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# A New Full Relativistic Escape Velocity and a New Hubble related Equation for the Universe

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## Abstract

The escape velocity derived from general relativity coincides with the Newtonian one. However, the Newtonian escape velocity can only be a good approximation when  $v \ll c$  is sufficient to break free of the gravitational field of a massive body as it ignores higher-order terms of the relativistic kinetic energy Taylor series expansion. Consequently, it does not work for a gravitational body with a radius at which  $v$  is close to  $c$ , such as a black hole. To address this problem, we re-visit the concept of relativistic mass, abandoned by Einstein, and derive what we call a full relativistic escape velocity. This approach leads to a new escape radius where  $v_e = c$  equal to a half of the Schwarzschild radius. Further, we show that one can derive the Friedmann equation for a critical universe from the escape velocity formula from general relativity theory. We also derive a new equation for a flat universe based on our full relativistic escape velocity formula. Our alternative to the Friedmann formula predicts exactly twice the mass density in our (critical) universe as the Friedmann equation after it is calibrated to the observed cosmological redshift. Our full relativistic escape velocity formula also appears more consistent with the uniqueness of the Planck mass (particle) than the general relativity theory: whereas the general relativity theory predicts an escape velocity above  $c$  for the Planck mass at a radius equal to the Planck length, our model predicts an escape velocity  $c$  in this case.

**Keywords:** Hubble constant, escape velocity, Schwarzschild radius, Hubble radius, Friedmann equation, Schwarzschild sphere, Planck scale.

## 1 The Newton escape velocity, the general relativity escape velocity, and a new full relativistic escape velocity

The Schwarzschild radius is simply the radius we have when a given mass is inside a radius so that the escape velocity at this radius is  $v_e = c$ . The idea of massive gravity objects, where not even photons could escape, was not first invented by Schwarzschild [1, 2] or from general relativity theory [3], but indirectly from Newton's [4] theory. Already in 1784, John Michell [5] wrote

*If the semi-diameter of a sphere of the same density as the Sun were to exceed that of the Sun in the proportion of 500 to 1, a body falling from an infinite height towards it would have acquired at its surface greater velocity than that of light and consequently supposing light to be attracted by the same force in proportion to it is vis inertia, with other bodies, all light emitted from such a body would be made to return towards it by its own proper gravity. This hypothesis assumes that gravity influences light in the same way as massive objects.*

This radius is basically identical to the Schwarzschild radius  $R_s = \frac{2GM}{c^2}$ , and also, the interpretation that light cannot escape is very similar to that suggested for black holes. Michell suggested dark stars, where not even light could escape. The Michell (dark star) radius was rooted in Newtonian mechanics. We can set the kinetic energy of the small mass to be equal to the gravitational potential energy to find the escape velocity from Newton mechanics, so we have:

$$\frac{1}{2}mv^2 - G\frac{Mm}{r} = 0 \quad (1)$$

and solving for  $v$ , this calculation gives the well known formula

$$v = \sqrt{\frac{2GM}{R}} \quad (2)$$

This equation is called the escape velocity, and the notation  $v_e$  is often used. If we now set  $v_e = c$  and solve the equation above with respect to  $R$ , we get

$$R = \frac{2GM}{c^2} \quad (3)$$

This equation is identical to the Schwarzschild radius but calculated from old Newton mechanics with no reliance on general relativity theory or the Schwarzschild solution. This finding is no big surprise because general relativity theory gives exactly the same escape velocity [6] formula as we have derived above from simple Newton mechanics. This similarity is also pointed out by Chandrasekhar, for example, [7]

*By a curious coincidence, the limit  $R_s$  discovered by Laplace is exactly the same that general relativity gives for the occurrence of the trapped surface around a spherical mass.*

Chandrasekhar and Hawking [8] have emphasized the similarity between Newton dark bodies and black holes, not only mathematically but also to a large degree in interpretation. On the other hand, for example, Loinger [9] claims “*The dark body of Michell-Laplace has nothing to do with the relativistic black hole.*” In several books on general relativity theory, the escape velocity even in the context of Schwarzschild black holes are derived from Newton mechanics. Further, some authors clearly point out that a more rigorous analysis (from general relativity theory) is needed for general relativity theory, but that it comes to the exact same equation [10] as the one derived from Newton mechanics. However, several researchers have correctly pointed out that the interpretation of the escape velocity from Newton and GR can be very different. To say that researchers agree on the interpretations of the “black hole mathematical framework” we think would be a mistake. In this paper, we will suggest that both general relativity theory and Newton mechanics both possibly are incomplete concerning their escape velocity formula and, therefore, likely also incomplete somewhere in their foundation. This hypothesis should naturally be discussed further before any final conclusions are made. However, already in this paper, we demonstrate good reasons to think that the standard escape velocity formula is incomplete.

When thinking in more depth about it, this idea seems a little strange that general relativity should give the same escape velocity as Newton mechanics because the Newton solution involves  $E_k \approx \frac{1}{2}mv^2$ , that we know is only a good approximation for kinetic energy when  $v \ll c$ . So we know the Newton solution cannot be correct for finding the radius where the escape velocity is  $v_e = c$ , even if it gives exactly the same mathematical end result as the Schwarzschild solution. However, we can ask how the Schwarzschild solution for general relativity theory can give exactly the same mathematical result as the Newton mechanical solution when we know that the Newton solution does not hold for  $v$  close to  $c$ . We will look closely at that point in this paper.

The small mass in the Newton formula should also be expected to be relativistic to incorporate Einstein’s relativistic kinetic energy. This point brings us to a discussion on relativistic mass. Already in 1899, Lorentz [11] among others, suggested that the mass of an object increased when it was moving, but that the effect was different for different directions relative to the observer. In 1903, Abraham [12] introduced the terms “*longitudinal* and *transverse* mass” for moving masses. Thomson [13] in 1904 also mentions that mass will increase as it moves, but that this effect would be directionally dependent. The correct relativistic mass formula was actually already given by Lorentz [14] in 1904, but he presented two formulas then, one for transverse relativistic mass  $m_T = m\gamma$ , and one for longitudinal mass  $m_L = m\gamma^3$ . The Lorentz transverse moving mass formula corresponded to what today is known as relativistic mass (for any direction). Einstein likely did not know about the Lorentz 1904 paper. In his [15] famous 1905 paper, he introduced a special relativity theory to derive formulas for relativistic mass, on which he is likely to have been incorrect. Einstein had derived the relativistic energy correctly and was the first to introduce this as

$$E = mc^2\gamma \quad (4)$$

By simply dividing by  $c^2$  on both sides, Einstein would have arrived at the correct relativistic mass. Instead, he followed the “speculative” tradition<sup>1</sup> laid out before him to try to perform separate derivations of longitudinal mass and transverse mass. In his 1905 paper on relativity theory, he gave the following relativistic mass results:

$$\text{longitudinal mass} = m\gamma^3 = \frac{m}{\sqrt{1 - \frac{v^2}{c^2}}} \quad (5)$$

This equation is the same longitudinal mass that Lorentz had suggested one year before, but without reference to Lorentz, so it is likely that Einstein was not aware of the paper written by Lorentz. For transverse mass, Einstein suggested

$$\text{transverse mass} = m\gamma^2 = \frac{m}{1 - \frac{v^2}{c^2}} \quad (6)$$

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<sup>1</sup>Many real ideas in theoretical physics started as speculative, almost per definition, as it can take many years to observe hard-to-observe phenomena and measure them accurately to decide if the speculative idea made sense or not.

