actual experience of drawing: draftsmen tend not to rely on such a feeling as minimizing a distance, using "dry" and straight segments of lines. On the contrary, appropriate strokes provide something more like a subtle nourishing feeling, related to some kind of generosity from which we can benefit, wherever it comes from.

Independently of this very new scientific means of access to the geometries of strokes, this concept already has a rich history outside of Western culture. Chinese painting (which is more precisely translated as "drawing with a brush" according to Fong, 2003) relied essentially on brush and ink for many centuries, perhaps making it easier to consider the concept of stroke (**Figure 9**). Shitao, one of the most famous Chinese painters, even wrote a treatise where the concept of stroke plays a major role (Shitao, 1710). Of course, accessing this information in translation from the original language brings its risks, but many of those issues have now been addressed, particularly by the philosopher François Jullien. Considering the importance given to transformations in ancient China (Jullien, 2009), this culture is worth considering here. Since the Chinese concept of stroke has now been made more accessible, what can we learn from it?

Two important aspects of strokes, addressed by Shitao, are worth highlighting here. The first is the importance given to the "empty wrist", when strokes are produced. This may give the impression that freedom of movement is completely embedded within strokes, but we must take care not to interpret this too quickly as "complete freedom in an Euclidian space": in the next part, we will see that simple composition processes in sketching suggest other kinds of spaces, with other kinds of freedom. A second important aspect of drawing with a brush, according to Shitao, is the distinction between "brush" and "ink" as two components of strokes. Both of them are equally important, and they act in a complementary way: "spirit dimension" is given to the ink by the movement of the brush, and ink provides "animation" to the brush by the use of different levels of darkness (Jullien, 2003). While remaining very cautious when it comes to adapting such ancient Chinese concepts in the present, this association between "brush" and "ink" can help us to escape from very deeply rooted habits when we address questions related to strokes, drawing and representation in general. Indeed, we are used to thinking in terms of "form" and "texture", and to considering that the latter is a secondary attribute to the first. Typically, in computer graphics, abstract surfaces will be defined first, followed by textures. In the next section, when I introduce the animation technique of morphing, I will show that a more balanced relationship is possible.

Part 3: The geometries of drawing

The first part of this chapter outlined how various transformations, as defined in modern mathematics, could be integrated into the way the brain works. The second part questioned how different transformations of the aspect of living bodies could be taken into account in the practice of drawing. This reflection led us to considering the geometries of strokes. This final part will address

the possibility of transforming the drawings themselves, and consequently accessing some of the geometries of drawing in a more general fashion.

In the first instance, I will stress the notion of composition, in the process of life sketching. I will infer the possibility of using a geometry “without” a point of view. This geometry will be mainly characterized by animation techniques such as “recomposition”, “travelling” or “scale change”, in an attempt to adapt the logic of mathematics to the domain of scientific drawing and film-making. Then I will suggest that a more advanced animation technique, known as “morphing” might provide us with deeper insight into the geometries of drawing.

From one body to the other: composition with space

Sketching living beings is a very ancient activity. For millennia, millions of draftsmen have been sharing a very simple problem: that of how to compose several views of living bodies on the same surface. Life sketching is a good example of this situation. Suppose, for example, that you are sketching in the presence of a living, moving subject. Say, a group of young children during a climbing lesson (**Figure 3b**). In such a context, it is helpful not to draw too big, since your memory does not necessarily allow you to develop a full page drawing. Furthermore, you don't want to use up too much paper on quick sketches. Consequently, it seems logical to draw one action here, and another action there, on the same page. Let's consider what happens when you draw first one child, and then what happens when you draw a second one (which may be the same child, in another situation)

In order to sketch children, you must certainly draw quickly, but speed alone will never be enough. The moment your pen touches the paper, you can be sure that what you just saw will have changed. So, you must use complementary strategies. Instead of beginning hurriedly, you can refine the way you look, in order to memorize aspects that will help you go on drawing what you wanted, when it is no longer visible. A knowledge of functional anatomy is very valuable in such cases: it allows you to feel how the body organizes itself in space, in relation to the physical surfaces on which forces are produced. Generally speaking, any consciousness of the structure of what you draw, whether it is a child or a mountain, will help you a great deal. Before the first stroke has even been drawn on the paper, it can be backed up by a whole spatio-temporal understanding of the situation, which can guide the following sequence of strokes, resulting in the drawing of one child, doing a certain action, on the paper.

Then, you can go on and make a second drawing. At this precise moment, a very interesting and fundamental aspect of drawing emerges: the second drawing can be made in relation to the first. More precisely, the spatio-temporal understanding that I just described can be used to find an interesting place for the second, not only in relation to the frame of an empty page, but also in relation to the first drawing. Then the third drawing can be made in relation to the former two, and so on.

