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Implementation of a dark hole maintenance algorithm for speckle drift in a high contrast space coronagraph

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\textbf{ABSTRACT}

Due to the limited number of photons, directly imaging planets requires long integration times. The wavefront must be stable on the same time scale which is often difficult in space due to thermal variations and vibrations. In this paper, we discuss the results of implementing a dark hole maintenance (DHM) algorithm (Pogorelyuk et al. 2019)\textsuperscript{1} on the High-contrast imager for Complex Aperture Telescopes (HiCAT) at the Space Telescope Science Institute (STScI). The testbed contains a pair of deformable mirrors (DMs) and a lyot coronagraph. The algorithm uses an Extended Kalman Filter (EKF) and DM dithering to predict the drifting electric field in the dark hole along with Electric Field Conjugation to cancel out the drift. The DM dither introduces phase diversity which ensures the EKF converges to the correct value. The DHM algorithm maintains an initial contrast of $8.5 \times 10^{-8}$ for 6 hrs in the presence of the DM actuator random walk drift with a standard deviation of $1.7 \times 10^{-3}$ nm/s.

\textbf{Keywords:} Exoplanets, Focal Plane Estimation, Drift

\section{1. INTRODUCTION.}

\subsection{1.1 Direct Observation of Exoplanets with a Space Telescope}

Direct imaging of exoplanets is challenging as the host stars of planetary systems are much brighter than the planets of interest. This means that even if the angular separation is large enough for the planet to be resolved, the lobes of the star’s point spread function (PSF) will hide the planet. To work around this, much effort has gone into developing sets of masks (coronagraphs) to manipulate and expel the starlight while allowing the planet light to pass freely. This creates a ‘dark hole’ region near the star where the planet can be observed. This is called a high contrast region as the starlight has been reduced by many orders of magnitude. For the purpose of this paper, contrast is taken to be the ratio of the intensity at a pixel to the maximum intensity of the unmasked stellar PSF and the mean contrast is the mean of the contrast values in the dark hole region. Imperfections and misalignments of the optical elements\textsuperscript{2} as well as diffraction effects introduced by the coronagraph\textsuperscript{3} degrade the contrast in the dark hole introducing the requirement of deformable mirrors (DMs) to achieve contrast levels below $10^{-5.4}$ Deformable mirrors correct the aberrated wavefront created by the surface defects and alignment errors and enable contrast limits up to $10^{-16}$ in a vacuum as shown by the Jet Propulsion Laboratory’s High

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Contrast Imaging Testbed (HCIT). Since the planet is still very dim, large numbers of exposures are required to observe and characterize the planet which puts strict stability requirements on the PSF. Often the planet can be dimmer than the static contrast obtained by the instrument so stability means stabilizing the image at levels that are a small fraction of the raw contrast so that one can calibrate that noise a posteriori.

The High-contrast imager for Complex Aperture Telescopes (HiCAT) is a scaled down representation of a space telescope capable of directly imaging exoplanets such as LUVOIR. It is used to test related hardware and software and operated with a classic Lyot coronagraph and two Boston Micromachine kilo deformable mirrors for this paper. The full layout of the testbed is discussed in Soummer et. al. 2018 and the architecture of the simulator used to emulate the testbed is summarized in Fowler et. al. 2020 and Moriarty et. al. 2018. HiCAT is operated on a vibration isolation table in a temperature and humidity controlled clean room with an additional enclosure on the table to reduce seeing effects. The current best mean dark hole contrast achieved during an experiment is \(4 \times 10^{-8}\) with HiCAT hitting contrast levels of \(6.5 \times 10^{-8}\) 90% of the time at 640 nm. This is on par with the requirements for the Coronagraph Instrument (CGI) on the Roman Space Telescope launching in 2025. Note that since HiCAT is not operated in a vacuum, it does not achieve the \(10^{-10}\) contrast limit required for future space telescopes (LUVOIR) or the \(4 \times 10^{-10}\) contrast limit shown by HCIT.

1.2 Quasi-static System Drifts

As previously stated, long integration times are required for directly observing exoplanets. Currently, there are multiple solutions available to deal with high spatial frequency, static wavefront errors (WFE) as well as low spatial frequency, fast WFE. Slow or quasi-static instabilities in the system degrade the contrast over an observation and can produce speckles that can be falsely identified as planets.

