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Nutation terms adjustment to VLBI and implication for the Earth rotation resonance parameters

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SUMMARY

The nutation harmonic terms are commonly determined from celestial pole offset series produced from very long baseline interferometry (VLBI) time delay analysis. This approach is called an indirect approach. As VLBI observations are treated independently for every session, this approach has some deficiencies such as a lack of consistency in the geometry of the session. To tackle this problem, we propose to directly estimate nutation terms from the whole set of VLBI time delays, hereafter referred as a direct approach, in which the nutation amplitudes are taken as global parameters. This approach allows us to reduce the correlations and the formal errors and gives significant discrepancies for the amplitude of some nutation terms. This paper is also dedicated to the determination of the Earth resonance parameters, named polar motion, free core nutation, and free inner core nutation. No statistically significant difference has been found between the estimates of resonance parameters based upon ‘direct’ and ‘indirect’ nutation terms. The inclusion of a complete atmospheric-oceanic non-tidal correction to the nutation amplitudes significantly affected the estimates of the free core nutation and the free inner core nutation resonant frequencies. Finally, we analyzed the frequency sensitivity of polar motion resonance and found that this resonance is mostly determined by the prograde nutation terms of period smaller than 386 d.

Key words: Core; Structure of the Earth; Earth rotation variations; Geodetic instrumentation.

1 INTRODUCTION

The Earth’s nutation can only be measured by very long baseline interferometry (VLBI). The common approach consists of adjusting session-wise nutation offsets (so-called celestial pole offsets) to an *a priori* model that is currently the model of Mathews *et al.* (2002), for the rest called IAU2000. This model is provided as a Poisson series of a thousand terms, largely dominated by some ‘outstanding’ terms such as the 18.6-yr, the semi-annual, the semi-monthly and the annual. As the IAU2000 nutation model is not perfect (e.g. Dehant *et al.* 2003), nutation offset time-series shows up with an rms of about 0.2 mas in terms of displacement at the Earth’s surface and are composed of several significant periodic signals and the free core nutation (FCN), all containing subtle information about the non-rigid Earth response to tidal forcing. The spectral analysis of these nutation offset time-series can be used to improve the IAU2000 nutation model by (i) correcting the amplitudes of the Poisson series and, more interestingly, by (ii) adjusting the basic Earth parameters of the underlying geophysical model used in IAU2000 (see, e.g. Koot *et al.* 2008; Rosat & Lambert 2009; Rosat *et al.* 2016; Dehant *et al.* 2017).

The common way to estimate the nutation components are relied upon least square analysis of nutation offset time-series that were produced by the analysis of a complete VLBI observational database

(see, e.g., Herring *et al.* 2002; Koot *et al.* 2008; Rosat & Lambert 2009; Rosat *et al.* 2016). This approach will be referred to as indirect approach in the following since it needs two steps: one global VLBI analysis to produce nutation offset time-series plus one specific analysis for estimating the nutation offset terms. In the present study, we aim at revisiting another approach called direct approach which has been already used by Himwich & Harder (1988) and Petrov (2007). In such an approach, the nutation components are estimated directly from VLBI delays, shortcutting, therefore, the traditional two-step approach. Also, it allows a rigorous propagation of the delay errors into the estimated parameters and the use of the full covariance information.

The Earth resonance parameters can be estimated from a comparison between IAU2000 nutation model corrected by nutation offset with rigid nutation model in the frequency domain (Mathews *et al.* 2002). We strive to characterize to what extent the results obtained by the direct approach modify the resonance parameters. Next, we study the effect of atmosphere and ocean to the Earth resonance parameters. The nutation contribution caused by diurnal circulation in the hydro-atmospheric layer perturbs the estimates of the resonance parameters and has to be removed from total nutation before inversion. Until now, only the prograde annual atmospheric effect was considered (Mathews *et al.* 2002). We extend this removal to the whole set of significant oceanic-atmospheric contribution to the

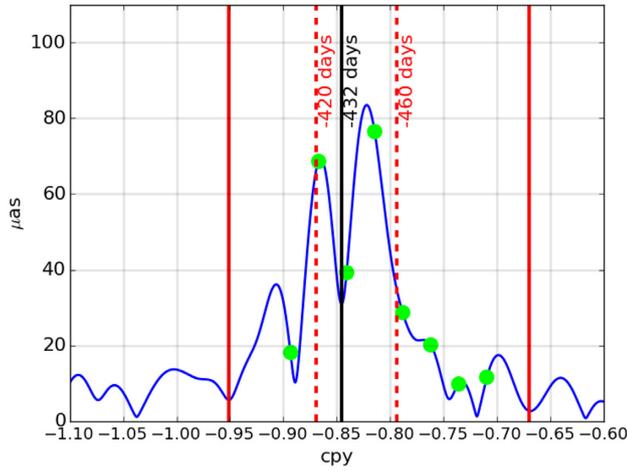


Figure 1. FFT spectrum of the nutation offset around the FCN frequency. The chosen frequencies are shown by green circles.

nutation. Finally, we investigate the sensitivity of polar motion (PM) resonance parameters to the frequency band of the selected set of nutation terms.

