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# A simplified non-linear modelling strategy to generate fragility curves for old masonry buildings

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## Abstract

In the context of historical seismology, studying the behaviour of historic masonry buildings is of great importance, as they are witnesses of past events. While interesting methods can be found in the literature to assess the seismic vulnerability of masonry structures subject to strong earthquakes, the topic of moderate seismicity, as encountered in many European countries, is still to be investigated. The present work proposes a global methodology to build fragility curves for existing masonry buildings. An efficient computational method to address the non-linear response of masonry structures is presented. The method is based on the modal decomposition of the structural response. An equivalent non-linear single degree of freedom oscillator is identified for the main modes. The modelling strategy enables to carry out numerous computations with a low computational effort, allowing for a probabilistic approach. To build fragility curves, a damage indicator based on the frequency shift is chosen. To validate the proposed approach, the simplified model of a real masonry building is compared to a full non-linear time history analysis. The method is eventually illustrated by the derivation of fragility curves for industrial masonry buildings.

*Keywords:* Masonry, Fragility Functions, Historical seismology, Damage

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## 1. Introduction

For design purposes, a level of seismic intensity based on the known past earthquakes is defined. In moderate seismic regions, the instrumental seismology is not old

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4 enough to rely on measured earthquakes only. The definition of a macro-seismic inten-  
5 sity for historical earthquakes is needed, often based on archival testimony. However,  
6 the data available to quantify these historical earthquakes may not be sufficient. In par-  
7 ticular, if no considerable damage is reported in the identified historical documents, the  
8 uncertainties related to the macro-seismic intensity and to the epicentre location of the  
9 earthquake may be high.

10 Recent methodologies [1] propose to introduce structural analyses in the process  
11 of defining the macroseismic intensity of historical earthquakes. The main idea is to  
12 update the distribution of intensities or magnitudes of the considered earthquake by  
13 means of a Bayesian approach, combining historical data, fragility curves and in-situ  
14 observed damages. This approach is very promising in the context of low to moderate  
15 seismic activity, such as encountered in France, associated to a large amount of old  
16 structures as well as cultural heritage. The method can be used as a way to reduce  
17 the uncertainties associated to the the process of assigning magnitudes to historical  
18 earthquakes. For this purpose, the building stock is classified in a number of typologies  
19 for which fragility curves are computed [2, 3]. This paper focuses on the derivation of  
20 such fragility curves for old masonry buildings subject to low to moderate earthquakes.

21 The structural analysis of historical masonry buildings can be handled with dif-  
22 ferent numerical strategies as presented in [4] and [5]. A global methodology to de-  
23 rive fragility functions for masonry structures can be found in [6], along with a set of  
24 fragility curves for specific building typologies. Macro-element models are commonly  
25 used for the generation of fragility curves [5, 7]. An exemple of numerical limit anal-  
26 ysis based on macro-elements can also be found in [8]. Nevertheless, finite element  
27 (FE) based modelling has several advantages, as explained below, if adequate amount  
28 of computational resources [9] is available. FE models are used to address the seis-  
29 mic vulnerability of historical structures exhibiting several local modes [10, 11, 12], or  
30 also within the framework of limit analysis [13, 14]. The macro-element approach is  
31 very interesting for a relatively high seismic level, when a global state of damage with  
32 large macro-cracks is reached for the structure. In the context of low seismic activity  
33 however, the seismic load generally leads to low damages. The cracks are rather small  
34 and distributed in the structures. In this case, a FE computation is more relevant for  
35 the structural analysis, and several masonry models have been developed based on a  
36 continuous approach [15, 16, 17, 18]. To build fragility curves based on this approach,  
37 a large number of computations must be carried out for a structure typology. Indeed,  
38 uncertainties on the structure as well as on the material must be taken into account.  
39 Thus, an efficient numerical method must be adopted.

40 In this paper, a global modelling strategy is presented, based on the modal decom-  
41 position of the structure and the identification of non-linear single degree of freedom  
42 (NLSDOF) oscillators. Despite its simplicity and the use of certain hypotheses, non-  
43 linear simple oscillators allow to obtain an accurate global response and information  
44 regarding the damage state of the structure considering global indicators. The method  
45 is suitable for structures for which local modes are not preponderant or do not lead  
46 to strong damage. After a detailed presentation of the developed modelling strategy,  
47 the method is applied to the case of a real historical masonry structure. The computed  
48 response is first compared to a full 3D non linear time history analysis (NLTHA) of  
49 the building. The probabilistic framework is then addressed. From the computed struc-

50 tural responses, the method used to derive fragility curves is presented. The definition  
 51 of a structural damage criterion is discussed, and fragility curves are derived for the  
 52 considered case-study.

## 53 2. Modelling strategy

54 The purpose of this work is to derive fragility curves for historical buildings for  
 55 cases where low damage is expected. To take into account the spatial and material  
 56 variability of structures within a typology, a large number of computations are needed.  
 57 In order to overcome the computational cost of such studies, specific tools need to be  
 58 used. A simplified modelling strategy of the non-linear response of the structure is  
 59 developed here. The overall methodology is based on the displacement modal decom-  
 60 position, described in section 2.1. A single degree of freedom oscillator is identified  
 61 for each mode of the considered structure by a pushover-like analysis. A 3D finite el-  
 62 ement model of the structure is used to compute its response. The model is described  
 63 in section 2.2, along with the corresponding developed numerical tools. The simplified  
 64 model and the identification of the parameters governing the response of the simple  
 65 oscillators are finally explained in section 2.3.

### 66 2.1. A simplified method to compute the nonlinear structural response

67 To carry out a fragility analysis accounting for the variability of the considered  
 68 building and seismic loading properties, efficient numerical tools are needed. In what  
 69 follows, the simplified modelling strategy of the nonlinear response of the structure  
 70 based on the modal decomposition of the response is presented.

71  
 72 The displacement field  $\mathbf{U}(t)$  can be expressed on the modal basis according to Eq.  
 73 (1).  $\phi_i$  and  $q_i(t)$  are respectively the eigenvector and the modal displacement of the  
 74 mode  $i$ .  $\mathbf{P}$  consequently denotes the matrix of the modal basis.

$$\mathbf{U}(t) = \sum_i q_i(t) \phi_i = \mathbf{P} \mathbf{q}(t) \quad (1)$$

75 In the context of low to moderate damage for regular buildings, the mode shape is  
 76 assumed to remain unaltered. Furthermore, the energies stored or dissipated in the sys-  
 77 tem are considered to evolve independently for each mode. This framework is similar  
 78 to the one proposed for modal pushover analysis [19, 20]. The free energy  $\psi$  in the  
 79 structure is decomposed on each mode according to Eq. (2), where  $V_i$  are the internal  
 80 variables associated to the chosen model of mode  $i$  (damage, plasticity. . .):

$$\psi(\mathbf{U}, \dots) = \sum_i \psi_i(q_i, V_i) \quad (2)$$

81 The dissipation  $\mathcal{D}_{vis}$  associated to viscous damping is also considered decomposed  
 82 on each mode shape as displayed in Eq. (3).  $c_i$  is the damping coefficient for mode  
 83  $i$ , modelled as a linear viscous damper. The dissipated energy is eventually written in  
 84 terms of the natural vibration frequency of mode  $i$   $\omega_i$  and the corresponding viscous



85 damping ratio  $\xi_i$ . Considering the modal displacement responding as a single degree  
86 of freedom vibrating in its linear elastic range,  $c_i = 2 \xi_i \omega_i$ , we have:

$$\mathcal{D}_{vis}(\dot{\mathbf{U}}, \dots) = \sum_i \mathcal{D}_{visi}(\dot{q}_i) = \sum_i \frac{1}{2} c_i \dot{q}_i^2 = \sum_i \xi_i \omega_i \dot{q}_i^2 \quad (3)$$

87 Since the mass matrix  $\mathbf{M}$  does not evolve, the kinetic energy  $\mathcal{T}$  can be decomposed  
88 independently on each mode (Eq. (4)).

$$\mathcal{T}(\dot{\mathbf{U}}, \dots) = \sum_i \mathcal{T}_i(\dot{q}_i) = \sum_i \frac{1}{2} \boldsymbol{\phi}_i^T \mathbf{M} \boldsymbol{\phi}_i \dot{q}_i^2 = \sum_i \frac{1}{2} m_i \dot{q}_i^2 \quad (4)$$

89 Furthermore, with  $a_{gk}(t)$  denoting the acceleration in the direction  $k$  and  $\mathbf{A}_k$  the  
90 vector associated to the direction  $k$ , the mathematical expression of the generalized  
91 force  $Q_i$  associated to the seismic loading on mode  $i$  can be expressed as :

$$Q_i = - \sum_k \boldsymbol{\phi}_i^T \mathbf{M} \mathbf{A}_k a_{gk}(t) \quad (5)$$

