A novel identification tool for rotors working under non-stationary conditions based on the IAS

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Abstract
The aim of this work is to provide a new model describing the dynamic behavior of the rotating machinery working under non-stationary conditions with no assumption on the IAS. This new model will provide an identification tool for the rotor parameters. An example of a simple rotor made of a flexible shaft, a rigid disk, bearings and a mass unbalance is considered. The model shows a coupling between the torsional and flexural behavior and it is proven that, owing to this coupled behavior, the signal of the IAS contains information about, not only the rotation frequency of the response of the rotor, but also about its flexural behavior, when operating under non-stationary conditions.

1 Introduction

Rotor dynamics are widely present in different engineering fields. In the automotive field for example, with the growing use of the electric motors rotating at very high speeds, it has become crucial to understand the dynamic behavior of rotors spinning at very high velocity especially when going through critical speeds. This critical working condition, as well as the presence of bearings or gears in the rotor structure, leads to a fluctuation of the instantaneous angular speed (IAS). However, conventional vibration-monitoring techniques are based on some assumptions on the IAS. The latter is often considered either constant or following a perfect linear or exponential law of variation [1]. Those assumptions lead to models which are not able to simulate rotating machinery under fluctuating load conditions.
Even though the major part of the scientific works on rotor dynamics focus on the study of the stationary working conditions, the transient response analysis is considered as the most inclusive analysis since it allows the study of different working conditions of the rotating machinery as the start up and shut down processes, stability, and impact [2].
Since many rotors operate above their critical speeds, a main interest was particularly given to the study of the behavior of rotating machinary when going through one or more critical speeds. Al-Bedoor [3] developed a model for the coupled torsional and lateral vibrations of unbalanced rotors that accounts for the rotor-to-stator rubbing. He showed that a split in resonance is observed due to the rubbing condition when the rotor torsional flexibility is considered. Zhou et Shi [4] gave an analytical solution for the unbalance response of the Jeffcott rotor during acceleration.They showed that, quantitatively, the motion consists of three parts: a transient vibration at damped natural frequency, a synchronous vibration with the frequency of instantaneous 'synchronous' frequency, and a suddenly occurring vibration at damped natural frequency. Li and Singh [5] studied also the analytical solutions of the transient response but focused on the envelopes of the lateral displacement, velocity and acceleration of a linear torsional oscillator excited by an instantaneous sinusoidal torque. They also gave a new analytical approximation to find the maximum amplifications and corresponding peak frequencies.

When the dynamic model takes as an input the induced driving torque and not the speed of rotation as a function of time, one must add an equation that describes how the energy source supplies the energy to the equations that govern the corresponding dynamical system. Mutual interaction between the driving source and the rotor motion is observed and we deal in this case with non-ideal problem of passage through resonance or what is also called sommerfeld effect [6]. This interaction manifests itself as a modification of the motor frequency or regime of operation near the resonance and changes in the stable-unstable portions of the dynamical system response [7].
In the following study, since we consider as a given of the dynamic problem the induced torque and as an unknown the speed of rotation (IAS), we will be dealing with a non-ideal system. A new dynamic model is then built to simulate the non-stationnary working conditions especially when going through one or more critical speeds. This model is developeed in order to serve as a tool for identification, based on the IAS, for certain parameters of the system as the mass unbalance and damping.