2 DATA AND THEIR PREPARATION

The data is made up of 6246 VLBI sessions between August 1979 to December 2017. We used the geodetic analysis software Calc/Solve (Ma *et al.* 1986) developed and maintained by the VLBI group at NASA/GFSC¹ in a standard configuration. We estimated station coordinates differences with respect to ITRF2014 (Altamimi *et al.* 2016) as global parameters with no-net rotation and no-net translation conditions applied to the positions and velocities of a group of 38 stations. All *a priori* station positions were corrected from tridimensional displacements due to oceanic and atmospheric tidal loading using FES2004 (Lyard *et al.* 2006) and output from the non-inverted-barometer version of Atmospheric Pressure Loading Service (APLO; Petrov & Boy 2004) as well as corrections for post-seismic relaxation for relevant stations as given in the ITRF2014. Radio source coordinates were estimated as global parameters for most of the sources except a set of 39 particularly active quasars (tagged as special-handling sources in the ICRF2 work, Fey *et al.* 2015) whose coordinates were estimated once per session. A no-net rotation condition was applied to the 295 ICRF2 defining sources. Antenna thermal deformations were obtained in Nothnagel (2009). *A priori* dry zenith delays were estimated from local pressure values and then mapped to the elevation using the Vienna Mapping Function (Böhm *et al.* 2006). The modeling of intraday variations of the troposphere wet delay, clocks, and troposphere gradients is realized through continuous piecewise linear functions whose coefficients are estimated every 10 min, 30 min and 6 hr, respectively. *A priori* Earth orientation parameters were taken from the IERS EOP 14 C04² data associated with the IAU2000/2006 nutation and precession models (Mathews *et al.* 2002; Capitaine *et al.* 2003). Offsets to polar motion and UT1 *a priori* together with polar motion and UT1 rates were estimated once per session.

In the indirect approach, corrections to the nutation and precession *a priori* values (i.e. nutation offset) were estimated once per

session, which is the classical way of processing by the VLBI analysis centers. In the direct approach, the nutation offset, dX and dY , are modeled as a series of harmonic functions incremented by a linear trend accounting for a possible correction to the precession

$$dX + idY = \sum_j A_j e^{i\phi_j} + at + b, \quad (1)$$

where $A_j = A_{(IP,j)} + iA_{(OP,j)}$ are complex amplitudes, ϕ_j are phases associated with periodical motion of the Moon and the Sun and generally given by a linear combination of the five Delaunay variables, and a and b are the complex coefficients of the linear term. The terms $A_{(IP,j)}$ are called in-phase terms and $A_{(OP,j)}$ are out-of-phase terms. VLBI method allows us to estimate time delay, τ , of the arrived signal. In order to obtain the nutation offset terms, we need to express the partial derivatives of τ with respect to the nutation offset amplitudes $A_{(IP/OP,j)}$. In the first approximation, these partial derivatives read

$$\frac{\partial \tau}{\partial A_{(IP/OP,j)}} = c^{-1} \cdot \underline{k} \cdot \frac{\partial M}{\partial A_{(IP/OP,j)}} \cdot \underline{b}, \quad (2)$$

where c is the speed of light, \underline{k} is the unit vector pointing in the source direction expressed in the ICRF, \underline{b} is the baseline unit vector expressed in the ITRF and M is the matrix of transformation from terrestrial to celestial reference (e.g. Petit & Luzum 2010). Matrix M can be represented as

$$M = Q(X, Y)R(-\theta)W(x_p, y_p), \quad (3)$$

with θ is the Earth rotation angle between the celestial and terrestrial intermediate origin, x_p and y_p are the terrestrial pole coordinates in the International Terrestrial Reference System, X and Y are the full Cartesian coordinates of the celestial pole in the International Celestial Reference System (i.e. the nutation-precession model plus the nutation offset). Matrix Q can be decomposed as

$$Q(X, Y) = dQ(dX, dY) \cdot Q(X_{IAU2000}, Y_{IAU2000}), \quad (4)$$

where

$$dQ(dX, dY) = \begin{bmatrix} 0 & 0 & dX \\ 0 & 0 & dY \\ -dX & -dY & 0 \end{bmatrix}. \quad (5)$$

Here $(X_{IAU2000}, Y_{IAU2000})$ is the nutation based on IAU2000 model. The derivation of dQ with respect to the in-phase (IP) and out-of-phase (OP) components of A_j gives

$$\frac{\partial dQ}{\partial A_{IP,j}} = \begin{bmatrix} 0 & 0 & \cos \phi_j \\ 0 & 0 & \sin \phi_j \\ -\cos \phi_j & -\sin \phi_j & 0 \end{bmatrix},$$

$$\frac{\partial dQ}{\partial A_{OP,j}} = \begin{bmatrix} 0 & 0 & -\sin \phi_j \\ 0 & 0 & \cos \phi_j \\ \sin \phi_j & -\cos \phi_j & 0 \end{bmatrix}.$$

We apply this modeling to finding corrections to the complex amplitudes of 42 nutation terms reported in Mathews *et al.* (2002).

Free core nutation

The FCN is the dominant contributor to nutation offset. Its period can be considered as strongly constrained by the fluid core properties (Mathews *et al.* 2002), its amplitude and phase, likely driven by surface fluid dynamics, remain unpredictable (Chao & Hsieh 2015), and still need to be accounted for by empirical modeling that does not address, however, the underlying physics.

¹<https://vlbi.gsfc.nasa.gov>.

²Data publicly available at <http://iers.obspm.fr/eop-pc>.

Table 1. The FCN complex amplitudes estimated by the direct and indirect approaches. Unit is days for the period and μas for the coefficients.