92 Let's introduce  $\mathcal{L}$  the Lagrangian of the structure, equal to  $\mathcal{L} = \mathcal{T} - \psi$ . According  
93 to the Lagrange equations, the equation of equilibrium on each mode reads:

$$\frac{d}{dt} \left( \frac{\partial \mathcal{L}}{\partial \dot{q}_i} \right) - \left( \frac{\partial \mathcal{L}}{\partial q_i} \right) = Q_i - \left( \frac{\partial \mathcal{D}_{vis}}{\partial \dot{q}_i} \right) \quad (6)$$

94 That is to say:

$$m_i \ddot{q}_i(t) + 2 \xi_i \omega_i \dot{q}_i(t) + f_{int}^i(q_i(t)) = - \sum_k \boldsymbol{\phi}_i^T \mathbf{M} \mathbf{A}_k a_{gk}(t) \quad (7)$$

95  $f_{int}^i(q_i(t))$  is the internal force associated to mode  $i$ . Its expression derives from the  
96 free energy  $\psi_i$  (Eq. (8)):

$$F_{int}^i(q_i(t)) = \frac{\partial \psi}{\partial q_i} = \frac{\partial \psi_i}{\partial q_i} \quad (8)$$

97 The model used to compute the modal response (ie  $F_{int}^i(q_i(t))$ ) can be more or less  
98 complex according to the nonlinear phenomena considered. Section 2.3 describes the  
99 model chosen for the present study. It is identified thanks to the global response of  
100 the structure submitted to a displacement field proportional to the mode shape. This  
101 global response is computed thanks to a 3D finite element analysis of the structure, as  
102 described in the following section.

## 103 2.2. 3D finite element analysis of the structure

104 In this section, the 3D numerical model used to compute the structural response  
105 is described. This model will be used to identify the simplified model behaviour in  
106 section 2.3. The numerical tools to generate a parametric mesh are developed first.  
107 The parametric mesh allows for handy propagation of uncertainties on geometrical  
108 parameters of the studied structures, using the SalomeMeca platform [21]. Then, the  
109 nonlinear material model to compute the masonry structural behaviour is explained.

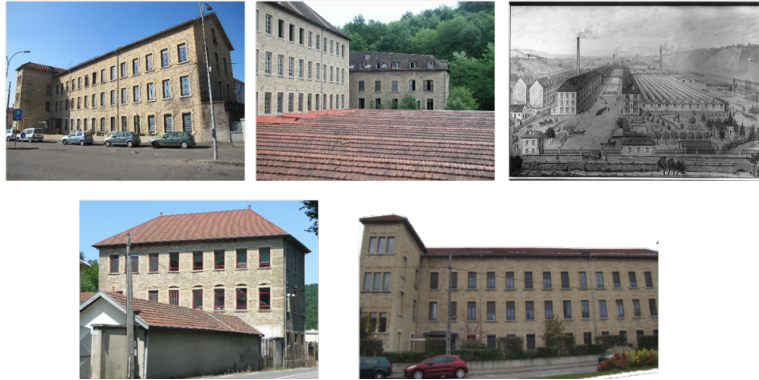


Figure 1: Examples of French historical industrial buildings in masonry from the 19th century. Pictures from <https://collections.isere.fr/> and [24]

110 The output of the global response used for the identification of the SDOF parameters  
 111 are eventually described.

112 *2.2.1. A parametric mesh to account for structural uncertainties*

113 In order to develop a tool capable of describing different buildings of a same ty-  
 114 pology, a parametric mesh generator has been developed. With the help of the Salome  
 115 mesh module [21], it is possible to generate a parameterised mesh. In this framework,  
 116 the following variables have been considered: base length to width ratio, storey height,  
 117 openings distribution, openings to wall ratio. These parameters have been chosen ac-  
 118 cording to the influence they can have in the seismic response of the structure, such  
 119 as the modal analysis, or the development of the nonlinearities. This choice has been  
 120 guided by the recommendation of the Perpetuate project [5] and the authors' experience  
 121 [22].

122 The code generates a mesh with 4 and/or 3 nodes multilayer shell elements, as  
 123 structured as possible. The NETGEN mesh engine is used [21]. The mesh refinement  
 124 is adapted according to the case study in order to reach an optimum between com-  
 125 putational resources and quality of output in a probabilistic framework (i.e. a good  
 126 description of the main modal shapes and a good estimation of the associated eigenfre-  
 127 quencies).

128 In this paper, the case of the industrial building typology will be addressed to il-  
 129 lustrate the proposed methodology. Figure 1 displays several examples of buildings,  
 130 all related to textile industry in France in the nineteenth century [23]. Figure 2 shows  
 131 one result of the mesh generator for the industrial building typology. For this exam-  
 132 ple of the industrial building typology, the global size (length, width and height of the  
 133 building) is different for each sample. As the opening distribution does not seem to  
 134 vary much from a building to another for this typology, the variability of regularity or  
 135 amount of perforation were not considered as stochastic variables for the present study.

136 The material models are described below.

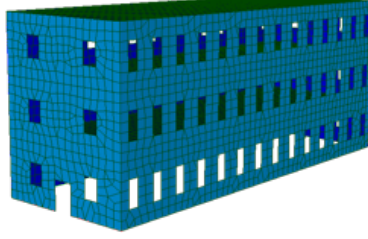


Figure 2: Example of a randomly generated mesh for the industrial building typology.

137 *2.2.2. Material modelling*

138 The masonry used for the buildings stock in the region of interest is mainly char-  
 139 acterized by a moderate aspect ratio, leading to a “weak” orthotropy. An isotropic  
 140 elastic model is thus considered to describe the linear behaviour of the masonry. When  
 141 subject to a monotonic loading, the nonlinear behaviour of the masonry is mainly char-  
 142 acterized by a progressive degradation of the stiffness with negligible dissipation by  
 143 hysteretic phenomena. These nonlinearities are well-reproduced with a damage model.  
 144 Due to the type of masonry (heterogeneous or with moderate aspect ratio), damage is  
 145 also considered as isotropic. The Mazars damage model [25] available in Code\_Aster  
 146 [26] has been used. This model is used to compute the response of the structure to a  
 147 pushover-like monotonic loading. Thus, the unilateral effect as well as complex hys-  
 148 teretic phenomena due to friction do not need to be described by the model. Examples  
 149 of application of Mazars damage model to masonry FE modelling can also be found  
 150 in the existing literature [27], and [28] for an application of the model with pushover  
 151 analysis over an old masonry tower.

152 The main equations of the model and the associated parameters are recalled in Eq.  
 153 (9) to (12).

154 The constitutive law is defined as:

$$\sigma = (1 - D) \cdot [\lambda \cdot \text{Trace}(\varepsilon) \cdot \mathbf{I} + 2 \cdot \mu \cdot \varepsilon] \quad (9)$$

155  $D$  is the scalar damage variable,  $\sigma$  denotes the stress tensor and  $\varepsilon$  the strain tensor.  
 156  $\lambda$  and  $\mu$  are the classical Lamé parameters. The threshold function  $\varphi$  associated to  
 157 damage is driven by an equivalent strain  $\varepsilon_{eq}$  depending on the principal positive strains  
 158 (Eq. (10)).

$$\varphi = \varepsilon_{eq} - \kappa(D) \quad (10)$$

159  $\kappa$  is a history variable corresponding to the maximum value reached for  $\varepsilon_{eq}$  along  
 160 history. It is equal to the strain at damage threshold  $\varepsilon_{D_0}$  for  $D = 0$  and up to damage  
 161 initiation.  $D$  is finally computed according to Eq. (11) and (12).

$$D = \alpha_C^\beta \cdot D_C + \alpha_T^\beta \cdot D_T \quad (11)$$

$$D_{C,T} = 1 - \frac{(1 - A_{C,T}) \varepsilon_{D_0}}{\varepsilon_{eq}} - A_{C,T} \exp(-B_{C,T} (\varepsilon_{eq} - \varepsilon_{D_0})) \quad (12)$$

162  $A_C$ ,  $B_C$  are parameters that define the evolution law in compression and  $A_T$ ,  $B_T$   
 163 are parameters that define the evolution law in tension. The variables  $\alpha_C$  and  $\alpha_T$   
 164 are computed thanks to the effective stress tensor. They weight the part of damage  $D_C$  due  
 165 to compression loading and  $D_T$  due to tension loading.  $\beta$  is a parameter that has been  
 166 originally introduced to better describe the response of quasi-brittle materials for shear  
 167 loading.

168  
 169 The constitutive laws are calibrated upon available data [29], [22].

170  
 171 For the simulation of the different structural elements, multilayer shell elements are  
 172 used. The nonlinear behaviour described above is used to model the vertical elements  
 173 representing the walls. A linear homogenized behaviour is considered for horizontal  
 174 elements modelling floors and roof, to optimize the computational time.

### 175 2.2.3. Structural global response

176 The 3D FE model described above is used to compute the structural response. The  
 177 computations of the 3D FE model have been performed with Code\_Aster [26]. After  
 178 generating a mesh corresponding to a typology of building, a modal analysis of the  
 179 structure is carried out. An example of response for one building for the main modes  
 180 in the x and y directions is displayed in figure 3. These modes are global modes of the  
 181 structure. The particular deformed shape that can be observed in figure 3 is due to the  
 182 stiffness of the floors, and to the relatively high number of openings.

