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# Nonlinear modelling of strongly coupled piezoelectric harvesters

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**Abstract**— This paper presents a nonlinear model that allows accurate consideration of the losses of piezoelectric harvesters with strong electromechanical coupling coefficient. The model takes mechanical and dielectric nonlinearities into account and is experimentally validated on a PMN-PT based cantilever at various acceleration levels.

## I. INTRODUCTION

Vibration energy harvesters based on piezoelectric resonators are promising for powering wireless sensors nodes. Yet, any mismatch between the resonant frequency of traditional harvesters and the vibration frequency can drastically decrease the scavenged power and make them ineffective. Electrical techniques able to tune the resonant frequency of piezoelectric harvesters have been proposed as a solution. This approach requires energy harvesters with very strong global electromechanical coupling coefficients  $k^2$  ( $>10\%$ ). Nevertheless, when strongly coupled materials are used, the presence of nonlinear dielectric losses limit the harvested power [1]. A model that takes into account nonlinear dielectric losses and predicts the harvester behavior in response to variable acceleration level is lacking from the literature. This paper proposes a new nonlinear model corroborated by experimental results for the losses consideration of strongly coupled energy harvesters. The studied system and the model are introduced in section II. The prototype configuration and the experimental validation are detailed in section III.

## II. MODELLING

### A. Studied device

The harvester design relies on a bimorph cantilever with a bulky proof mass at tip end (Figure 1.a). The two piezoelectric layers share the same length as the substrate (central layer), have the same thicknesses and are entirely covered by electrodes at the bottom and the top surfaces, which are connected in parallel. Figure 1.b depicts the proposed cantilever during bending. We model the proof mass with an equivalent point mass defined by a mass  $M_t$  a rotary inertia  $I_t$  at a distance  $D_t$  from the “free” end of the beam on the neutral axis.  $\ddot{w}_b$  is the base acceleration and  $r$  is the relative transverse displacement of the beam end.

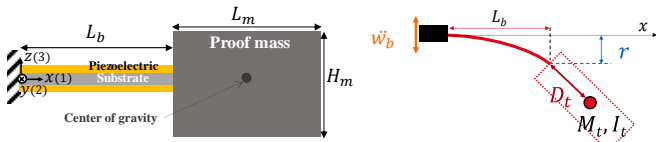


FIGURE 1. A) CANTILEVER WITH PROOF MASS AND B) CANTILEVER DURING BENDING.

### B. Model introduction

The model introduced here is derived from the two degree of freedom linear model that we presented in [2] with a focus on the first resonant mode. The Rayleigh method is used to determine the equation of motion (1) and the equation of current (2). The parameter definitions are given in TABLE 1. In contrast to [2], the nonlinear terms  $K_2$  and  $\Theta_2$  are introduced to consider the mechanical softening and piezoelectric coefficient nonlinearity respectively. Moreover, linear and nonlinear dielectric losses are considered thanks to the coefficients  $d_1$  and  $d_2$ .

$$M\ddot{r} + Kr + K_2r^2\text{sgn}(r) - \Theta v - \Theta_2vr\text{sgn}(r) + (b_1r\text{sgn}(r) + b_2r^2)\text{sgn}(\dot{r}) = -B_f\ddot{w}_B \quad (1)$$

$$\Theta\dot{r} + \Theta_2r\dot{r}\text{sgn}(r) + C_p\dot{v} + \frac{v}{R_{load}} + (d_1\dot{v}\text{sgn}(v) + d_2v\dot{v})\text{sgn}(\dot{v}) = 0 \quad (2)$$

## III. EXPERIMENTAL VALIDATION

### A. Device presentation and experiment

Experiments have been performed on the strongly coupled ( $k^2 = 16\%$ ) cantilever based on PMN-PT single crystals represented in Figure 2.

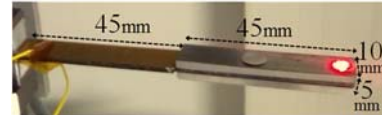


FIGURE 2. PMN-PT BASED PROTOTYPE

The prototype has been tested under vibration excitation to determine the output power and the displacement amplitude for various accelerations levels. The experiments have been done for 60 resistive loads  $R_{load}$  between 1 k $\Omega$  and 2 M $\Omega$  over 100 excitation frequencies.

The equivalent model parameters were determined by fitting the power and displacement amplitude to the measured values. The experimental harvested power and results from the model are given in Figure 3. The model parameters are given in TABLE 1.