2 New model describing the rotordynamics

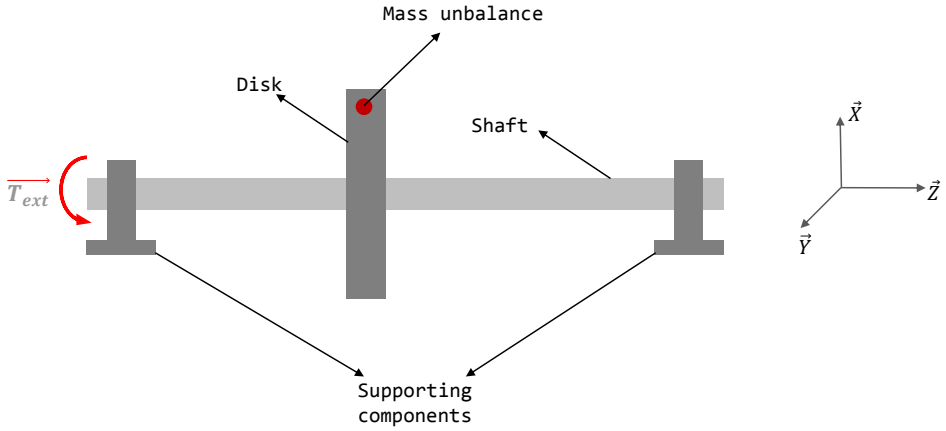

Figure 1: Rotor simplified model

Euler angles are used to discribe the motion of the rotor. The coordinate systems used in developing the model are shown in the following figure, wherein, (XYZ) is the fixed reference frame and (UVW) is the rotating reference frame which coincides with the principle axis of the cross section of the shaft. A first rotation by an angle Φ about the $Z-$axis is done then by an angle θ about the new X'-axis and finally by an angle Ψ about the final W'-axis. The latter, combined to the torsional deformation, is considered as an unknown of the dynamic problem.

The angular displacement about the X and Y axis as well as the spinning angle about the Z axis are calculated using Euler angles as follows:

$$\theta_x = \theta.cos(\phi) \tag{1a}$$

$$\theta_y = \theta.sin(\phi) \tag{1b}$$

$$\theta_z = \phi + \psi \tag{2}$$

It is very important to notice here that θ_z accounts for both the free body rotating motion as well as for the torsional deformation.

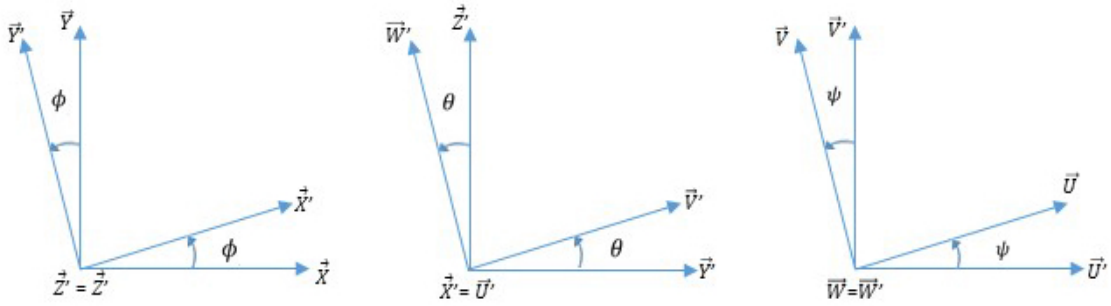

Figure 2: Euler angles

First the kinetic energy calculation is done. This energy is obtained by the sum of the contribution of the different components of the rotor, namely the disk, the shaft and the mass unbalance.

$$T = T_D + T_s + T_u \tag{3}$$

The finite element method is used and the dsiplacement fields are approximated using shape functions. We denote by $\{\delta\}$ the nodal displacement vector. Then the Lagrange equations are applied, which leads to the following matrix form:

$$\begin{aligned} \frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\delta}}\right) - \frac{\partial T}{\partial \delta} = &([M_{Dconst}] + [M_{Dvar}(\{\delta_c\})])\left\{\ddot{\delta}\right\} + \left[C_D(\dot{\theta}_z)\right]\left\{\dot{\delta}\right\} \\ &+ [M_u(\theta_z)]\left\{\ddot{\delta}\right\} + \{F_{nl_u}(\theta_z)\} \\ &+ [M_s]\{\ddot{\delta}\} + [S_{gyr}(\{\delta\})]\{\ddot{\delta}\} + \{Fnl_{gyr}(\{\dot{\delta}\})\} \end{aligned} \tag{4}$$

where:

- $[M_{Dconst}]$ and $[M_s]$ are the classical mass matrices of the disk and the shaft respectively.
- $\left[C_D(\dot{\theta}_{z_c})\right]$ is the classical skew-matrix related to the gyroscopic effects of the disk.
- $\{F_{nl_u}(\theta_{z_c})\}$ is the centrifugal force resulting from the mass unbalance and proportional to the square of the spinning speed.