This result is different from the Lorentz transverse mass, and the Einstein relativistic mass is known today as likely incorrect. None of Einstein's relativistic mass formulas correspond to the well-known relativistic mass as we know it today as given by (see, for example, [16–18])

$$m_r = m\gamma = \frac{m}{\sqrt{1 - \frac{v^2}{c^2}}} \quad (7)$$

This formula is often used with somewhat different notation, some researchers use the following notation

$$m = m_0\gamma = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}} \quad (8)$$

where  $m_0$  is the rest-mass. In 1908, Lewis [19] presented the relativistic mass formula for any direction as we know it today. It is the same as the equation 7 and 8.

Furthermore, in 1909, Lewis and Tolman [20] correctly derived the relativistic mass formula from mechanics. In 1912, Tolman [21] insisted that the relativistic mass given by the equation 8 was the right and relevant relativistic mass. In 1934 [22], Tolman argues that only the transverse mass  $m = m_o\gamma$  makes sense “*since this is the quantity that will give momentum multiplied by the velocity of the particle. It is the quantity that is conserved when particles interact by collision*”. This view was also held by Vereide [23], for example, that in 1921 claimed physicists now had agreed on that the relevant moving mass formula was  $m_o\gamma$  and that the suggestions by Bucherer and Abraham and others had been excluded since only this expression of moving mass seemed consistent with principles of conservation of moving masses (and momentum). This debate was, however, in no way fully settled by then (or now). Several prominent authors kept referring to Einstein's likely incorrect 1905 moving mass formulas. For example Wien [24] in 1921 refers to Einstein's transverse mass as  $\frac{m}{1 - \frac{v^2}{c^2}}$ , without

any critics of it. Wien is behind the *Wien's displacement law* as it is called after him. He is a very prominent physicist who got the Noble prize in 1911.

Einstein [25] in 1906 publishes a paper where he suggests an experimental method to distinguish between the different theoretical ideas of transverse and longitudinal mass by different researchers. He distinguishes between the theories of Bucherer and Abraham. He also mentions the theory of Lorentz and Einstein as though they were the same. This understanding is a bit strange because the Lorentz and Einstein theories were different with respect to moving masses, as pointed out above. As well, they were the same for longitudinal mass but differed for transverse mass. This 1906 paper does not seem to have led to much.

In 1907 Einstein [26] writes “*Thus, a system of moving mass points — taken as a whole — has the more inertia, the faster the mass points move relative to the other*”. Einstein here clearly indicates that at least the inertia of mass is relativistic.

Actually, Max Born in 1920 [27] was possibly the first person to coin the formula 8 *relativistic mass*. Lewis [28] in 1925 called the relativistic mass formula for the Lorentz mass formula, in other words correctly not the Einstein mass formula, as it was clearly Lorentz that had invented the relativistic mass formula many today uses.

Max Planck [29] had derived the relativistic momentum in 1906,  $p = mv\gamma$ , which is something Einstein [30] first mention in 1907 without reference to Planck. Standard theory today typically relies on relativistic momentum instead of relativistic mass, but the relativistic momentum is simply the relativistic mass multiplied by  $v$ . Actually, the standard momentum is likely not valid for the rest masses, see [31], and is partly why the four-vector approach is used in our view. The rest mass momentum should be zero since we have  $p = mv\gamma$  it is replaced with rest-mass energy divided by  $c$  to get the correct four-momentum when the mass is at rest, the so-called time component of the four-momentum. The time component is identical to what Haug recently coined Compton momentum when the particle is at rest, see [31]. The Compton momentum,  $p_t = mc\gamma$ , is well defined for any velocity from  $v$  to  $v < c$ , unlike the standard momentum that not is defined for  $v = 0$ , or one could try to argue it is then zero, but it is then replaced by rest-mass energy divided by  $c$  in the fourth momentum framework. However, an in-depth discussion on this is beyond the remit of this paper.

Many researchers have strongly criticized the use of relativistic mass as a mathematical artifact that should not be used. See for example [32–35]. For example, Adler has claimed:

*Anyone who has tried to teach special relativity using the four-vector space-time approach knows relativistic mass and four-vectors make for an ill-conceived marriage. In fact, most of the recent criticism of relativistic mass is presented in the context of the four-vector formulation of special relativity.*  
– Adler 1987

So, relativistic mass seems to mainly causing interpretation challenges due to Minkowski space-time (four-vector interpretation). Einstein had adopted Minkowski [36] space-time by 1922, and it seems he had abandoned the relativistic mass concept by then. Possibly, he was also happy to do so too since Lorentz had the correct relativistic mass as early as 1904, one year before he published his own relativity theory. Einstein also got the relativistic mass likely wrong, and other relativity theories, like that of Lorentz, were still considered competitors of special relativity theory. Still, it would seem a little strange that we cannot divide two sides of an equation

with a constant that is already in the formula, namely to divide the relativistic mass formula of Einstein with  $c^2$ , and call the result relativistic mass. Is it forbidden to divide both sides of his formula with a constant that already is there? In a letter to Lincoln Barnett, an American journalist, dated 19 June 1948, Einstein wrote,

*It is not good to introduce the concept of the mass  $M = m/\sqrt{1 - \frac{v^2}{c^2}}$  of a moving body for which no clear definition can be given. It is better to introduce no other mass concept than the “rest mass”,  $m$ . Instead of introducing  $M$ , it is better to mention the expression for the momentum and energy of a body in motion.*

This claim by Einstein has fueled critics of the relativistic mass concept. See, for example Hecht [34]. However, the arguments against the use of relativistic mass seem rather weak. Perhaps critics should instead take a closer look at the Minkowski space-time concept that, for example, as it is potentially inconsistent with quantum mechanics [37]. Also, a close look at the corresponding four-momentum and how the rest mass “momentum” suddenly is energy divided by  $c$  is worth thinking about, see [31].

