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# Predictive-reactive strategy for identical parallel machine rescheduling 

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#### Abstract

Through new technologies development, customers can create or cancel an order in real time, which disrupts the established production schedule. This realty forces many companies to quickly react for dealing with this situation through rescheduling processes. When efficiency usually measures the performance of a scheduling system, in dynamic environments, stability measures the impact of jobs deviation. Differently from previous works, this paper investigates a new approach to measure simultaneously efficiency by total weighted waiting times, and stability by weighted completion time deviation. This mix could be a very helpful and significant criterion in industrial and health care environments. This paper considers a parallel machine rescheduling problem with jobs arriving over time. Based on predictive-reactive strategy, a mixed integer linear programming model is developed, as well as an online iterative methodology to solve this rescheduling problem. Finally, numerical results are performed for studying the impact of efficiency and stability coefficient, as well as the computing time to solve the described problem.


Keywords: Rescheduling, Parallel Machine, Waiting time, Stability, Completion time deviation, Disruption.

## 1. Introduction

Due to the fourth revolution experiencing, today's industry is developing new technologies and solutions for the production process. In addition, the customers' practices are also changed. Henceforth, a customer can, at any time, create or cancel an order in real time. Indeed, these changed situations have a direct impact on work organization, especially production scheduling (Ivanov et al. 2016; Uhlmann, and Frazzon 2018; Da Silva et al., 2019). In fact, production planning is subject to unexpected events that disrupt the established schedule, such as: new jobs arrival, cancelation of jobs, machine failures, quality problems, shortage of raw materials, etc. (Li et al., 1993; Sabuncuoglu and Karabuk 1999; Katragjini et al., 2013). Hence, the decision makers are forced to quickly react to deal with these disruptions. Therefore, rescheduling is necessary to revise the initial schedule in a cost-effective manner (Vieira et al., 2003).

Rescheduling is the process of updating an established schedule in response to disruptions (Vieira et al., 2003). In the rescheduling literature, the classical efficiency criteria are often used to measure the performance of a schedule, including: the Makespan, the total completion time, the maximum lateness, etc. Differently from previous researches, this work considers a new efficiency criterion, which is the Total Weighted Waiting Times (TWWT). On the scheduling literature, the waiting time is the period that the job has waited before its execution in a system (Jurčišin and Šebo 2015). Indeed, the $T W W T$ could be a very helpful and significant criterion in industrial and health care environments. For instance, in production systems, it can represent
the waiting time of a job in front of a workstation, considering the importance of each job as the weight. In hospital systems, the waiting time can represent the delay between a patient's arrival and his actual treatment, considering the emergency level as the weight. Guo et al., (2013) is among the first works that considered the waiting time on rescheduling problems. The authors formulated a single machine rescheduling problem to minimize the sum of waiting times of rework jobs and the original loads with release time. They proved that the problem is NP-hard and designed a dynamic insert heuristic algorithm of polynomial-time. To fill the void in existing literature, in our study, we considered the $T W W T$ as an efficiency scheduling measure, which has been defined for the first time by Kan (1976), and classified as NP-hard for a single machine. To the best of our knowledge, there is not any rescheduling problem in the literature which discussed this criterion.

The efficiency usually measures the performance of a scheduling system, but, in dynamic environments, the impact of jobs deviation is also studied in the literature, known as stability measure (Wu et al., 1993; Pfeiffer et al., 2007; Zhang et al., 2013). This criterion measures the impact of disruptions induced by moving jobs during a rescheduling event (Rangsaritratsamee et al., 2004). In real life systems, when the sequence is changed, this matter may generate additional costs, such as reallocation costs, raw-materials reordering costs, etc (Rahmani and Ramezanian 2016). Ahmadi et al., (2016) studied a multi-objective jobshop rescheduling problem, in which the disruptions were caused by random machine breakdown. The authors considered the makespan as the efficiency measure, and calculated the stability with the average difference between the completion times of the predicted schedule and the realized completion time. In fact, when a disruption occurs, initial completion times of jobs may change and are replaced by real completion times obtained after disruption. Two evolutionary algorithms are applied to solve this problem, with the objective simultaneously combines makespan improvement and stability. Indeed, this paper presents a new approach to measure the schedule stability, which consists in calculating the difference between the completion times of the job in the initial schedule and in the new one, associating for each job a weight. We denote by the Total Weighted Completion Time Deviation (TWCTD) this stability criterion. In fact, when a job has a high weight (i.e. more important), it is difficult to disrupt it. This is the case of an urgent patient in a hospital, for example.

According to (Vieira et al., 2003; Herrmann 2006), there are two basic rescheduling strategies. The first is the dynamic scheduling strategy which consists in dispatching jobs when it is necessary and using the available information at the moment of dispatching. This strategy uses dispatching rules or other heuristics to prioritize jobs waiting to be processed at a workstation (El-Bouri 2012; Nurre and Sharkey 2018; Bektur, and Saraç 2019; Riahi et al., 2019). The second is the Predictive-reactive scheduling strategy that consists in generating an initial schedule in a first step, and to update the schedule in response to a disruption (Vieira et al., 2003; Duenas and Petrovic 2008). In our approach, we firstly generate an initial schedule with the objective of minimizing the $T W W T$. After a disruption, we generate a second schedule to minimize a multi-criterion combining simultaneously efficiency and stability, with the stability measured with the Total Weighted Completion Time Deviation.

The analysis of scheduling performance often requires more than one aspect (Loukil et al., 2005). Therefore, in the rescheduling literature, several papers deal with multi-objective function. For instance, Wang, D et al., 2015 considered a rescheduling problem disrupted by machine breakdown and deterioration effect. The authors aim to minimize a multi objective function, considering initial operational cost assessed by the total completion time and resource consumption, as well as the rescheduling cost assessed by match-up time and additional resource cost. In this paper, we established a relationship between both parts of the objective function, by associating a coefficient for each measure (i.e. $\alpha$ efficiency $+(1-\alpha)$ stability). We called $\alpha$ the efficiency-stability coefficient, and studied the influence of this coefficient on the system.

According to Alagöz and Azizoğlu (2003), in the flexible manufacturing systems, changes in jobs assignments to machines may generate some additional costs, like setup and tooling costs. Thus, the authors considered the number of jobs processed on different machines after the disruption as the stability measure in parallel machines rescheduling problem. In the operating rooms scheduling case, this matter can also generate additional costs related to the preparation phase before the operation. In our study, we have used the lexicographic optimization to reduce the number of jobs processed on different machines after disruption. This method is investigated in many papers dealing with the multi objective problems. It always provides a Pareto optimal solution (Chang 2015). It consists in obtaining an arbitrarily small improvement for the most important criterion through any loss in other less important criteria (Zykina 2004). In our work, we have assumed the bi-objective function considering simultaneously the efficiency and the stability as the priority function. From the optimal value obtained for this objective function, we minimize then the number of jobs processed on different machines, taking as constraint the optimal value of the priority function.

This work addresses a rescheduling problem on identical parallel machines, when the disruption is caused by the arrival of new jobs. A mixed integer linear programming model is developed to describe the problem, and an online iterative methodology to solve it. The proposed methodology provides, for each step, the optimal solution. The presented rescheduling problem has not been studied in the research literature before. The main contributions of this work are as follows:

- We implement a mathematical model based on sequence formulation, after we have improved it to match the parallel machines problem. Besides, we develop an online iterative methodology adapting this model to our rescheduling problem, and we discuss its limitation in function of the number of jobs.
- We consider for the first time ' $T W W T$ ' as a criterion in a parallel machine rescheduling problem, which is a very significant criterion in several environments.
- We adopt a new approach in parallel machine rescheduling problem by simultaneously considering the efficiency defined by the TWWT, and the stability as the Total Weighted Completion Time Deviation.
- We establish a lexicographic optimization, by minimizing the number of jobs which are processed on different machines after disruption.