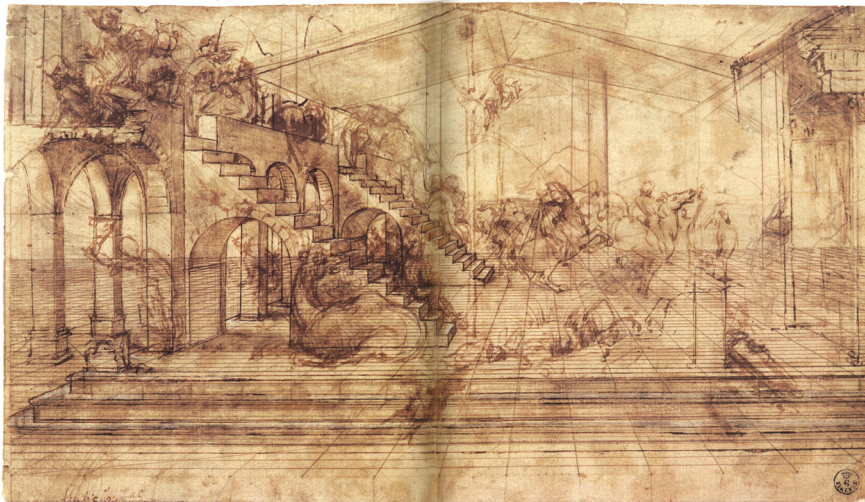
In general, whether the draftsman is working from a fixed model, from a moving model, or from his imagination, the process we have just described is always possible: it simply adapts to different durations. This composition work seems simple, maybe even trivial. However, it is far more than what today we would call a “graphic composition” in two dimensions, because the drawings, like the strokes they are made of, are not “flat graphic objects”. Each of them generates its own spatiality, and this spatiality can be connected to the spatialities of the others. Thus, in order to find a proper place for each new drawing on the page, one has to evaluate the spatiality involved in the former



a



b



d



c

Figure 4: Geometry without a point of view has been extensively used in both occidental and oriental art, typically for the study of living beings.

a: Leonardo da Vinci, movement study with men at work, 1506-1508.

b: Hieronymus Bosch, various figures study, mid-16th century.

c: Katsushika Hokusai, Manga vol.9, 1819.

Geometry with a point of view, i.e. linear perspective, has been used alongside by Renaissance artists for other composition purposes.

d: Leonardo da Vinci, linear perspective study for The Adoration of the Magi, 1481.

ones, and make choices in order to adapt the new drawing to its context. All this mental effort relies on the capacity of our brains to imagine transformations. Now we will look at how this composition process might be used to characterize an unexpected geometry in the brain, with a direct application in the epistemology of representations.

Geometries with or without a point of view

The compositions of life sketches described above highlight a very interesting characteristic, which has so far gone largely unremarked: although the compositions have a highly developed spatiality, they do not use a “point of view”. Indeed, in such compositions, it is impossible to say that the “observer” was “here”. It is certainly possible to say from which direction each body was “represented” or “viewed”. But it is simply not possible to use this information to define a particular point or place from which the entire scene has been viewed. In a similar way, this type of drawing clearly escapes the notion of “instant”, since it does not represent an action taken at a particular moment: it opens up instead a range of possible temporal interpretations. Compared to the geometry “with” a point of view, as will be further evidenced, the geometry without a point of view prevents the image from being identified with what a viewer can see from a particular place, at a precise moment.

Such a geometry has been extensively used historically. Cave art, in the grotte Chauvet or in the grotte de Lascaux, shows this kind of composition. Much later, Chinese painters mastered this absence of point of view. Interestingly, this may perhaps be linked to the fact that the verb “to be” does not exist in the Chinese language. Indeed, in Western philosophy and aesthetics, the verb “to be” plays a central role in the unending quest for “identity” and “essence”, with unbroken reference back to Aristotle. The epistemologist Alfred Korzybski characterized this use of “is”, in the form of an identification between two concepts, when we say “something is something else”. He showed that this way of speaking tends to induce confusion between levels of abstraction (Korzybski, 1933). In our case, the identification of a painting with the particular object that it represents, or with the point of view that it uses in time and space, is strongly related to a philosophy built around the verb “to be”. This might simply have no meaning in the Chinese cultural context, before the end of nineteenth century.

The nineteenth century was an extremely rich period of cultural exchange between East and West, whether welcome or not. While China was discovering some of the new constraints of Western civilizations, including central perspective, Japanese artists like Hokusai produced renowned collections of woodcuts, where compositions are clearly based on a geometry without a point of view (**Figure 4c**). Such woodcuts became an important influence on Western modern artists such as Monet or Rodin, who were beginning to break free from the established academic rules.

¹ The reader with a background in mathematics might think, with reason, that what I am describing here is simply linear perspective with a point of view at infinity, or parallel perspective. This kind of perspective is largely used to produce very neutral representations of space in an engineering context. By introducing a “geometry without a point of view” my intention is to stress the composition freedom it provides and to stay close to the experience of drawing, rather than to identify this process with a special case of central perspective. Indeed, fundamental aspects of drawing tend to be discarded when the idea of “point of view at infinity” is introduced, especially texture, which is strongly related to proximity.

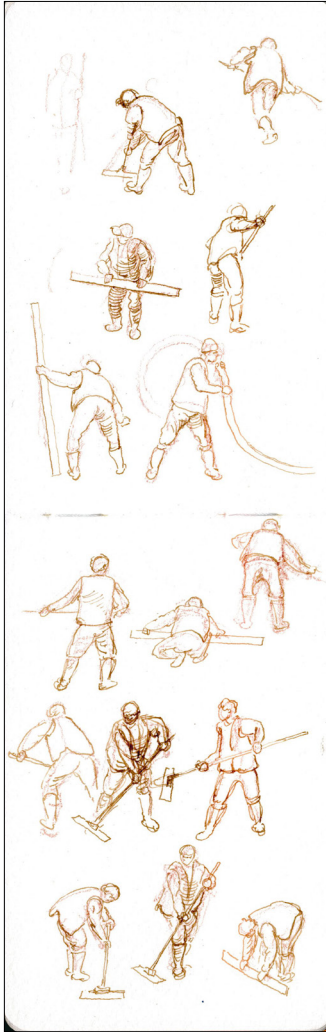


Figure 5: Compositions without a point of view can be easily recomposed, while maintaining or enhancing their spatiality. Drawing by R. Chabrier.