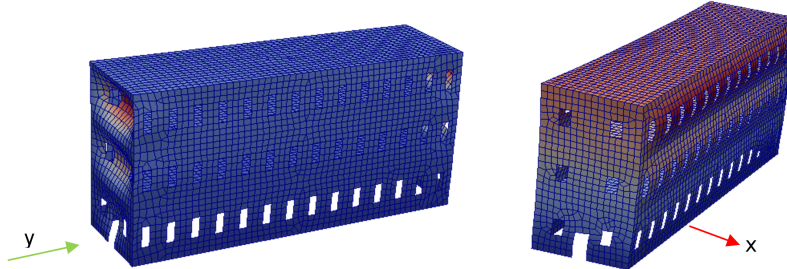


Figure 3: Modal analysis displacement fields for principal modes in y and x direction.

183 A nonlinear pushover-like analysis is then performed for the first natural modes.  
 184 Previous studies [11, 30] have successfully used this pushover approach to compute  
 185 the response of masonry structures. The structure is submitted to a displacement field  
 186 proportional to the mode shape:  $\delta_{load} = \lambda \phi_i$ . This means that the displacement control  
 187 coefficient  $\lambda$  is given by the distribution of displacements obtained with the modal  
 188 analysis, and can be directly identified as the modal displacement  $q_i$  defined in Eq.  
 189 (1). To identify the internal force  $F_{int}^i$ , a conversion factor is used. By considering the

190 direction  $k$  as the main direction of the considered mode,  $F_{int}^i$  can be identified with the  
 191 base shear force  $V_{b_k}$  in direction  $k$ , and the modal participation factor  $p_{i_k}$  of mode  $i$  in  
 192 the direction  $k$ . Eq. (13) provides this coefficient for the example of a linear behaviour  
 193 for mode  $i$ :

$$V_{b_k} = \Delta_k^T (\lambda \mathbf{K} \boldsymbol{\phi}_i) = \lambda \sum_i p_{i_k} \boldsymbol{\phi}_i^T \mathbf{K} \boldsymbol{\phi}_i = p_{i_k} k_i \lambda \rightarrow F_{int}^i(q_i) = k_i q_i = \frac{V_{b_k}}{p_{i_k}} \quad (13)$$

194 The computed response  $(V_{b_k}, q_i)$  is then used to identify the parameter of a single  
 195 degree of freedom (SDOF) model for each mode. This model and the identification  
 196 process are described in what follows.

### 197 2.3. Identification of the single degree of freedom system behaviour for each mode

#### 198 2.3.1. Damage model used for the single degree of freedom system

199 For the nonlinear SDOF behaviour of a mode, a unilateral damage model is con-  
 200 sidered. Eq. (14) gives the free energy of this model.  $k_i$  is the initial modal stiffness.  
 201  $D_i^+$  and  $D_i^-$  represent the damage associated with the positive and negative modal dis-  
 202 placements, respectively.

$$\psi_i(q_i, D_i^+, D_i^-) = \frac{1}{2} k_i (1 - D_i^+) < q_i >_+^2 + \frac{1}{2} k_i (1 - D_i^-) < q_i >_-^2 \quad (14)$$

203 The internal force is obtained by deriving the free energy according to the modal  
 204 displacement (Eq. (15)).

$$f_{int}^i[q_i(t)] = \frac{\partial \psi_i}{\partial q_i} = k_i (1 - D_i^+) < q_i >_+ + k_i (1 - D_i^-) < q_i >_- \quad (15)$$

205 A threshold function is defined in Eq. (16) for each damage variable as a function  
 206 of the elastic energy release rate ( $Y_+ = \frac{1}{2} k_i < q_i >_+^2$  and  $Y_- = \frac{1}{2} k_i < q_i >_-^2$ ).  $d_\infty$  and  
 207  $b$  govern the evolution of the threshold function.  $Y_0$  corresponds to the limit energy  
 208 of the linear behaviour. These parameters characterize the global damage both in the  
 209 positive and negative directions of the displacement. This assumption of taking the  
 210 same value of  $d_\infty$ ,  $b$ , and  $Y_0$  seems reasonable for simple modes without complex  
 211 triaxial modeshape. It must be remarked here that the methodology cannot detect a  
 212 damage related to torsion.

$$\varphi_+ = Y_+ - Y_0 \left( \frac{d_\infty}{d_\infty - D_i^+} \right)^{\frac{1}{b}} \leq 0 \quad | \quad \varphi_- = Y_- - Y_0 \left( \frac{d_\infty}{d_\infty - D_i^-} \right)^{\frac{1}{b}} \leq 0 \quad (16)$$

213 Damage evolves by respecting the Kuhn-Tucker conditions. By considering the  
 214 condition  $\varphi_i = 0$  when damage evolves, the damage evolution law can be derived (Eq.  
 215 (17)).

$$D_i^+ = d_\infty \left[ 1 - \left( \frac{Y_0}{Y_+} \right)^b \right] \quad | \quad D_i^- = d_\infty \left[ 1 - \left( \frac{Y_0}{Y_-} \right)^b \right] \quad (17)$$

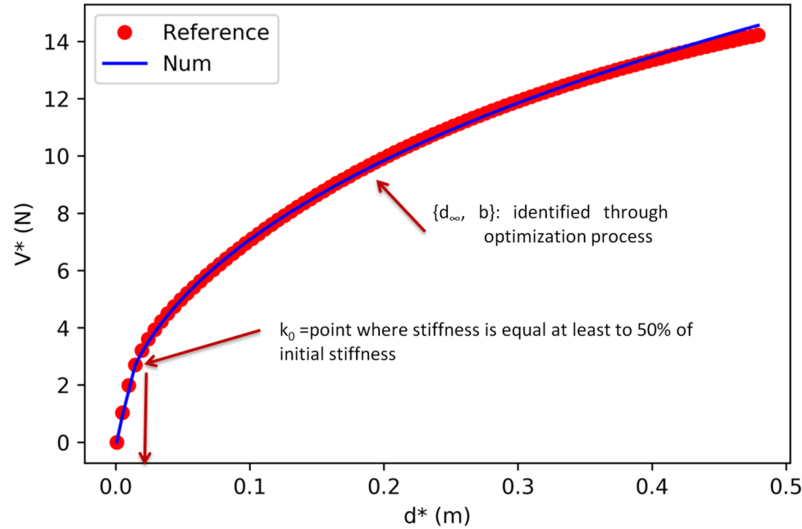


Figure 4: Example of identification of the SDOF nonlinear model for one mode.

216 **2.3.2. Model identification**

217 From the nonlinear response ( $V_{b_k}, q_i$ ) of the full 3D model of the structure (see  
 218 section 2.2.3), the parameters  $Y_0$ ,  $d_{\infty}$ , and  $b$  of the single degree of freedom model  
 219 for mode  $i$  can be identified. The error between the response obtained by the damage  
 220 model for the SDOF system and the response of the 3D model is minimized using the  
 221 least-square method. Figure 4 displays the identification process, with  $V^* = V_{b_k}/p_{ik}$   
 222 and  $d^* = q_i$ . Figure 5 summarizes the modelling strategy, from the identification of a  
 223 structural typology to the identification of the SDOF behaviour.

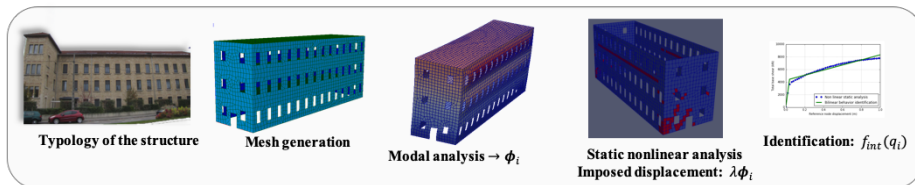


Figure 5: Modelling strategy.

224 **2.4. Modal recombination**

225 The time history analysis of the structure subject to a seismic loading is carried out  
 226 on the modal basis. In order to limit the number of 3D non-linear analysis, only the  
 227 first main modes are considered as non-linear. The higher modes are assumed to remain  
 228 linear, with properties directly obtained from the modal analysis of the 3D structure.

229 Once the parameters governing the behaviour of the SDOF systems are identified for  
 230 each mode, their response to a seismic loading can be easily computed by following Eq.  
 231 (7). The integration method is the classical Newmark method. The non-linear equa-  
 232 tions are solved using a Newton-Raphson procedure. The displacement of each point  
 233 of the structure is then obtained by modal recombination of the computed responses.  
 234 This simplified modelling allows to get a complete response of a structure using few  
 235 computational resources.