The rest of the paper is organized as follows: we present in section 2 the literature review. Our research motivation is given in section 3. In section 4 the problem description, as well as the strategy that we propose to tackle it. In section 5, the mathematical model is presented. We then analyze the numerical results in section 6 . We conclude and propose some perspectives in section 7.

## 2. Literature review

In this section, we focus on the different issues studied in the rescheduling. The rescheduling literature is very rich. Vieira et al. (2003), Li and Ierapetritou (2008), Ouelhadj and Petrovic (2009), and recently Uhlmann, and Frazzon (2018) provided interesting reviews of the state-of-the-art of developing research on rescheduling at those times. In the rescheduling literature, the classical efficiency criteria are often used to measure the performance of a schedule, including the makespan (Yuan and Mu 2007; Zakaria and Petrovic 2012; Tang et al., 2016; Li et al. 2017; Wu et al., 2018), the total completion time (Lu et al., 2003; Yang 2007), the total weighted completion time (Hall and Potts 2010; Luo et al., 2018), the average flowtime, the average tardiness and the percentage tardy (Kunnathur et al, 2004), the total weighted tardiness (Mason et al., 2004; Kaplan and Rabadi 2015), the total flow time (Ozlen and Azizoğlu 2011), and the maximum lateness (Hall and Potts 2004). As well, the rescheduling literature has taken interest in studying different types of machine environments such as: single machine (Unal et al., 1997; Hall and Potts 2004; Hall et al., 2007), flowshop (Yan-hai et al., 2005; Katragjini et al., 2013), jobshop (Bierwirth and Mattfeld 1999; Mason et al., 2004; Salido et al., 2017; Yan et al., 2018; Larsen and Pranzo 2019), or openshop (Liu and Zhou 2013b). Moreover, there are also several papers dealing with parallel machines rescheduling problems. Yin et al., (2016) studied a rescheduling problem on identical parallel machines, and considered the total completion time as the original scheduling objective and the maximum time deviation or the total virtual tardiness with respect to the original schedule as the deviation cost associated with a disruption caused by machine breakdowns. The complexity of the described problem is studied and a pseudo-polynomial time solution algorithm is proposed. Hamzadayi and Yildiz (2016) treated a dynamic identical parallel machines scheduling problem, in which sequence dependent setup operations are performed by a single server. An event driven rescheduling strategy is proposed to control the dynamic system, with the objective of minimizing the makespan. The authors indicated that the simulated annealing based complete rescheduling approach produces the best performance. Alagöz and Azizoğlu (2003) addressed a rescheduling problem in parallel machine environments under machine eligibility constraints. The authors considered the total flow time as efficiency measure, and the number of jobs processed on different machines after disruption as stability measure. Several heuristics are proposed to provide a set of approximate efficient schedules. For other related papers, dealing with parallel machine rescheduling, the reader may refer to Church and Uzsoy (1992); Vieira et al., (2000); Azizoglu and Alagöz (2005); Curry and Peters (2005); Duenas and Petrovic (2008); Arnaout (2010); ShangGuan et al., (2012); Liu and Zhou (2013a), and recently Wang et al., (2018); Ferrer et al.,(2018); Kovalyov et al., (2019); Cunha et al., (2020).

The waiting time is the period during which a job is waiting before its execution in a system (Jurčišin and Šebo 2015). In the scheduling literature, this criterion is used to measure the schedule efficiency. Guo et al., (2016) treated a rescheduling problem with the objective of minimizing the maximum waiting time, inspired from quartz manufacturing industry, where waiting times represent the waiting for some materials before the reheating step in oven, where both the rework stream and the regular production stream meet. Thus, the authors studied and analyzed the complexity of a rescheduling problem on a single machine, with the objective of deciding where to insert the rework jobs in an initial sequence of regular jobs to minimize maximum waiting time. Guo and Xie (2017) formulated a two mixed integer programming models for a single machine rescheduling problem with the objective of minimizing the total waiting time. The studied problem came also from quartz glass factory, considering that the waiting time is the waiting for materials before the welding step, and its minimization saves the energy consumption. However, in dynamic environments, stability criteria are also studied. Several papers in the literature deal with this criterion. Akkan (2015) considered a multi objective rescheduling problem that simultaneously measures the scheduling efficiency with the maximum tardiness, and the stability with the sum of absolute starting time deviations, which are the sum of absolute difference between starting times of jobs before and after rescheduling. The author demonstrated that his algorithm improves the stability of schedules with no degradation in the maximum tardiness. Gao et al. (2018) studied a rescheduling problem on a flexible jobshop, with newly arrived priority jobs which are inserted in the existing schedule. The authors considered a bi-objective rescheduling criterion. The first is the efficiency, and the second is the stability which is defined as the percentage of machineschanged operations of the existing jobs during rescheduling. Pfeiffer et al., (2007) proposed a schedule stability measure calculated with two penalties. The first is the starting time deviation penalty, which is the difference between starting times of jobs at the new and previous rescheduling points. The second is the penalty related to the deviation of job starting time from the current time. Moreover, we found other related works dealing with stability measure in rescheduling problems, such as starting time deviation and total deviation penalty (Rangsaritratsamee et al., 2004), starting time deviation (Cui et al., 2018), the amount of operations and starting times operations which have been altered (Li et al., 2015, Peng et al. 2018), the absolute positional disruption, the positional disruption and absolute completion time disruption (Hoogeveen et al., 2012), the total sequence disruption (Yuan et al., 2007), the maximum time disruption (delivery times of jobs to customers changes) (Liu and Ro 2014). For considering the both aspects, efficiency combining with stability, the most of the treated mathematical models use a multi objective function, such as (Wu et al., 1993; Rangsaritratsamee et al., 2004; Qi et al., 2006; Wang et al., 2017; Peng et al., 2019).

In the rescheduling literature, we found also several papers dealing with operating rooms rescheduling, modeling the problem as a machine scheduling problem. For instance, Stuart and Kozan (2012) addressed a rescheduling problem of a day-to-day running of a day surgery unit, modeling the problem as a single machine scheduling problem, with sequence dependent processing time and due date. They proposed a branch and bound algorithm to solve the problem. Wang, B et al., (2015) discussed a surgery scheduling problem for single surgical suite, where each patient should be treated in same order by three stages. The system is
disrupted by the arrival of random emergency surgeries. The authors handled the system as a no-wait permutation flowshop scheduling problem and developed a predictive reactive approach to tackle it. For other related papers, the reader may refer to Fei et al., (2010), Farrokhi et al., (2014), M'Hallah and Al-Roomi (2014), Addis et al., (2016), Zhong and Shi (2018).

The Predictive-reactive scheduling strategy consists in generating an initial schedule in a first step, and to update it in response to a disruption (Vieira et al., 2003). In the literature, several researchers have adopted predictive-reactive approach. For example, Yang and Geunes (2008) considered a predictive-reactive scheduling on a single machine with uncertain future jobs. The authors determined the amount of planned idle time for uncertain jobs and their positions in the predictive schedule. Then, the schedule reacts when a disruption occurs, through including the new jobs in the idle times. Gürel et al. (2010) developed an anticipative scheduling approach based on a predictive reactive strategy, on parallel non-identical machines with controllable processing times. Firstly, an initial schedule is designed to minimize the total manufacturing cost of the jobs. When a machine breakdown occurs, a reactive schedule is generated and the remaining schedule is repaired.

## 3. Motivation

Our research motivation is inspired from real cases which can be encountered, either in hospital environment or in industrial environment. In what follows, we present an illustration of our problem in the context of hospital systems and in the context of industrial systems.