The two matrices $[M_{Dvar}]$ and $[M_u]$ respectively related to the disk and the mass unbalance result from the assumption of non-stationnary working condition where:

$$[M_{Dvar}(\{\delta\})] = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{I_{Dw}}{2}\theta_{y_c} \\ 0 & 0 & 0 & 0 & 0 & -\frac{I_{Dw}}{2}\theta_{x_c} \\ 0 & 0 & 0 & \frac{I_{Dw}}{2}\theta_{y_c} & -\frac{I_{Dw}}{2}\theta_{x_c} & 0 \end{bmatrix} \tag{5}$$

$$[M_u(\theta_z)] = d\, m_u \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & cos(\theta_{z_c}) \\ 0 & 0 & 0 & 0 & 0 & -sin(\theta_{z_c}) \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ cos(\theta_{z_c}) & -sin(\theta_{z_c}) & 0 & 0 & 0 & d \end{bmatrix} \tag{6}$$

We notice that, by considering non-stationnary working conditions, matrices with extra-diagonal terms appear in the equation of dynamics of the rotor. Those terms clearly include coupling between the flexural and torsional degrees of freedom.
Finally, the matrix $[S_{gyr_e}]$ and the force vector $\{Fnl_{gyr_e}\}$ derive from the new formulation of the gyroscopic effect of the shaft under non-stationnary regime such as:

$$\begin{aligned} [S_{gyr_e}(\{\delta\})] = &- \langle N_3(l)\rangle^t \{\delta\}^t [M_{67}^g(l)]^t - [M_{67}^g(l)]\{\delta\}\langle N_3(l)\rangle \\ &+ \left\langle \frac{\partial N_3}{\partial z} \right\rangle^t \{\delta\}^t [M_{67}^{g*}(l)]^t + [M_{67}^{g*}(l)]\{\delta\} \left\langle \frac{\partial N_3}{\partial z} \right\rangle \end{aligned} \tag{7}$$

and,

$$\begin{aligned} \{Fnl_{gyr}(\{\dot\delta\},\{\dot\delta\})\} = &- \langle N_3(l)\rangle^t \left(\{\dot\delta\}^t [M_{67}^g(l)]\{\dot\delta\}\right) \\ &- 2\left(\langle N_3(l)\rangle\{\dot\delta\}\right)\left([M_{67}^g(l)]\{\dot\delta\}\right) \\ &+ \left\langle \frac{\partial N_3}{\partial z} \right\rangle^t \left(\{\dot\delta\}^t [M_{67}^{g*}(l)]\{\dot\delta\}\right) \\ &+ 2\left(\left\langle \frac{\partial N_3}{\partial z} \right\rangle\{\dot\delta\}\right)\left([M_{67}^{g*}(l)]\{\dot\delta\}\right) \end{aligned} \tag{8}$$

where $\langle N_3\rangle$ is a vector with polynomial shape functions of first order. $[M_{67}^g]$ and $[M_{67}^{g*}]$ are skew-symmetric matrices with constant terms obtained by the multiplication of shape function vectors.

Once the strain energy of the shaft and the virtual works of the bearings, assumed to be with linear stiffness and damping are calculated, the classical stiffness matrix is obtained and we finally write the equation of the dynamics of rotating shaft element such as:

$$\begin{aligned} ([M_s] + [M_{D_{const}}])\{\ddot\delta\} + [C_s]\{\dot\delta\} + [K_c]\{\delta\} = \\ - ([S_{gyr}(\{\delta\})] + [M_{D_{var}}(\{\delta\})] + [M_u(\theta_{z_c})])\{\ddot\delta\} \\ - \left[C_D(\dot\theta_{z_c})\right]\{\dot\delta\} \\ - \left(\{Fnl_{gyr}(\{\dot\delta\})\} + \{F_{nl_u}(\theta_{z_c})\}\right) \\ + \{F_{ext}\} \end{aligned} \tag{9}$$

3 Results and discussions

We consider an example of an idealized rotor with the properties detailed in table 1. The rotor is made of a flexible shaft, a rigid disk and is excited by the presence of a mass unbalance.