However, geometry without a point of view was not unknown to Western art at all. In fact, Leonardo da Vinci shows a thorough practice of it when he works on cats, horses or workmen (**Figure 4a**). The drawings and paintings of Hieronymus Bosch are evidently based on a similar process (**Figure 4b**). In this type of drawing, the direct observation of successive moments is often combined with imaginary developments on the same subject. To appreciate them properly, we must pay attention to all the possibilities of spatial and temporal combination between bodies. Depending on whether he attributes the different sketches to a single body seen at different times, or to several bodies that coexist, the spectator's gaze can appreciate the passage from one posture to another, or alternatively, he can travel in a common space.

As we have seen, a geometry is defined by a set of possible transformations. Geometry “without a point of view¹” offers considerable possibilities for recomposition by simple displacement of the represented bodies: a double-page of one of my own sketchbooks can be recomposed in another format (**Figure 5**). This last transformation does not allow any random reorganization, however: one must be careful to maintain a sufficient dialogue between the spaces occupied by the characters. It is also possible to move your gaze onto the composition, to bring it closer or move it further away, to focus or expand, without losing the spatiality it offers. All these observations would of course be trivial if we were interpreting the composition in a two-dimensional space: they become interesting if we adopt a spatial interpretation of the drawing.

A great paradox, as far as Renaissance artists are concerned, is that they are often considered as discoverers and promoters of “the” central perspective, while they were actually mastering both techniques: using a point of view, or not. Indeed, geometry “without” a point of view can be compared to geometry “with” a point of view when comparing two drawings by Leonardo da Vinci (**Figure 4a** and **4d**). In the first case, as we have already seen, the space is structured by the relation between the horse bodies. In the second case, the space is primarily structured by a grid. In this instance, we can say that bodies do not structure space: they are placed, in the second stage, “in” a well-defined three-dimensional space that pre-existed them.

It can be noted, in the drawing with the grid, that the lines converge in depth towards a vanishing point. The existence of the latter is due to the laws of projection of a scene on a plane, from a particular point of view. Opposite the vanishing point is thus a privileged “place” for the draftsman and the spectator, which is simply the place that provides the best illusion of space (similarly, there was once a specific place for the king in theatres of the eighteenth century, which determined the design of the scenography). According to their place relative to this privileged observer, the representations of the different visible objects will then undergo specific deformations. Thus, in theory, objects drawn in the context of a central perspective cannot simply be relocated elsewhere: we must apply a new perspective transformation to adapt them to their new place. Similarly, if we want to pan above the picture: we should be able to update the entire transformation in order to maintain the illusion of depth in the context of central perspective.

Consequently, operations such as “recomposition” and “travelling” are more complex if a point of view is introduced into the geometry. The increase in complexity is even greater when it comes to “scale change”. This has huge consequences for the range of transformations that the brain can interpret, when it processes an image. In a previous study, I demonstrated that the absence of central perspective was fundamental in scientific drawing, since this geometry would limit the required

imagination (Chabrier, 2016). Since the present study is dedicated to space-time representations, I will focus here on the issue of animating drawing: in this case, the imagination of transformations in the brain has to be converted into methods and algorithms, in order to produce an animation that provokes the same effects (recompositions, travelling, scale change, or more complex phenomenon) in the brains of as many people as possible.

Transforming drawings: from compositing to morphing animation

Animated movies can use many different media: drawings but also clay, paper cuts, and so on. Here, I will focus on animated movies which are mainly based on drawings (or paintings) that were initially made on a physical background. In this instance, the animation consists of transforming those drawings. Consequently, knowing if those transformations are related to a geometry with or without point of view is essential, since this will alter the effect of the animation in the viewer's brain.

As we have already seen, the absence of point of view appears natural in scientific drawing. Thus, in animated scientific movies based on drawing, many transformations can take advantage of this absence. The film “Birth of a Brain” (**Movie [1]**) can be considered as a demonstration of those possibilities: drawings are composed and recomposed, the spectator's gaze travels across the compositions, and important scale changes occur without inducing confusion or an excessive vertigo sensation. In practice, all those transformations are produced today in the context of “compositing” software. This word gives name to an important category of production tools, which offer the possibility of manipulating multi-layer images and altering those layers with special effects and transformations. Such software gives concrete access to the fundamental operations that we began to distinguish in a geometry without a point of view: animating a form along a trajectory, zooming inside a composition, and so on. Another important feature is the possibility of making layers appear or disappear, by controlling their opacity and the way their textures are mixed. Consequently, both movement and texture can have equal importance in the context of compositing software, if used properly.