Among the hospital departments, the Operating Theater $(O T)$ represents the largest hospital cost category by $33 \%$ (Macario et al., 1995). Hence, operating rooms scheduling has been identified as an interesting potential area by hospital decision makers. Moreover, among the heard complaints in the health care services, the long waiting lists are the most received (Cardoen et al., 2010). Therefore, it is relevant to react on the minimization of the patient waiting time, to reduce hospital costs and improve patient's satisfaction. In practice, there are two types of operations, the elective operations that are known in advance, and the emergency operations caused by the arrival of an urgent patient. These ones are unknown in advance, and represent a disruption for the already established schedule. To cope with this uncertainty, Wullink et al. (2007) proved that the approach consisting to share $O T$ between elective and emergency operations is better than the approach of dedicating one or several OTs for emergency operations. That justifies our approach of modelling the system as a parallel machine rescheduling problem. In addition, according to surgery priorities, the patients can be divided to five majors' groups (Abedini et al., 2016). Thus, we have considered for each job a weight representing its emergency level. The aim of our study is to model this practical example as a parallel machine rescheduling problem, consisting in the predictive (elective) step to minimize the patient weighted waiting times, and in the reactive step (i.e. when the emergency operations occur), the same criterion combined with the stability, in order not to deviate too much from the initial schedule.

This described system can also be an illustration of industrial cases, as described by Guo et al. (2016), where the waiting time is the waiting period for some materials before the reheating
step in oven, with disruptions caused by the arrival of new orders. The weight can also be considered, representing the customer priority.

## 4. Problem description

We study a parallel machine rescheduling problem, under disruptions caused by the arrival of new jobs. For each job $j$, we associate a processing time $p_{j}$, a release date $r_{j}$ and a weight $w_{j}$. When a job $j$ starts at time $S_{j}$, it is processed to completion time $C_{j}$ without interruption (i.e. $C_{j}$ $=S_{j}+p_{j}$ ). The waiting time $W_{j}$ of job $j$ is the period during which the job has waited, from its entry into the system $r_{j}$, until its execution at the starting time. Then, it is defined as $W_{j}=S_{j}$ -$r_{j}=C_{j}-r_{j}-p_{j}$ (see Fig.1).


Fig. 1 The waiting time of job $\boldsymbol{j}$
In the studied system, we supposed that we have a set of $n$ jobs, $N=\{1,2, \ldots, n\}$, already available to be scheduled in the set of parallel machines $M=\{1,2, \ldots, n m\}$, all the jobs information $\left(p_{j}, r_{j}, w_{j}\right)$ is available at the beginning. An initial schedule should be determined with the objective of minimizing the $T W W T$, i.e. $\sum_{j \in N} w_{j} W_{j}$. Indeed, a mathematical model is developed to solve the initial problem, that can be represented in standard notation by $P\left|r_{j}\right| \sum w_{j} W_{j}$. During execution of the jobs, the initial schedule can be disrupted by the arrival of new jobs. Consequently, a new set of jobs is generated, $N^{\prime}=\left\{1,2, \ldots, n^{\prime}\right\}$, which contains the unexecuted jobs (i.e. the jobs that haven't started) as well as new arrival jobs. The methodology is to omit the already executed jobs, and to combine the unexecuted jobs with new arrival ones to create an updated schedule. Therefore, a new reactive schedule is generated to update the existing one, simultaneously considering efficiency measured by original objective $\sum_{j \in N^{\prime}} w_{j} W_{j}$ and stability measured by the deviation from the original schedule, as described hereafter.

Stability measure: If no disruption occurs, the job $j$ will finish processing at completion time of original schedule $C o j$. Else, if a disruption occurs and a new job is inserted inside the old schedule, some jobs will change their positions. Hence, a new schedule will be established. Consequently, the job $j$ will really finish in the real completion time $C_{j}$. Then, the difference between $C o_{j}$ and $C_{j}$ can be used to assess the deviation cost. In addition, we used the weight $w_{j}$ for each job, which makes it difficult to deviate important jobs. Hence, the stability objective is defined as $\sum_{j \in N} w_{j}\left(C_{j}-C o_{j}\right)$. We called it the Total Weighted Completion Times Deviation (TWCTD). In this approach, we only calculate the stability criterion for the jobs existing in the previous schedule $(j \in N)$. However, once the new jobs are scheduled in their first sequence,
they will be concerned by the stability criterion in the ulterior schedules. When $w_{j}\left(C_{j}-C o_{j}\right)>0$, a deviation has occurred and an instability penalty is generated.

Along with that, we established a relationship between the both parts of objective functions, by integrating $\alpha$ as an efficiency-stability coefficient. Thus, $\alpha$ is associated to efficiency part and $(1-\alpha)$ to stability, where $\alpha$ is a real number between zero and one. Finally, we defined the new objective function that simultaneously measures efficiency and stability as $\alpha \sum_{j \in N^{\prime}} w_{j} W_{j}+(1-\alpha) \sum_{j \in N} w_{j}\left(C_{j}-C o_{j}\right)$. Then, the dynamic rescheduling problem can be represented in standard notation by: $P\left|r_{j}\right| \alpha \sum w_{j} W_{j}+(1-\alpha) \sum w_{i}\left(C_{j}-C o_{j}\right)$.

In the rescheduling process, the disruptions occur while the machine executes jobs. Therefore, we noted by $t_{d}$ the current time when a disruption occurs (see Fig.2).


Fig. 2 Disruption occurred at time $\boldsymbol{t}_{\boldsymbol{d}}$
Indeed, all executed jobs before $t_{d}$ must keep the same position. Thus, these jobs are omitted, and the new set $N$ ' will be scheduled, then a new sequence will be established, it begins, for each machine, from the completion time of the last executed job.

## Online iterative methodology

Firstly, we have discretized the finite time horizon [0, T], into periods $\Delta t$ (see Fig.3). Indeed, $\Delta t$ is the time period length which represents the time step. The occurrence of disruptions may be possible only at these times. In fact, when a job occurs at time $t$, this date will be its release date. In order to simplify the calculations, we assumed that $\Delta t=1$ unit of time, the same time unit that we used for the jobs data $p_{j}, r_{j}$, and $w_{j}$.


Fig. 3 Time discretization
We then defined $\theta(t)$ a binary variable, equals to 1 if a job (or several jobs) occurs at time $t$, and 0 otherwise. The generation of its values is explained in the numerical results section. The algorithm below describes the online iterative methodology.

Solve the initial problem
LIST $=\left\{J_{1}, \ldots, J_{n}\right\}$
Me morize solution
For (tin 1 to $T$ )
If $(\theta(t)=1)$

- Add new job(s) to LIST.
- O mit the already executed jobs from LIST (comparing their me monized starting time with $t$ )
- Reschedule the new problem with jobs in IIST
- Memorize solution

End If

## End For

The following flowchart describes the predictive-reactive strategy that we used to tackle the problem (see Fig.4).


Fig. 4 The flowchart of predictive-reactive strategy

## 5. Mathematical model

A mixed integer linear programming model is implemented in this section, it is based on predictive-reactive strategy, Thus, in the first subsection, we present the mathematical model before disruption. In the second subsection, we present the mathematical model after disruption.