Period	Indirect			Direct		
	IP	OP	\pm	IP	OP	\pm
-408.58	64.2	-24.1	3.8	58.1	-20.7	1.3
-420.97	-87.5	107.9	5.1	-81.4	102.4	1.8
-434.12	49.8	-64.1	6.1	44.1	-54.3	2.1
-448.12	21.6	-60.8	6.5	22.1	-69.1	2.3
-463.06	-46.2	-29.6	6.5	-43.0	-20.0	2.2
-479.03	0.5	-10.8	5.9	-8.3	-17.1	2.0
-496.14	-6.4	6.4	4.9	3.4	6.4	1.7
-514.51	3.5	-9.9	3.7	-3.5	-6.3	1.3

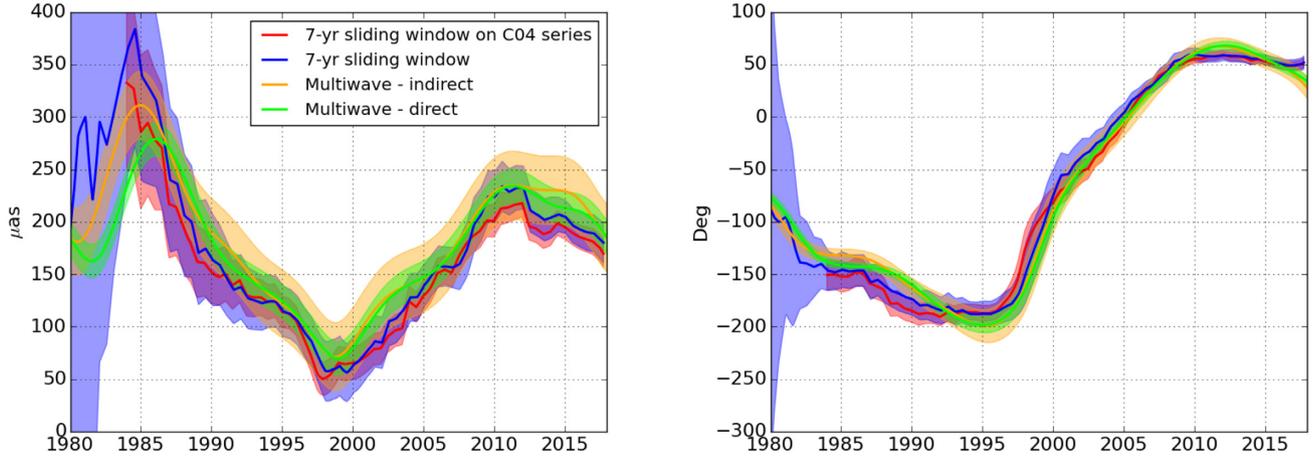


Figure 2. The amplitude (left-hand panel) and phase (right-hand panel) of the FCN in time domain. The thinner color corresponds to their 1σ .

As shown in Fig. 1, the FCN appears as a broad band spectral process (e.g. Vondrák *et al.* 2005; Chao & Hsieh 2015), hinting that its amplitude and phase are variable. The spectrum of the Fig. 1 was obtained by FFT of the nutation offset series regularized at 10-d intervals by a running average, and padded with zeros to get 2^{12} points. It suggests that the power relevant to the FCN lies between approximately -0.95 and -0.67 cpy, that is, -386 to -545 d. This large band contains the periods of the two ‘extreme’ peaks at -420 and -460 d showed by FFT spectra realized over different time spans by Chao & Hsieh (2015) and earlier with little differences by Vondrák *et al.* (2005).

Several empirical models of the FCN have been obtained by different methods (e.g. least-squares fit over sliding windows, wavelet decomposition, and singular spectral analysis) in the recent years, all being generally in good agreements (Malkin 2007; Krásná *et al.* 2013; Belda *et al.* 2016). A direct adjustment of the FCN usually has been made via a sliding window, by selecting a subset of data similarly to what is done in the indirect approach. However, as the window is restricted to some years, the sliding window method introduces correlations with other nutation terms. Here, we propose the adjustment of an empirical harmonic model composed of close frequencies for representing the broad band spectral peak in accordance with the formula

$$dX_{\text{FCN}} + i dY_{\text{FCN}} = \sum_j A_{\text{FCN},j} e^{i\sigma'_{\text{FCN},j}t}, \quad (6)$$

where $A_{\text{FCN},j}$ is the complex amplitude of the j th FCN component and $\sigma'_{\text{FCN},j}$ is its frequency.

We found that eight equally-spaced frequencies within the interval $[-0.95, -0.67]$ cpy describe at best the FCN over the period

1979–2017, in the sense that a more dense model does not drop significantly the residuals. Then, the $A_{\text{FCN},j}$ parameters are estimated by direct and indirect approaches. In contrast to estimates obtained by sliding window adjustment, the formulation (6), referred to as harmonic model in the following, constitutes a compact description that can be extrapolated into the future. The corresponding results are reported in Table 1 and Fig. 2. It matches the estimates obtained by the more traditional sliding-windowed approach within one sigma standard errors, except before 1985 and after 2016, where the sliding window results are perturbed by 3.5-yr edge effect.

3 PERFORMANCE AND CONSISTENCY OF THE DIRECT AND INDIRECT APPROACHES

The global solutions returned a postfit rms of 26.75 picoseconds and a χ^2 per degree of freedom of 0.94. Table 2 displays the adjusted corrections to the complex amplitudes of the 42 nutation terms following the indirect and the direct approaches. The largest differences between the two approaches reside in the 18.6-yr term for which the complex amplitude change is larger than $10 \mu\text{as}$. The direct approach provides formal errors that are smaller than for the indirect approach by a factor of two to three. The correlations between the various estimated complex amplitudes are reported in Fig. 3. The direct approach permits to lower the correlations between 18.6- and 9.3-yr terms and between 6.86- and 346.64-d terms by 5 and 10 per cent. For the other terms, there are only marginal changes in correlation between the two approaches. On average, direct approach permits to lower the correlation by 7 per cent.

Table 2. In-phase and out-phase coefficients of 42 nutation terms estimated by the indirect and direct approaches. Unit is day for periods and μas for the coefficients.