For high contrast imaging space telescopes without deformable mirrors, such as the Hubble Space Telescope and the James Webb Space Telescope, slow system drifts must be corrected for solely through modelling and post processing. The Roman Space Telescope (RST) will be the first NASA mission to use deformable mirrors in space for high contrast imaging. The Observing Scenario 9 for RST is to point at a bright reference star to dig the dark hole, ie find the DM command that corrects the static wavefront error (WFE) induced by the optical surfaces etc, and then perform a slew to point at the science target at which point the DM command is kept constant (Krist et. al. 2020). The roll of the telescope is changed four times while on the science target and then RST will slew back to the reference star to re-dig the dark hole that has degraded do to the various drifts in the system. This reference star–target star–reference star cycle is repeated three times for each target if RST is using the lyot coronagraph. Each slew and roll change the thermal environment of the telescope and cause a slow drift in the WFE. If the DHM algorithm presented here was implemented on RST, it would not require slews to the reference star.

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**Figure 1:** HiCAT summer 2020 lab instabilities due to climate control issues. A constant DM command was applied repeatedly and the thermal/humidity drifts caused a contrast degredation of a factor of two over the course of 14 hrs.
In the HiCAT lab, the main drivers of quasi-static WFE are temperature and humidity. This is captured by Figure 1 which shows the mean contrast on HiCAT degrading by a factor of two over the course of 14 hours while repeatedly applying a constant DM command. We acquired this data during the summer of 2020 when there were issues with the temperature and humidity control in the lab. As a result, the stability and performance shown in Figure 1 are an order of magnitude worse than what is possible using the current HiCAT environment.

We added a manual drift for the initial implementation of DHM because we wanted to control the perturbation to quantify the algorithm performance as a function of drift. We used random walk of each DM actuator to create a high order, slow drift of the electric field at the focal plane. For Figure 2, the drift added to the initial DM command was chosen from a normal distribution \( N(0, \sigma_{\text{drift}}^2 I) \) where \( \sigma_{\text{drift}} = 0.01 \text{ nm/iteration} \). The left panel shows the final image after 1.5 hours and the right panel shows the growing mean contrast in the dark hole region over time.

In this paper, we demonstrate a Dark Hole Maintenance (DHM) algorithm on HiCAT that compensates for a DM actuator random walk drift and maintains a contrast of \( 8 \times 10^{-8} \) for 6 hours. Section 2 discusses the DHM algorithm and how it is implemented. In Section 3, the results of the experiments on HiCAT as well as complementary simulations are covered. Finally, Section 4 summarizes the work accomplished so far as well as future plans.

2. DARK HOLE MAINTENANCE.

2.1 Dark Hole Creation and Metrics

The goal of the DHM is to maintain the contrast in the dark hole meaning first we must create or ‘dig’ the dark hole. HiCAT uses a pair-wise probe estimator\(^{20}\) combined with the stroke-minimization\(^{20}\) controller as discussed in N’Diaye et al 2015\(^{21}\) and Soummer et al 2019.\(^{22}\) The standard control region is an annulus from \( 4 - 12 \lambda/D_{\text{apod}} \) and the contrast is defined as the intensity at each pixel divided by the peak intensity when the focal plane mask is not in place,

\[
\text{contrast} = \frac{I(i,j)}{I_{\text{open}}(0,0)} = \frac{I(i,j)}{I_0}
\]

The mean contrast is defined as

\[
\text{mean contrast} = \mu = \frac{1}{n} \sum_{k,l} \frac{I(k,l)}{I_{\text{open}}(0,0)}
\]

where \( n \) is the number of pixels in the dark hole and \( k,l \) are the indices of the pixels in the dark hole.
Figure 3: HiCAT lab data comparing a dark hole dig experiment and a dark hole maintenance experiment. The pair-wise probe estimator and the stroke-minimization controller are used to dig the dark hole. No drift is injected during the dark hole dig. For the DHM experiment, the Extended Kalman Filter combined with DM dithering is used to estimate the electric field and Electric Field Conjugation is used to correct for the injected drift. The injected drift is characterized by the standard deviation of the random walk of each actuator, \( \sigma_{drift} = 1.7 \times 10^{-3} \text{ nm/s} \), and the dither is characterized by the standard deviation of the random command added to the EFC control, \( \sigma_{dither} = 0.2 \text{ nm} \).