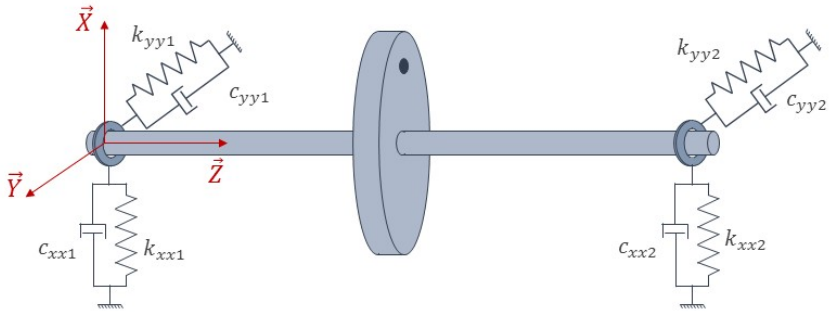

Figure 3: rotor exemple

Symbol	Quantity	Value
L	shaft length	$0.6\ m$
R_1	shaft radius	$0.01\ m$
R_2	disk radius	$0.08\ m$
h	thickness of the disk	$0.03\ m$
ρ	mass per unit volume	$7800\ kg.m^{-3}$
E	young modulus	$2.10^{11}\ N.m^{-2}$
m_u	mass unbalance	1% of the mass of the disk
d	eccentricity of the mass unbalance	0.1m
k_{xx1}, k_{xx2}	stiffness along x-axis of the left and right bearings	$1.e^8\ N.m^{-1}$
k_{yy1}, k_{yy2}	stiffness along y-axis of the left and right bearings	$1.e^8\ N.m^{-1}$
c_{xx1}, c_{xx2}	damping along x-axis of the left and right bearings	$2e2\ N.s.m^{-1}$
c_{yy1}, c_{yy2}	damping along y-axis of the left and right bearings	$2e2\ N.s.m^{-1}$

Table 1: Proprieties of the studied rotor

In this exemple we simulate the start up of a rotor. Therefore a driving torque following the law shown in figure 4 is induced to the system. The aim of applying a constant torque before the linear time-varying one is to better intialize the simulation for the non-stationary regime. It is important to mention that, for this exemple of rotor with the mentioned properties, the first critical speed is at 2527 rpm and the second one is at 26364 rpm. During the accelerarion period the rotor will go through the two mentioned critical speeds.

The simulation results are used for time-frequency analysis for both lateral displacement and instantaneous angular speed.
Figure 5 shows the results for the lateral displacement. As we can see, both the excitation frequency f_e and the first natural frequency f_1 contributes to the motion of the rotor. The results are qualitatively coherent with the analytical solution proposed by Zhou et Shi [4] saying that, during acceleration, the motion consists of three parts: a transient vibration at damped natural frequency, a synchronous vibration with the frequency of instantaneous 'synchronous' frequency, and a suddenly occurring vibration at damped natural frequency. We can see that the rotor has more difficulties crossing the second natural frequency than the first one.

The spectrogram of the instantaneous angular speed (fig.6) shows a coupling between the excitation frequency f_e and the first eigen frequency f_1. This simulation result proofs that the IAS, whose analyses is