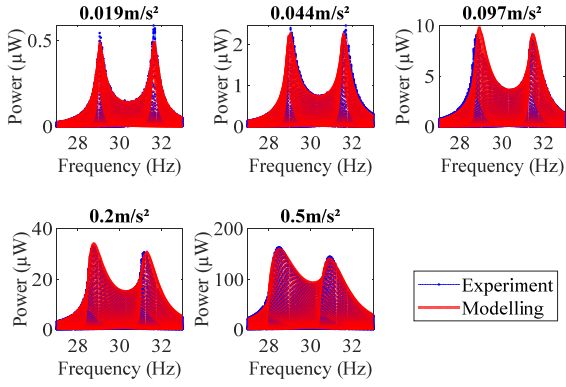


FIGURE 3. OUTPUT POWER AS A FUNCTION OF THE EXCITATION FREQUENCY FOR RESISTIVE LOADS AND VARIOUS ACCELERATION LEVEL

TABLE 1. MODEL PARAMETERS

Name	Coeff.	Value
Mass	$M$	35,2 g
Forcing term	$B_f$	23,1 g
Linear stiffness	$K$	1 178 N/m
Coupling term	$\Theta$	$-3,05 \times 10^{-3}$ N/V
Capacitance	$C_p$	$41,9 \times 10^{-9}$ F
Nonlinear stiffness	$K_2$	$-2,44 \times 10^5$ N/m <sup>2</sup>
Nonlinear coupling term	$\Theta_2$	1,22 N/Vm
Linear mechanical loss	$b_1$	12,8 N/m
Nonlinear mechanical loss	$b_2$	$8,57 \times 10^4$ N/m
Linear dielectric loss	$d_1$	0 F
Nonlinear dielectric loss	$d_2$	$2,03 \times 10^{-10}$ F/V

### B. Discussion

The presented model allows a good evaluation of the experimental behavior of our prototype. The model presented is therefore capable of predicting the behavior of strongly coupled harvesters with varying levels of acceleration.

One can notice that the linear dielectric loss coefficient is taken null in the model (TABLE 1). The nonlinear dielectric losses coefficient appears to be more relevant to consider the harvester behavior than linear ones. In order to analyze the influence of the dielectric losses coefficient on the harvested power, the power at optimal resistive loads is represented in Figure 4 as a function of the frequency for 3 acceleration levels. The power is represented with the consideration of the dielectric loss coefficient  $d_2$  (in red) and without its consideration (in blue). Not taking the dielectric loss coefficient into account is equivalent to using the model presented in [3]. The power for optimal capacitive and resistive loads is also plotted to deduce the expected results when the resonant frequency of the harvester is electrically tuned. Indeed, the harvested power from resistive and capacitive loads allows us to know the expected power using a non-linear electrical interface such as FT-SECE or regenerative SECE [4].

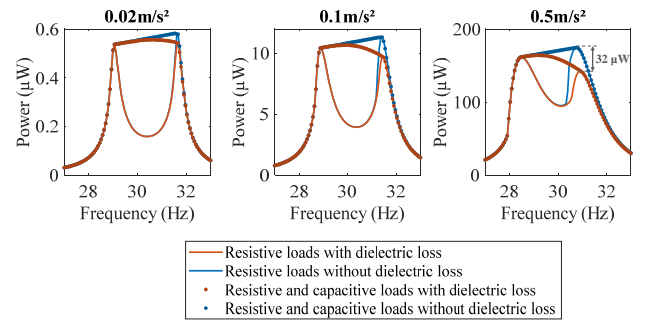


FIGURE 4. POWER DEDUCED FROM THE MODEL FOR OPTIMAL RESISTIVE LOADS AND OPTIMAL PARALLEL CAPACITANCE AND RESISTIVE LOADS

We notice that nonlinear dielectric loss coefficient has an important influence on the harvested power close to the open-circuit resonant frequency (around 31.5 Hz). Indeed, the load impedance magnitudes required to maximize the power are higher near the open-circuit resonant frequency and lower near to the short-circuit resonant frequency. This means that the optimal voltage close to 31.5 Hz is higher than the optimal voltage close to 29 Hz. As a result, as the energy loss per cycle increase with the voltage amplitude when we consider dielectric losses, lower output power around 31.5 Hz is expected compared to the case when these losses are not considered.

We can also notice from Figure 4 that the influence of dielectric losses increases with the acceleration level. While the influence is difficultly perceptible at low level (0,02m/s<sup>2</sup>), strong influence is noticed at high acceleration level (0,5m/s<sup>2</sup>). Considering dielectric losses, the maximal harvested power at the high optimal restive load is 32µW lower than the maximal power harvested at the high optimal restive load without considering them at 0.5m/s<sup>2</sup>. This represents a relative diminution of more than 18% of the expected power at the optimal resistive load.

### IV. CONCLUSION

This work presents a new model to consider the influence of nonlinear dielectric losses on the power harvested by strongly coupled vibration harvesters. The model correlates well with experiments. As dielectric losses strongly affect the harvested power, such nonlinear modelling is necessary in order to analyze the harvester behavior with varying acceleration level.

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