### 5.1 Mathematical model before disruption

The proposed model is based on sequence position formulation (assigning a job to a position). In the scheduling literature, several papers used this formulation. For instance, Eren (2009), Baker and Keller (2010), Kooli and Serairi (2014) and Guo and Xie (2017) in single machine scheduling problems. As well, other authors, like Abdel-Jabbar et al. (2014) and Beezão et al. (2017), used this formulation for parallel machine scheduling problems. However, most of this works consider the Makespan as a criterion. Indeed, our problem consists in minimizing the $T W W T$. Hence, supplementary data and variables are considered, such as (weights, release dates, waiting time). In fact, we were inspired by Beezão et al. (2017) model and have adapted it to our variables and constraints. The parameters, variables and constraints of the model are given below:

Parameters:
$N$ : set of jobs $\{1,2, \ldots, n\}$
$K$ : set of positions $\{1,2, \ldots, n\}$
$M$ : set of machines $\{1,2, \ldots, n m\}$
$j$ : index of job, $j=1,2, \ldots, n$
$k$ : index of position, $k=1,2, \ldots, n$
$m$ : index of machine, $m=1,2, \ldots, n m$
$w_{j}$ : weight of job $j$
$r_{j}$ : release date of job $j$
$p_{j}$ : processing time of job $j$
bigM: big value, bigM $=\sum_{j=1}^{n} p_{j}+\max r_{j}$
Decision variables:
$x_{j k m}=\left\{\begin{array}{lc}1 & \text { if the job } j \text { is assigned tomachine } m \text { in } k^{\mathrm{th}} \text { position } \\ 0 & \text { Otherwise }\end{array}\right.$
$C P_{k m}$ : Completion time of job in $k^{t h}$ position in machine $m$.
$S P_{k m}$ : Starting time of job in $k^{\text {th }}$ position in machine $m$.
$C_{j}$ : Completion time of job $j$.
$W_{j}$ : Waiting time of job $j$.
Objective function: $\quad \operatorname{Min} \sum_{j \in N} w_{j} W_{j}$
s.t:
$\sum_{m=1}^{n m} \sum_{k=1}^{n} x_{j k m}=1 \forall j \in \underset{N}{358}$

$$
\begin{equation*}
\sum_{j=1}^{n} x_{j k m} \leq 1 \quad \forall k \in K, \forall m \in M \tag{2}
\end{equation*}
$$

$$
\begin{equation*}
\sum_{j=1}^{n} x_{j(k+1) m} \leq \sum_{j=1}^{n} x_{j k m} \quad \forall k=1 . . n-1, \forall m \in M \tag{3}
\end{equation*}
$$

$S P_{k m} \geq \sum_{j=1}^{n} r_{j} x_{j k m} \quad \forall k \in K, \forall m \in M$
$S P_{(k+1) m} \geq S P_{k m}+\sum_{j=1}^{n} p_{j} x_{j k m} \quad \forall k=1 \ldots n-1, \forall m \in M$
$C P_{k m}=S P_{k m}+\sum_{j=1}^{n} p_{j} x_{j k m} \quad \forall k \in K, \forall m \in M$
$C_{j} \geq C P_{k m}-\operatorname{big} M\left(1-x j_{j k m}\right) \forall j \in N, \forall k \in K, \forall m \in M$
$C_{j} \leq C P_{k m}+\operatorname{big} M\left(1-x_{j k m}\right) \forall j \in N, \forall k \in K, \forall m \in M$
$W_{j}=C_{j}-r_{j}-p_{j} \quad \forall j \in N$

$$
C P_{k m}, S P_{k m}, C_{j}, W_{j} \geq 0 \forall j \in N, \forall k \in K, \forall m \in M
$$

$x_{j k m} \in\{0,1\} \quad \forall j \in N, \forall k \in K, \forall m \in M$
Constraint (1) specifies that each job is affected to one position in a machine. Constraint (2) consists in having only one or no job per position in a machine. Constraint (3) specifies that, if the position $k$ is not occupied, the position $k+1$ is not occupied too. Constraint (4) consists in making, for all machines, the starting time of $k^{\text {th }}$ position greater than or equal to assigned release date. Constraint (5) consists in making, for all machines, the starting time of $(k+1)^{\mathrm{th}}$ position greater than or equal to the previous position completion time. Constraint (6) specifies that, for all machines, the completion time of $k^{\text {th }}$ position is equal to starting time of $k^{\text {th }}$ position plus the assigned processing time. Constraint (7) defines, for all machines, the completion time of job $j$, which is greater than or equal to completion time of assigned position, where bigM must be sufficiently large. For that reason, we defined for our model a value of bigM according to the total completion times and maximum value of release dates to take into account jobs availability. Thus, $\operatorname{big} M=\sum_{j=1}^{n} p_{j}+\max r_{j}$. Constraint (8) is a cut that allows reducing the computing time. Constraint (9) defines the waiting time of job $j$, according to its completion time, release date, and processing time, Constraint (10) consists in making all variables greater than or equal to zero. Constraint (11) consists in having a binary decision variable.

### 5.2 Mathematical model after disruption

This second model is used after a disruption caused by the arrival of new jobs. We denoted by $n t$ the number of the unexecuted jobs at time $t$, and $n j$ the number of new arrived jobs. Thus, the new jobs have the same entry parameters as the first ones. For each job, we have $w_{j}, p_{j}$, and $r_{j}$ which correspond to the time when the job occurs. So, the new parameters are as follows:

New parameters:
$n^{\prime}=n t+n j$
$N^{\prime}$ : set of jobs $\left\{1,2, \ldots, n^{\prime}\right\}$
$K^{\prime}$ : set of positions $\left\{1,2, \ldots, n^{\prime}\right\}$
$j$ : index of job, $j=1,2, \ldots, n$,
$k$ : index of position, $k=1,2, \ldots, n$,
$C o s_{j}$ : original completion time of job $j$ (obtained by the resolution of the initial problem).
$\alpha$ : efficiency-stability coefficient.
$x o_{j k m}$ : variable assigning jobs to positions in the initial problem
We notice that the parameters $x o_{j k m}$, and $C o_{j}$ are not decision variables. They are memorized when the initial model is solved. we recall that we only calculate the stability criterion for the jobs existing in the previous schedule $(j \in N)$. The decision variables $x_{j k m}, C P_{k m}, S P_{k m}, C_{j}, W_{j}$ are the same for this new model. However, the new objective function is as follows:

$$
\operatorname{Min} \alpha \sum_{j \in N^{\prime}} w_{j} W_{j}+(1-\alpha) \sum_{j \in N} w_{j}\left(C_{j}-\operatorname{Co}_{j}\right)
$$

The constraints (1), (2), (3), (4), (5), (6), (7), (8), (9), (10) and (11) are also used for this new model.

In addition, when new jobs are added, the completion time of a job will be greater than its original completion time, because the original completion time have been obtained through minimization of the principal criterion " $T W W T$ " only, since it is not concerned by the stability. Indeed, the job is placed in its ideal position, and it will not be moved forward after the rescheduling. Thus, in all rescheduling steps, we assumed that:

$$
\begin{equation*}
C_{j} \geq C o_{j} \quad \forall j \in N \tag{12}
\end{equation*}
$$

## 6. Numerical results

The proposed mixed integer linear programming model has been designed on FICO Xpress IVE and has been performed on a Core i5 2.40 GHz laptop. The programed model is composed of two parts. The first contains the predictive approach, and consists in solving the initial problem normally, with the objective of minimizing the TWWT. The second contains the reactive approach, and consists in rescheduling at each time the jobs occur, accordingly to the online iterative methodology previously described. The system omits the already executed jobs,
and combines the still unexecuted jobs with the new ones to create an updated schedule. The model implementation has been developed accordingly to the chart presented on Fig.4. This proposed methodology has been tested on the instances described hereafter, and it allowed giving, at each step, the optimal solution.