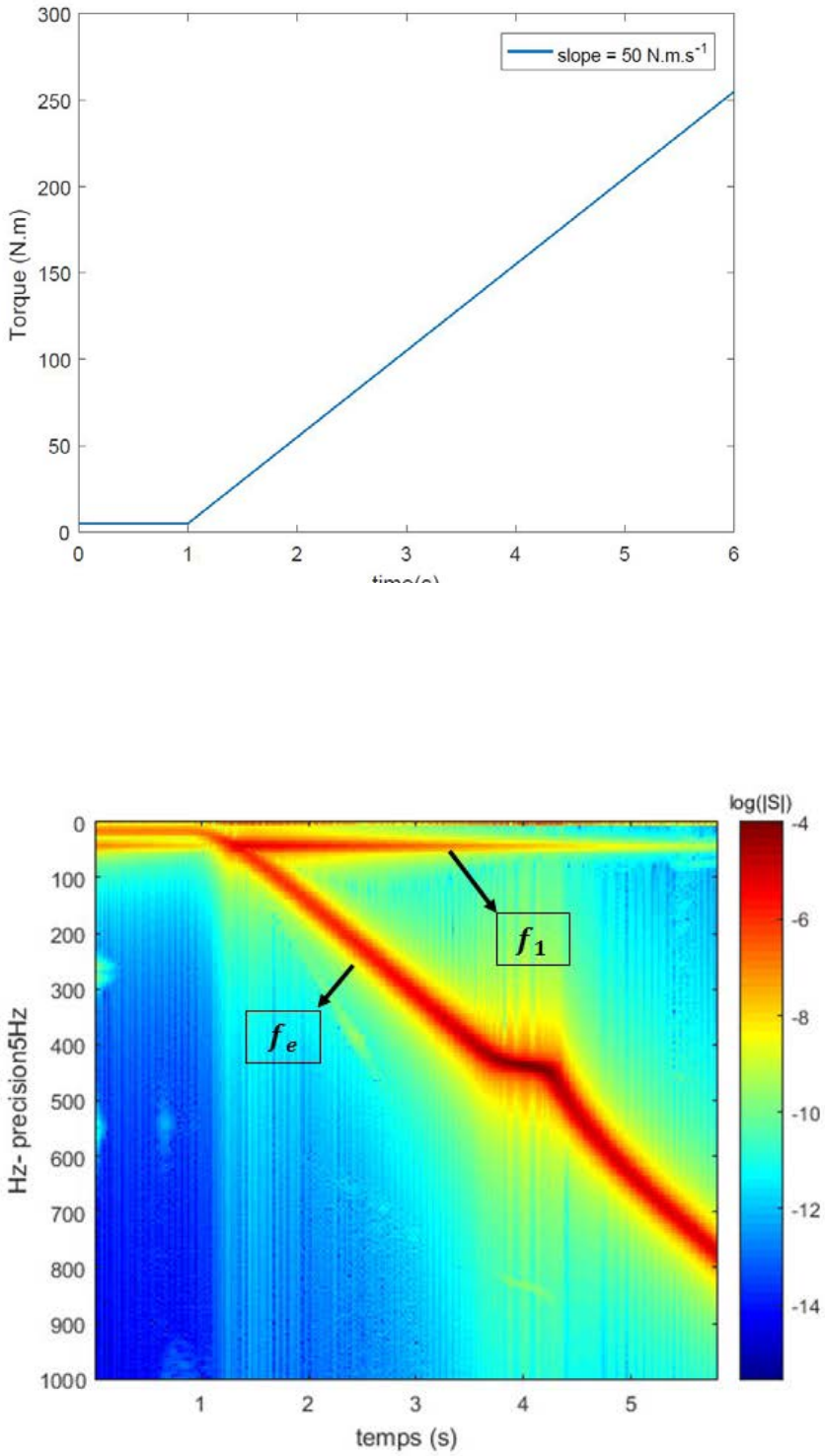

Figure 5: Spectrogram of the lateral displacement

neglected by previous research works, contains informations about the system dynamic properties. Considering the IAS as an unknown of the dynamic problem and not as a given law as a fonction of time provides us with a new source of informations about the rotor dynamics. Further analysis using Kalman filters will allow a better exploitation of the IAS in order to do a better tracking of the frequencies in the time-frequency

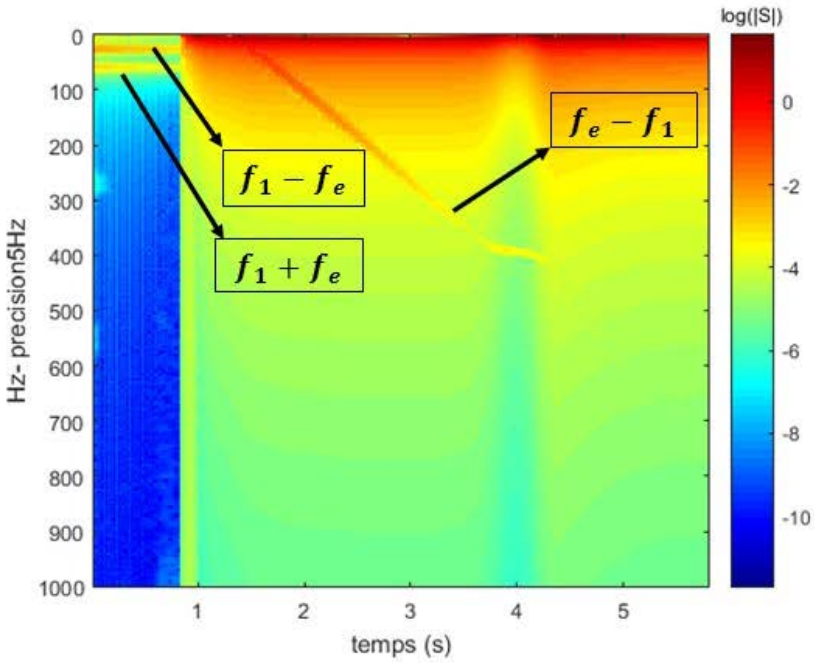

Figure 6: Spectrogram of the instantaneous angular speed

4 Conclusion

The big majority of previous works on the non-stationary rotordynamics were conducted under the assumption of a given law of the rotational speed. In our study, a new dynamic model is built considering no assumption on the IAS. It was shown, for a simple exemple of rotors, that when the IAS is considered as an unknown of the dynamic problem,it is highly influenced by the rotor dynmic properties. Based on the obtained results, we assume that the IAS is a physical quantity which is rich enough with informations about the studied system and that a better analysis using kalman filters is of big intesrest in order to identify the system parameters using the new proposed dynamic model for non-stationnary rotor dynamics.

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