Figure 3 shows the mean contrast vs time in blue for digging the dark hole on HiCAT (with no drifts added other than the natural drifts of the testbed) using the pair-wise probe estimator combined with the stroke-minimization controller. Once the mean contrast is suitably low, the digging process is terminated and the final state of the dig \((E_0, u_0)\) is used as the initial condition for the Dark Hole Maintenance (DHM) algorithm. Figure 3 also shows a DHM run overlaid in purple where each DM actuator is now performing a random walk with \( \sigma_{drift} = 1.7 \times 10^{-3} \text{ nm/s} \); see Figure 2 for the open loop behaviour of this drift. The two curves in Figure 3 cannot be compared with respect to performance since the purple curve contains a drift and the blue curve does not, but it is important to note the increase in cadence of science images when the DHM algorithm is used. For pair-wise probing, none of the probed images used to obtain the electric field estimate can in principle be used for science. We interleave un-probed images at the end of each iteration as shown by the blue dots on the blue curve in Figure 3 and these are the science images. When using the EKF with DM dithering shown in purple, there is only one image taken each iteration and every image is a science image. The dithering technique used in the DHM algorithm to calculate the estimate can also be used in post-processing to locate a planet in the image as discussed in Pogorelyuk et. al.\(^1\)

### 2.2 Dark Hole Maintenance Algorithm

The high level Dark Hole Maintenance (DHM) algorithm is shown in Figure 4. An aberrated electric field arrives at the DMs and is partially corrected to create the closed loop (CL) electric field at the science camera, \( E_{CL}^{f} \). The camera measures the intensity, \( z = I = |E_{CL}^{f}|^2 \), which is used to estimate the open loop (OL) electric field, \( X = E_{OL}^{f} \), at the camera using an Extended Kalman Filter (EKF). Once the estimate is obtained, the Electric Field Conjugation (EFC) algorithm is used to determine the next DM command. A small random dither command is added to the EFC command to increase the phase diversity at the focal plane which improves the estimate. The drift is added to the DM command when the image is taken but that command is not passed to the estimator. For this paper, the drift is injected as a random walk of each DM actuator, it can be expressed as

\[
u_{drift}^{k+1} = u_{drift}^{k} + \mathcal{N}(0, \sigma_{drift}^2 I)
\]

where \( k \) is the iteration and \( \sigma_{drift} \) is the standard deviation of the normal distribution. Any drift value quoted is to be taken as \( \sigma_{drift} \) in units of nanometers.
Figure 4: The light passes through the telescope optics which introduce small static aberrations due to surface imperfections and misalignments. In addition, there are transient aberrations $\delta E_{ab}$ which come from thermal drifts and pointing jitter. The intensity of the closed loop electric field $E_f$ is detected at the science camera after interacting with the deformable mirrors and the set of masks that make up the coronagraph. The measured closed loop intensity, $y = I$ is used to estimate the open loop electric field at the camera, $E_{OL}$. The estimate is used to find the change in the DM command that will correct for the open loop field in the dark hole region, $\Delta u_{opt}$. Before being applied to the DMs, a random dither ($\delta u_{dith}$) and drift ($\delta u_{drift}$) are added to the DM command.1

Table 1: System definitions.

<table>
<thead>
<tr>
<th>Description</th>
<th>Variable</th>
<th>Expression</th>
<th>Dimensions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of dark hole pixels</td>
<td>$n$</td>
<td>$n=2544$</td>
<td>$1 \times 1$</td>
</tr>
<tr>
<td>Number of DM actuators</td>
<td>$m$</td>
<td>$m = 2 \times 952$ act/DM</td>
<td>$1 \times 1$</td>
</tr>
<tr>
<td>Complex OL Electric Field</td>
<td>$X$</td>
<td>$x_{Re} + x_{Im}$</td>
<td>$n \times 1$</td>
</tr>
<tr>
<td>State vector: stacked OL Electric Field</td>
<td>$x$</td>
<td>$[x_{Re} \ x_{Im}]$</td>
<td>$2n \times 1$</td>
</tr>
<tr>
<td>Measured CL Intensity</td>
<td>$z$</td>
<td></td>
<td>$n \times 1$</td>
</tr>
<tr>
<td>DM command</td>
<td>$u$</td>
<td></td>
<td>$m \times 1$</td>
</tr>
<tr>
<td>Jacobian</td>
<td>$G$</td>
<td></td>
<td>$2n \times m$</td>
</tr>
</tbody>
</table>