In this section, we present the numerical results obtained through the model application, on two parallel machines. The data used in this section are as follows:

Table 1 parameter values

| Parameters | Values |
| :---: | :---: |
| $w_{j}$ | $\sim U(1,5)$ |
| $p_{j}$ | $\sim N(2.5,0.5)(u t)$ |
| $T$ | $24(u t)$ |

The data that we have chosen can be adapted for real cases, either in a hospital or in an industrial environment. The simulation will be over a 6 -hours' time horizon. The weight values can represent the 5 -emergency levels or 5 priority levels of customers. We assumed also that $\Delta t$ $=1$ unit of time $(u t)$, and $1 u t$ is equivalent to 15 minutes. The processing times can represent the durations of operations or a product manufacturing time, and follow a normal distribution with mean 2.5 and standard deviation 0.5 , obtaining operations of 1 ut ( 15 minutes) to 4 ut ( 60 minutes), with more probability to have a processing time of $2 u t$ or $3 u t$. The values of the variable $\theta(t)$ are randomly generated with the Bernoulli distribution which gives, at each time $t$, the value 1 with probability $p_{\theta}$, and 0 with probability $1-p_{\theta}$. At first, we considered $p_{\theta}=0.8$ (high appearance probability), then we tested also for $p_{\theta}=0.5$ (medium appearance probability) and $p_{\theta}=0.2$ (small appearance probability).

We established three main studies. Firstly, we examined the influence of efficiency-stability coefficient $\alpha$ on the optimal schedules. Secondly, we used the lexicographic method to minimize the number of altered jobs. Finally, we tested the computing time of the model resolution with different instances, to show its limitations according to the problems' size.

### 6.1 Influence of efficiency-stability coefficient on the optimal schedules

In this study, we vary the value of $\alpha$ from 0.5 to 1 to discuss its impact on the optimal schedule. We have assumed that the efficiency criterion is the principal one, while the stability criterion is the secondary. Thus, it is not necessary to test the values of $\alpha$ inferior to 0.5 .

We first calculate the initial objective obtained by solving the offline problem. After each disruption, we update the problem, then we calculate the new objective function. The number of altered jobs is also calculated in this study, which represents the number of jobs which changed of machine after rescheduling. For that, we included the supplementary parameters: ${ }^{i} A_{j m}$ to assign jobs to machines in iteration $i$, and $D i f_{j}$ that represents the deviation of a job from its machine.

$$
{ }^{i} A_{j m}=\left\{\begin{array}{l}
1 \text { if } \sum_{k=1}^{n} x_{j k m}=1  \tag{13}\\
0 \quad \text { Otherwise }
\end{array} \quad \forall j, \forall m, \forall i\right.
$$

$$
\operatorname{Dif}_{j}=\left\{\begin{array}{l}
1 \quad \text { if } \sum_{m=1}^{n m}\left|{ }^{i} A_{j m}-{ }^{i-1} A_{j m}\right| \neq 0  \tag{13}\\
0 \quad \text { Otherwise }
\end{array} \forall j\right.
$$

Therefore, the number of altered jobs is given by: $\sum_{j \in N} \operatorname{Dif} j_{j}$. We note that $A_{j \underline{m}}$ and $D i f_{j}$ are not considered as decision variables in this study, but they will be considered as such in the lexicographic optimization sub section.

The study was conducted and analyzed for 25 different instances, generated accordingly to the parameters presented in Table 1. We present, in the following example, the data and the results obtained on a particular instance with two parallel identical machines.

## Example

In this example, the processing times $p_{j}$, release dates $r_{j}$ and the weights $w_{j}$ for 5 jobs are given in the Table 2.

Table 2. Initial problem data

| $j o b$ | 1 | 2 | 3 | 4 | 5 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $r_{j}$ | 0 | 1 | 0 | 1 | 2 |
| $p_{j}$ | 1 | 2 | 2 | 3 | 4 |
| $w_{j}$ | 5 | 1 | 4 | 2 | 1 |

The problem is solved with the objective to minimize the total weighted waiting times. So, the obtained optimal value is $\sum_{j=1}^{5} w_{j} W_{j}=3$. The optimal solution is given in Fig.5, we noted it "the first schedule".


Fig. 5 The optimal solution for the initial problem
In the reactive approach, we assumed that the disruptions occur by the arrival of new jobs. In this example, a total of 20 jobs are treated online. The obtained results are given in the Table 3.

In the Table 3, we assumed that a disruption occurs at each step with appearance probability $p_{\theta}=0.8$ ( 20 disruptions appeared in the total). At each disruption, we reschedule the problem, and we extract four information: the value of the efficiency criterion "TWWT", the value of the stability criterion "TWCTD", the number of jobs which are executed on a different machine after the disruption " $N^{\circ}$ of altered jobs", as well as the value of the objective function that combines TWWT and TWCTD thanks to $\alpha$. We have also numbered the schedules with their