### 236 2.5. Damage state indicator

237 In order to build fragility curves for existing structures, a relevant damage indicator  
 238 needs to be defined. The damage model chosen to compute the SDOF oscillator re-  
 239 sponse allows to easily compute a criterion based on frequency shift, called the Eigen-  
 240 Frequency Drop Off (EFDO). In this way, it is not necessary to compute the global  
 241 behaviour of the structure using the modal recombination. This indicator is common  
 242 to detect global damage of a structure [31]. Its relevancy has been evidenced by exper-  
 243 imental testing [32].

244 The damaged frequency  $f_D^i$  of mode  $i$  and the damage level  $D$  corresponding to a  
 245 frequency shift  $\Delta f$  are obtained with Eq.(18).  $D$  is the maximum damage level reached  
 246 by mode  $i$  ( $D = \max[D_i^+, D_i^-]$ ).  $f_0^i$  is the non damaged frequency of mode  $i$ .

$$f_D^i = \sqrt{1-D}f_0^i \mid \Delta f = \frac{f_0^i - f_D^i}{f_0^i} \rightarrow D = 2\Delta f - \Delta f^2 \quad (18)$$

247 The results of an analysis carried out on a SDOF equivalent oscillator are shown in  
 248 figure 6. It is interesting to note that the damage variable evolves through time. This  
 249 leads to a shift in the frequency response of the structure that can be seen by comparing  
 250 the peaks of the fast Fourier transform (FFT) for the elastic response and the non-  
 251 linear response. The FFT is carried out on the displacement of the SDOF system.  
 252 For this example, the evolution of damage of the SDOF oscillator leads to an EFDO  
 253 equal to 70%. The advantage of the adopted damage model is that some softening  
 254 can be considered as well as an evolution in the damage law. As expected, damage  
 255 is concentrated in the strong motion phase of the signal, approximately between 5s  
 256 and 10s. A simple windowing algorithm can be used to calculate the EFDO for each  
 257 oscillator and then for each frequency. It has been observed that higher frequency  
 258 modes are less prone to step into a damaged state, because a small amount of energy  
 259 is introduced by the earthquake in this frequency range. This is why the results will  
 260 focus on the EFDO of the main longitudinal and transversal modes. It has been also  
 261 observed that, in the vast majority of cases, damage concentrates in the first and second  
 262 modes. The EFDO of higher modes is then a less interesting indicator of structural  
 263 performance, although it might be an evidence of possible local phenomena activation.

264 The damage state 0 (DS0), characterized by negligible to slight damage (EMS98  
 265 description [33]) is defined by a range from 0% EFDO up to 15% EFDO (Limit State 1  
 266 – LS1). The average damage map observed with the pushover analysis for LS1 corre-  
 267 sponds to distributed cracks that do not affect at all the integrity of the structure. This  
 268 15% EFDO criterion for reaching LS1 is consistent with results obtained by shaking  
 269 table tests on masonry houses. For example, [32] obtained a drop in frequency of 11%

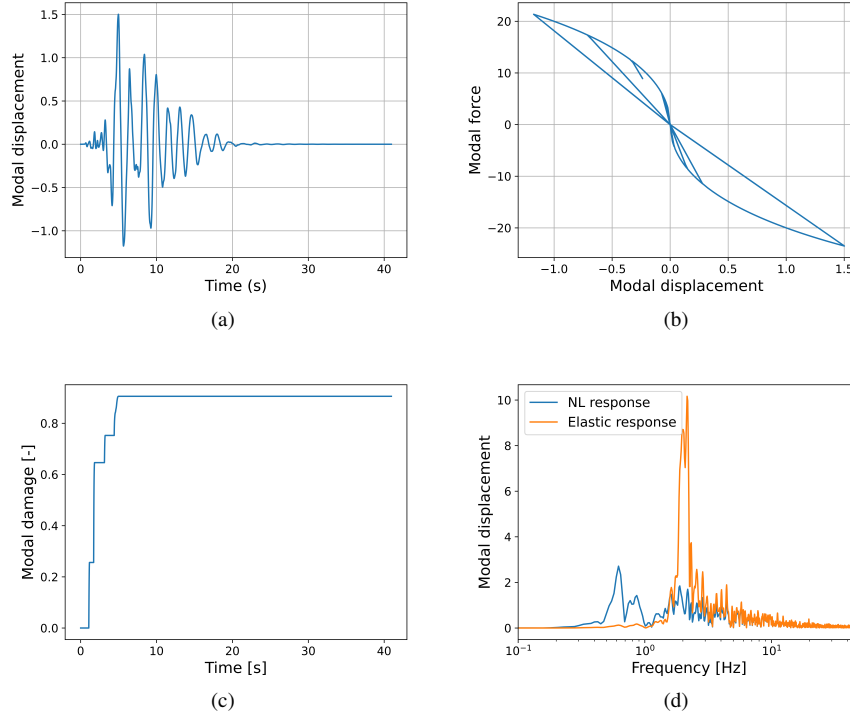


Figure 6: NLTHA of a SDOF oscillator: 6(a): displacement vs time response, 6(b): force vs. displacement response 6(c): damage evolution. 6(d): identification of the EFDO by means of a FFT calculated for the elastic oscillator (orange) and on the non linear one (blue).

270 for the calcium silicate masonry structure and 18% for the clay masonry structure, at  
 271 the beginning of the nonlinear phase of the behaviour. [34] observed a frequency drop  
 272 off of 19 % at the beginning of the first damage limit state (denoted by DL2 in [34]).  
 273 For DS1 (15% to 30% EFDO), cracks are more widely distributed on the buildings,  
 274 corresponding to moderate damage (EMS98 description [33]). This corresponds to an  
 275 intermediate limit state between the DL2 and DL3, as defined by [34]. For larger damage  
 276 states, the authors recommend to consider other modelling strategies (e.g. Structural  
 277 Element Method or Macro Blocks Method [5]), that better describe the occurrence  
 278 of mechanisms associated to large macro-cracks. Nevertheless, these damage states are  
 279 not expected to occur for the earthquake intensities considered. Figure 7 describes the  
 280 damage states for the response of a SDOF calculated directly with the damage variable  
 281 (Eq. (18)).



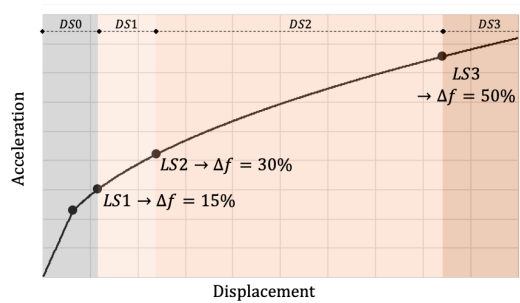


Figure 7: Definition of the damage states on the capacity curve of a SDOF (ADRS format)

### 282 3. Validation of the simplified modelling strategy

283 The interest of the presented approach is the possibility to perform calculations at  
 284 the scale of a real structure. To illustrate this, the method is applied to a real building,  
 285 from the typology shown in figure 1. This industrial building is representative of histor-  
 286 ical masonry structures built in France in the nineteenth century. This type of building  
 287 is characterized by a regular shape with a homogeneous distribution of the openings  
 288 and a regular story height. These simple characteristics allow to develop a parametric  
 289 model with a relatively small number of parameters in order to cover a large panel of  
 290 buildings in this typology.

291 To validate the simplified modelling strategy, one structure of the typology is con-  
 292 sidered. It is 40 meters long, for a width of 14 meters, with 3 stories of 6 meters each.  
 293 Figure 8 specifies the dimensions and axes used for this reference structure. The be-  
 294 haviour resulting from the approach detailed in section 2 is compared to a non-linear  
 295 time history analysis (NLTHA) of a 3D model of the structure. Walls are modelled with  
 296 multilayer quadrangle shell elements and slabs with linear quadrangle shell elements.  
 297 The multilayer shell elements have 6 layers, with 1 integration point per layer. The  
 298 element size of the mesh is 60 cm. The corresponding mesh is represented in figure 9.

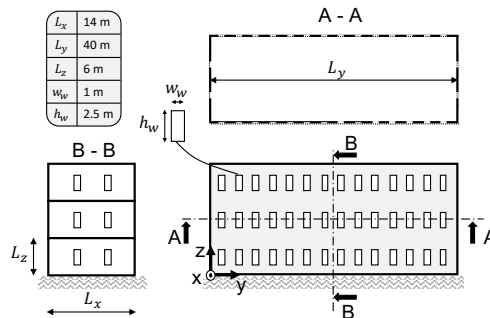


Figure 8: Geometry of the reference structure.