Period	Indirect			Direct		
	IP	OP	\pm	IP	OP	\pm
-6798.38	26.2	-15.7	2.4	30.8	-26.9	0.9
6798.38	2.8	-33.2	2.4	15.2	-32.8	0.9
-3399.19	7.3	-13.1	2.4	3.0	-10.9	0.9
3399.19	20.2	-17.2	2.4	14.1	-11.5	0.9
-1615.75	-0.8	-2.2	2.4	-5.3	-5.7	0.8
1615.75	-1.8	-12.8	2.4	2.6	-8.0	0.8
-1305.48	-0.9	10.0	2.4	-3.6	8.8	0.9
1305.48	-5.2	1.5	2.4	-4.0	3.3	0.8
-1095.18	-3.4	4.8	2.4	1.1	4.1	0.8
1095.18	-4.1	6.1	2.4	-4.1	1.1	0.8
-386.00	-9.7	1.5	2.6	-9.1	-4.5	0.9
386.00	-1.0	-1.2	2.3	0.6	-0.0	0.8
-365.26	35.7	4.5	2.5	33.5	7.6	0.9
365.26	-4.1	-0.2	2.4	-9.7	-0.8	0.8
-346.64	-16.2	-2.8	2.5	-13.3	0.8	0.9
346.64	1.3	-0.0	2.5	-0.0	-3.0	0.9
-182.62	-11.5	5.5	2.2	-12.6	5.9	0.8
182.62	7.8	-5.7	2.2	9.0	-4.3	0.8
-121.75	-4.2	4.7	2.2	-1.7	4.4	0.8
121.75	4.6	1.4	2.2	3.8	-1.2	0.8
-31.81	0.8	-3.9	2.2	0.7	-2.8	0.7
31.81	-2.0	2.6	2.2	-3.2	2.8	0.8
-27.55	-17.6	-9.4	2.2	-17.8	-7.6	0.7
27.55	1.1	-3.9	2.2	1.0	-2.6	0.8
-23.94	-0.6	-1.4	2.2	-0.9	-0.2	0.7
23.94	-0.8	-1.1	2.2	-1.9	-0.7	0.8
-14.77	-3.1	2.2	2.1	-1.9	2.4	0.7
14.77	1.0	-0.0	2.1	1.3	-0.9	0.8
-13.78	-2.4	-2.0	2.2	-2.4	-1.4	0.7
13.78	1.4	1.1	2.2	0.5	0.4	0.8
-13.66	-6.7	-7.7	2.2	-8.5	-5.6	0.7
13.66	-3.4	9.0	2.2	-5.6	8.6	0.8
-9.56	-0.9	-3.7	2.2	-1.7	-1.5	0.7
9.56	3.0	-3.7	2.2	2.0	-1.1	0.8
-9.13	-1.2	1.1	2.2	-1.5	1.5	0.7
9.13	-1.3	3.8	2.2	-3.3	5.7	0.9
-9.12	2.1	4.2	2.3	2.2	3.7	0.7
9.12	1.5	-0.9	2.3	1.4	-1.6	0.9
-7.10	-3.8	-1.2	2.2	-5.0	0.5	0.7
7.10	1.4	1.9	2.4	-1.3	2.3	1.0
-6.86	1.3	-0.1	2.3	0.2	-0.6	0.7
6.86	-1.3	2.9	2.3	-2.1	6.4	0.9

The solutions associated with the indirect and direct approaches produced obviously different Earth rotation parameters and radio source positions, raising the question of to which extent the differences are significant. Rms differences in polar motion remain within 0.03 mas with a χ^2 of about 0.1 indicating no systematics. We reached the same conclusions for UT1 and length-of-day with rms differences of about 1 ms. Median errors for polar motion are slightly lowered (by 2 μas) in the direct approach with respect to the indirect approach. The celestial reference frames obtained from the two approaches and composed of 4118 radio sources were compared in terms of global rotation and deformations up to degree 2 (see, e.g. Mignard & Klioner 2012). They only differ by small rotations of less than 3 μas . The modeling of nutation in the direct approach has, therefore, no significant impact on other EOP and celestial reference frame.

The robustness of the results is examined by estimating the nutation components from two independent session lists which have

approximately the same number of observations. We divided the session list into two lists by taking the ‘even’ and ‘odd’ sessions of the initial list in chronological order. For a long period, the nutation amplitudes are not expected to be so sensitive to the number of observation divided by two. For a short period (say few times the mean sampling rate), one could expect an increased sensitivity due to the rapidly changing network characteristics (size, geometry and data quality) for one session to another. We found that ‘even’ and ‘odd’ solutions return χ^2 per degree of freedom in good agreement with the initial session list. Fig. 4 shows the differences between the results from ‘even’ and ‘odd’ sessions in each nutation component. These differences can be interpreted as an ‘empirical error’. Such error appears much less homogeneous than the initial standard error when contrasted with the frequency. The less robust nutations are generally short-period nutations (e.g. 13.78 d). But the empirical error does not decrease as the period increases: a possible reason for this is the inhomogeneity of the VLBI data quality and error along