Basic system definitions are shown in Table 1. Note that the state vector $x$ consists of the stacked coefficients of the real and imaginary components of the electric field to have a purely real state vector. Before developing the estimator or the controller, a model of the system is required. The full nonlinear state space model can be expressed as

\[
x^{k+1} = f(x^k) + w^k
\]

\[
x^k_{CL} = x^k + Gu^k
\]

\[
y^{k+1} = h(x^k, u^k) + n^k = x^k_{CLRe} \circ x^k_{CLRe} + x^k_{CLIm} \circ x^k_{CLIm} + n^k
\]

where $G$ is the Jacobian, $x^k_{CL}$ is the closed loop electric field, $\circ$ is the Hadamard operator, $u^k$ is the DM command for the current iteration, and $w^k, n^k$ are the process and observation noise terms. The full model is then linearized.
at each time step to produce

\[ x^k = F^k x^{k-1} + w^k \]  \hspace{1cm} (7)

\[ z^k = H^k x^k_{CL} + Gu^k + n^k \]  \hspace{1cm} (8)

\[ F^k = \frac{\partial f}{\partial x^k} \bigg|_{x^k = \hat{x}^{k-1}} \]  \hspace{1cm} (9)

\[ H^k = \frac{\partial h}{\partial x^k} \bigg|_{x^k = \hat{x}^{k-1}} \]  \hspace{1cm} (10)

where \( F^k = I \) is the state transition matrix and \( H^k \) is the observation matrix. There are no dynamics in this system as we are modeling the open loop electric field which is only affected by the drift which comes from the process noise term. This result is then used to put together the estimator. The Extended Kalman Filter solves a maximum likelihood estimation problem which takes into account the current estimate residual as well as the bias from the previous time steps.\(^{12}\) First, the estimate and variance for the current time step are calculated given the estimate for the previous time step

\[ \hat{x}^{k|k-1} = F^k \hat{x}^{k-1} \]  \hspace{1cm} (11)

\[ P^{k|k-1} = F^k P^{k-1}(F^k)^T + Q^k \]  \hspace{1cm} (12)

where \( Q^k \) is the process noise. In the presence of a random walk drift of each DM actuator, the process noise is expressed as

\[ Q^k = GG^T \sigma_{\text{drift}}^2 t_{\text{exp}} I_0 \]  \hspace{1cm} (13)

as discussed in.\(^ {19}\) Here \( \sigma_{\text{drift}} \) is the standard deviation of the random drift command applied to each actuator as shown in Equation 3, \( t_{\text{exp}} \) is the exposure time of the image, and \( I_0 \) is the maximum intensity of the point spread function (PSF) when the focal plane mask is not in place. Continuing on, the Kalman gain is calculated which is then used to determine the electric field estimate and covariance for the current iteration

\[ K^k = P^{k|k-1}(H^k)^T \left( H^k P^{k|k-1}(H^k)^T + R^k \right)^{-1} \]  \hspace{1cm} (14)

\[ \hat{x}^k = \hat{x}^{k|k-1} + K^k \left[ z^k - h(\hat{x}^{k|k-1}, u^k) \right] \]  \hspace{1cm} (15)

\[ P^k = P^{k|k-1} - K^k H^k P^{k|k-1} \]  \hspace{1cm} (16)

where \( R^k = \text{diag}(\hat{y}^{k|k-1}) = \text{diag}(h(\hat{x}^{k|k-1}, u^k)) \). Note for EKF calculations shown,

\[ u^k = \Delta u^k_{\text{opt}} + \delta u^k_{\text{dither}} \]  \hspace{1cm} (17)

\[ \delta u^k_{\text{dither}} = \mathcal{N}(0, \sigma_{\text{dither}}^2 T) \]  \hspace{1cm} (18)

where \( \Delta u^k_{\text{opt}} \) is the optimal DM command determined by EFC. As discussed in Pogorelyuk et al 2019\(^{1} \) and Riggs et all 2016,\(^ {23} \) using an EKF without any probe images can cause the estimate to converge to the wrong value. The addition of the dither can be thought of as a probe to increase the phase diversity of the electric field between iterations to improve the estimate.