Actually, we think that both the abandonment of the relativistic mass and the interpretation of special relativity in the form of Minkowski space-time (four vectors) was a mistake that has slowed the progress in physics for many years. We have recently shown how a modified relativistic theory can be unified with gravity theory and that quantum mechanics only is consistent with a 3-dimensional space-time (three time-dimensions and three space-dimensions), [38]. Inside this framework, relativistic mass leads to no conceptual problems as it possibly does in Minkowski space-time, but then we are also working with a more complete mass definition. We like to call this 3-dimensional space-time, as the space and time dimensions are just two faces of the same coin. That is, in our model, we cannot move, for example, only along the  $x$  axis in space and at the same time along the  $y$  axis in time, when only moving along the  $x$  axis in time, we can only move along the  $x$  axis of time  $t_x$ , see the last section before the conclusion.

Adler in 1987 also claimed:

*It should also be pointed out that there is no reason to introduce relativistic mass in general relativity theory.*

This view possibly explains to a greater extent why there is no relativistic mass in today’s gravity theory. Also, prominent figures in gravity like Taylor and Wheeler [35] have been speaking out against relativistic mass. A series of well-known specialists on general relativity theory have largely ignored the investigation on what a gravity theory incorporating relativistic mass would look like in terms of predictions relative to observations. We have [39] recently shown, for example, that by introducing relativistic mass in Newtonian gravity, we can explain supernova data without the need of the dark energy hypothesis, this should naturally be carefully studied before accepted, but it should also not be excluded based on prejudice.

Other prominent physicists such as Rindler [40, 41] who have worked for much of his career with relativity theory have defended the use of relativistic mass, criticizing the critics of it, see also [42] who seem to be positive in terms of its use. Here, we will go back and claim that relativistic mass is essential and a part of relativity theory. It goes hand in hand with relativistic energy. It must be introduced in all parts of physics, including gravity. We can start by deriving the relativistic escape velocity from relativistic modified Newton theory, and this must be given by solving the following equation:

$$\begin{aligned} E_k - G \frac{Mm\gamma}{R} &= 0 \\ mc^2\gamma - mc^2 - G \frac{Mm\gamma}{R} &= 0 \end{aligned} \quad (9)$$

We use Einstein’s relativistic kinetic energy, but we also must ensure that the small mass is relativistic in the Newton gravity formula. That is, we need relativistic mass in gravity theory. Solved with respect to  $v$ , this result gives:

$$v_e = \sqrt{\frac{2GM}{R} - \frac{G^2M^2}{c^2R^2}} \quad (10)$$

If we set  $v_e = c$  and solve for  $R$  we get:

$$R = \frac{GM}{c^2} \quad (11)$$

We can call this the corrected Schwarzschild radius. However, it is not derived from general relativity theory or the Schwarzschild metric, so humble as we are, we can call it Haug radius and use notation  $R_{h,s} = \frac{GM}{c^2}$  (not to be confused with the Hubble radius that we later will use the notation  $R_H$  for.) because we have been the first to show this derivation, see [31, 43]. So, it is simply the radius where the velocity in kinetic energy from a small mass offsets the gravitational energy when the velocity is  $v = c$ . The Schwarzschild radius is the double of this radius, or for a fixed radius, the mass alternatively has to be twice in our model.

The relativistic ad-hock adjustment of the Newton formula:  $F = G \frac{Mm\gamma}{R^2}$  was actually suggested in 1981 and 1986 by Bagge [44] and Phipps [45]. However, Peters [46] showed that it only predicted half of the observed Mercury precession, so the idea of using relativistic modified Newtonian mechanics was basically abandoned and not fully investigated. However, recently, Corda [47] claims that we can get the correct Mercury precession if we take the relativistic effect into account, as well as consider the Mercury and the Sun as a real two-body problem, so there is much in favor of adding relativistic masses to the Newton framework to see what it can explain, see also [48]. Also, as mentioned above, adding relativistic effects to masses in the right way means we can predict supernova data without the need for the dark energy hypothesis.

It is worth mentioning that in the case  $v_e = c$  where we replace  $R$  with  $\frac{GM}{c^2}$  as it must have this value at this escape velocity, then we end up with

$$c = \sqrt{\frac{2GM}{R} - \frac{G^2 M^2}{c^2 \left(\frac{GM}{c^2}\right)^2}} = \sqrt{\frac{GM}{R}} \quad (12)$$

Our escape velocity in the special case when it is equal to  $c$  is identical to the standard orbital velocity. Therefore, one possible interpretation is that a mass with a radius where the escape velocity is  $c$  perhaps means it has a spherical boundary of light — a light wall (fire wall). This phenomenon would mean no information can likely pass through this wall. As we will see, we perhaps live inside a gigantic sphere with a light shell, known as the Hubble sphere. However, it could be that relativistic adjustments are needed to the orbital velocity, so they are not really the same. This area should be investigated further. More likely, we think much based on the findings we soon will come to that the universe is infinite, but that each point in the universe maximum can be interacted over time by a distance given by the Hubble radius. In other words, also our model gives an information horizon, as we will see.

## 2 The Mass Density and Mass of the Critical Universe

For example, Weinberg [49] in 1972 gives<sup>2</sup> the critical mass density of the observable universe as

$$\rho_c = \frac{3H_0^2}{8\pi G} \quad (13)$$

where  $H_0$  is the Hubble constant, and  $\rho_c$  is the critical mass density. Here, the critical mass density is when the cosmological constant  $\Lambda$  is set as equal to zero as it is for all basic Friedman universes [51]. This point means that before we can introduce such “matters” as dark energy. For refreshment, the Friedmann equation (one of the two) is given by

$$\frac{\dot{a}^2 + kc^2}{a^2} = \frac{8\pi G\rho + \Lambda c^2}{3} \quad (14)$$

by setting the  $\Lambda$  and  $k$  both equal to zero and solve with respect to  $\rho$  we get the well known critical mass density equation 13. The mass in a sphere with this mass density  $\rho_c$  is then given by:

$$M_c = \rho_c V = \rho_c \frac{4}{3}\pi R_o^3 = \frac{3H_0^2}{8\pi G} \frac{4}{3}\pi R^3 \quad (15)$$

Furthermore, if we set the radius of the observable (critical) universe equal to the Hubble radius  $R = R_H = \frac{c}{H_0}$ , we can re-write and simplify the equation above as:

$$M_c = \frac{3H_0^2}{8\pi G} \frac{4}{3}\pi \left(\frac{c}{H_0}\right)^3 = \frac{1}{2} \frac{c^3}{GH_0} \quad (16)$$

Another less known way to derive the critical mass density is from the escape velocity by replacing  $M$  with a mass density of a sphere. The mass can be described as the mass density multiplied by the volume of the sphere that contains this mass density. That is, we have  $M = \rho \frac{4}{3}\pi R^3$ , now replacing  $M$  with this in the GR escape velocity formula (or Newton escape formula), we get

$$v_e = \sqrt{\frac{2G\rho \frac{4}{3}\pi R^3}{R}} = R\sqrt{2G\rho \frac{4}{3}\pi} \quad (17)$$

Next, remember the Hubble radius is given by  $R_H = \frac{c}{H_0}$ , so if the escape velocity is  $c$  and we divide it by  $R$  on both sides of the equation and set  $R$  equal to the Hubble radius, we get

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<sup>2</sup>Page 476. See also [50].

$$\begin{aligned}
c &= R_H \sqrt{2G\rho \frac{4}{3}\pi} \\
\frac{c^2}{R_H^2} &= 2G\rho \frac{4}{3}\pi
\end{aligned} \tag{18}$$

next we solve the equation above with respect to  $\rho$  and get

$$\rho = \frac{3H_0^2}{8\pi G} \tag{19}$$

Which is the same as the critical mass we got from the Friedmann equation. Even if this is a less known way to find the critical density, it is described in some sources, see for example [52].