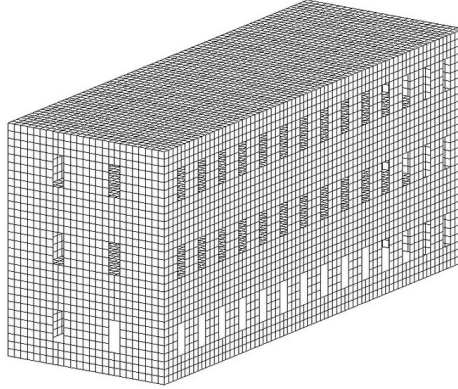


Figure 9: Mesh of the reference structure

299 To carry out the NLTHA, the seismic acceleration is applied as inertia forces con-  
 300 sidering a study in the relative frame. The structure is embedded at its base. Two  
 301 synthetic accelerograms have been generated for this first study with target PGA of  
 302 0.1 g and 0.6 g. The method used to generate the accelerograms is detailed in section  
 303 4.2. The accelerograms have been applied to the 3D model of the structure, in the x di-  
 304 rection (weak direction of the structure). Even though the level of acceleration of 0.6 g  
 305 is too high for the purpose of the study, dedicated to low to moderate seismicity, this  
 306 test case is interesting to evaluate the robustness of the proposed modelling technique  
 307 for larger non-linearities.

308 The time-history computation of the structure is performed with the CAST3M fi-  
 309 nite element software [35]. Because of the dynamic loading, a damage model able to  
 310 account for the unilateral effect needs to be used. A suitable isotropic damage model,  
 311 called RICCOQ in the CAST3M software, is thus used here for the reference model.  
 312 It corresponds to a simplified version of the model proposed by [36], keeping only the  
 313 nonlinear mechanisms associated to damage. Its parameters have been calibrated at the  
 314 material scale to provide the same average tensile behaviour curve as the Mazars damage  
 315 age model. The tensile strength  $f_t$ , compressive strength  $f_c$ , and the corresponding peak  
 316 deformations  $\varepsilon_t^u$  and  $\varepsilon_c^u$  are given in table 1. The corresponding material parameters for  
 317 the Mazars' model, described in section 2.2.2, are listed in table 2.

Model	E [GPa]	$f_t$ [MPa]	$f_c$ [MPa]	$\varepsilon_t^u$ [-]	$\varepsilon_c^u$ [-]	$\rho$ [kg/m <sup>3</sup> ]
Unilateral	2.59	0.4	10	0.001	0.01	2500

Table 1: Material parameters of the unilateral damage model used for the masonry walls for the seismic response of the reference 3D model.

318 The classical Newmark time integration scheme is used, with parameters  $\gamma = 1/2$ ,  
 319  $\beta = 1/4$ . A Newton-Raphson procedure is used to compute the non linear response of

Model	E [GPa]	$B_T$ [-]	$\epsilon_{D0}$ [-]	$\rho$ [kg/m <sup>3</sup> ]
Mazars	2.59 GPa	6250	0.00016	2500

Table 2: Material parameters of the the Mazars' model used for the masonry walls used in the proposed approach for the pushover-like analysis.

320 the structure.

321 As explained in section 2.3, a modal analysis is also carried out on the 3D structure.  
 322 An imposed displacement following the first mode shape enables the identification of  
 323 a SDOF behaviour. The corresponding force-displacement curve is displayed in figure  
 324 10.  $d^*$  is the control coefficient for the displacement, and  $V^*$  the base shear force  
 325 normalized by the modal participation factor.

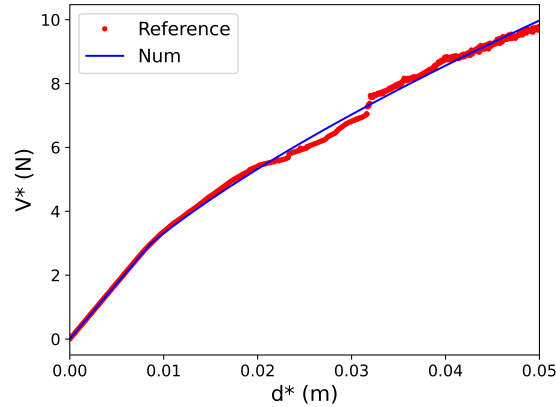


Figure 10: Identification of the nonlinear SDOF model (Num, in blue) from the global response of the pushover analysis (Reference, in red), following the procedure described in section 2.3.

326 The dynamic response of the Non Linear Equivalent Single Degree of Freedom  
 327 (NLESDOF) oscillator is compared to the full NLTHA of the reference model. Figures  
 328 11 and 12 display the response in displacement of the middle point at the roof level  
 329 and the normalized FFT of both models. The normalized FFT is obtained by dividing  
 330 the FFT by the maximum of the signal. The direction of the loading has been chosen  
 331 as it is the weakest direction for the building. Therefore, it corresponds to the direc-  
 332 tion in which damage develops the most. The results for three acceleration levels are  
 333 presented. The response to the 0.1 g PGA seismic input has been chosen to display  
 334 the results around the elastic limit. The 0.6 g PGA response corresponds to extended  
 335 damage.

336 It can be observed in figure 11 that both models provide the same maximum dis-  
 337 placement in the medium low acceleration domain (PGA = 0.1 g). For larger PGA  
 338 (figure 12), the NLESDOF tends to overestimate the maximum displacement of the  
 339 middle point at the roof level, regardless the number of modes recombined to get the

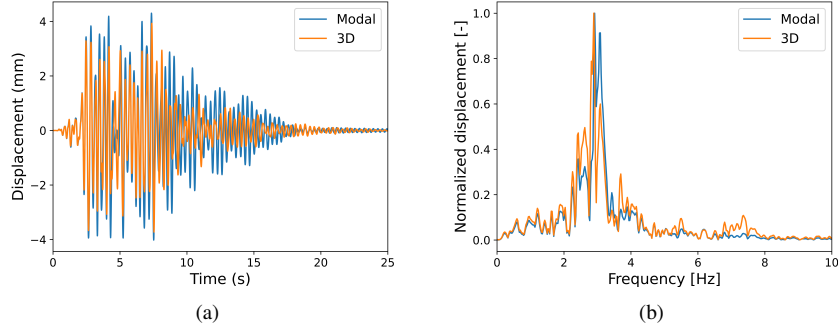


Figure 11: Comparison of the responses obtained by the simplified and the global FE model, for an acceleration level of 0.1 g: displacement of the middle point at the roof level and normalized FFT.

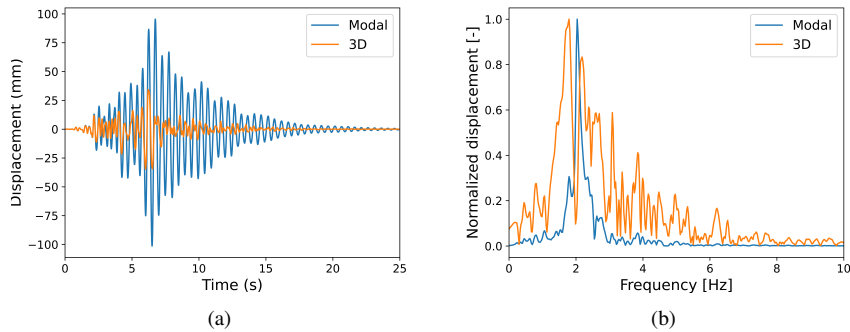


Figure 12: Comparison of the responses obtained by the simplified and the global FE model, for an acceleration level of 0.6 g: displacement of the middle point at the roof level and normalized FFT.

340 response. This difference regarding the maximum displacement may be explained by  
 341 the fact that the evolution of damping for the first mode can be observed in the 3D  
 342 model considering the Rayleigh damping matrix. This effect is not considered in the  
 343 NLESDOF model. In figure 13, one can see that with a modification of the damping  
 344 ratio associated to the first mode (2%  $\rightarrow$  10%), the maximum displacement is recover-  
 345 ed with the simplified modelling. Nevertheless, without considering modification of  
 346 damping ratio for the NLESDOF, one can see from the normalized FFT of the same  
 347 response that the frequency drop off is well captured by the NLESDOF. The frequency  
 348 drop off obtained for both models and the two signals are detailed in table 3. The  
 349 damaged frequency is computed according to equation 18, using the maximum level  
 350 of damage (quantified by the local damage variable  $D$ ) encountered by the considered  
 351 structure during the seismic loading. The NLESDOF model is so sufficiently robust  
 352 to follow the damage evolution and consequently the eigenfrequency drop off up to

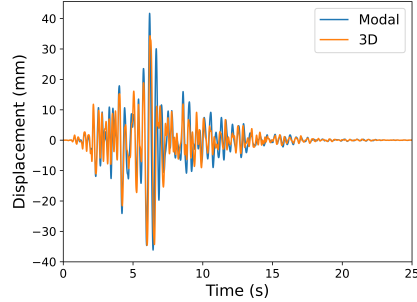


Figure 13: Comparison of the responses obtained by the simplified (with a modification of the damping) and the global FE model, for an acceleration level of 0.6 g: displacement of the middle point at the roof level and normalized FFT.

	0.1 g	0.6 g
Global FE	2%	39%
NLESDOF	2%	31%

Table 3: Eigenfrequency drop off for the global FE model and the NLESDOF for the two PGA levels.

353 0.6 g. This quantity is an important engineering performance indicator, that can be  
 354 accurately reproduced by the simplified approach. The efficiency of the proposed nu-  
 355 merical method enables extensive computations in a probabilistic framework, which is  
 356 useful to build fragility curves.