For \( \sigma_{\text{dither}} = 0.2 \text{ nm} \) and \( \sigma_{\text{drift}} = 0.01 \text{ nm/iteration} \) the estimator converges in approximately 60 iterations as shown by Figures 5 and 6. Convergence is taken to be the iteration when the absolute intensity estimate error reaches its steady-state value; this will be discussed more in Section 3.3. The HiCAT hardware experiment shown in Figures 5 and 6 is a typical DHM run but takes an open loop image every iteration to track the open loop estimate error. The open loop estimate and DM command are calculated as

\[ \hat{I}^k_{\text{OL}} = \hat{x}^k_{\text{Re}} \circ \hat{x}^k_{\text{Re}} + \hat{x}^k_{\text{Im}} \circ \hat{x}^k_{\text{Im}} \]  \hspace{1cm} (19)

\[ u^k_{\text{open loop}} = u_0 + u^k_{\text{drift}} \]  \hspace{1cm} (20)
where \( \hat{x} \) is calculated in Equation 15 and \( u_0 \) is the initial DM command obtained from the dark hole dig as discussed in Section 2.1. At iteration one, Figure 5, the larger features of the image (left panel) are captured by the estimate (right panel) but there are various holes that are severely under-estimated. The constant background evident in the image is also not seen in the estimate but that is expected as the EKF is not currently formulated to estimate the incoherent electric field caused by stray light on the testbed. At iteration 60 (Figure 6) the estimator is now over-estimating the main features such as the ringing and the hot-spots but is now capturing the finer structure present in the image.

Figure 5: Log contrast open loop image and estimate for first iteration of DHM algorithm on HiCAT testbed. For this experiment, \( \sigma_{\text{dither}} = 0.2 \text{ nm} \) and \( \sigma_{\text{drift}} = 0.01 \text{ nm/iteration} \) and an open loop image was taken every iteration for comparison purposes. The estimate (right panel) captures the main features of the image (left panel) such as the ringing and the hot spots in the upper right and lower left quadrants but does not capture the finer structure. The estimate also does not show the incoherent background seen in the image.

Figure 6: Log contrast open loop image and estimate for 60th iteration of DHM algorithm during same experiment as Figure 5 on HiCAT testbed. The estimate (right panel) is over-estimating the main features of the image (left panel) such as the ringing and the hot spots in the upper right and lower left quadrants but is also capturing some of the finer structure. The incoherent background is evident in the image and is not being estimated by the EKF.

Once the estimate of the open loop electric field is obtained, EFC is used to determine the optimal DM command to minimize the closed loop field present via

\[
\Delta u_{\text{opt}}^{k+1} = -(G^T G + \alpha T) G^T \hat{z}^k
\]

\[
u_{\text{control}}^{k+1} = u_0 + \Delta u_{\text{opt}}^{k+1} + \delta u_{\text{dither}}^{k+1}
\]

\[
u_{\text{tot}}^{k+1} = u_{\text{control}}^{k+1} + u_{\text{drift}}^{k+1}
\]
where $\alpha$ is the Tikhonov regularization parameter to avoid excessively large DM commands, $u_0$ is the DM command obtained from the dark hole dig, $u_{control}^{k+1}$ is the total closed loop DM command, and $u_{tot}^{k+1}$ is the total command sent to the DMs including the drift. The derivation of this control law can be found in Pogorelyuk 2019. This is slightly different than when EFC is used to dig the dark hole as derived in Sun 2019 where the state variable is the closed loop electric field under the assumption that the open loop electric field is constant.

3. LABORATORY RESULTS.

For all subsections below, any quoted drift or dither values are the standard deviation of the random command in nanometers ($\sigma [nm]$) and all hardware / simulated experiments are at 640 nm.

3.1 Behaviour on Long Runs

In this section we demonstrate the ability of this algorithm to maintain high contrast levels ($8.5 \times 10^{-8}$) for a long duration of time (6 hrs, 4000 iterations). Figure 7 shows the mean contrast of the dark hole region (red) over time in the presence of a $\sigma_{drift} = 1.7 \times 10^{-3} \text{ nm/s}$ which is equivalent to $\sigma_{drift} = 0.01 \text{ nm/iteration}$. For this experiment, a dither of 0.2 nm was chosen. This choice of dither will be discussed in detail in Sections 3.2–3.4. The solid black line in Figure 7 is the mean of the red curve and the dotted black line is the standard deviation of the red curve values. To track the drift at the focal plane, six images were taken throughout the experiment with the open loop DM command as shown by the blue asterisks in Figure 7. The DM command used for the open loop images is shown in Equation 20. The open loop mean contrast degrades by a factor of four while the closed loop mean contrast remains at the initial value of $8.5 \times 10^{-8}$ within a standard deviation of $2.4 \times 10^{-8}$. Using this data with the post-processing method discussed in Pogorelyuk et. al. can further reduce the contrast to $2.4 \times 10^{-8}$.