Using our new relativistic escape velocity  $v_e = \sqrt{\frac{2GM}{R} - \frac{G^2 M^2}{c^2 R^2}}$ , to do the same, we instead end up with a critical mass density off

$$\rho_{h,c} = \frac{3H_0^2}{4\pi G} \tag{20}$$

The critical mass density predicted by our model is twice that of the critical mass density of the Friedmann equation. The mass of the critical universe based on our full relativistic escape velocity must therefore be

$$M_{h,c} = \frac{3H_0^2}{4\pi G} \frac{4}{3}\pi \left(\frac{c}{H_0}\right)^3 = \frac{c^3}{GH_0} \tag{21}$$

This result is twice the mass in the universe given by the Friedman model. A series of researchers have pointed out that there seems to be missing about 50% of the baryonic matter, see for example [53–55], in addition, one have the missing dark matter, that is, one has not detected the dark matter yet. We also have the so-called flatness problem that potentially also could have a new explanation. If our alternative model also could be related to an explanation to any of these partly outstanding questions should be worth investigating further. Uncertainty in the Hubble constant also lead to some uncertainty in our mass density, Soltis et al. [56] measured the Hubble constant to  $H_0 = 72 \pm 2$  (km/s)/Mpc, while for example Mukherjee et al. [57] in 2020 measured the Hubble constant to be  $67.6 \pm 4.2$  (km/s)/Mpc, so there is a considerably uncertainty in this parameter still, see also [58–61]. For example, the predicted mass with Hubble constant 67 would be approximately  $1.85 \times 10^{53}$  kg. In comparison, a Hubble constant of 72 would mean a mass of approximately  $1.72 \times 10^{53}$  kg, but more importantly, always twice the expected mass as the Friedmann model for the critical universe.

### 3 Deriving the Friedmann equation and also an alternative equation for the universe from escape velocity

The Friedmann equation for a critical universe is given by (by setting the  $\Lambda$  and  $k$  both equal to zero)

$$\frac{\dot{a}^2}{a^2} = H_0^2 = \frac{8\pi G\rho}{3} \tag{22}$$

That is in a flat universe with no expansion. This Friedmann equation was originally derived from the Einstein field equation. Also, the standard escape velocity formula is derived from the general relativity theory. What is not widely known to our knowledge is that the Friedmann equation can be found from the escape velocity formula when setting  $v_e = c$ , and the radius equal to the Hubble radius  $R = R_H$ , from this we get

$$\begin{aligned}
v_e &= \sqrt{\frac{2GM}{R}} \\
c &= \sqrt{\frac{2GM}{R_H}} \\
c &= \sqrt{\frac{2G4\pi \frac{M}{\frac{4}{3}\pi R_H^3} R_H^2}{3}} \\
c &= \sqrt{\frac{8\pi G\rho R_H^2}{3}} \\
c^2 &= \frac{8\pi G\rho R_H^2}{3} \\
\frac{c^2}{R_H^2} &= \frac{8\pi G\rho}{3}
\end{aligned} \tag{23}$$



And since  $H_0 = \frac{c}{R_H}$ , we get

$$H_0^2 = \frac{8\pi G\rho}{3} \quad (24)$$

which is the well-known Friedmann equation for the flat universe (critical universe). We see this way to derive the Friedmann equation for the critical universe is fully consistent mathematically with the more standard way of deriving it from Einstein's field equation. This result is not a big surprise since both the Friedmann equation and the escape velocity formula given above can be derived from general relativity theory. So also deriving it from the escape velocity can be seen as another to derive it from Einstein's field equation. However, this way of deriving the Friedmann equation for the critical universe makes it easy to see that exactly the same solution can be derived from standard Newton mechanics. It has the same escape velocity as the general relativity theory. Still, we know the Newton solution involves deriving it from a kinetic energy approximation and non-relativistic mass assumption that only can be valid for  $v \ll c$ , namely  $\frac{1}{2}mv^2 - G\frac{Mm}{R} = 0$ . With non-relativistic mass, we are pointing to that the mass in the Newton gravity force formula has no relativistic adjustments, despite  $m$  must be moving close to  $c$  or even at  $c$  when  $v = c$  (escape velocity equal to the speed of light). So how can it be that the Friedmann solution gives exactly the same result for a critical universe as a solution from standard non-relativistic Newton mechanics that we know not can hold if taking into account relativistic effects when  $v$  is significant to that of the speed of light. We think the reason perhaps is that Einstein ignored relativistic mass (see section 1), something that naturally should be studied more carefully before any final conclusion is made.

Einstein's field equation gives much of the same results as Newton in a weak gravitational field. The Hubble sphere is a very interesting case. Or, in more general terms, supermassive black holes are interesting here. The Hubble sphere has the mathematical properties of one. The gravitational acceleration for supermassive black holes and the Hubble sphere is extremely weak at the Hubble radius, equal to the Schwarzschild radius. For example, if the Hubble constant is 70, then the gravitational acceleration at the Hubble radius is only  $3.40 \times 10^{-10} \text{ m/s}^2$  in general relativity theory and twice of that in our theory. This result is very small compared to gravitational acceleration, for example, at the Earth's surface, which is about  $9.8 \text{ m/s}^2$ , which even is a weak gravitational field. The escape velocity is also insignificant to the speed of light at the Earth's surface; however, the escape velocity at the Hubble radius is  $c$  in the critical universe. The Hubble sphere and any super large Schwarzschild sphere have properties of a weak gravitational field (the gravitational acceleration) and at the same time properties where relativistic effects should be of great importance, namely the escape velocity.