357 The damage fields after the applied seismic signals corresponding to an intensity  
 358 level of 0.1 g and 0.6 g are shown in figures 14 and 15 respectively. A light out-of plane  
 359 flexure damage of the longitudinal walls can be observed in the case of the 0.1 g seismic  
 360 loading. The damage profile of the 0.6 g case shows the activation of the in-plane shear  
 361 failure mechanism in the transverse walls, starting from the window corners.

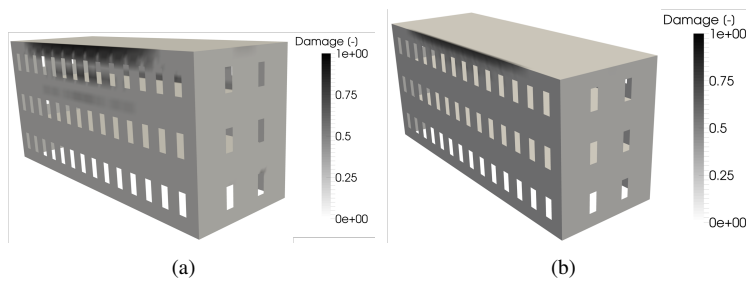


Figure 14: Damage field obtained by: (a) the global FE model, (b) the pushover analysis at the maximum displacement of the NLESDOF for mode 1, for an acceleration level of 0.1 g

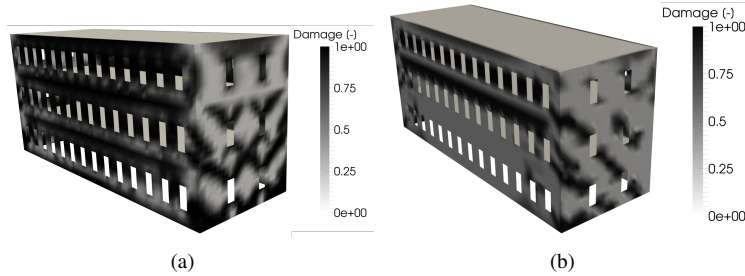


Figure 15: Damage field obtained by : (a) the global FE model, (b) the pushover analysis at the maximum displacement of the NLESDOF for mode 1, for an acceleration level of 0.6 g.

362 Qualitatively, these damage profiles compare well with the failure modes observed  
 363 in masonry shearwalls after an earthquake. Furthermore, the damage fields are compar-  
 364 able for the global FE model and for the damage state reached at the maximum  
 365 displacement obtained with the NLESDOF for mode 1. The main difference observed  
 366 for an acceleration level of 0.6 g is due to the fact that a monotonic loading is con-  
 367 sidered for the pushover analysis. As a consequence, the cracks due to shear loading  
 368 are only observed for one direction (i.e. no cross crack in shear wall for monotonic  
 369 loading).

## 370 4. Computation of fragility curves

### 371 4.1. Methodology

372 Fragility curves express the conditional probability  $P_{(DS \geq DS_i)}$  for the damage state  
 373 (DS) of the structure to reach a damage state level  $DS_i$ , for a given seismic intensity  
 374 measure (IM), denoted by  $\alpha$ . To compute fragility curves, damage states (DS) need to  
 375 be defined, as well as the associated limit states (LS). The damage criterion is already  
 376 discussed in section 2.5. For this study, a classical intensity measure is chosen with  
 377 the peak ground acceleration (PGA). A synthetic seismic database has been used, as  
 378 explained in section 4.2. To reduce the number of calculations of the structural response  
 379 under seismic loadings, the classical lognormal fragility model is used [37]. Eq. (19)  
 380 recalls the shape of the fragility curve, where  $\Phi$  is the cumulative distribution function  
 381 of the standard normal distribution.

$$P_{(DS \geq DS_i)}(\alpha) = \Phi\left(\frac{\ln(\alpha/A_m)}{\beta}\right) \quad (19)$$

382 The parameters are  $A_m$  the median capacity and  $\beta$  the logarithmic standard devia-  
 383 tion. The computation of these two parameters is performed with the maximum like-  
 384 lihood method [38]. More information regarding the derivation of fragility curves can  
 385 be found in [39]. The validity of the lognormal fragility model is analysed in section  
 386 4.3. The methodology is applied to derive fragility curves for the industrial building  
 387 typology.

388 *4.2. Seismic database*

389 To compute the fragility curves, a synthetic seismic ground motion database is  
390 considered. The signals are characterized by their Power Spectral Density (PSD) [40,  
391 41], which is a filtered white noise parameterised by the central frequency and the  
392 bandwidth. Uncertainty is accounted for by considering that the fundamental frequency  
393  $f_0$  follows a lognormal distribution. For this study, the mean value is set to 5 Hz. The  
394 coefficient of variation is taken equal to 10%. The bandwidth is characterized by the  
395 critical damping ratio of the filter. It is considered fixed for the PSD model and equal to  
396 0.5. The PSD is corrected for the low frequencies with a high-pass filter at a frequency  
397 0.1 Hz [42]. Finally, the shape of the accelerogram in the time domain is obtained with  
398 a Gamma modulation function, parameterised by the strong motion duration [43]. The  
399 mean value of the strong motion duration is equal to 9 seconds with a coefficient of  
400 variation equal to 10%. The start of the strong motion part for the modulation function  
401 is equal to 3 s.

402 *4.3. Analysis of the fragility curve model*

403 In order to validate the choice of the lognormal fragility model, a set of 2000 syn-  
404 thetic accelerograms is used, regularly distributed by packet of 100 each 0.05 g from  
405 0.005 g up to 1 g. To compute a reference fragility curve, the response of a NLSDOF  
406 structure is computed using these signals. Using the maximum damage experienced by  
407 the structures, the EFDO is obtained (Eq. 18). Figure 16 shows the EFDO computed  
408 for each accelerogram of the database.

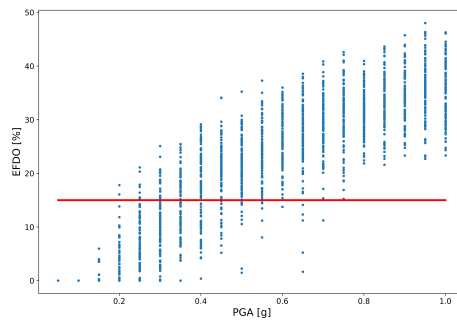


Figure 16: Cloud data representing the damage measure (EFDO) vs. the intensity measure (PGA). The 15% EFDO criterion is materialised by the red line.

409 This set is used to explicitly compute points of the fragility curve for an EFDO  
410 criteria of 15%. Figure 17 shows these point estimates as well as the lognormal fragility  
411 curve identified by maximum likelihood estimation (MLE).

412 As can be seen in figure 17, a very good agreement is obtained between the discrete  
413 points obtained by direct computation and the lognormal model. This confirms the  
414 relevance of using this model for the fragility analysis carried out in this study.

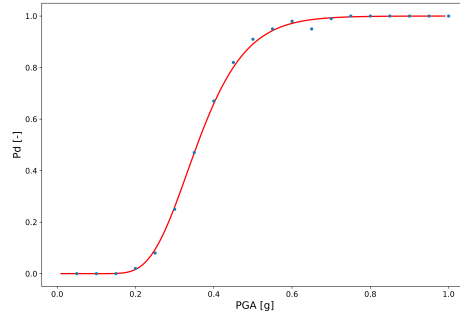


Figure 17: Discrete points and lognormal fragility curve.

415 Before computing fragility curves for the masonry buildings, a convergence analysis  
 416 of the lognormal fragility model versus the number of synthetic accelerogram is per-  
 417 formed. For this analysis, subsets of accelerograms are randomly picked in a database  
 418 of 2900 accelerograms with PGA from 0.04 g up to 2.3 g. Figure 18 shows the proba-  
 419 bility density of the pga for the synthetic database created for the study. One can see a  
 420 relative uniform distribution up to 1.5 g.

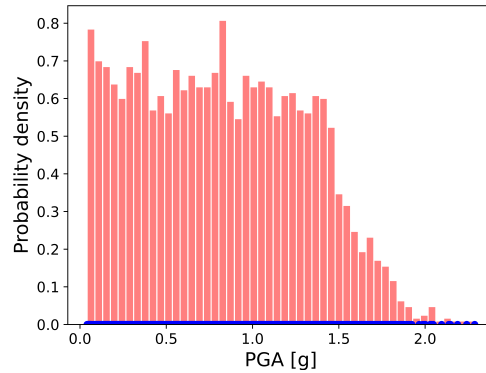


Figure 18: Probability density of the pga for the synthetic database.

421 Each sample of accelerograms is defined by picking signals randomly in  $N_s$  regu-  
 422 larly distributed subsets with  $N_s$  equal to the sample size. For the convergence analysis,  
 423 ten different sampling are considered for each sample size. The distribution of the value  
 424 identified by MLE for the median capacity  $A_m$  and the logarithmic standard deviation  $\beta$   
 425 versus the sample size is studied. Figures 19 and 20 show the decrease of the standard  
 426 deviation of the estimates and the convergence for  $\beta$  and  $A_m$  respectively. In accordance  
 427 with these results, a sampling size of 500 seismic signals has been adopted for



428 the last part of this study.

