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# EVIDENCE FOR DETERMINISTIC CHAOS IN LONG-TERM HIGH-RESOLUTION KARSTIC STREAMFLOW TIME SERIES

David Labat, B. Sivakumar, A. Mangin

► **To cite this version:**

David Labat, B. Sivakumar, A. Mangin. EVIDENCE FOR DETERMINISTIC CHAOS IN LONG-TERM HIGH-RESOLUTION KARSTIC STREAMFLOW TIME SERIES. *Stochastic Environmental Research and Risk Assessment*, 2016, 30 (8), pp.2189-2196. hal-03239245

**HAL Id: hal-03239245**

**<https://hal.science/hal-03239245>**

Submitted on 27 May 2021

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1 **EVIDENCE FOR DETERMINISTIC CHAOS IN LONG-TERM HIGH-RESOLUTION**

2 **KARSTIC STREAMFLOW TIME SERIES**

3 Labat D.<sup>1</sup>, Sivakumar B.<sup>2,3</sup>, Mangin A.<sup>4</sup>

4 1 - Géosciences Environnement Toulouse

5 2 - School of Civil and Environmental Engineering, The University of New South

6 Wales, Sydney, Australia

7 3 – Department of Land, Air and Water Resources, University of California,

8 Davis, USA

9 4 – Station d'Ecologie Expérimentale de Moulis - CNRS

10

11

12 Corresponding author :

13 David LABAT

14 Géosciences Environnement Toulouse - GET

15 14, Avenue Edouard Belin 31400 TOULOUSE FRANCE

16  (+33) 5.61.33.26.12

17 Fax: (+33) 5.61.33.25.60

18 E-mail: [labat@get.obs-mip.fr](mailto:labat@get.obs-mip.fr)

19

20 **Abstract** One of the major challenges in hydrology consists in the conception of models to  
21 predict runoff evolution in time, as this is of crucial importance in water resource assessment and  
22 management. These models are required to provide estimations of high flows and low flows, so that  
23 appropriate short-term (flood) emergency measures and long-term (drought) management activities  
24 can be undertaken. However, due to the inherent nonlinearity of climate inputs (e.g. rainfall) and the  
25 heterogeneous nature of watersheds, understanding and modeling the catchment hydrologic  
26 response is tremendously challenging. This is particularly the case for karstic watersheds that are  
27 generally highly nonlinear and also sensitive to initial conditions. Investigation of the dynamic nature  
28 of hydrologic response is an important first step towards developing reliable models for such  
29 watersheds. To this end, this study examines the dynamic nature of streamflow discharge from  
30 karstic watersheds, especially the short-term variations. A nonlinear dynamic method, the  
31 correlation dimension method, is employed to unique long, continuous, and high-resolution (30-min)  
32 streamflow data from two karstic watersheds in the Pyrénées Mountains (Ariège) of France: the  
33 Aliou spring and the Baget spring. The results reveal the presence of deterministic chaos in the  
34 streamflow dynamics of the two watersheds, with attractor dimension values below 3. These results  
35 have great significance regarding the presence of deterministic chaos in karstic flows and in the issue  
36 of data size regarding chaos studies in hydrology.

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38 **Keywords** : Karstic springs, streamflow, nonlinearity, chaos, time series analysis, correlation  
39 dimension