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| $\alpha$ | (1) | (2) | (3) | (4) | (1) | (2) | (3) | (4) | (1) | (2) | (3) | (4) | (1) | (2) | (3) | (4) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $2^{\text {nd }}$ Schedule |  |  |  | $3^{\text {rd }}$ Schedule |  |  |  | $4^{\text {th }}$ Schedule |  |  |  | $5^{\text {th }}$ Schedule |  |  |  |
| 1 | 5 | 2 | 2 | 5 | 8 | 5 | 1 | 8 | 13 | 6 | 0 | 13 | 16 | 9 | 2 | 16 |
| 0,9 | 5 | 2 | 2 | 4,7 | 8 | 2 | 2 | 7,4 | 13 | 3 | 0 | 12 | 16 | 3 | 1 | 14,7 |
| 0,8 | 5 | 2 | 2 | 4,4 | 8 | 2 | 2 | 6,8 | 13 | 3 | 0 | 11 | 16 | 3 | 1 | 13,4 |
| 0,7 | 5 | 2 | 2 | 4,1 | 8 | 2 | 2 | 6,2 | 13 | 3 | 0 | 10 | 16 | 3 | 1 | 12,1 |
| 0,6 | 5 | 2 | 2 | 3,8 | 8 | 2 | 2 | 5,6 | 13 | 3 | 0 | 9 | 16 | 3 | 1 | 10,8 |
| 0,5 | 5 | 2 | 2 | 3,5 | 8 | 2 | 2 | 5 | 13 | 3 | 0 | 8 | 16 | 3 | 1 | 9,5 |
|  | $6^{\text {th }}$ Schedule |  |  |  | $7^{\text {th }}$ Schedule |  |  |  | $8^{\text {th }}$ Schedule |  |  |  | $9^{\text {th }}$ Schedule |  |  |  |
| 1 | 18 | 10 | 1 | 18 | 23 | 11 | 0 | 23 | 27 | 13 | 0 | 27 | 29 | 13 | 0 | 29 |
| 0.9 | 19 | 3 | 0 | 17,4 | 25 | 4 | 0 | 22,9 | 28 | 4 | 1 | 25,6 | 29 | 5 | 2 | 26,6 |
| 0.8 | 19 | 3 | 0 | 15,8 | 25 | 4 | 0 | 20,8 | 28 | 4 | 0 | 23,2 | 29 | 5 | 0 | 24,2 |
| 0.7 | 19 | 3 | 0 | 14,2 | 25 | 4 | 0 | 18,7 | 28 | 4 | 0 | 20,8 | 29 | 5 | 0 | 21,8 |
| 0.6 | 19 | 3 | 0 | 12,6 | 25 | 4 | 0 | 16,6 | 28 | 4 | 0 | 18,4 | 29 | 5 | 0 | 19,4 |
| 0.5 | 19 | 3 | 0 | 11 | 25 | 4 | 0 | 14,5 | 28 | 4 | 1 | 16 | 29 | 5 | 2 | 17 |
|  | $10^{\text {th }}$ Schedule |  |  |  | $11^{\text {th }}$ Schedule |  |  |  | $12^{\text {th }}$ Schedule |  |  |  | $13^{\text {th }}$ Schedule |  |  |  |
| 1 | 30 | 14 | 1 | 30 | 32 | 16 | 0 | 32 | 36 | 16 | 0 | 36 | 38 | 16 | 0 | 38 |
| 0.9 | 30 | 5 | 0 | 27,5 | 32 | 7 | 0 | 29,5 | 36 | 8 | 1 | 33,2 | 38 | 10 | 1 | 35,2 |
| 0.8 | 30 | 5 | 0 | 25 | 32 | 7 | 0 | 27 | 36 | 8 | 1 | 30,4 | 38 | 10 | 1 | 32,4 |
| 0.7 | 30 | 5 | 0 | 22,5 | 32 | 7 | 0 | 24,5 | 36 | 8 | 1 | 27,6 | 38 | 10 | 1 | 29,6 |
| 0.6 | 30 | 5 | 0 | 20 | 33 | 5 | 0 | 21,8 | 39 | 5 | 0 | 25,4 | 43 | 5 | 0 | 27,8 |
| 0.5 | 30 | 5 | 0 | 17,5 | 33 | 5 | 0 | 19 | 39 | 5 | 0 | 22 | 43 | 5 | 0 | 24 |
|  | $14^{\text {th }}$ Schedule |  |  |  | $15^{\text {th }}$ Schedule |  |  |  | $16^{\text {th }}$ Schedule |  |  |  | $17^{\text {th }}$ Schedule |  |  |  |
| 1 | 41 | 16 | 0 | 41 | 42 | 18 | 0 | 42 | 44 | 18 | 2 | 44 | 46 | 20 | 0 | 46 |
| 0.9 | 41 | 10 | 0 | 37,9 | 42 | 11 | 0 | 38,9 | 44 | 11 | 2 | 40,7 | 46 | 13 | 0 | 42,7 |
| 0.8 | 41 | 10 | 0 | 34,8 | 42 | 11 | 0 | 35,8 | 44 | 11 | 2 | 37,4 | 46 | 13 | 2 | 39,4 |
| 0.7 | 41 | 10 | 0 | 31,7 | 42 | 11 | 0 | 2 | 44 | 11 | 2 | 34,1 | 46 | 13 | 2 | 36,1 |
| 0.6 | 45 | 5 | 0 | 29 | 47 | 7 | 0 | 31 | 49 | 7 | 2 | 32,2 | 54 | 9 | 0 | 36 |
| 0.5 | 45 | 5 | 0 | 25 | 47 | 7 | 2 | 27 | 49 | 7 | 2 | 28 | $21^{\text {th }}$ Schedule |  |  |  |
|  | $18^{\text {th }}$ Schedule |  |  |  | $19^{\text {th }}$ Schedule |  |  |  | $20^{\text {th }}$ Schedule |  |  |  |  |  |  |  |
| 1 | 50 | 20 | 0 | 50 | 52 | 21 | 0 | 52 | 55 | 24 | 1 | 55 | 57 | 26 | 0 | 57 |
| 0.9 | 49 | 16 | 0 | 45,7 | 50 | 17 | 2 | 46,7 | 52 | 19 | 2 | 48,7 | 53 | 20 | 2 | 49,7 |
| 0.8 | 49 | 16 | 2 | 42,4 | 50 | 17 | 0 | 43,4 | 52 | 19 | 0 | 45,4 | 53 | 20 | 2 | 46,4 |
| 0.7 | 49 | 16 | 2 | 39,1 | 50 | 17 | 0 | 40,1 | 52 | 19 | 0 | 42,1 | 53 | 20 | 2 | 43,1 |
| 0.6 | 58 | 9 | 0 | 38,4 | 60 | 10 | 2 | 40 | 63 | 13 | 2 | 43 | 65 | 15 | 2 | 45 |
| 0.5 | 58 | 9 | 1 | 33,5 | 60 | 10 | 2 | 35 | 63 | 13 | 1 | 38 | 66 | 14 | 0 | 40 |

step number. Different values of $\alpha$ are tested, to discuss the impact of this parameter on the obtained solutions.

Table 3. Influence of $\alpha$ on the optimal schedules
(1) $T W W T$ : Total Weighted Waiting Times $\sum_{J \in N^{\prime}} w_{j} W_{j}$
(2) TWCTD: Total Weighted Completion Time Deviation $\sum_{j \in N} w_{j}\left(C_{j}-C o_{j}\right)$
(3) $N^{\circ}$ of altered jobs
(4) Objective function

For $\alpha=1$, the objective function only considers the first criterion, TWWT. We noted that, when the new jobs arrive, they have been scheduled according to their weights and their processing times. Generally, when $\alpha=1$, and if all jobs are available, the $w S P T$-rule is optimal for this problem.

Otherwise, for $\alpha$ values between 0.9 and 0.7 , the stability is also considered. In this case, we have noted two observations:

- Short term: The optimal schedule has the same value of $T W W T$ as when $\alpha=1$, but the value of the second criterion TWCTD is smaller. Indeed, the schedule is more stable. In this case, during the model execution, if there are two solutions which give the same value of $T W W T$, the system chooses the one that minimizes the stability.
- Long term: The model gives a better value of both criteria (TWWT and TWCTD) compared to $\alpha=1$. This is a result of the proactive effect induced by the stability criterion. Indeed, when we have two similar solutions in terms of efficiency, the system prioritizes the one that minimizes $\sum_{j \in N} w_{j}\left(C_{j}-C o_{j}\right)$. This decision impacts the efficiency criterion in the next steps.

For $\alpha=0.6$ and 0.5 , the $T W W T$ increases, because we give more importance to the stability criterion. Figure 6 shows, for each step, the variation of $T W W T$ according to $\alpha$.


Fig. 6 TWWT in function of $\alpha$ with $p_{\theta}=0.8$
In the short term, the value of $T W W T$ is stable in most of cases. In the long term, the value of $T W W T$ is smaller when $\alpha=0.7,0.8$ and 0.9 compared to others values as result of proactive effect induced by the stability criterion. To clearly illustrate this phenomenon, we present the Gantt diagram (Table 5) of the optimal sequences given by model resolution for $\alpha=0.8$, compared to those obtained with $\alpha=1$, on the eleven first modified schedules (first eleven steps) for the instance below (Table 4).

Table 4. Problem data

| $\mathrm{Job} j$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $r j$ | 0 | 1 | 0 | 1 | 2 | 2 | 3 | 4 | 6 | 7 | 9 | 10 | 11 | 12 | 13 | 14 |
| $p j$ | 1 | 2 | 2 | 3 | 4 | 3 | 4 | 1 | 1 | 4 | 1 | 2 | 1 | 3 | 2 | 4 |
| $w j$ | 5 | 1 | 3 | 2 | 1 | 1 | 1 | 5 | 3 | 1 | 5 | 3 | 4 | 1 | 1 | 1 |



From the first step until step 10, in both cases, the model provides the same value of the principal criterion $T W W T$. However, when $\alpha=0.8$, the model provides more a stable schedule compared to $\alpha=1$. This difference has impacted the schedule in step 11. For instance, in the step 9 , with $\alpha=1$, the system places the job 14 before the job 7 . Whereas, if we consider the stability criterion (TWCTD), the system proposes the opposite to have a stable schedule. Consequently, this choice impacts the value of $T W W T$ in the step 11. Thus, the difference of one unit of time between the schedule when $\alpha=1$ and $\alpha=0.8$.

To confirm this behavior, we present in Figure 7, the average of TWWT in function of alpha, on the 25 instances generated.