In “Birth of a Brain”, in addition to these basic operations, other more complex transformations have been used. At certain times, cells are moving or differentiating, so the movie has to show significant transformations of their structures. At other times, it is necessary to turn the embryo around and slightly change the view. In order to address such transformations related to changes in the aspect of living forms, we have two complementary possibilities. The first is to draw a sequence of drawings and to display it at the appropriate frame rate. This is the strategy in traditional animation, which is a natural extension of the sequence drawing that we described earlier. The second possibility is to compute intermediate images between drawings, using a technique called morphing. I will briefly describe these two options in the context of “Birth of a Brain”, before analysing morphing in more detail.

“Birth of a Brain” uses traditional animation for a few very brief actions, for example when a microglial cell phagocytises a dead neuron. In this case, displaying several drawings from a sequence, one after the other, produces an obvious appearance of motion. However, an image sequence can be used with different timing. The display of the sequence of embryo drawings (**Figure 2b**) is spanned throughout the movie, using cross-fades for transition between one step and another: here, we rely

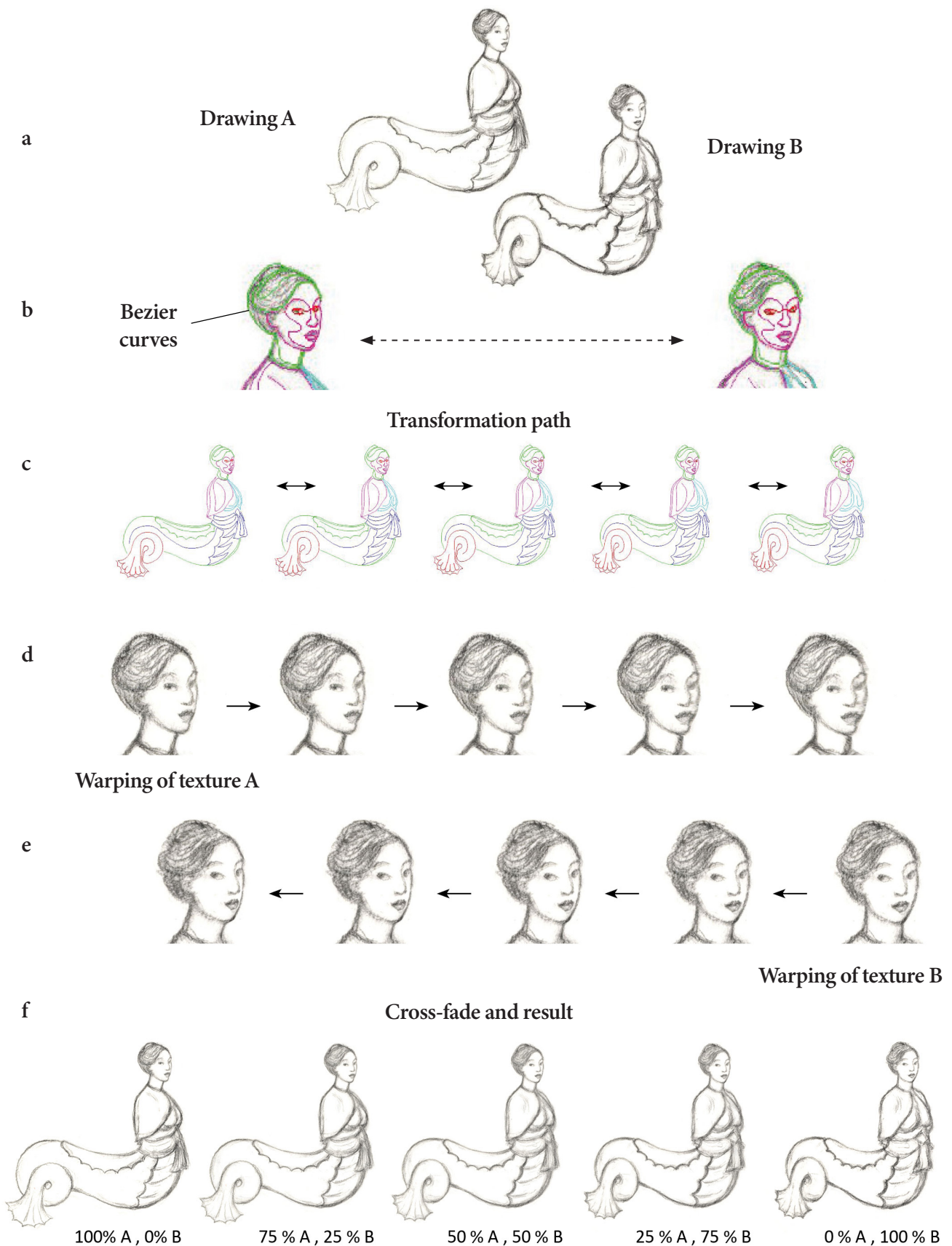


Figure 6: Step-by-step description of morphing between two drawings: in this case, Bezier curves allow fine control of the transformation of strokes. a: A draftsman creates 2 drawings. **b:** The animator specifies corresponding curves on the 2 drawings. **c:** A transformation path is computed on the basis of interpolation between those curves. **d:** The drawing A is transformed (warped). **e:** The drawing B is transformed (warped). **f:** Finally, a transformation from animated texture A to animated texture B is computed (Cross fade).

on the more long-term memory of the spectator to induce the concept of a slow, progressive transformation. In any case, fast or slow, the feeling of transformation induced by such image sequences cannot be entirely continuous or fluid: if displayed at a slow rate, one feels each different step, and if displayed fast, one feels a vibration of strokes. This vibration, by the way, is not a defect: it can give a sense of energy to animated drawings.