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## 44 **Introduction**

45 If we consider a watershed as a hydrological “system”, then we may consider rainfall as the  
46 principal input to the system and streamflow discharge (or a hydrobiogeochemistry index,  
47 such as conductivity, temperature, or bacteria distribution) as a representative output from  
48 the system. Therefore, study of discharge time series should give us relevant information on  
49 the hydrological behavior of the watershed that integrates the rainfall inputs and the  
50 different pathways of water in the several compartments of the watershed. This approach,  
51 based on the study of an input-output temporal relationship, known as the systemic analysis  
52 (Mangin, 1984), appears as a valuable paradigm in hydrology and hydrogeology. For a long  
53 time, this input-output relationship had been considered as linear (Labat et al., 2000).  
54 However, due to the co-existence of saturated or diphasic conditions with surface free or  
55 under pressure flow conditions, the dynamics of water and, hence, the rainfall-runoff  
56 relationship is inherently complex and nonlinear. As a result, estimation and prediction of  
57 discharge is very difficult. The problem is further complicated due to our inability to provide  
58 a precise knowledge of the initial conditions as well as the spatial and temporal variability of  
59 the physical parameters that control the transfer of water. This insufficient information  
60 limits our understanding and predictive ability.

61 Among all the different types of hydrological watersheds, arguably the least predictable  
62 remain the watersheds with underground karstic water pathways. Karstic aquifers constitute  
63 a freshwater resource that emerges from complex carbonate geological systems  
64 characterized by a high degree of heterogeneity. On one hand, these watersheds are  
65 characterized by unexpected flash floods, such as those occurred over the cities of Nimes (in  
66 the year 1988) or Vaison La Romaine (in the year 1992) in France, **that cause human losses**

67 and severe economic damages. On the other hand, they will certainly play a key role in the  
68 future, as water demands continue to grow and the quantity of freshwater available (per  
69 capita) continues to shrink. The exploitation of these aquifers is much more difficult than  
70 porous, fissured and even fractured aquifers. This is a direct consequence of the dissolution  
71 of the limestone. When water infiltrates in the upper zone of the aquifer, it becomes acid  
72 because of the contact with the microfauna and the roofs that produce the acid carbonic.  
73 Therefore, when water percolates in the fractured carbonate media, the dissolution process  
74 favored by the existence of fractures or fissures involves the development of a high  
75 permeability together with the presence of micro- and macro-porosity. Water penetrates in  
76 the system via boreholes or sinks and then flows into drains and conduits connected to large  
77 water reserves (Labat et al., 2000). But this rapid infiltration contrasts with delayed  
78 infiltration via epikarstic soil. Saturated or diphasic conditions, together with surface free or  
79 under pressure flow conditions, generally co-exist and the dynamics of water in a watershed  
80 remains nonlinear and complex. The concomitance of both flows leads to a nonlinear  
81 response reflecting the large diversity of pathways connecting the surface with springs that  
82 involves complex hydraulic connections in the saturated zone. Karstic watersheds are then  
83 essentially spatially heterogeneous groundwater systems characterized by an inherent  
84 temporal nonstationarity and nonlinearity of their hydrological response that should be  
85 naturally investigated by nonlinear techniques, especially those that address the sensitivity  
86 to initial conditions (chaos).

87 First described by Lorenz in the context of meteorological models (Lorenz, 1963) and then  
88 connected to the concepts of fractal geometry by Mandelbrot (e.g. Mandelbrot, 1983), the  
89 theory of chaos has been successfully applied in many areas of engineering and natural  
90 sciences during the last three decades. Basically, a chaotic system is defined as a nonlinear

91 deterministic system in which small changes in the initial conditions could lead to severe  
92 divergence in its future behavior. This property is obvious, for example, in global climate  
93 system, where it is popularly known as the “butterfly effect”. Therefore, it is quite difficult to  
94 distinguish a chaotic deterministic system from a purely random system, since both of them  
95 can produce irregular and apparently unpredictable temporal and/or spatial variability. The  
96 theory of chaos in hydrological sciences was introduced in the late 1980s (e.g. Hense, 1987;  
97 Rodriguez-Iturbe et al., 1989). Since then, applications of chaos theory in hydrological  
98 sciences have been significantly advanced, including some very recent studies (e.g.  
99 Jayawardena et al., 2010; Kyoung et al., 2011; Sivakumar and Singh, 2012; Tongal et al.,  
100 2013; Tongal and Berndtsson, 2014); see, for example, Sivakumar (2000, 2004, 2009) for  
101 detailed reviews.