An interesting question is what type of equation similar to Friedmann will we get when to derive it from the relativistic escape velocity given in section 1 (Eq. 10), by simply replacing  $R$  with the Hubble radius that must be identical to the radius where the escape velocity is  $c$  we get

$$\begin{aligned} v_e &= \sqrt{\frac{2GM}{R} - \frac{G^2 M^2}{c^2 R^2}} \\ c &= \sqrt{\frac{2GM_c}{R_H} - \frac{G^2 M_c^2}{c^2 R_H^2}} \\ c &= \sqrt{\frac{8\pi G\rho R_H^2}{3} - \frac{16\pi^2 G^2 \rho R_H^6}{9c^2 R_H^2}} \\ c &= \sqrt{\frac{8\pi G \frac{M}{\frac{4}{3}\pi R_H^3} R_H^2}{3} - \frac{\frac{16}{9}\pi^2 G^2 \frac{M^2}{\frac{16}{9}\pi^2 R^6} R^6}{c^2 R_h^2}} \\ c &= \sqrt{\frac{8\pi G\rho R_H^2}{3} - \frac{16\pi^2 G^2 \rho R_H^4}{9c^2}} \\ c^2 &= \frac{8\pi G\rho R_H^2}{3} - \frac{16\pi^2 G^2 \rho R_H^4}{9c^2} \\ c^2 &= \frac{8\pi G\rho R_H^2}{3} - \frac{16\pi^2 G^2 \rho R_H^4}{9c^2} \\ \frac{c^2}{R_H^2} &= \frac{8\pi G\rho}{3} - \frac{16\pi^2 G^2 \rho R_H^2}{9c^2} \\ H_0^2 &= \frac{8\pi G\rho}{3} - \frac{16\pi^2 G^2 \rho^2}{9H_0^2} \end{aligned} \quad (25)$$

This can be simplified further to

$$H_0^2 = \frac{4\pi G\rho}{3} \quad (26)$$



The last simplification is perhaps easiest seen by study section 2 (equation 13), where we know that when  $v_e = c$  we can simplify our escape velocity formula  $v_e = c = \sqrt{\frac{2GM}{R} - \frac{G^2 M^2}{c^2 R^2}} = \sqrt{\frac{GM}{R}}$ , and therefore we have

$$\begin{aligned} c &= \sqrt{\frac{GM_{c,h}}{R_H}} \\ c &= \sqrt{\frac{4\pi G \rho R_H^2}{3}} \\ \frac{c^2}{R_H} &= \frac{4\pi G \rho}{3} \\ H_0^2 &= \frac{4\pi G \rho}{3} \end{aligned} \quad (27)$$

This result can be seen as an alternative to the Friedman critical universe equation (and therefore even to general relativity theory). The equation above is not from Newton's theory because Newton gives the same escape velocity as general relativity and the same universe solution as the Friedmann solution for a critical universe. The equation above is from relativistic adjusted Newtonian theory, where we take into account relativistic kinetic energy and relativistic mass. Our result should first be carefully investigated at this stage, for example, the implications that it predicts. As we can measure the Hubble constant, we can solve for  $\rho$ . This procedure gives

$$\rho_{h,c} = \rho = \frac{3H_0^2}{4\pi G} \quad (28)$$

This result is naturally the same result we got in equation 20. Again we highlight that this result predicts that the mass and mass density inside the Hubble sphere without any inflation is twice that of what is predicted by the Friedmann equation. Both equations have the same Hubble radius, and in both models, the Hubble radius is the radius where the escape velocity is  $c$ . But our new theory gives an escape velocity of  $c$  at a radius equal to  $R_{h,s} = \frac{GM}{c^2}$  while general relativity predicts that this is at the Schwarzschild radius  $R_s = \frac{2GM}{c^2}$ . This result can only be true if the mass density in our model is twice that in the Friedmann solution, which is the case as we have demonstrated to be mathematically consistent with our theory.

From our new universe equation, we can understand that the so-called scaling parameter in the Friedmann equation, likely in reality, only represents how the redshift is a function of how far we are from the Hubble sphere radius. The Hubble sphere radius is likely to represent an information horizon where we not can get any information from beyond it. That is about all. We think the interpretation that it means the universe is expanding could be incorrect.

## 4 More Formal derivation of the Friedmann equation from Newton mechanics and the Haug equation from relativistic modified Newton theory

It is well known the Friedmann equation also can be derived from Newton mechanics, this likely because general relativity theory is considered to be Newtonian in the weak field limit. The Friedmann equation can from Newtonian mechanics be derived as

$$U = T + V = \frac{1}{2}m\dot{R}^2 - \frac{GMm}{R} = \frac{1}{2}m\dot{R}^2 - \frac{4\pi}{3}G\rho R^2 m \quad (29)$$

Assume  $\vec{R}(t) = a(t)\vec{x}$ , and substitute this in the equation above and we get

$$U = \frac{1}{2}m\dot{a}^2 x^2 - \frac{4\pi}{3}G\rho a^2 x^2 m \quad (30)$$

we can re-arrange this and we get

$$\begin{aligned} \frac{\dot{a}^2}{a^2} &= \frac{8\pi}{3}G\rho - \frac{kc^2}{a^2} \\ H_0^2 &= \frac{8\pi}{3}G\rho - \frac{kc^2}{a^2} \end{aligned} \quad (31)$$

where  $kc^2 = -\frac{U}{x^2 m}$ . The equation above is the Friedmann equation including the constant  $k$ . When setting  $k = 0$  we get the critical universe solution that we also got from the escape velocity formula.

If we take into account relativistic energy as well as relativistic mass we get

$$\begin{aligned}
U &= T + V = m\dot{R}^2\gamma - m\dot{R}^2 - \frac{GMm\gamma}{R} = m\dot{R}^2\gamma - m\dot{R}^2 - \frac{4\pi}{3}G\rho R^2m\gamma \\
U &= m\gamma\dot{a}^2x^2 - m\dot{a}^2x^2 - \frac{4\pi}{3}G\rho a^2x^2m\gamma \\
\gamma\frac{\dot{a}^2}{a^2} - \frac{\dot{a}^2}{a^2} &= \frac{4\pi}{3}G\rho\gamma - \frac{kc^2}{a^2}
\end{aligned} \tag{32}$$

where  $\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$  and  $v$  is the velocity of  $m$ , and where  $kc^2 = -\frac{U}{x^2m}$ . Divide by  $\gamma$  on both sides and we get

$$\frac{\dot{a}^2}{a^2} - \frac{\dot{a}^2}{a^2}\sqrt{1-\frac{v^2}{c^2}} = \frac{4\pi}{3}G\rho - \frac{kc^2}{a^2}\sqrt{1-\frac{v^2}{c^2}} \tag{33}$$

when  $v_e = v = c$ , that is when we are at the Hubble radius, we get

$$\begin{aligned}
\frac{\dot{a}^2}{a^2} &= \frac{4\pi G\rho}{3} \\
H_0^2 &= \frac{4\pi G\rho}{3}
\end{aligned} \tag{34}$$

This is the same solution we got from the escape velocity formula. That is unlike in the Friedmann solution where it in the formula is an open question what to set  $k$  to when solved this way. In our full relativistic framework it seems we have no choice other than that the constant  $k$  has no impact on our universe when our theory is linked to the Hubble scale. This proves the universe not can be expanding in a full relativistic Newton mechanics. The Big Bang theory is likely just a hypothesis that now have got competition.