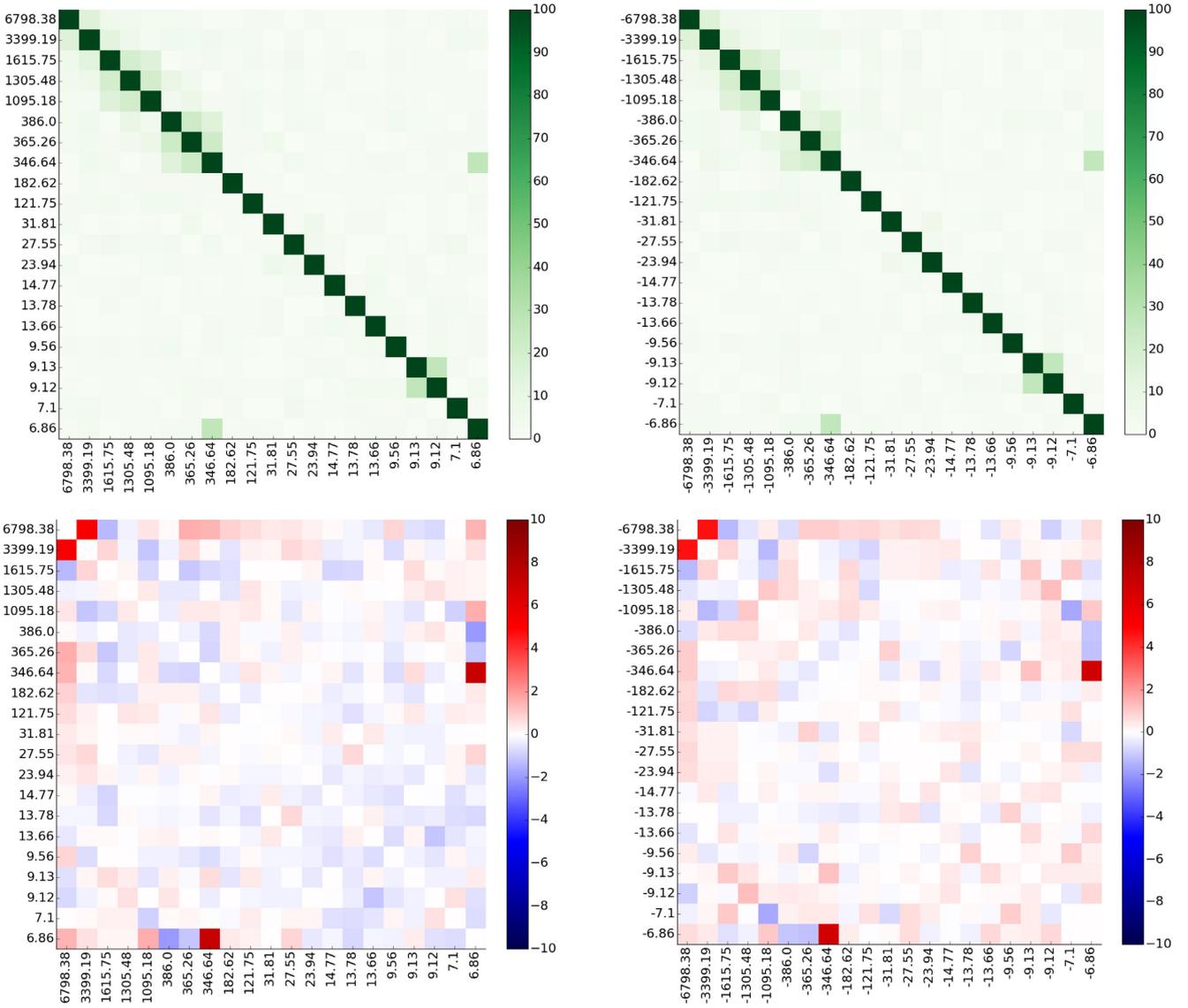


Figure 3. Top panels: the absolute correlation coefficients between several in-phase and out-phase amplitudes for indirect approach. Bottom panels: the absolute correlation coefficients of indirect approach minus the absolute correlation coefficients of direct approaches. A positive value means that the correlation is diminished when using the direct approach. Left- and right-hand side are for prograde and retrograde terms, respectively. The unit of correlation is in (per cent).

the observational period, especially the early data (before 1990) compared with the more recent ones.

4 ESTIMATES OF THE EARTH RESONANCE PARAMETERS

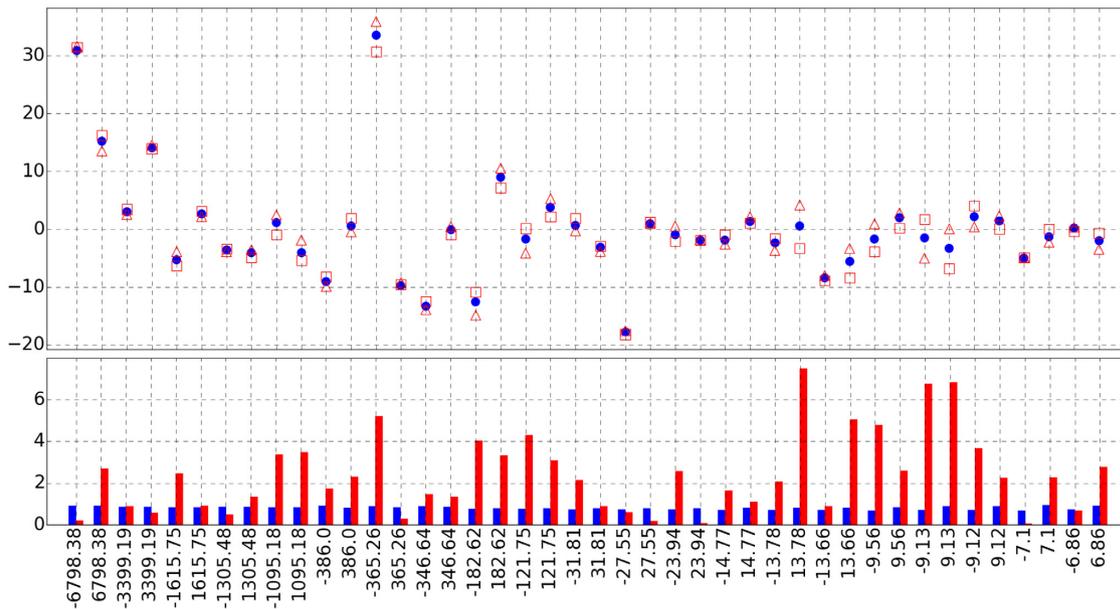
In order to estimate the Earth resonance parameters through VLBI data, in a first approach, we strictly follow the procedure of Mathews *et al.* (2002). It is based on a transfer function that expresses the ratio between a non-rigid Earth nutation term ($\tilde{\eta}_{NR}(\sigma')$) and its corresponding term for a rigid Earth ($\tilde{\eta}_R(\sigma')$). This transfer function has a resonance expansion of the form

$$\tilde{T}(\sigma') = \frac{(e - \sigma'/\Omega)}{(e + 1)} \left[1 + (1 + \sigma'/\Omega) \left(\frac{-\tilde{\sigma}'_1/e}{\sigma' - \tilde{\sigma}'_1} + \frac{\tilde{N}_2\Omega}{\sigma' - \tilde{\sigma}'_2} + \frac{\tilde{N}_3\Omega}{\sigma' - \tilde{\sigma}'_3} \right) \right]. \quad (7)$$