102 A few important points must be noted from the literature on chaos studies on streamflow  
103 (and other hydrological) processes.

104 1. Thus far, chaos applications to streamflow processes have been largely limited to  
105 classical surface hydrological watersheds. However, as mentioned earlier, streamflow  
106 discharge from karstic springs may exhibit very different dynamics from that in  
107 surface hydrological watersheds. Therefore, whether streamflow from karstic springs  
108 exhibit chaotic behavior is an important question to study. To our knowledge, the  
109 study by Lambrakis et al. (2000) has been the only study that has attempted such an  
110 analysis, and further exploration is certainly worthwhile.

111 2. Chaos studies on streamflow time series (e.g. Lambrakis et al. (2000)) have largely  
112 focused on daily or coarser- resolution dynamics, with a few rare exceptions (e.g.  
113 Stehlik, 1999). While understanding the dynamics of streamflow at daily and coarser

114 resolutions is indeed useful for various water resources and environmental  
115 applications, study of streamflow at much finer resolutions (e.g. in the order of hours  
116 and minutes) also serves important purposes, especially in the context of flood  
117 forecasting and emergency measures.

118 3. In almost all the chaos studies on streamflow, the length of data considered is less  
119 than  $10^4$ , which amounts to roughly 25 years of consecutive daily data at best; for  
120 instance, Lambrakis et al. (2000) have used just over 8000 values of daily data (22  
121 years). Although reliable identification of chaotic behavior of streamflow can be  
122 made even with a few hundreds or just over a thousand of data representing a  
123 system (e.g. Jayawardena and Lai, 1994; Sivakumar et al., 2002b; Sivakumar, 2005), it  
124 is obviously desirable to have a much longer time series to eliminate any misgivings  
125 about the analysis and results.

126 These observations provide the motivation for the present study. In this study, we  
127 investigate the dynamics of long-term high-resolution streamflow data from two karstic  
128 systems in the Pyrénées Mountains (Ariège) of France: the Aliou spring and the Baget spring.  
129 We use streamflow data observed at every 30-minute resolution over a period of about 20  
130 years, which amount to a total of over 350,000 values. We believe that these long-term,  
131 high-resolution discharge data provide the hydrological community the best opportunity for  
132 a highly reliable analysis of nonlinear and chaotic dynamics in karstic systems, and  
133 streamflow dynamics in general.

## 134 **Method**

135 A popular approach for detection of chaotic patterns in time series is based on the  
136 estimation of the correlation dimension, introduced by Grassberger and Procaccia (1983).

137 The correlation dimension method uses the correlation integral to distinguish between  
138 chaotic and stochastic systems. Let us assume that the time series under investigation is  
139 generated by a nonlinear dynamical system with  $m$ -degree of freedom. It is first necessary to  
140 construct a series of state vectors  $X^m(t)$  (e.g. Packard et al., 1980). With observations of only  
141 the variable of interest available, one way to deal with this reconstruction is by using delay  
142 coordinates in an  $m$ -dimensional phase space (Takens, 1981):

$$143 \quad X^m(t) = [X(t), X(t + \tau), \dots, X(t + (m-1)\tau)] \quad (1)$$

144 where  $m$  is called the embedding dimension and  $\tau$  is an appropriate delay time. These  
145 vectors describe the trajectory of the system in the phase space. If the system is  
146 characterized by a low-dimensional chaotic behavior, the trajectories of the system will  
147 converge towards the subset of the phase space called "attractor." When dealing with  
148 natural processes, which are inherently nonlinear, convergence with a cyclic trajectory  
149 cannot be observed. Rather, nonlinear natural dynamical systems tend to converge on  
150 attractors on which the motion is periodic and unpredictable over long time, called "strange  
151 attractors."