Figure 7: Long DHM Experiment. The mean contrast of the dark hole is shown in red with the mean and standard deviation of the red curve shown by the solid and dashed black lines respectively. The blue asterisks are the mean contrast in the dark hole for the open loop DM command. The open loop mean contrast degrades by a factor of four while the closed loop mean contrast remains at the initial value of $8.5 \times 10^{-8}$ within a standard deviation of $2.4 \times 10^{-8}$.
3.2 Varying Dither

Adding a random dither to a DM command degrades the contrast as shown in Figure 8. Here the lines with the dots are produced using the HiCAT simulator and the solid lines are produced using the HiCAT testbed. A DM command producing a high contrast dark hole is used to start \( u_0 \) and then a random realization of the dither normal distribution is added to it for each image. Note that this random command does not accumulate and as a result at the contrast degradation is minimal and more importantly controlled (vis the amplitude of dither). As shown by the green, blue, and purple lines in Figure 8 the simulation and lab data agree very well. For the red line, where the dither is the smallest, the initial contrast on the testbed was too high to be able to see a significant change in the contrast due to the dither.

Figure 8: Random realization of mean contrast in dark hole due to varying DM dither standard deviations \((\sigma_{\text{dither}} \text{ [nm]})\). Solid lines are hardware data and the lines with dots are simulated data. For the red hardware curve the initial contrast was not sufficiently high to see the effect of the dither.

The trend seen in Figure 8 is expected as we are in a small aberration regime where the contrast scales with the square of the wavefront error. In this case, the wavefront error is caused by the dither. The dither-contrast relationship is modelled using a second order polynomial by averaging the mean contrast values over the 50 images taken. The zero dither point is added in to be the initial contrast as if there is no change in the DM command, the contrast should remain constant. The data and models for simulated and hardware experiments are shown in Figure 9 where the data points include error bars to show the standard deviation of the contrast in the dark hole region over the 50 images. As shown in the left two panels of Figure 9 the data and models are well matched. The hardware and the simulation models agree well at higher dithers but deviate at the smaller dithers as shown by the right panel in Figure 9. At small dithers the contrast change is commensurate with the raw tested contrast and thus dither induced modulation is not sufficient as shown by the red curves in in Figure 8.
Figure 9: Second order model of mean dark hole contrast vs dither for HiCAT simulator and hardware. Data points for the simulated and hardware experiments include error bars to show the standard deviation of the contrast in the dark hole region over the 50 images. At smaller dithers, the hardware does not reliably produce high enough contrasts to capture the behaviour.

In some ways, Figure 9 shows the limiting contrast achievable for a certain dither, but this is not the entire story as there is an estimation and control loop as well. This will be discussed next.

3.3 Effect of Dither on Estimator Convergence (Simulations)

As previously stated, the dither increases the phase diversity of the electric field at the focal plane. One can think of the EKF as a pair-wise probe estimator with memory that acquires one probed image per iteration. The larger the dither magnitude, the larger the change between images and the better the estimator will perform. We cannot assume we have access to the open loop image thus the the closed loop estimate error must be used as a proxy as it is in the EKF. The closed loop intensity estimate error is calculated as

$$
\epsilon_{CL} = |z^k - \hat{y}^k| = |z^k - h(\hat{x}^k, u^k)|
$$

(24)

where $z$ is the measured closed loop intensity and $\hat{y}^k$ is the estimate of the closed loop intensity as defined in Section 2.2. The mean and standard deviation of the closed loop intensity estimate error for a given iteration are then

$$
\mu_{err} = \frac{1}{n} \sum_{i=1}^{n} |z_i^k - \hat{y}_i^k|
$$

(25)

$$
\sigma_{err} = \sqrt{\frac{1}{n-1} \sum_{i=1}^{n} \left| |z_i^k - \hat{y}_i^k| - \mu_{err} \right|^2}
$$

(26)

where $n$ is the number of pixels in the dark hole and $k$ is the current iteration. Figure 10 shows the mean intensity estimate error in units of contrast for a simulated DHM run on HiCAT with a random walk DM drift of 0.01 nm/iteration. As shown in Figure 10, the mean estimate error initially gets worse and then turns over as the estimator accumulates enough information to improve the estimate. It improves for a number of iterations and then asymptotes to a steady state mean estimate error caused by discrepancies between the model and the actual system combined with the effectiveness of the dither. Estimator convergence is defined as the iteration where the absolute estimate error reaches the steady state absolute estimate error; for the orange curve in Figure 10 this would be iteration 98. Since we are looking at the absolute error and not the relative error, the estimate error will increase as the contrast degrades.