## 5 Any mass density above zero in a large area of space always has a Schwarzschild radius (sphere)

That our observable universe is the interior of a gigantic black hole has been suggested already in 1972 by [62], that suggested “*the universe may not only be a closed structure (as perceived by its inhabitants at the present epoch) but may also be a black hole, confined to a localized region of space which cannot expand without limit.*”. Several others have also published about this possibility [63–65], for example, Kip Thorn has said we have enough mass inside a sphere somewhat larger than 10 billion light-years out for this to be the case and that we therefore theoretically could live inside a black hole or what he calls a reverse black hole (white hole), but that he still thinks this is rather improbable [66]. This idea that we live inside the sphere with mathematical properties equal or similar to a black hole we think is still too early to be excluded. However, our theories about the interior of such “structures” have been very limited. We would say speculative, and are often based on only interpretation from general relativity theory despite spheres with escape velocity  $c$  can also be predicted from other types of modified gravity theories. In our theory, we get different results, and it seems very different interpretations of how such structures are linked to the observable universe and the Planck scale.

Assume a very large area of the universe or even an infinite universe with a given average density. The mass density in the surface or center of the Earth is clearly much higher than the mass density at the mid-point between Earth and the Moon, for example, known as outer space. However, inside an enormous space volume covering millions of galaxies, we can calculate an average density that is basically the same if we split that large volume in two or even ten. Basically, this is the cosmological principle; that is empirically justified on scales larger than 100 Mpc. For a given universe mass density, even if it is very small, the mass will increase as a function of the volume at which we look. If we look at the volume inside a sphere shape, the mass for a given density will increase by  $R^3$  as we increase the radius. This occurrence is naturally because the volume of a sphere is  $V = \frac{4}{3}\pi R^3$ . On the other hand, the Schwarzschild radius  $R_s$  is a linear function of  $M$ , which means any large space area with a given mass density must have a Schwarzschild radius (and a Haug radius), something we will look at in detail here. One can try to argue that gravity bends space, so that we must go beyond sphere shapes to discuss this. However, in a critical universe, even from general relativity theory, the Friedmann solution for the critical universe, we are still operating in Euclidean geometry.

The mass of a given density for a given sphere filled with that mass density is given by:

$$M = \rho \frac{4}{3}\pi R^3 \tag{35}$$

The escape velocity for a sphere filled with a given density of mass is given by:

$$\begin{aligned}
v_e &= \sqrt{\frac{2GM}{R}} \\
v_e &= \sqrt{\frac{2G\rho\frac{4}{3}\pi R^3}{R}}
\end{aligned} \tag{36}$$

The escape velocity of  $c$  is the maximum escape velocity, if we set  $v_e = c$ , and, at the same time, keep the mass density  $\rho$  as a constant and solve this with respect to  $R$  to get the radius of a sphere with a given mass density that must be to have a Schwarzschild radius. This is given by:

$$\begin{aligned}
v_e = c &= \sqrt{\frac{2G\rho\frac{4}{3}\pi R_s^3}{R_s}} \\
R_s^2 &= \frac{c^2}{2G\rho\frac{4}{3}\pi} \\
R_s &= \sqrt{\frac{c^2}{2G\rho\frac{4}{3}\pi}}
\end{aligned} \tag{37}$$

If we input  $\rho$  equal to the critical mass density of the observable universe, we get:

$$R_s = \sqrt{\frac{c^2}{2G\frac{3H_0^2}{8\pi G}\frac{4}{3}\pi}} = \sqrt{\frac{c^2}{H_0^2}} = \frac{c}{H_0} \tag{38}$$

This equation is the Schwarzschild radius of a universe with a mass density equal to the critical mass density, which is equal to the Hubble radius. The Haug radius based on relativistic mass adjustments is  $R_{h,s} = \frac{GM}{c^2}$ , but this will also be identical to the Hubble radius, as the mass density in our model is twice that of the standard model.

$$\begin{aligned}
v_e = c &= \sqrt{\frac{2GM_{h,u}}{R} - \frac{G^2M_{h,u}^2}{c^4R^2}} \\
c &= \sqrt{\frac{2G\frac{c^3}{GH_0}}{R} - \frac{G^2\left(\frac{c^3}{GH_0}\right)^2}{c^2R^2}} \\
c &= \sqrt{\frac{2c^3}{H_0R} - \frac{c^4}{R^2H_0^2}}
\end{aligned} \tag{39}$$

This gives  $R = R_{h,s} = \frac{GM_{h,c}}{c^2} = \frac{c}{H_0}$ , which is identical to the Hubble radius for the Hubble sphere. We also have that  $R_s = \frac{2GM_u}{c^2} = \frac{c}{H_0}$  from general relativity that is consistent with the Friedmann solution and the Schwarzschild solution. Since  $H_0$  is empirically observed, this point can only be true if the predicted mass inside the Hubble sphere is twice in our new model as in the Friedmann model. This result also explains why the Hubble radius is so special. One possible interpretation is that every point in an infinite universe can only be reached by light (information) that comes at the Hubble radius distance. Each point in space has an information horizon.

## 6 Half the Schwarzschild time is the maximum acceleration time for the Schwarzschild radius acceleration

What we can call the Schwarzschild time is simply the Schwarzschild radius divided by the speed of light. In other words, the time it takes for light to travel a distance equal to the Schwarzschild radius is

$$T_s = \frac{R_s}{c} = \frac{2GM}{c^3} \tag{40}$$

Interestingly the Hubble constant is also one divided by the Schwarzschild time, that is

$$H_0 = \frac{1}{T_s} = \frac{1}{\frac{2GM_u}{c^3}} = \frac{c}{R_s} = \frac{c}{R_H} \tag{41}$$

That is, the Hubble constant can also possibly be seen as a frequency. At the Schwarzschild radius, the gravitational acceleration field strength is given by

$$g = \frac{GM_c}{R_s^2} = \frac{GM_c}{R_H^2} \quad (42)$$

This phenomenon we can call the Schwarzschild acceleration. Interestingly it takes twice the Schwarzschild time, which is identical to twice the Hubble time to accelerate a particle from zero to  $c$  at this acceleration. That is, we have

$$c = 2T_s a_s = \frac{2}{H_0} a_s = 2T_s \frac{GM}{R_s^2} = 2 \frac{2GM}{c^3} \frac{GM}{4G^2 M^2 / c^4} \quad (43)$$