However, in certain cases, transforming the drawings in a continuous way, without vibration, can prove very useful. This is one of the main purposes of morphing-based animation. With this technique, an interpolation between two consecutive drawings in a sequence can be computed, in order to produce a fluid transition. This opens up new possibilities for guiding the attention of the audience along a scientific narrative. For example, in the early stages of embryo development, in “Birth of a Brain”, the body of stem cells moves up and down while they divide. With conventional animation techniques, such repeated movement would cause a lot of visual distraction, and make the narration difficult. Morphing animation helps by creating a fluid movement for a whole population of cells, without over-burdening the attention of the audience.

Morphing can also be used to create transitions between two views of a body. In that case, the resulting animation is a turning movement. This operation can be very important in scientific transmission, since the most efficient view for exploring an object is often slightly different to the most efficient view for explaining a more detailed structure or process related to it. Providing such a transition between two views to an audience can be critical when explaining the early stages of the embryo, because most spectators will not know where to direct their attention. In that case, a slight turning movement, coordinated with pan and zoom, can be essential in order to orient properly the attention of the viewer. The fluidity of morphing is very important here, because it drives the attention far more efficiently than a simple cut transition between two images, due to the optic flow induced by continuous texture transformations.

Inside morphing: warping and cross-fade

Technically speaking, morphing appears to be a way of calculating intermediate images between two images. In practice, it breaks down into two operations: the first is an image distortion, called “warping” and the other is a cross-fade. Interestingly, warping was invented in the 1980s during the rise of digital animation (Heckbert, 1989) while crossfade was already being used by Daguerre in 1822. Before he invented his photography technique, he had the idea to paint the two sides of a canvas and to mix both images by illuminating the back. Strictly speaking, morphing cannot be reduced to either old or new technology: it uses an interesting combination of the two.

In order to understand how warping and cross-fade are combined, consider for example (**Figure 6a**) two drawings of a mermaid inspired by a vintage merry-go-round, seen from a slightly different angles. From these two views, we will create a slow turning drawing². First, using appropriate software, we will define deformation constraints on each of the two drawings, in the form of Bézier curves as shown in colour (**Figure 6b**). Each of these curves is two-dimensional, so the curves on one drawing include no information about depth a priori. However, those curves can be animated:

² This merry-go-round can be seen in the Musée des Arts Forains in Paris. In the late nineteenth century, adults could take a ride on it for the simple pleasure of feeling the world turn slowly.

coaxial circles, as in Fig. 520, and then filling into the new system,

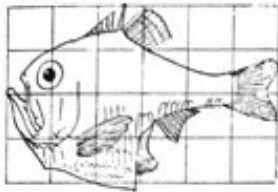


Fig. 517. *Argyropelecus Olfersi*.

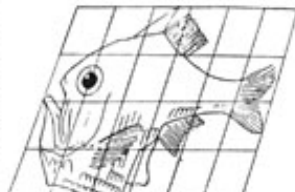


Fig. 518. *Sternopyx diaphana*.

space by space and point by point, our former diagram of *Scarus*, we obtain a very good outline of an allied fish, belonging to a neighbouring family, of the genus *Pomacanthus*. This case is all the more interesting, because upon the body of our *Pomacanthus* there are striking colour bands, which correspond in direction very closely

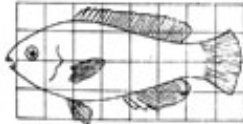


Fig. 519. *Scarus* sp.



Fig. 520. *Pomacanthus*.

to the lines of our new curved ordinates. In like manner, the still more bizarre outlines of other fishes of the same family of Chaetodonts will be found to correspond to very slight modifications of similar

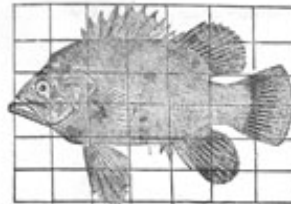


Fig. 521. *Polyprion*.

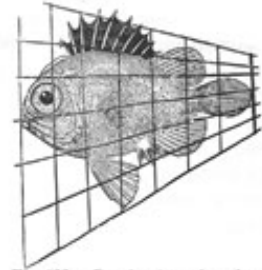


Fig. 522. *Pseudopriacanthus altus*.

of triangular, or radial, coordinates for the rectangular ones in which we had inscribed *Polyprion*. The very curious fish *Antigonia capros*, an oceanic relative of our own boar-fish, conforms closely to the peculiar deformation represented in Fig. 524.

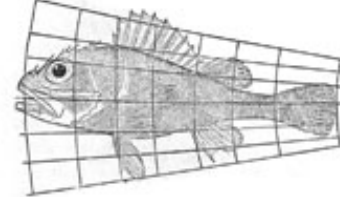


Fig. 523. *Scorpaena* sp.

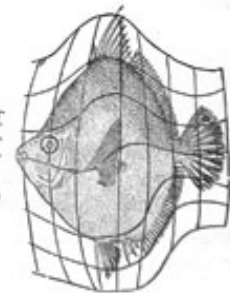


Fig. 524. *Antigonia capros*.