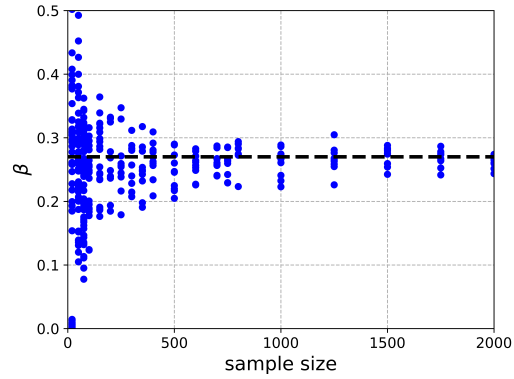


Figure 19: Evolution of the distribution of  $\beta$  according to the sample size.

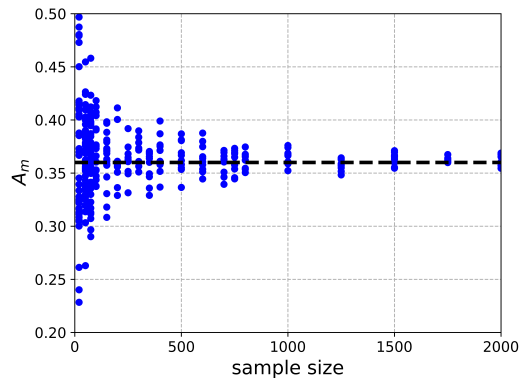


Figure 20: Evolution of the distribution of  $A_m$  according to the sample size.

429 *4.4. Derivation of fragility curves for a building typology*

430 The fragility model described and analysed above is used here to compute fragility  
431 curves for the industrial building typology. The global methodology to build fragility  
432 curves for a given typology is summarized in figure 21.

433 In order to consider the uncertainties in the characteristics of structures and mater-  
434 als, a probabilistic approach is adopted. Monte Carlo simulation is chosen. Each  
435 building is defined by the size of a story (length  $L_y$ , width  $L_x$ , and height  $L_z$ ). All the  
436 random parameters are considered to be log-normally distributed. Table 4 summarizes  
437 the random parameters used for the industrial building typology, with their mean  $\mu$  and

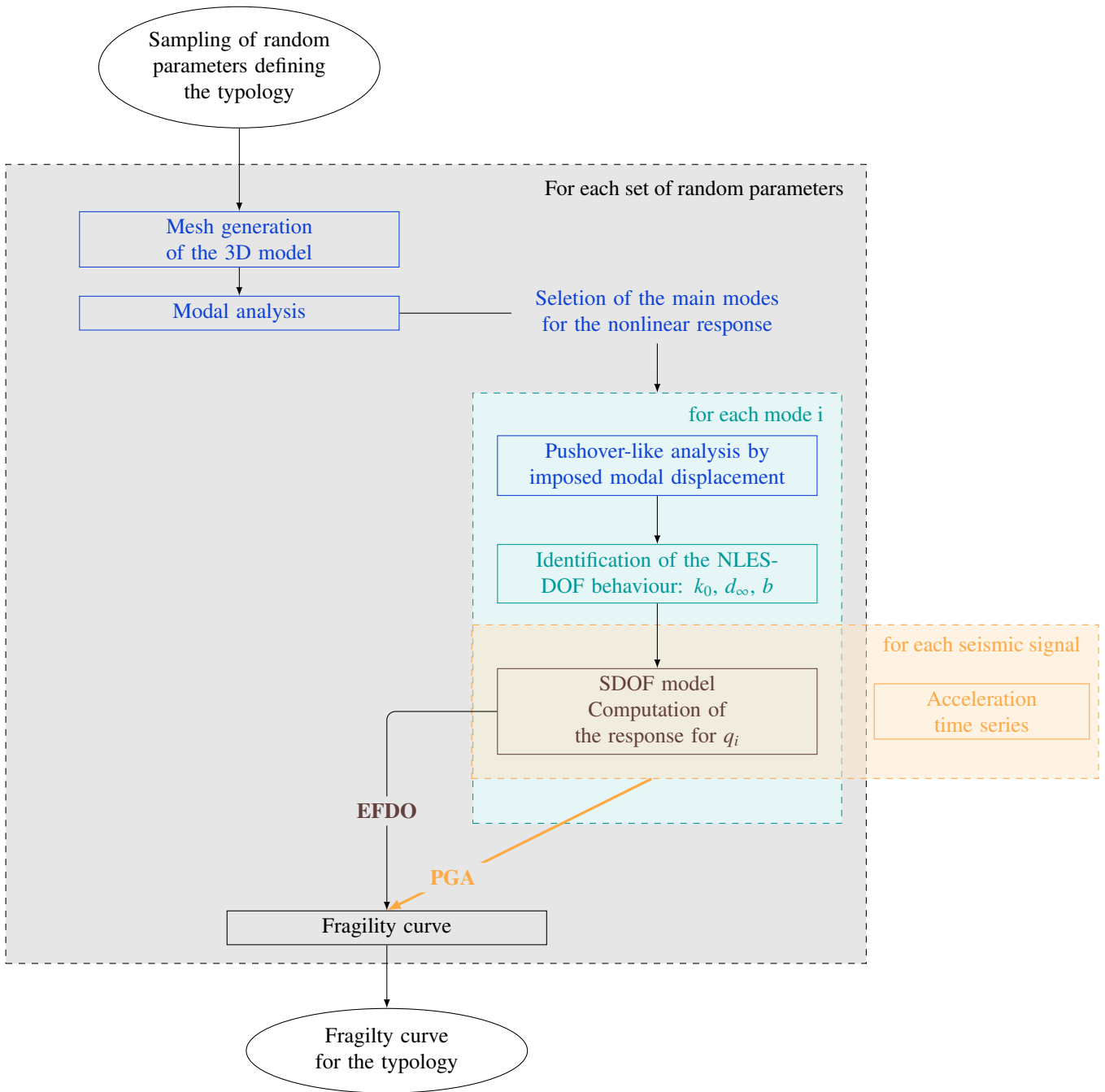


Figure 21: Global methodology to obtain the set of fragility curves for one typology.

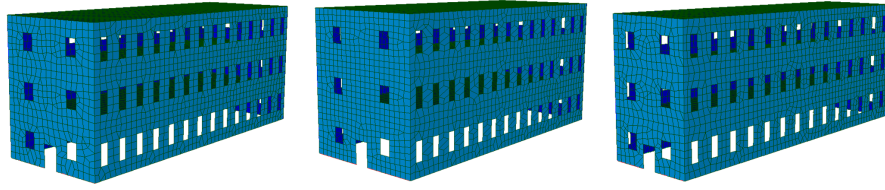


Figure 22: Geometrical mesh support: random generation of 3 models.

438 coefficient of variation COV values. Each building has three stories. The location and  
 439 size of the openings are related to the length and width of the building.

440 Wood floors are considered for the building typology. No specific information is  
 441 available as regards the floor stiffness, or their connection to the walls. In the model, the  
 442 connections between floors and walls are considered as rigid, and the high COV applied  
 443 to the floor stiffness is intended to take these uncertainties into account. Besides, the  
 444 chosen mean wood floor density is lower than the practical values, and the masonry  
 445 wall density higher. The choice has been made to lump part of the floor's masses to the  
 446 walls, to avoid the excitation of local vertical floor modes before the structural one.

Geometrical global parameters

	$\mu$	COV
Lx	14 m	20%
Ly	40 m	20%
Lz	6 m	5%

Material parameters for the masonry walls

	$\mu$	COV
Young modulus E	2.5E9 N/m <sup>2</sup>	20%
Damage model param. $B_{t1}$	6250	10%
Strain limit $\epsilon_{D0}$	0.00016	5%
Density $\rho$	2500 kg/m <sup>3</sup>	5%

Floor parameters

	$\mu$	COV
Equivalent Young modulus E	1E9 N/m <sup>2</sup>	50%
Equivalent density $\rho$	100 kg/m <sup>3</sup>	20%

Table 4: Characteristics of the random distribution for the structural parameters

447 The geometry and the mesh of each structure is generated using the parametric  
 448 mesh generator described in section 2.2. Multilayer shell elements are used to mesh  
 449 the structures. Each mesh is generated by the NETGEN mesh engine, as explained in

450 section 2.2.1. The mesh engine uses 3 and/or 4 nodes multilayer shell elements, DKT  
451 and DKQ, based on the "Discrete Kirchhoff" kinematics [21]. The elements have 5  
452 layers, which corresponds to 11 integration points in the element thickness. Figure  
453 22 displays an example of 3 randomly generated meshes for the considered building  
454 typology.