Here $\Omega = 7.292115 \times 10^5 \text{ rad s}^{-1}$ is the mean angular velocity of the Earth, σ' is the frequency in cycle per day (cpd) as seen from a celestial frame, e is the dynamical ellipticity of the Earth, $\tilde{N}_{(2,3)}$ are complex coefficients and $\tilde{\sigma}'_{(1,2,3)}$ are the complex frequencies specifying the resonance modes. These later ones are put under the form $\tilde{\sigma}'_j = 2\pi/P'_j(1 + n_j(i/2Q'_j))$ where n_j is '+1' for prograde and '-1' for retrograde modes, respectively, P'_j and Q'_j are the corresponding space-referred period and quality factor, respectively. The upper script '~' denotes a complex variable. Index 1 until 3 is for representing the resonances associated with PM, FCN and free inner core nutation (FICN), respectively. It should be noted that PM resonance parameters are characteristic of the Earth response to the pole tide potential in the retrograde diurnal band, therefore they are not the ones prevailing in the seasonal band of the polar motion, namely the Chandler Wobble parameters.

Geophysicists favor the terrestrial frame for expressing the rotation modes. Therefore, the celestial frequencies $\tilde{\sigma}'_j$ are mapped into terrestrial frequencies $\tilde{\sigma}_j = \tilde{\sigma}'_j - \Omega$ with corresponding period and

(a) In-Phase



(b) Out-Phase

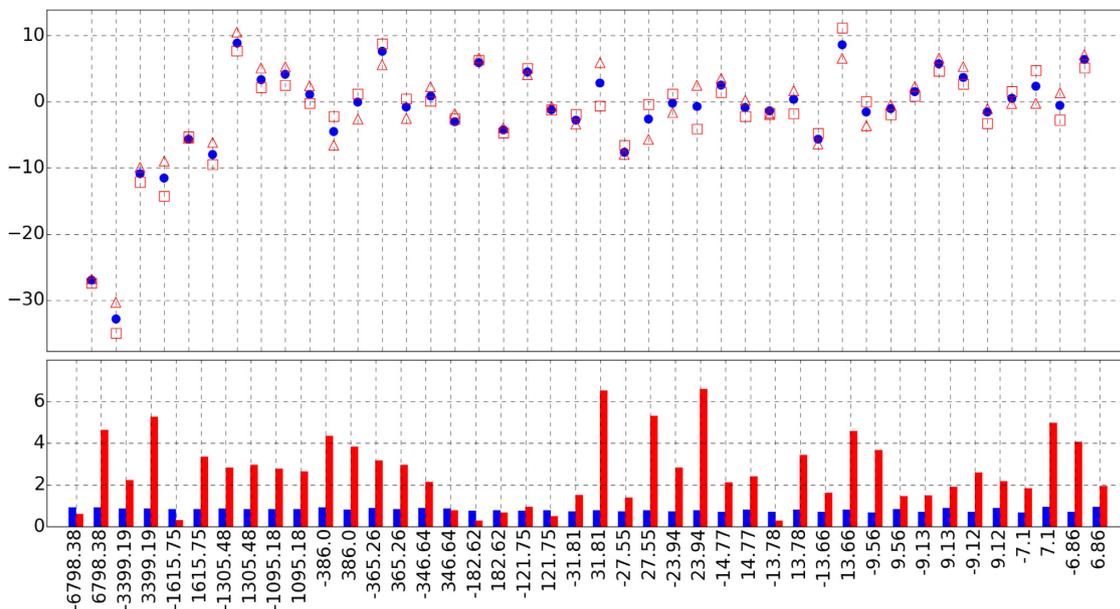


Figure 4. The in-phase and out-phase amplitudes of nutation components estimated by direct approach. Here ● corresponds to the nutation terms from all VLBI sessions and △ and □ are the nutation terms from even and odd sessions, respectively. The blue bars correspond to the formal error of ● and the red bars are the different between the nutation terms from odd and even sessions. The units in y-axis are in μas.

quality factor P_j and Q_j . In our study, we ignore the contribution of Inner Core Wobble in (7) since it is negligible.

We estimate the complex parameters $\tilde{\sigma}'_j$ and \tilde{N}_2 from a set of observed nutation terms $\tilde{\eta}_{NR}(\sigma')$ and the corresponding set of rigid

Earth terms by performing a weight least square inversion. As N_3 is correlated with $\tilde{\sigma}'_3$, N_3 is fixed to its theoretical value ($2.95844 \times 10^{-4} - i9.57705 \times 10^{-5}$) as given by Mathews *et al.* (2002). Here $\tilde{\eta}_{NR}(\sigma')$ are constituted by adding the nutation term corrections

Table 3. Atmospheric-oceanic contribution to the Earth nutation. Unit is μas .

Period (d)	Atmosphere			Ocean		
	IP	OP	\pm	IP	OP	\pm
-365.26	84.3	-50.6	3.9	-5.5	19.4	5.5
+365.26	43.4	-59.4	0.4	32.5	-22.2	0.5
+182.62	9.3	-43.6	0.3	32.8	7.3	0.3
+121.75	0.8	-3.4	0.2	2.1	-2.5	0.2
+13.66	3.9	-1.8	0.1	-0.1	-0.3	0.0

Table 4. Resonance parameters associated with PM, FCN and FICN in the terrestrial frame (P, Q). For FICN and FCN, the period is also given in the celestial frame (P'). Periods are given in mean solar days. AO is an abbreviation for *atmospheric-oceanic*.