It takes twice the Hubble time to accelerate to the speed of light can easily lead one to the incorrect conclusion that space must expand. When using the Haug radius that is calculated from the full-relativistic escape velocity, then the Haug acceleration is given by

$$g = \frac{GM_{h,c}}{R_{h,s}^2} = \frac{GM_{h,c}}{R_H^2} \quad (44)$$

It takes the Haug time  $T_{h,s} = \frac{R_{h,s}}{c} = \frac{GM}{c^3} = \frac{R_H}{c}$ , at the Haug acceleration to accelerate a particle from zero to  $c$ , that is we have

$$T_{h,s} a_{h,s} = \frac{1}{H_0} a_{h,s} = T_{h,s} \frac{GM_{h,u}}{R_{h,s}^2} = \frac{GM_{h,u}}{c^3} \frac{GM_{h,u}}{G^2 M_{h,u}^2 / c^4} = c \quad (45)$$

This procedure means our new model predicts that a mass located at or very close to the corrected Hubble radius leaving the Hubble radius will be reaching speed  $c$  in the Hubble time (Hubble age of the universe). This result perhaps indicates that each point in an infinite universe only can be affected by information at the Hubble distance apart. Please pay close attention to our new model that one gets from zero to speed  $c$  at the Hubble time, while in the standard model, this takes twice the Hubble time. In our view, the standard model is likely incomplete. That a particle not can reach  $c$  at this acceleration at the Hubble time has likely been misinterpreted to think the universe is expanding. Our new model does not seem to need any expansion of space and still be internally consistent. However, these theories require further investigation, and we encourage other researchers to look into this.

The cosmological redshift has been interpreted as space is expanding at the following velocity (the Hubble flow where  $D \ll R_H$ )

$$v_H = H_0 D \quad (46)$$

This again is linked to cosmological red-shift by

$$z \approx \frac{v}{c} \quad (47)$$

However we can also re-write this as

$$z \approx \frac{v}{c} = \frac{R}{M_{h,c} c} = \frac{D}{R_s} = \frac{D \bar{\lambda}_H}{l_p^2} = \frac{D c^2}{GM_{h,c}} = \frac{1}{\frac{GM_{h,c}}{D c^2}} \quad (48)$$

where  $M_{h,c}$  is here the Haug critical mass. So, one possibility is that the observed cosmological redshift is not related to expanding space. Some will possibly recognize the denominator as the formula for gravitational redshift. But it will be a type of inverse gravitational redshift as observed from inside the Hubble sphere.

## 7 Similarity challenges between the Hubble scale and the Planck scale in general relativity theory

While Max Planck introduced the Planck mass (and the Planck length and Planck time) already in 1899 and thought the Planck mass was important, he was not very clear on what it could represent in reality, could it, for example, be linked to a particle? Loyd Motz [67, 68] while working at Rutherford laboratory, was possibly the first to suggest the Planck mass could be linked to a particle, the Planck mass particle or Uniton, as he coined it. Motz naturally understood that the Planck mass, approximately  $2.17 \times 10^{-8}$  kg, was enormous in mass compared to any particle observed, so he suggested that such particle only had existed just after the big bang and then radiated most of its energy into today's observed particles, such as the electron and the proton. Some years later, both he [69] and Hawking [70] suggested that the Planck mass instead could be a micro black hole, something that we even could call a Planck mass particle. Again, a black hole is defined as something where the mass is enclosed inside a small enough radius to make the escape velocity at this radius to be exactly  $c$ . The mass that gives an escape velocity radius equal to the Planck length is half the Planck mass in general

relativity theory (at least under the Schwarzschild solution). In our new theory the Planck mass gives an escape velocity radius (where  $v_e = c$ ) equal to the Planck length. In the general relativity theory, the Planck mass does not seem that unique. The Planck mass particle has a reduced Compton wavelength equal to the Planck length  $\bar{\lambda} = \frac{h}{m_p c} = l_p$ , but the escape velocity at this radius for the Planck mass is above  $c$  in general relativity theory, it is namely  $v_e = \sqrt{\frac{Gm_p}{l_p}} = c\sqrt{2}$ . This point seems to make it impossible for the Planck mass particle to exist under general relativity theory or get as close to it as the Planck length or the reduced Compton wavelength of the particle. Hawking [70] likely purposely indicated that the micro black hole is only approximately equal to the Planck mass. He could possibly see that general relativity theory did not perfectly match the Planck mass's mathematical properties. Motz and Eppstein [69] suggested the micro black hole to have half the Planck mass. This likely because half the Planck mass has the properties of a black hole at a distance equal to the Planck length in general relativity theory and not the Planck mass.

Papers discussing micro black holes are often diffuse on the exact mass of this object. They say it is close to the Planck mass, but clearly, it seems the standard theory does not make the Planck mass unique. In our theory, the escape velocity at the Planck length for the Planck mass is exactly  $c$ . That is, we have

$$v_e = \sqrt{\frac{Gm_p}{l_p} - \frac{G^2 m_p^2}{c^2 l_p^2}} = c \quad (49)$$

We find the Hubble sphere's mass density to be half in the standard theory to what we find when using our new relativistic escape velocity formula. We can also find the Hubble equivalent constant of a Planck mass particle sphere. We just do similar to when we derived our Hubble sphere equation, we set  $R = l_p$  and  $v_e = c$ , first in our new escape velocity formula and get

$$H_{0,p} = \frac{4\pi c^3}{3G\rho_p} = \frac{1}{t_p} = \frac{c}{l_p} \quad (50)$$

so the Hubble equivalent constant for the Planck particle sphere is the Planck frequency. Based on the standard escape velocity from general relativity theory one get

$$H_{0,p} = \frac{8\pi c^3}{3G\rho_p} = \frac{1}{\frac{1}{2}t_p} = \frac{2c}{l_p} \quad (51)$$

So it is here twice the Planck frequency which again is likely impossible based on the idea that the Planck length is the minimum length and the maximum speed of light is  $c$ , so the maximum frequency should be  $\frac{c}{l_p}$ . Alternatively, one must indeed introduce the expansion of space to make the formula to make sense. A third alternative is not to have the Planck mass density but half the Planck mass density inside the Planck sphere. General relativity, in our view, gives strange predictions when approaching the Planck scale. This concern is, in our view, much of the same issues as one has with the Hubble sphere in general relativity theory. However, it is more easily seen theoretically at the Planck scale because one sees that the Planck mass cannot be such a unique object in general relativity theory. In general relativity theory, the Planck mass needs alterations to fit the formula, either one must go away from the Planck length as unique, or one must go away from maximum speed is  $c$  (and introduce expansion) and or one must reduce the unique mass to half the Planck mass. Our new theory lines perfectly up at the same time with the Planck mass, the Planck length, and the Planck time. Our theory seems to make full sense from the Planck scale to the Hubble sphere. Table 1 summarize similarities between the Planck scale issues and Hubble scale issues in general relativity theory. Table 2

