

Appendix 1

Mathematical proof of the equivalence between Rényi entropy and Shannon's H given $\alpha \rightarrow 1$

rasterdiv - an Information Theory tailored R package for measuring ecosystem heterogeneity from space: to the origin and back

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Theorem (De l'Hôpital). *Let $f, g : (a, b) \mapsto \mathbb{R}$ be two functions such that*

- $\lim_{x \rightarrow a} f(x) = \lim_{x \rightarrow a} g(x) = 0$
- f and g are derivable in (a, b) with $g'(x) \neq 0$ for every $x \in (a, b)$
- the limit $\lim_{x \rightarrow a} \frac{f'(x)}{g'(x)} = L$ with $L \in \mathbb{R}$

then

$$\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = L.$$

We now verify if the hypothesis hold in our case. Let $f : (1, +\infty) \mapsto \mathbb{R}$ and $g : (1, +\infty) \mapsto \mathbb{R}$ be functions such that $f(x) = \log(\sum_{i=1}^N p_i^x)$ and $g(x) = 1 - x$ for every $x \in (1, +\infty)$. Since p_i is a fixed number in $(0, 1)$ for every $i \in \{1, \dots, N\}$ with $\sum_{i=1}^N p_i = 1$, then

$$f(1) = \log\left(\sum_{i=1}^N p_i\right) = 0.$$

Moreover, since f is a composition of derivable functions in $(1, +\infty)$, f is derivable in the definition domain. It is trivial that $g(1) = 0$, that g is

21 derivable in the domain set and that $g'(x) \neq 0$ for every $x \in (1, +\infty)$. The
22 limit of the derivatives ratio is:

$$\lim_{x \rightarrow 1} \frac{f'(x)}{g'(x)} = \lim_{x \rightarrow 1} \frac{(\sum_{i=1}^N p_i^x)^{-1} (\sum_{i=1}^N p_i^x \log p_i)}{1} = \sum_{i=1}^N p_i \log p_i$$

23 which is finite due to the hypothesis over p_i . Since all the hypotheses of De
24 l'Hôpital's Theorem hold, the thesis follows, i.e.

$$\lim_{\alpha \rightarrow 1} H_\alpha = H,$$

25 or Rényi 's entropy equals to Shannon's H for $\alpha \rightarrow 1$.