Procedure	AO correction	Resonance parameters		
		P_{PM}	Q_{PM}	-
Indirect	-	(380.9, 382.5 , 384.0)	(-11.0, -10.4 , -9.8)	-
Indirect	+365.26 d only	(381.3, 382.6 , 383.9)	(-11.0, -10.5 , -10.0)	-
Indirect	Complete	(380.8, 382.1 , 383.4)	(-10.9, -10.4 , -9.9)	-
Direct	Complete	(380.7, 382.0 , 383.4)	(-11.0, -10.4 , -9.9)	-
	Ziegler <i>et al.</i> (2019)	(381.3, 382.4)	(-10.9, -10.3)	-
		P_{FCN}	Q_{FCN}	P'_{FCN}
Indirect	-	(-0.994961, -0.994960 , -0.994959)	(16461, 16907 , 17379)	(-429.8, -429.6 , -429.4)
Indirect	+365.26 d only	(-0.994961, -0.994960 , -0.994959)	(16592, 16967 , 17358)	(-429.8, -429.6 , -429.4)
Indirect	Complete	(-0.994964, -0.994963 , -0.994962)	(16958, 17361 , 17785)	(-430.3, -430.2 , -430.0)
Direct	Complete	(-0.994964, -0.994963 , -0.994962)	(16946, 17332 , 17736)	(-430.3, -430.1 , -430.0)
	Ziegler <i>et al.</i> (2019)	-	(15600, 16000)	(-429.5, -429.3)
		P_{FICN}	Q_{FICN}	P'_{FICN}
Indirect	-	(-0.998492, -0.998261 , -0.998031)	(387, 443 , 518)	(770.8, 1003.8 , 1236.9)
Indirect	+365.26 d only	(-0.998422, -0.998240 , -0.998059)	(424, 476 , 543)	(833.9, 1025.6 , 1217.4)
Indirect	Complete	(-0.998538, -0.998312 , -0.998087)	(319, 355 , 400)	(748.4, 954.7 , 1161.0)
Direct	Complete	(-0.998559, -0.998329 , -0.998099)	(304, 338 , 380)	(735.6, 939.5 , 1143.5)
	Ziegler <i>et al.</i> (2019)	-	$<6 \times 10^4$	(600, 1200)

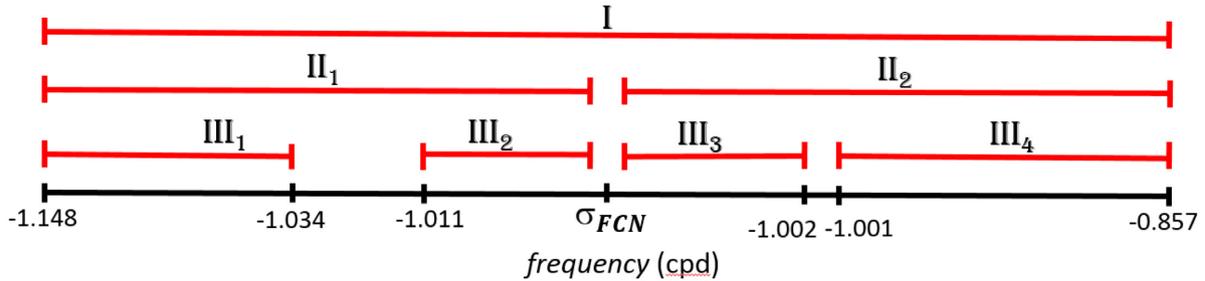


Figure 5. Selected frequency band in the terrestrial frame for least-square adjustment of the PM resonance parameters. 18.6-yr nutation term is belong to the Band III₃. Their limits are precisely reported in Table 5.

Table 5. The period and quality factor of polar motion resonance determined over certain band of frequencies.

Band	Frequency (cpd)	P_{PM}	Q_{PM}
I	$(-\Omega - 1/6.86 \leq \sigma \leq -\Omega + 1/6.86)$	382.0 ± 1.3	-10.4 ± 0.5
II ₁	$(-\Omega - 1/6.86 \leq \sigma \leq -\Omega - 1/386)$	418.5 ± 7.2	-8.24 ± 1.7
II ₂	$(-\Omega - 1/1095.18 \leq \sigma \leq -\Omega + 1/6.86)$	381.8 ± 1.2	-10.4 ± 0.5
III ₁	$(-\Omega - 1/6.86 \leq \sigma \leq -\Omega - 1/31.81)$	415.1 ± 3.3	-7.7 ± 0.7
III ₂	$(-\Omega - 1/121.75 \leq \sigma \leq -\Omega - 1/386)$	486.8 ± 58.4	$+13.4 \pm 30.7$
III ₃	$(-\Omega - 1/1095.18 \leq \sigma \leq -\Omega + 1/1095.18)$	381.7 ± 7.6	-10.2 ± 2.9
III ₄	$(-\Omega + 1/386 \leq \sigma \leq -\Omega + 1/6.86)$	381.8 ± 1.3	-10.4 ± 0.5

(Table 2) to their modeled part (from IAU2000 nutation model). The weight itself is purely taken from the formal error which is displayed in Table 2.

As mentioned by Mathews *et al.* (2002), before the inversion is performed, some corrections have to be applied. First, the nutation

terms have to be referred to a dynamical celestial reference frame by removing the geodetic nutation and then non-linear effects are suppressed [see table 7 of Mathews *et al.* (2002)]. Finally, we eventually get rid of the atmospheric-oceanic contribution, which cannot be related to rigid Earth nutation caused by luni-solar tides.