The mean estimate error for the smallest dither in Figure 10 (red curve) takes much longer to peak and much longer to reach the same error level as the $\sigma_{dither} = 0.1 \text{ nm}$ case. The largest dither case (teal curve)
‘converges’ the fastest but the steady state mean estimation error is very high and does not to improve. The EKF is performing well but the $\sigma_{dither} = 0.4 \ nm$ case is at the dither limit meaning the contrast remains constant at the value determined in Section 3.2. Since this is a higher, constant contrast, the absolute error will scale appropriately and follow a similar trend. This dither limiting case is also shown in Figure 11 and will be discussed more in Section 3.4. The two medium dithers (blue and orange curves) have similar profiles but the smaller of the two asymptotes to a lower error. From this we can see that there is an ‘optimal’ dither if the goal is to minimize the mean estimate error.

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$\sigma_{dither}$

Figure 10: Mean absolute error between the estimated closed loop intensity and the measured closed loop intensity at each pixel in the dark hole for 200 iterations during active control sequence for various dither values, $\sigma_{dither} \ [nm]$. For all cases, $\sigma_{drift} = 0.01 \ nm/iteration$.

3.4 Choosing an Optimal Dither

Figure 11 shows the mean contrast evolution for both simulation and hardware data for different dithers and a $\sigma_{drift} = 0.01 \ nm/iteration$. The two plots are well matched for $\sigma_{dither} = 0.2, 0.4 \ nm$. The lab result (right panel) for the yellow curve, $\sigma_{dither} = 0.1 \ nm$, initially behaves like the simulation result (left panel) for the red curve, $\sigma_{dither} = 0.05 \ nm$. These experiments are currently performed with no real-time tip-tilt correction so this is likely due to small lab instabilities that hinder the effect of the small dither making it harder for the estimator to converge. Once the estimate for the lab case peaks, around iteration 30, the contrast for $\sigma_{dither} = 0.1 \ nm$ returns to the original contrast with a slope similar to the simulated $\sigma_{dither} = 0.1 \ nm$. In the future when the real-time tip-tilt correction is incorporated into the DHM algorithm the smaller dithers are expected to better match the simulations.

Figure 11 illustrates the combination of the described effects in Figures 9 and 10. The contrast initially degrades as the estimate error is large and thus the optimal command provided by EFC is not cancelling out the electric field very well. As the estimate improves, so does the quality of the EFC command and the contrast trends back to the original state. The steady state contrast is then determined by a combination of the dither contrast limit as shown in Figure 9 and the estimate error shown in Figure 10. For larger dithers, the dither contrast limit dominates and for smaller dithers, the estimate error dominates. It is important to note that the red curve in Figure 11 does return to the original contrast after ~200 iterations. The teal curve ($\sigma_{dither} = 0.4 \ nm$) stays relatively constant at in both the simulation and on the hardware. The contrast is at the ‘dither limit’ as shown in Figure 9 and the EFC cannot do much to improve it.

To choose an optimal dither there are three main parameters of interest: number of iterations required for the estimator to converge, steady-state mean estimate error, and steady-state contrast. The goal is to pick the dither that minimizes all three. For a drift of 0.01 nm/iteration the effect of varying dither values is shown in Figure 12. Note that the dashed red line representing the asymptotic contrast vs dither agrees with Figure 9 for the larger dither values. It depends on how each of the parameters in Figure 12 are weighted, but the optimal dither lies in the grey region highlighted where $0.1 \ nm \leq \sigma_{dith} \leq 0.2 \ nm$. 