Figure 7: The principle of morphing anticipated by D'Arcy Thompson in his book «On growth and form» (1917). In practice, controlling morphing with such a grid allows insufficient control of the transformation of strokes.

we thus specify how the curve associated to the left cheek in drawing A must be transformed to go to land on the left cheek in drawing B. While this transformation occurs in the plane, it integrates some information about space, since the transformation of the curve, while we turn the object around, depends on the depth. The set of all these curves allows us to define a global planar transformation of the image (**Figure 6c**) which brings all the aspects represented in the drawing A to the place which corresponds to them (more or less precisely) in drawing B. We thus obtain two sequences of images: one which distorts drawing A in order to put it in the position of drawing B, and the other doing the opposite (**Figure 6d** and **6e**).

However, half-way through the respective transformation (warping) of A and B, the textures of the strokes begin to lose their adaptation to the space they should suggest. To compensate for this problem, we have to make a crossfade between the two sequences: while A is deformed, its texture is gradually replaced by that of B, so that at the end of the transformation the strokes of B have taken the place of those of A, with the correct texture for this angle of view. The whole of this transformation induces the sensation that the drawing turns like a rigid object, according to a vertical axis (**Figure 6f**; see **Online resource [a]** for an animated visualization).

However, all mathematical operations have been performed in the two-dimensional plane, where ordinarily only rotations with an axis perpendicular to the plane are permitted. By introducing a well-controlled non-linear deformation (i.e. warping) and a change of texture (i.e. cross-fade), the morphing thus makes it possible to evoke tri-dimensional transformations of drawings.

What does morphing enable access to?

In the above example, I showed that morphing could produce a visualization of rigid movements in three dimensions, using two drawings. Keeping in mind that drawings are made of strokes, we can try to establish if the concept associated with stroke in China, as we saw at the end of the previous section, could be used in that situation. I, personally, am tempted to say that both “brush” and “stroke” are taken into account in that technical process, thanks to the association of curve movements and texture changes. This might indicate that morphing can be considered not only as a special effect, but may give access to fundamental aspects of space-time geometries in the brain. We will consider here the kind of imagination that morphing gives access to, and progressively turn to more geometric aspects of the control of deformation, which may have an interesting scientific significance.

To begin with, morphing gives access to metamorphosis. This concept has a long history, from poetry to science. In his famous book “On Growth and Forms” (1917), long before digital imaging existed, D’Arcy Thompson demonstrated the possibility of geometric transformations between the bodies of two species. Planar transformations between the side view of various fishes or animal skulls, visualized through a grid, allowed him to question evolution and development (**Figure 7**). The scientific concept had limited scientific success, but the concept of transforming one image into another remained. As we just saw, it was realized in the 1980s with morphing, largely outside of any scientific endeavour. A typical example would be the film “Willow”, in which the viewer could watch a tiger turning into a woman. Because of the difficulty of controlling transformations precisely, morphing and warping were usually associated with transformations defying physical reality: significantly, one of the first software packages specifically for creating these effects was named “Elastic Reality”.



Figure 8: Raw material for an animation of cave drawings with warping and morphing. Photography Marc Azéma, drawing R. Chabrier.

a: The (planar) texture of the image can be transformed in such a manner that the animation remains coherent with the spatiality of the representation.

b: Working on the computer is not sufficient: new drawings on paper, with real strokes, are often mandatory to specify this transformation correctly. Note that this association of three figures generates a composition without a point of view.

In addition, in line with the work that I was able to carry out between 2000 and 2016, morphing can be used to visualize and control other kinds of changes in the form and structure of a body: rotations, articulations, rigid movement, and of course deformation when necessary. Ultimately, according to my experience of film-making, a multi-layered approach in morphing appears to be the most complex transformation in the compositing repertoire. Without it, it is very difficult to find any coherence in the disparate set of so-called “2D” animation techniques. Conversely, since the concept of morphing is closely related to drawing, it allows us to better integrate hand-made drawings in the context of fluid digital animation, from paper to screen. This approach gives access not only to spectacular metamorphosis, but to a subtler kind of imagination, based on the propensity of the drawings to be transformed by the imagination of their viewers.

Before the advent of morphing, there was no standard technique for visualizing what we imagine when we assess the possible transformations of a drawing. Depending on the animation approach, either the movement or the textures had to be greatly simplified, despite the fact that both contribute strongly to the spatiality of the stroke. With the use of morphing, during the 2010s, it was possible for me to animate Leonardo da Vinci’s drawings (**Movie [2]**), as well as the drawings from the Chauvet cave (**Movie [3]**), while respecting the texture and geometry of the original strokes. These animations are not intended to develop a story that goes beyond the intention of the original artists. They simply seek to make visible a propensity of the drawing to be transformed in a certain way. But for this to happen, the animation process must not distort the geometry of the drawing. In particular, strokes with a spatial value must not be treated as lines or areas in a flat frontal plane.

A remarkable aspect of morphing is that it forces the animator to become much more aware of the transformations he uses. Indeed, rotations in three dimensions raise perceptual problems, depending on the way they are projected on a plane: a rotational movement whose axis is in the plane of the image will result in a linear trajectory of textural elements, as in the case of the mermaid. Alternatively, a rotational movement whose axis is perpendicular to the plane will move the texture of the image along a circular trajectory around a point of rotation. Those two particular types of spatial rotations are quite easy to specify from inside the projection plane, but general cases are more puzzling. If we show a head rising from a three-quarter view, for instance, the axis of this rotation (in space) will be neither in the image plane nor perpendicular to it. In such cases, if we want to animate the texture of the image according to this rotation, it will be necessary to specify a combination of rectilinear trajectories and circular trajectories. The morphing animation has to deal with this difficulty of specifying the appropriate transformations while staying in a plane. In practice, for example if we want to animate cave art like in **Movie [3]**, specifying the transformation is often impossible without hand-made reference sketches on paper (**Figure 8**).