455 195 building models have thus been generated, corresponding to the industrial  
456 building typology. For each generated mesh, the identification procedure described  
457 in section 2 is applied. The parameters of the single degree of freedom model for the  
458 first mode along the weakest (x) axis are identified for each structure according to the  
459 process explained in section 2.3.2. This identification provides information relative to  
460 the probability density for each parameters of the single degree of freedom model:  $k_0$   
461 ( $=\omega_0^2$ ),  $Y_0$ ,  $d_\infty$ ,  $b$ , represented in figure 23. These distributions show how the uncer-  
462 tainties propagate from the geometrical and material parameters of the 3D model to the  
463 parameters governing the SDOF model of the first mode.

464 During this process of identifying the SDOF models for each oscillator, the com-  
465 putational effort is mainly taken by the pushover analysis. Several structures can be  
466 computed in parallel, which reduces the computation time. For this case-study, 10  
467 computation cores have been used. Doing so, the 195 models have been generated and  
468 identified within 2 days. Once the SDOF oscillators have been identified, the seismic  
469 response is very fast to compute.

470 500 seismic signals are randomly generated, according to the method presented  
471 in section 4.2. Using these signals and the 195 identified SDOF models, the fragility  
472 curves ( $DS \geq DS1$ , for the case corresponding to a first limit state LS1=15% EFDO,  
473 and  $DS \geq DS2$ , for the case corresponding to the second limit state LS2=30% EFDO)  
474 are plotted in figure 24. The distribution of the value identified by MLE for the median  
475 capacity  $A_m$  and the logarithmic standard deviation  $\beta$  versus the sample size is shown  
476 in figures 25 and 26.

477 The median capacity and the shape of the presented fragility curves compare well  
478 with the ones presented in [44] for similar damage levels. [44] studied a three-story  
479 masonry building in Benevento (southern Italy). Although only the order of magnitudes  
480 of the fragility curve parameters can be compared, since the building typologies are  
481 slightly different, as well as the used damage indicator, this result seems to confirm the  
482 relevancy of the obtained fragility curves.

483 The dispersion in the fragility curves obtained for the considered typology shows  
484 the influence of the structural uncertainties on the fragility analysis. The set of struc-  
485 tures is large enough to allow for an adequate representation of the variability, in order  
486 to illustrate the performance of the proposed methodology. Further studies would be  
487 required in order to evaluate the sensitivity related to each parameters.

## 488 5. Concluding remarks

489 An efficient modelling strategy to assess the seismic vulnerability of historical ma-  
490 sonry buildings has been presented. In order to integrate a large number of data coming  
491 from the observations on the masonry buildings, an automatic random mesh generator  
492 has been developed. To investigate the vulnerability of these buildings for a large num-  
493 ber of seismic scenarios, a simplified modelling strategy is proposed. A 3D FE model

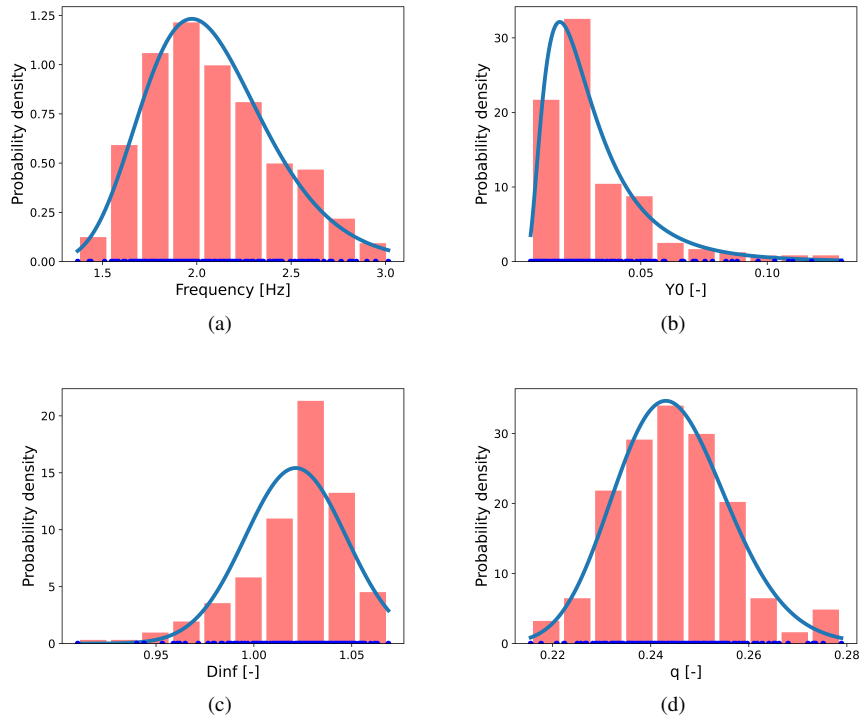


Figure 23: Probability density for : (a)  $f_0$ ; (b)  $Y_0$ ; (c)  $d_{\infty}$ ; (d)  $b$ .

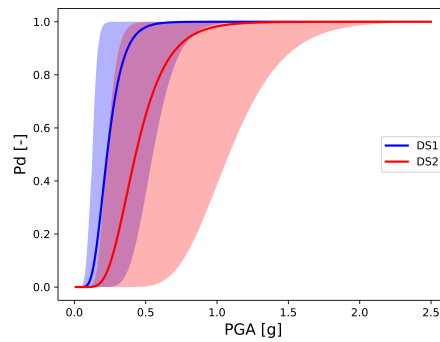


Figure 24: Fragility curves for the industrial building typology.

494 is built and used to determine the parameters of the NLESDOF model associated to the  
 495 main modes. The nonlinear time history analysis for each earthquake loading is then  
 496 performed with the simplified model. This modelling strategy has proven to be relevant

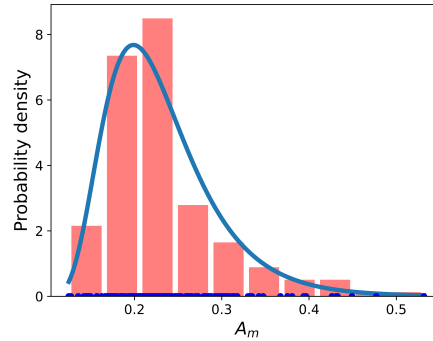


Figure 25: Median capacity  $A_m$  of the fragility curves for the industrial building typology for damage state DS1.

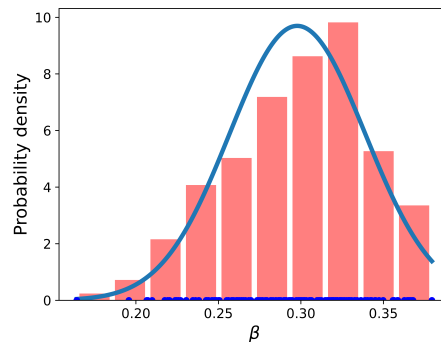


Figure 26: Standard deviation  $\beta$  of the fragility curves for the industrial building typology for damage state DS1.

497 for the identification of engineering structural response indicators, such as the maxi-  
 498 mum displacement at low level or the frequency shift. The model is valid as long as the  
 499 level of damage in the structure is sufficiently low to allow the approximation of un-  
 500 changed modal shapes to be relevant. It is thus an interesting approach in the context of  
 501 low to moderate seismicity, when macro-modelling strategies are not adapted to model  
 502 the structural behaviour. The modal basis is assumed not to be modified here, which  
 503 is a reasonable assumption when the seismic loading is not too high. A comparison of  
 504 the proposed strategy to a 3D full time-history analysis show that identifying the first  
 505 mode enables an accurate computation of the EFDO, chosen as a damage criterion.

506 The modelling strategy is applied to build fragility curves for one building typology.  
 507 The case of industrial masonry buildings, representative of French constructions in the  
 508 nineteenth century, is addressed. Thanks to its computational efficiency, the method  
 509 can be applied to several typologies to cover a large panel of data available from in-situ  
 510 observations. More case-studies can be addressed in the future, to further address the

511 advantages and drawbacks of the proposed modelling strategy. It would be interesting  
512 to compare the results of the proposed approach to other modelling strategy, such as  
513 the one used in "3-walls" [45]. Structural typologies for which fragility curves have  
514 already been derived, such as the ones proposed by the SYNER-G project [6], could  
515 be addressed with the proposed method for low damage levels. Sensitivity analyses on  
516 the mechanical and geometrical variables can also be conducted in order to identify the  
517 most relevant parameters, and then to eventually reduce the number of computations.  
518 In particular, it would be interesting to further investigate the influence of the floor  
519 stiffness and of the floor to wall connections on the resulting fragility curves. Once  
520 the statistical distribution of the resulting parameters of simple oscillators identified  
521 for a typology is characterized, new oscillators corresponding to the typology could  
522 also be randomly drawn to complete the set of fragility curves, taking advantage of the  
523 very fast computation of their seismic response. The derivation of fragility curves for  
524 different typologies is a step towards the update of historical earthquake intensities by  
525 a probabilistic approach.

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