Fig. 7 The average of $T W W T$ in function of $\alpha$ with $p_{\theta}=0.8$
Among the 25 instances that we have tested, for 8 of them, the proactive effect appears. In Fig 7, we observe that the average of $T W W T$ is stable in the short term. However, in the long term, it is increasing when $\alpha$ goes down to 0.7 .

In the case of $p_{\theta}=0.8$ (high appearance probability), and when alpha equals 1 , the system does not consider the stability. So, if we have various solutions that gives the same value of $T W W T$, the system chooses arbitrarily one of them, as observed in the step 5 on Table 5, between the jobs 7 and 10 . Consequently, this arbitrary choice impacts the next schedules. In the example of Table 5, the difference is shown in the step 11. Therefore, as result of this behaviour, we have concluded that the stability criterion can generate a proactive effect, when $\alpha$ is between 0.7 and 0.9 , providing better results compared to a mono-criterion considering only efficiency. This behaviour, that we called "proactive effect" appears when we have a various choice in some steps, i.e. when the set of jobs that have to be executed is bigger.

To confirm this conclusion, we have tested other sets of instances, with $p_{\theta}=0.5$ (medium appearance probability) and $p_{\theta}=0.2$ (small appearance probability) on the same time horizon $T=24 u t$. The average of $T W W T$ in function of $\alpha$ is respectively presented in figures 8 and 9 .


Fig. 8 The average of TWWT in function of $\alpha$ with $p_{\theta}=0.5$
In figure 8 , the average of $T W W T$ increases when $\alpha$ becomes inferior to 0.7 . Moreover, we have observed the proactive effect in only one out of the 25 instances we have tested. In fact, when $p_{\theta}=0.5$, the jobs appear with a medium frequency. Then, the set of jobs that have to be executed at each step is small compared to $p_{\theta}=0.8$. Hence, the system does not have various choices at each step. Thus, the proactive effect does not appear as often as for $p_{\theta}=0.8$.


Fig. 9 The average of $T W W T$ in function of $\alpha$ with $p_{\theta}=0.2$
In figure 9 , the average of $T W W T$ decreases with $\alpha$. Indeed, when $p_{\theta}=0.2$, the jobs appear with a small frequency. Then, the number of disruptions is very small, and we have not observed the proactive effect. However, in the last steps $(4,5,6)$, the average of $T W W T$ is identical,
because the disruptions occur after the execution of existing jobs. Thus, the jobs do not wait, and $T W W T$ does not increase. Therefore, we have also tested this case by increasing the number of initial jobs to seven jobs. The results are shown in Figure 10.


Fig. 10 The average of $\boldsymbol{T W W T}$ in function of $\alpha$ for 7 initial jobs and $p_{\theta}=\mathbf{=} .2$
With 7 initial jobs, we observed the proactive effect in 2 instances out of the 25 tested. The increase of the number of initial jobs allows to increase the set of jobs that have to be executed at each time, and the probability to observe a proactive effect. This is what we have observed.

### 6.2 Lexicographic optimization for minimizing the number of altered jobs

The lexicographic optimization consists in optimizing the first objective function (of higher priority). Then, among the possible alternative optima, we optimize the second objective and so on (Mavrotas 2009). In this part, we performed as follows: we first optimize the multi objective function that simultaneously measures efficiency and stability criteria, $f_{1}=\alpha \sum_{j \in N^{\prime}} w_{j} W_{j}+(1-\alpha) \sum_{j \in N} w_{j}\left(C_{j}-C o_{j}\right)$, obtaining the $\min f_{l}=f_{l} *$. Then, we secondly optimize the number of altered jobs $f_{2}=\sum_{j \in N} \operatorname{Dif} f_{j}$ by adding the constraint $f_{1} \leq f_{1} *$, in order to keep the optimal solution of the first optimization. To do this, we put the constraints $A_{j m}$ and $D i f_{j}$ into a linear form:

$$
\begin{gather*}
A_{j m}=\sum_{k=1}^{n^{\prime}} x_{j k m} \quad \forall j \forall m  \tag{15}\\
\operatorname{Dif}_{j} \geq{ }^{1} A_{j m}-{ }^{0} A_{j m} \quad \forall j \forall m \tag{16}
\end{gather*}
$$

Constraint (15) specifies that each job is affected to machine $m$. Constraint (16) consists in having the binary variables $D i f_{j}$ positive when the job changes of machine during an iteration, and equal to zero otherwise.

In the solution presented in Table 3, the number of altered jobs is not minimized. Therefore, at each iteration, the model gives a different value of the number of altered jobs. In the following table, we present the lexicographic optimization of the same example. The aim is to minimize $f_{2}$ with $f_{1} \leq f_{1} *$ as a constraint. However, if the model cannot reduce the number of altered jobs to zero, we relaxed the constraint in order to obtain a better solution for $f_{2}$. Therefore, the constraint becomes $f_{1} \leq f_{1} *(1+\varepsilon)$, with $\varepsilon$ a relaxation coefficient. In this study, we vary the value of this coefficient until we reduce $f_{2}$ to zero. For that purpose, we fixed the value of $\alpha$ at 0.8 , the value for which we observed the most the proactive effect in the last sub section. The results we have obtained are given in the Table 6.

Table 6. Optimal solution of number of altered jobs, with $f_{i} \leq f_{1}{ }^{*}(1+\varepsilon)$.
(1) TWWT: Total Weighted Waiting Times $\sum_{J \in N^{\prime}} w_{j} W_{j}$
(2) TWCTD: Total Weighted Completion Time Deviation $\sum_{j \in N} w_{j}\left(C_{j}-\operatorname{Co}_{j}\right)$
(3) $f_{l} *(1+\varepsilon)$
(4) $f_{2}: \sum_{j \in N} D i f_{j}$

| $\varepsilon$ | (1) | (2) | (3) | (4) | (1) | (2) | (3) | (4) | (1) | (2) | (3) | (4) | (1) | (2) | (3) | (4) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $2^{\text {nd }}$ Schedule |  |  |  | $3^{\text {rd }}$ Schedule |  |  |  | $4^{\text {th }}$ Schedule |  |  |  | $5^{\text {th }}$ Schedule |  |  |  |
| 0 | 5 | 2 | 4,4 | 0 | 8 | 2 | 6,8 | 0 | 13 | 3 | 11 | 0 | 16 | 3 | 13,4 | 0 |
|  | $6^{\text {th }}$ Schedule |  |  |  | $7^{\text {th }}$ Schedule |  |  |  | $8^{\text {th }}$ Schedule |  |  |  | $9^{\text {th }}$ Schedule |  |  |  |
| 0 | 19 | 3 | 15,8 | 0 | 25 | 4 | 20,8 | 0 | 28 | 4 | 23,2 | 0 | 29 | 5 | 24,2 | 0 |
|  | $10^{\text {th }}$ Schedule |  |  |  | $11^{\text {th }}$ Schedule |  |  |  | $12^{\text {th }}$ Schedule |  |  |  | $13^{\text {th }}$ Schedule |  |  |  |
| 0 | 30 | 5 | 25 | 0 | 32 | 7 | 27 | 0 | 36 | 8 | 30,4 | 1 | 42 | 7 | 35 | 0 |
| 0.01 | - | - | - | - | - | - | - | - | 36 | 8 | 30.7 | 1 | - | - | - | - |
| 0.02 | - | - | - | - | - | - | - | - | 36 | 8 | 31.0 | 1 | - | - | - | - |
| 0.03 | - | - | - | - | - | - | - | - | 36 | 8 | 31.3 | 1 | - | - | - | - |
| 0.04 | - | - | - | - | - | - | - | - | 36 | 8 | 31.6 | 1 | - | - | - | - |
| 0.05 | - | - | - | - | - | - | - | - | 38 | 7 | 31.9 | 0 | - | - | - | - |
|  | $14^{\text {th }}$ Schedule |  |  |  | $15^{\text {th }}$ Schedule |  |  |  | $16^{\text {th }}$ Schedule |  |  |  | $17^{\text {th }}$ Schedule |  |  |  |
| 0 | 44 | 7 | 0 | 0 | 46 | 9 | 38.6 | 0 | 48 | 9 | 40.2 | 0 | 53 | 11 | 44.6 | 0 |
|  | $18^{\text {th }}$ Schedule |  |  |  | $19^{\text {th }}$ Schedule |  |  |  | $20^{\text {th }}$ Schedule |  |  |  | $21^{\text {th }}$ Schedule |  |  |  |
| 0 | 57 | 11 | 47.8 | 0 | 59 | 12 | 49.4 | 0 | 62 | 15 | 52.6 | 1 | 64 | 17 | 54.6 | 0 |
| 0.01 | - | - | - | - | - | - | - | - | 62 | 15 | 53.1 | 1 | - | - | - | - |
| 0.02 | - | - | - | - | - | - | - | - | 63 | 16 | 53.6 | 0 | - | - | - | - |