In 3D synthetic images, this kind of problem does not arise: the animator has three parameters of rotation in space to control the movement of the objects he manipulates. Morphing therefore introduces a difficulty, due to the fact that it uses coordination between transformations “in the plane” and texture changes. Here we can observe that the brain largely uses a system in the form of maps, which appears more adapted to planar deformations than to tri-dimensional transformations. An appreciation of the specific difficulty of morphing could therefore strengthen the idea that within the brain, taking into account rotations in space requires a more heterogeneous process than what one might imagine.

Finally, we should bear in mind that the morphing of drawings does not use a camera, and conse-

quently does not necessarily use a point of view. We have seen above that geometry without a point of view was based on the possible relations between the drawings, without any need to explicitly structure the space with a three-dimensional grid. Similarly, the morphing technique that I described is based on the use of curved lines to specify the transformation between drawings, without using a “deformation grid”. It should be noted that a morphing action can be specified with an explicit grid (as in the images of D’Arcy Thompson), but in practice this approach turns out to be far too imprecise. Consequently, the effectiveness of Bézier curved lines in specifying the transformation of the main strokes in a drawing might make sense. As Tamar Flash (2007) remarked, Bézier curves are parabola, and such parabola seem to be used by the brain to generate tracing movements, because of their curvature properties in equi-affine geometry. Thus, morphing animation could have an epistemological role in addition to its applications in film making: it could also highlight the role of curvature in the representation of space in the brain.

Redefining the epistemological role of drawing: a work in progress

Since drawing began to be replaced by photography, its epistemological role has not been clear. The digital revolution introduced even more tools, seemingly enabling the same tasks of representation as drawing, but without the constraints of materiality or lengthy practical training. But during the last decade, through a number of publications, workshops, movies and exhibitions, it has become evident that drawing gives access to a form of knowledge, or concentration, that could not be achieved through other media. In this new context where animated images are at least as important as static ones, redefining the epistemological role of drawing seems necessary.

Such a task can be expressed more simply: what does drawing give access to? In this text, I focused on how drawing could provide access to various geometries of living beings. As I have demonstrated, thinking in terms of transformations, thanks to the model provided by modern geometry, makes it easier to compare a number of drawing practices, with different historical and epistemological backgrounds. The concept of stroke seems a mandatory means to unify them. But if we want to think and investigate further, in order to grant to the concept of stroke the importance it deserves, differential geometry as well as Chinese painting treatises appear to be required. Acting as a bridge between such distant domains, in terms of space, time and logic, might be one of the major epistemological roles of drawing.

This role is more than that of an academic pastime, since today the representation of life raises important issues. As far as geometry is concerned, we currently live in a paradox: while projective geometry is no longer taught to anyone apart from specialists in mathematics and computer graphics, central perspective and flat projection have never been so present in the images and films that are available to the public. By reducing the world of representations to a dialectic between “2D” and “3D”, it is likely that we have lost in the sciences, arts and architecture, an important aspect of our ability to take into account the spatiality and diversity of living beings in our representations. Since the aspects of life that are not properly represented tend to be neglected in a world dominated by images, this may have contributed to a serious lack of awareness of environmental problems in recent decades. On this basis, emphasising a geometry without a point of view, where curvature plays a major role, seems necessary if we want to re-develop the ability to represent the spatiality of life in an accessible way.

Designing appropriate training that integrates the practice of drawing might be the beginning of a solution for re-developing access to this spatiality. For three years now, I have been running an experimental course for engineering students at the Ecole Polytechnique (Palaiseau) entitled “Drawing and Animation for Scientific Transmission”, where the basics of representation are addressed simultaneously through drawing and animation. Short courses for researchers in cell biology and biophysics have also been successfully tested at Institut Curie, as well as a re-introduction of drawing by hand at different levels of scientific transmission (Chabrier, Janke, 2017). Similarly, the scientific iconography department of the National Museum of Natural History recently established, under the direction of Didier Geffard-Kuriyama, a doctoral module in scientific drawing that had not existed for twenty years. Thus an interesting dynamic seems to be emerging in France in the scientific field. In addition, interest in drawing today cuts across the arts as well as design, architecture, medicine and other disciplines. In the United States and the UK, the Thinking through Drawing network, initiated by Andrea Kantrowitz, Angela Brew and Michelle Fava (2011), leads meetings, workshops and interdisciplinary debates that testify to the vivacity of the field.

As far as I know, the way in which a stroke produces a sensation of space, and the way in which several strokes cooperate to construct a spatial representation has not yet been modelled more widely. Today, several approaches seem to be converging towards a better understanding of this phenomenon using scientific methods: one can quote again the works of Viviani and Flash (1995), which introduced a power law in the neuronal planning of trajectories by connecting curvature and tangential velocity, and the subsequent advances when a more extensive connection with geometry was made (Flash, Handzel, 2007; Bennequin et al., 2009). Creative robotic research currently aims at reproducing the process and the structure of the stroke in a more complete way, with a goal of developing drawing robots (Tresset, Fol Leymarie, 2013; Gülzow et al., 2018). In this field, it is worth noting that the team of Frederic Fol Leymarie at Goldsmiths, University of London works alongside on the modelization of calligraphy (Berio et al. 2017), and on life sciences problems involving 3D representations, such as the visualisation of folding and docking of proteins (Tod et al. 2015).