Escape velocity	$v_e = \sqrt{\frac{2GM}{R}}$ GR	$v_e = \sqrt{\frac{2GM}{R}}$ GR
Radius where $v_e = c$	$R_s = \frac{2GM}{c^2}$ GR	$R_s = \frac{2GM}{c^2}$ GR
	Planck mass	Half Planck mass
Mass	$m_p = \sqrt{\frac{\hbar c}{G}}$	$\frac{1}{2}m_p = \frac{1}{2}\sqrt{\frac{\hbar c}{G}}$
Schwarzschild radius	$R_s = 2l_p$	$R_s = l_p$
Gravitational acceleration in Planck time	$g_s t_p = \frac{1}{4}c$	$g_s t_p = \frac{1}{2}c$
Gravitational acceleration in Schwarzschild time	$g t_s = \frac{1}{2}c$	$g_s t_s = \frac{1}{2}c$
Escape velocity	$v_e = c\sqrt{2}$	$v_e = c$
Planck scale equation	$\frac{1}{t_p^2} = H_p^2 = \frac{32\pi G\rho_{p,s}}{3}$	$\frac{1}{t_p^2} = H_p^2 = \frac{8\pi G\rho_{p,s}}{3}$
	$\rho_{p,s} = \frac{m_p}{\frac{4}{3}\pi(2l_p)^3}$	$\rho_{p,s} = \frac{\frac{1}{2}m_p}{\frac{4}{3}\pi l_p^3}$
Conclusion:	Do not match Planck scale	Do not match Planck scale
Possible (but “absurd”) solution	Expanding space?	Expanding space?
Solution 2	GR incomplete	GR Incomplete
	Universe mass	Universe mass
	Alternative	Standard theory:
Mass	$M_c = \frac{c^3}{GH_0}$	$M_c = \frac{1}{2}\frac{c^3}{GH_0}$
Schwarzschild radius	$R_s = R_H$	$R_s = \frac{c}{H_0} = R_H$
Gravitational acceleration in Hubble time at $R_H$	$g_H t_H = c$	$g_H t_H = \frac{1}{2}c$
Gravitational acceleration in Schwarzschild time at $R_H$	$g_H t_s = c$	$g_H t_s = \frac{1}{2}c$
Gravitational acceleration in Hubble time at $R_s$	$g_s t_H = \frac{1}{4}c$	$g_s t_H = \frac{1}{2}c$
Gravitational acceleration in Schwarzschild time at $R_s$	$g_s t_s = \frac{1}{2}c$	$g_s t_s = \frac{1}{2}c$
Escape velocity at $R_s$	$v_e = c\sqrt{2}$	$v_e = c$
Universe equation	$\frac{1}{\frac{R_H^2}{c^2}} = \frac{1}{t_H^2} = H_0^2 = \frac{32\pi G\rho}{3}$	$\frac{1}{t_H^2} = H_0^2 = \frac{8\pi G\rho}{3}$
		Friedmann critical universe
Conclusion	Do not match up	Do not match up
Solution 1	Expanding space?	Expanding space?
Solution 2	GR incomplete	GR Incomplete

**Table 1:** General relativity and the Planck scale and the Hubble scale

Escape velocity	$v_e = \sqrt{\frac{2GM}{R} - \frac{G^2 M^2}{c^2 R^2}}$
Radius where $v_e = c$	$R_{h,s} = \frac{GM}{c^2}$
Haug time	$t_{h,s} = \frac{R_{h,s}}{c}$
	Planck mass
Mass	$m_p = \sqrt{\frac{\hbar c}{G}}$
Haug radius	$R_{h,s} = l_p$
Gravitational acceleration in Planck time	$g_s t_p = c$
Gravitational acceleration in Haug time	$g t_s = c$
Escape velocity	$v_e = c$
Planck scale equation	$\frac{1}{t_p^2} = H_p^2 = \frac{4\pi G \rho_p}{3}$
Planck mass density inside $R_{h,s}$	$\rho_p = \frac{m_p}{\frac{4}{3}\pi l_p^3}$
Conclusion:	Match Planck scale
	Universe mass
Mass	$M_{h,c} = \frac{c^3}{GH_0}$
Haug radius	$R_{h,s} = R_H$
Gravitational acceleration in Hubble time at $R_H$	$g_H t_H = \frac{GM_h}{R_H^2} \frac{R_H}{c} = c$
Gravitational acceleration in Haug time at $R_H$	$g_H t_{h,s} = \frac{GM_h}{R_H^2} \frac{R_{h,s}}{c} = c$
Gravitational acceleration in Hubble time at $R_{h,s}$	$g_{h,s} t_H = \frac{GM_h}{R_{h,s}^2} \frac{R_H}{c} = c$
Gravitational acceleration in Haug time at $R_{h,s}$	$g_{h,s} t_{h,s} = \frac{GM_h}{R_{h,s}^2} \frac{R_{h,s}}{c} = c$
Escape velocity at $R_{h,s}$	$v_e = c$
Universe equation	$H_0^2 = \frac{4\pi G \rho}{3}$
	Haug universe
Conclusion:	Match Hubble scale

**Table 2:** Full relativistic escape velocity and the Planck scale and the Hubble scale.



## 8 Conclusion

The escape velocity from general relativity is identical to that one gets from Newtonian mechanics. This escape velocity seems to ignore the possibility of relativistic mass. Since the introduction of the relativistic mass by Lorentz, there has been a long discussion of whether a relativistic mass should be allowed. Einstein was negative to the relativistic mass concept and seemed to have avoided it in developing his gravity theory. We show that if one allows relativistic mass in the Newtonian framework, one gets a different escape velocity from Newton mechanics and from general relativity theory. We call our new escape velocity for full relativistic escape velocity, as it also considers relativistic mass. We have shown how one can derive the Friedmann equation for the critical universe from the standard escape velocity and derive a similar equation from our full relativistic escape velocity formula. Our formula predicts that the critical universe's mass (energy) density is twice that what the standard model predicts. Our escape velocity also shows that the Planck mass particle has an escape velocity of  $c$  at the Planck length. At the same time, general relativity here gets an escape velocity above  $c$  in this case. Therefore, the so-called micro black holes are different from the Planck mass, or must have a radius twice that of the Planck length, which implies changes to Planck units. In short, the general relativity solution for micro black holes does not match up with all the Planck units simultaneously, while our theory does so. Consequently, our model leads to a different interpretation of cosmology than the standard model. In particular, our new universe equation is consistent without the need for cosmic expansion or a Big Bang event. Although further work is needed to establish our approach, we think that our findings are already interesting enough to be presented to the scientific community for discussion and feedback.

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