As can be seen in Table 6, from the $2^{\text {nd }}$ schedule until the $11^{\text {th }}$, the model reduced the value of $f_{2}$ to zero, with $\varepsilon=0$. For the $12^{\text {th }}$ schedule, we have increased the value of $\varepsilon$ up to 0.05 to obtain $f_{2}=0$, and $f_{1} *$ becomes $f_{1} *(1+\varepsilon)$. For the $13^{\text {th }}$ schedule, we proceed in the same way, considering the last solution obtained in the previous schedule.

Among the 20 schedules, we varied $\varepsilon$ in only two of them. It means that, in the majority of cases, we can minimize $f_{2}$ without impacting $f_{1} *$. However, when we reduce $f_{2}$ to zero by varying $\varepsilon$, the value of $f_{1} *(1+\varepsilon)$ increases as well as the values of TWWT and TWCTD. This causes an increase in $f_{1} *$ in the following iterations, as in the $13^{\text {th }}$ schedule for example, in which $f_{1} *=35$ compared to its value in Table 3, where $f_{1} *=32.4$.

### 6.3 Computing time study

In this study, we tested on 10 different instances the Maximum Duration of Iteration (MDI) and the Computing Time ( $C T$ ) for the execution of all iterations. The sets of tested instances contain respectively 5 and 7 initial jobs. They are disrupted by the arrival of new jobs over a horizon of $T=24 u t$ and $\alpha=0.8$. We varied the appearance probability $p_{\theta}$ with different values $(0.2 ; 0.5 ; 0.8 ; 1)$ to analyze its impact on the $M D I$ and $C T$. Then, we calculated the minimum,
maximum, average and standard deviation of $M D I$ and $C T$. The obtained results are respectively presented in Table 7 and Table 8.

Table 7 Maximum Duration of Iteration

| Initial jobs | $p_{\theta}$ | Min MDI (s) | Max $M D I(s)$ | Avg MDI (s) | $\operatorname{Std} \operatorname{dev} M D I(s)$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | 0.2 | 0.12 | 0.26 | 0.17 | 0.05 |
| 5 | 0.5 | 0.13 | 0.38 | 0.21 | 0.07 |
|  | 0.8 | 0.21 | 1.14 | 0.44 | 0.31 |
|  | 1 | 0.24 | 112 | 19.25 | 38.4 |
| 7 | 0.2 | 1.37 | 2.88 | 1.97 | 0.51 |
|  | 0.5 | 1.58 | 3.66 | 2.03 | 0.69 |
|  | 0.8 | 4.37 | 542 | 64.1 | 167 |
|  | 1 | 403 | 6324 | 3606 | 2069 |

When $p_{\theta}$ increases, the set of jobs that have to be executed at each iteration increases too much, as well as the average of MDI. As well, the average MDI increases also when the set of initial jobs is bigger.

In the rescheduling system, we have to establish a planning after each disruption. This one should be made as soon as possible, and at latest before the appearance of another disruption. In our study, according to the time discretization, the disruption can be possible at each $\Delta t$. Thus, if the MDI exceeds $\Delta t$, it is unacceptable. In our case, based on the assumption that $\Delta t$ is equivalent to 15 minutes, and according to Table 7, the proposed model is effective in solving the problems of 7 initial jobs subject to disruptions which appear at each period by a probability with up to 0.8 . It is equivalent to 27 jobs in the total on average.

Table 8. Computing time

| Initial jobs | $p_{\theta}$ | Min $C T(s)$ | Max $C T(s)$ | $\operatorname{Avg} C T(s)$ | $\operatorname{Std} \operatorname{dev} C T(s)$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | 0.2 | 0.27 | 0.51 | 0.35 | 0.07 |
| 5 | 0.5 | 0.42 | 0.69 | 0.51 | 0.07 |
|  | 0.8 | 1.01 | 2.31 | 1.57 | 0.4 |
|  | 1 | 2.39 | 460 | 65 | 142 |
| 7 | 0.2 | 3.98 | 6.12 | 4.96 | 0.68 |
|  | 0.5 | 5.71 | 9.86 | 6.52 | 1.2 |
|  | 0.8 | 16.48 | 8012 | 829 | 2523 |
|  | 1 | 1231 | 20945 | 10622 | 6933 |

We can remark that the computing time depends on the set of initial jobs and the appearance frequency of jobs. If these ones are bigger, the average and the standard deviation of $C T$ increase, which makes difficult to estimate the computing time for solving the proposed model.

As mentioned previously, the set of jobs that have to be rescheduled at each step contains the unexecuted jobs combined with the new ones. Thus, when a job with a long processing time occupies the machine, if disruptions still appears during this period, the set of jobs that have to be rescheduled increases, because it will contain a lot of unexecuted jobs that are waiting since the machine is busy. This matter appears when the disruptions coincide with a long processing
time's job. This explains the big values of the average and standard deviation of $C T$ when $p_{\theta}=$ 0.8 and $p_{\theta}=1$, observed in Table 8.

## 7. Conclusion and perspectives

In this paper, we have proposed a new multi criterion objective function for a parallel machine rescheduling problem. The proposed measure combines simultaneously efficiency represented by total weighted waiting times, and stability as the weighted completion time deviation. This multi criterion has never been studied in the literature before, and it can be a very helpful and significant criterion in industrial and hospital environments. We also measured the number of jobs processed on different machines after disruption. To tackle this problem, we adopted a predictive-reactive strategy which consists in updating at each time the schedules, in response to a disruption caused by the arrival of new jobs. Consequently, we developed a mixed integer linear programming model, as well as an iterative methodology to solve the rescheduling problem. The resolution of the problem provides, for each step, an optimal solution. The numerical results we have presented in this paper allow us to observe that:

- The stability criterion can generate a proactive effect, when $\alpha$ is between 0.7 and 0.9 , providing better results compared to a mono-criterion considering only efficiency.
- Using the lexicographic optimization, in the majority of cases, we can minimize the number of jobs which are processed on different machines after disruption without impacting the principal objective function.
- The computing time and maximum duration of iteration depend on the set of initial jobs and on the appearance frequency of the new arrival jobs. When these ones are bigger, $C T$ and $M D I$ increase.

These research works can help decision makers to provide, at each step, an optimal schedule in response to a disruption caused by the arrival of new jobs in a parallel machines scheduling problem. Although the proposed model provides optimal solutions, it is limited in terms of number of jobs. Therefore, in our future works, we intend to propose heuristics that allow to browse more jobs in a short time. We also intend to take interest in flowshop rescheduling problem.

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