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Figure 11: HiCAT simulation and hardware comparison of the mean dark hole contrast response for $\sigma_{\text{drift}} = 0.01 \, \text{nm/iteration}$ and varying dither. The contrast response as a function of dither follows a similar trend to the estimate error with the smallest and largest dithers performing worse than the dithers in between. The simulations match the hardware for $\sigma_{\text{dither}} = 0.2, 0.4 \, \text{nm}$ but deviate at $\sigma_{\text{dither}} = 0.1 \, \text{nm}$.

Figure 12: Summary plot of Figures 10 and 11. Simulated effect of dither on iterations until estimator converges (blue), steady state closed loop contrast estimate error (solid red), and steady state contrast (dashed red). The grey region shows the region in which these three parameters are minimized.

The behaviour of the DHM algorithm varies with a number of factors including starting contrast, observation noise, and drift. All plots shown in Sections 3.1–3.4 vary the dither while keeping all other parameters constant. Repeating the process in Sections 3.1–3.4 for a variety of $\sigma_{\text{drift}}$ values shows that for a DM actuator random walk drift, the optimal dither is $\sigma_{\text{dither}} \sim 15 \times \sigma_{\text{drift}}$. 
3.5 Effect of Estimator Reset

The Extended Kalman Filter is a nonlinear estimator that linearizes about each time step. Since the DM command, and thus the estimate, are continuously accumulating, this can lead to large linearization errors. A way around this is to shift the DM command and electric field estimate periodically throughout the experiment. Essentially you adjust so that

\[
\hat{E}_0 = \hat{E}^k_r \tag{27}
\]
\[
u_0 = u^k_r \tag{28}
\]

where \(k_r\) is the estimator reset iteration.

Figure 13 shows the effect of an estimator reset on HiCAT at iterations 180 and 300 as indicated by the black dots. The closed loop mean dark hole contrast is shown in red and the open loop mean dark hole contrast is shown by the dashed blue curve. Note that for this experiment, the open loop image was taken every iteration. When the estimator resets, it loses the previous knowledge it had and has to build up images and converge again. This causes the addition bumps in the contrast seen after iterations 180 and 300. Despite these oscillations, this is still preferable behaviour to the open loop case as the contrast returns to the original value each time and the closed loop data can be used in post processing to reject speckles and obtain a higher signal-to-noise image of the planet.\(^{1}\) From Figure 7 it was shown that the linearization errors are not sufficiently large to cause issues at the \(8.5 \times 10^{-8}\) level for at least 6 hours / 4000 iterations. Combining the results of Figure 13 and Figure 7 it is clear that estimator resets should only be performed if the estimator begins to diverge as they create periodic contrast degradation. It also may be more advantageous to use the data accumulated to perform system identification\(^{12}\) and periodically update the jacobian instead of, or in addition to, the estimator resets.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure13.png}
\caption{Mean dark hole contrast vs iteration with periodic estimator resets. Closed loop contrast is shown in red with the estimator reset iterations noted in black. The blue dashed line shows the open loop mean dark hole contrast for \(\sigma_{\text{drift}} = 0.01\) nm/iter.}
\end{figure}

4. CONCLUSIONS AND FUTURE WORK.

Quasi-static wavefront error drifts degrade the contrast in the dark hole and reduce the fidelity of the exoplanet observation. The Dark Hole Maintenance Algorithm described in Section 2 is proven to be able to correct for a
DM dither random walk drift and maintain or improve on the starting contrast as shown by the various results in Section 3. This is encapsulated by Figure 7 where a contrast of $8.5 \times 10^{-8}$ is maintained for six hours. The dither has a large impact on the performance algorithm and can be efficiently tuned for a DM actuator random walk drift using the results from Section 3.4.

With this initial proof of concept, the next step is to expand DHM capabilities. There are many sources of drifts on large high contrast imaging instruments thus this algorithm should be tested with each potential source. Recently, an Iris AO PTT111L mirror was installed on HiCAT as a proxy for a segmented primary mirror. This deformable mirror has 37 segments which can each be controlled in piston, tip, and tilt. The primary mirror on a space telescope is the largest and thus the most susceptible to thermal drifts. Applying a drift to the PTT111L will capture primary mirror instabilities and is the next milestone for this DHM algorithm. Another planned upgrade includes defining drifts as zernike modes which will allow HiCAT to take predicted or measured drifts from other experiments (such as RST) and try to correct for them. Finally, we plan to add broadband capabilities and to try the suggested optimal dither technique (rather than the random dither used in this paper) described in Sun 2019.12

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