Atmospheric-oceanic contribution

The analysis done in Mathews *et al.* (2002) was restricted to the atmospheric effect on the annual prograde nutation. Actually, a more complete treatment of the fluid layer perturbations has to include other nutation components (-365 , $+182.6$, $+121.75$, and $+13.66$ d) and consider the contribution of the non-tidal circulation in the ocean. The corresponding effects, as calculated by Brzeziński (2010), were reevaluated by using the non-inverted barometer version of atmospheric angular momentum time-series from TU Vienna³ (based upon ECMWF model, Schindelegger *et al.* 2011) and oceanic angular momentum series from ERA40 and ERA operational (based upon OMCT model, Dobsław & Thomas 2007) from 1984 until 2017. This calculation is done through Celestial Angular Momentum Functions, and their effects on the nutation as formulated by Brzeziński (1994) from the theory exposed in Wahr & Sasao (1981). Our estimates are reported in Table 3.

The resonance parameters are estimated by weighted least square based upon the formal error of the nutation terms as reported in Table 2, starting from *a priori* values given by Mathews *et al.* (2002). The impact of atmospheric-oceanic circulation is investigated by considering three cases: (i) no correction at all, (ii) correction restricted to the annual prograde term and (iii) correction involving all terms listed in Table 3.

The results are displayed in Table 4 for the resonance parameters (P , Q) for PM, FCN and FICN in the terrestrial frame. For FCN and FICN we also provide the equivalent parameters (P' , Q') in the non-rotating frame. It should be noted that the negative value Q_{PM} is not associated with the damping of the resonance, but stems from the dynamical response of the oceans to the pole tide in the retrograde diurnal band, as modeled in Bizouard *et al.* (2019).

Restricting the atmospheric-oceanic correction to the prograde annual term has no significant impact on the resonance parameters. In contrast, the resonance parameters are significantly changed if we consider all terms reported in Table 3. Whereas PM resonance parameters are not modified significantly, the absolute value of the FCN period in celestial frame increases by 0.6 d and the corresponding quality factor in the terrestrial frame by 400. As well, the FICN parameters are affected: the period decreases by 50 d and the terrestrial quality factor by 90. The Table 4 also shows that the nutation estimates obtained by the direct and indirect approaches lead to almost the same results. By way of comparison, we report in Table 4 the results of Ziegler *et al.* (2019), which used a Bayesian inversion technique to estimate the nutation components. It shows that the results match ours except for the quality factor of FCN.

5 ANALYSIS OF POLAR MOTION RESONANCE

The estimated parameters of the PM resonance ($P_{\text{PM}} = 382$ d, $Q_{\text{PM}} = -10$) confirm the fit of Mathews *et al.* (2002) based upon 20 yr of VLBI observation, who had obtained ($P_{\text{PM}} = 383$ d, $Q_{\text{PM}} = -10$). These values differ from the ones corresponding to the common polar motion ($P_{\text{PM}} = 430 - 432$ d, $Q_{\text{PM}} = 56 - 255$, Nastula & Gross 2015). This modification results from the dynamical oceanic response in the diurnal band, as reconstructed from the modeling of the diurnal ocean tide at K_1 frequency (Mathews *et al.* 2002).

Mathews *et al.* (2002) assumed that the estimated values ($P_{\text{PM}} = 383$ d, $Q_{\text{PM}} = -10$) are mostly constrained by the 18.6-yr (σ

$= -1.00288$ cpd) nutation term. To check this assumption, $\tilde{\sigma}_1$ is re-estimated (with FICN and FCN parameters fixed to the estimated values, ‘direct-complete’ case, see Table 4) over restricted sets of nutation terms sweeping frequency bands listed in Table 2 and illustrated in the Fig. 5. It turns out that the estimates $\bar{P}_{\text{PM}} = 382$, $\bar{Q}_{\text{PM}} = -10$ over the whole set (band I) are at discrepancy with the ones obtained over restricted bands, as shown by Table 5. In particular, for frequencies smaller than σ_{FCN} (Band II₁, III₁, III₂) we get P_{PM} significantly larger than 415 d. Although the 18.6-yr retrograde nutation prevails, the estimates of PM parameters are quite loose in the K_1 band (band III₃) ($P_{\text{PM}} = 382 \pm 8$, $Q_{\text{PM}} = -10 \pm 3$) and better constrained by the prograde short period terms (band III₄) ($P_{\text{PM}} = 382 \pm 1$, $Q_{\text{PM}} = -10 \pm 1$).

So, a frequency dependence of the PM resonance parameters is observed in the diurnal band, especially in the vicinity of the FCN. This is in accordance with the theoretical modeling of Bizouard *et al.* (2019), providing more insights on this issue.

6 CONCLUSION

In this study, we performed the adjustment of the main nutation terms and of the forced-free motion associated with the FCN by applying both direct and indirect approaches. The direct approach returned lower formal errors, lower correlations between nutation estimates, and, for some of them, amplitudes differing significantly from those of the indirect approach. The obtained nutation terms were used to determine the resonance frequencies of PM, FCN, and FICN. No significant discrepancies are noted between the estimates based upon the ‘direct’ nutation terms and estimates based upon ‘indirect’ nutation terms. The inclusion of the complete atmospheric and ocean non-tidal corrections has increased the FCN period in the celestial frame by 0.6 d and the corresponding quality factor in the terrestrial frame by 400. The FICN parameters have decreased by 50 d and 90 for its period and quality factor, respectively. More importantly, the PM resonance parameters are mostly determined by the prograde nutation terms of period smaller than 386 d. The PM period and quality factor seem to be frequency dependent in the diurnal band in reason of the FCN resonance, in agreement with the theoretical prediction of Bizouard *et al.* (2019).

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³ <ftp://ftp.iers.org/products/geofluids>.

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