In general, modern scientific drawers have to use 3D information. Hence, even though they can be distant, the worlds of drawing and computer-assisted projective geometry (i.e 3D), are no longer separated: in fact, they are complementary, and a large amount of time can be devoted to mixing the two types of information. In computer graphics, attempts to create a more direct link between planar representation and 3D also exist. The “sketch-based modelling” approach consists of interpreting as 3D information diagrams or drawings made in two dimensions. Several specific cases have been approached in this way, for example the spatial interpretation of blood vessel schematics (Palombi et al. 2011), notably by postulating that ellipses can be interpreted as sections of vessels. Currently, at Ecole Polytechnique, I am working with Marie-Paule Cani’s team on a broader spatial interpretation, from strokes on paper to their display on an autostereoscopic screen.

Given these activities in various complementary fields, a “true” modelization of strokes may appear as possible in short term. Indeed, modelling a phenomenon is often considered a positive step for scientists. In artistic practice, however, nothing is more dangerous than thinking about the stroke through modelization. In my experience, the drawer always runs the risk of focusing his attention on an aspect of the stroke that could have been important yesterday, but which is no longer relevant today. If we want to develop robust research on the spatio-temporality of the stroke,



Figure 9: The epistemological role of drawing appears to be related to the “stroke”. This concept has been developed primarily in Chinese civilization, on the basis of painting (or drawing) with brush and ink. The stroke was noticeably used for painting landscapes which do not use a point of view. Shitao, lake Cao, 1695.

in terms of transformations, we must therefore rely on a safeguard that prevents the concept from closing up and becoming sterile. The mere practice of drawing in real life, on real paper, provides such a safeguard. But once again, it is a Chinese concept which seems to best account for how the world can be connected to the stroke (and how the wealth of associated geometries in the brain can harmonize, if we can risk using such modern ideas). This is the “Oneness of brush stroke”, also translated as “Unique brush stroke”, expressed by the painter Shitao at the beginning of his treatise (1710, translation by Coleman, 1978):

“The principle of oneness of strokes is such that from non-method method originates; from one method, all methods harmonize [...] With regards to the delicate arrangement of mountains, streams, and human figures, or the natural characteristics of birds, animals, grass, and trees, or the proportions of ponds, pavilions, towers, and terraces, if one’s mind cannot deeply penetrate into their reality and subtly express their appearance, one has not yet understood the fundamental meaning of the oneness of strokes.”

With or without a brush, the whole treatise of Shitao can still be read today as an invitation to access the diversity and the dynamics of the world through the use of strokes. Although this citation should not be interpreted without its historic and Taoist context, it seems a rather clear warning of the risks of reducing the stroke too quickly to one or other of its applications. Methods and rules can be useful, but non-method is really at the core of drawing, especially in the daily use of a sketchbook.

Conclusion

The main goal of this text was to introduce a new, more effective and relevant vocabulary, in order to understand the epistemological role of drawing. Thanks to the work done by mathematicians, the word “transformation” now signifies an extremely powerful and general concept that covers events related to both movements and changes. If we use this concept of transformation in the context of drawing, we find that it is closely associated to the concept of “stroke”.

“Stroke” provides an excellent alternative to the word “line” in many situations. Thanks to its use in the field of Chinese painting, it embraces the materiality, the spatiality and the temporality that develop in drawing, and especially in the sketch. Using the word “line” often results in implicitly reducing the stroke to a graphic object in two dimensions. Where possible, I suggest reserving “line” and “curve” for more explicitly abstract and/or mathematical uses, since in such contexts those concepts are far more clearly defined.

Finally, the word “texture” puts a necessary emphasis on some of the dimensions of the stroke that are currently the most neglected because of the dematerialization of tools for design and creativity. Animation approaches such as morphing can be used to make the role of textures in the representation of space more evident.

Thus, using more adapted concepts, we have seen that it is possible to shed light on a geometry without point of view, which has not previously been fully described. It appears to have constituted a fundamental basis for the drawing of living beings since the very origins of this practice. The characteristics of this geometry in terms of composition, recomposition and movements also allows us

to understand better how to develop the potential of modern drawing approaches, for scientific or non-scientific purpose. In animation, museography or design, an important challenge today is to complete and balance the extensive development of tools and images based on flat projection and central perspective with a geometry that does not rely on a unique point of view. This geometry is easy to access thanks to drawing, and its maintenance may be equally important for the human brain. Its re-development might create the opportunity for a new association between mathematics, drawing and neurosciences.

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Methods

The morphing animations were mainly created with the Combustion compositing software and the associated Re:Flex morphing plugin.

Movie references

- [1] Birth of a brain (2016): <https://vimeo.com/183004485>
- [2] Leonardo da Vinci – Motion Picture (2013): <https://vimeo.com/84689530>
- [3] Cave art animation studies (2011-2016): <https://vimeo.com/171478742>

Online resource

- [a] Chabrier, R (2014) Le Morphing, l'art de transformer les images:
<https://images.math.cnrs.fr/Le-Morphing.html>
- [b] Popescu-Pampu, P (2016) De la Topographie à la Géométrie:
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