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APPROXIMATION OF THE EFFECTIVE FLUID PROPERTIES OF A WAVEGUIDE SIDE-LOADED BY HELMHOLTZ RESONATORS

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ABSTRACT

One dimensional waveguides are often modelled with the transfer matrix method. This requires splitting the waveguide system into layers and building the system of transfer matrices associated with these layers. Multiplied together, these matrices can be used to find the effective material properties of the waveguide system. However, the resulting analytical expressions requires the full TMM approach and provide no explicit information about the waveguide structure. Here, a waveguide side loaded by three Helmholtz resonators is modelled using two methods; the transfer matrix method and an effective property model which will be available in a future publication. This effective property model utilises the linear superposition of effective property components obtained via the transfer matrix method. The effective properties obtained via these two methods are compared to assess the validity of the effective property model, with excellent agreement being found.

1. INTRODUCTION

The transfer matrix method (TMM) is a simple and powerful method to model acoustical systems. Using this method it is possible to analyse the sound absorption/transmission properties of one and two port systems [1] and derive effective property expressions for porous layers [2]. It has been used to model sound absorbing acoustic metamaterials consisting of waveguide structures side-loaded by Helmholtz resonators [3, 4].

In this extended abstract, a rectangular waveguide side-loaded by Helmholtz resonators is modelled using two methods. Namely, the TMM and a general effective model based upon the linear superposition of components obtained via the TMM, which will be available in a later publication. Using these methods, simple general expressions for the effective dynamic density and complex compressibility were derived.

2. THEORY

Consider a waveguide section of length L, side-loaded by M Helmholtz resonators, periodically spaced by \( l = \frac{L}{(M - 1)} \), as shown in Figure 1.

The transfer matrix for the whole system is expressed as

\[
T = M_{WG} \cdot M_{HR}^{(1)} \cdot M_{WG} \cdot M_{HR}^{(2)} \cdots, \quad (1)
\]

where the waveguide transfer matrix is

\[
M_{WG} = \begin{bmatrix} \cos(k_w l) & i Z \sin(k_w l) \\ i Z \sin(k_w l) & \cos(k_w l) \end{bmatrix}. \quad (2)
\]

\( Z_w \) is the waveguide characteristic impedance for plane wave propagation, written as \( Z_w = \rho_w c_w / S_w \), with \( S_w \) being the cross sectional area of the waveguide, and \( k_w \) is the characteristic wavenumber. The resonators are introduced as point scatterers within the transfer matrix, which is facilitated for the \( m^{th} \) resonator by the following matrix:

\[
M_{HR}^{(m)} = \begin{bmatrix} 1 & 0 \\ \frac{1}{Z_{HR}^{(m)}} & 1 \end{bmatrix}. \quad (3)
\]

Here \( Z_{HR} \) is the Helmholtz resonator impedance and is defined within [5]. To calculate the effective material properties using the traditional method of matrix multiplication, it is a simple manner of utilising the equations set out in [2] upon the final transfer matrix, \( T \), of the system.

Using the aforementioned general model based upon the linear superposition of effective property components obtained utilising the TMM, the effective dynamic density and complex compressibility of a waveguide side-loaded by M Helmholtz resonators can be obtained using the following expressions, respectively. These expressions are valid within the low frequency regime below the first Bragg and cross-sectional mode frequency.

\[
\rho_{eff}(\omega) = \rho_w(\omega), \quad (4)
\]

\[
C_{eff}(\omega) = C_w(\omega) + \frac{1}{i \omega L S_w} \sum_{m=1}^{M} \frac{1}{Z_{HR}^{(m)}}. \quad (5)
\]
Here $\rho_w(\omega)$ is the dynamic density of the waveguide and $C_w(\omega)$ is the complex compressibility of the waveguide. Expressions for these can be found within [6] for a rectangular cross-section.

3. RESULTS

To assess the validity of equations (4) and (5), a system of three equispaced cylindrical Helmholtz resonators side-loading a square waveguide is modelled. This system contains three distinct resonances resulting from variation in geometry between the three Helmholtz resonators. Namely, a difference in the cross sectional area of the HR necks.

The length of the system is $L = 88\, mm$, the width, $a$, and height, $b$, of the waveguide are $a = b = 60\, mm$. The geometry of the modelled Helmholtz resonators can be seen in Table 1.

<table>
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<th>$r_c$</th>
<th>$l_n$</th>
<th>$l_c$</th>
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<td>20</td>
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<td>3</td>
<td>1.5</td>
<td>20</td>
<td>15</td>
<td>45</td>
</tr>
</tbody>
</table>

Table 1. Geometric properties of the three modelled Helmholtz resonators. All units are [mm].

The plots of the effective density and bulk modulus using the effective property model and the TMM multiplication method can be seen in Figure 2.

![Figure 2](image-url)

Figure 2. The effective density ($\rho(\omega)$), bulk modulus ($K(\omega) = 1/C(\omega)$), impedance ($Z(\omega)$), wavenumber ($k(\omega)$) and speed of sound ($c(\omega)$) computed using the effective property model (TMM summation) and the traditional TMM method (TMM Multiplication).

From this figure it is evident that there is good agreement in both terms, although slight fluctuations within the effective density are evident from the TMM multiplication model which have not been captured with the TMM summation model. Whilst pronounced, the fluctuations are small with a MAE of 0.0012kg/m$^3$ between the two models for the dynamic density. These fluctuations are not captured with the TMM summation model due to the HRs being introduced as point scatterers, which contradicts assumptions within the derivation of the expression. Whilst this is effect is noticeable, the effect it plays on the accuracy of terms derived using the dynamic density is negligible, meaning the validity of the effective property model still holds. The MAE when comparing the bulk modulus obtained by using the two models is 33kPa, highlighting again the good agreement between the models.

4. CONCLUSION

The effective fluid properties of a waveguide sideloaded by three HRs obtained were obtained with the traditional TMM method and a by a new model yet to be published. These were compared and it was found that there is excellent agreement in all terms except the effective dynamic density. It is apparent through the TMM multiplication method that there are slight disturbances due to resonance which the proposed effective property model fails to capture. However, these fluctuations can be considered negligible and therefore the effective property model is valid.

5. REFERENCES


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