152 The choice of the delay time is a very important step in phase space reconstruction and,  
153 hence, in the correlation integral estimation. It is well known that if the delay time is too  
154 small, the phase-space coordinates will not be independent enough to produce new  
155 information about the evolution of the system, whereas if it is too large then all the relevant  
156 information is lost because of diverging trajectories. Additionally, if the delay time is smaller  
157 than necessary to unfold the attractor, then the estimated correlation dimension from the  
158 Grassberger-Procaccia algorithm will be smaller (i.e. underestimation) than the "true"  
159 dimension of the attractor. In chaos studies, two functions are commonly used to determine

160 a proper delay time: the autocorrelation function (Holzfuss and Mayer-Kress, 1986) and the  
 161 mutual information function (Fraser and Swinney, 1986); see Islam and Sivakumar (2002) for  
 162 an example of their applications for streamflow time series. In the autocorrelation function  
 163 method, a common rule consists of setting the delay time as the lag time at which the  
 164 autocorrelation function first reaches zero, but some authors have also suggested 0.1 (Tsonis  
 165 and Elsner, 1988) or 0.5 (Schuster, 1988). In the mutual information method, the lag time  
 166 corresponding to the first minimum of the mutual information is generally chosen as the  
 167 delay time.

168 Considering the set of points in the attractor, one can define the correlation integral (or  
 169 function) in order to distinguish between stochastic and chaotic behavior. In practice, when  
 170 dealing with the sampled (not continuous) time series, the radius cannot tend to zero and  
 171 the correlation integral is approximated numerically and computed as follows:

$$172 \quad C(r) = \frac{1}{N_{ref}} \sum_{j=1}^{N_{ref}} \frac{1}{N} \sum_{i=1}^N \Theta(r - |X^i - X^j|) \quad (2)$$

173 where  $N$  is the number of points  $X_N(t)$ ,  $N_{ref}$  is the number of reference points taken from  $N$   
 174 and  $\Theta(r)$  is the Heaviside function. Therefore, the correlation integral consists of an estimate  
 175 of the number of points out of the data set which are closer than a radius  $r$  or within an  
 176 hypersphere of radius  $r$ . Then, when the radius tends towards zero, the correlation integral  
 177 will be characterized by an asymptotic power-law behavior

$$178 \quad C(r) \propto r^v \quad (3)$$

179 where  $v$  is the correlation exponent given by the slope of correlation integral versus radius in  
 180 a log-log plot. In order to detect chaotic behavior, the correlation exponent has to be plotted

181 against the embedding dimension  $m$ . If the correlation exponent  $\nu$  increases with the  
182 embedding dimension, then no chaotic behavior is present in the dynamical system. For  
183 example, a purely random process, where there is no determinism, is characterized by a  
184 linear increase of the correlation exponent with the embedding dimension without  
185 saturation value. If the correlation exponent  $\nu$  reaches a saturation value, the asymptotic  
186 value is called the correlation dimension of the attractor. ~~The correlation dimension is often~~  
187 ~~fractal, meaning that it is a non-integer dimension typical from a chaotic system.~~ The nearest  
188 integer above the correlation dimension is generally assumed to be the number of degrees  
189 of freedom of the system. This information is of great importance in the modeling and  
190 prediction process. The effective region of scaling in the  $\log C(r)$  versus  $\log r$  plot is dictated  
191 principally by the possible presence of noise and so the intrinsic quality of the data (Lai and  
192 Lerner, 1998) and also by the proper determination of the delay time.

193 The correlation dimension method has been very widely used to distinguish between chaotic  
194 and stochastic time series in almost all fields. However, there have also been strong  
195 criticisms on the use of the method to real time series. Extensive details of the associated  
196 issues, including temporal correlations, data size, data noise, and presence of zeros, are  
197 already available in the literature (e.g. Osborne and Provenzale, 1989; Nerenberg and Essex,  
198 1990; Theiler, 1990; Tsonis et al., 1993; Schreiber and Kantz, 1996; Sivakumar, 2005). The  
199 issue of data size, however, has often dominated criticisms on chaos studies in hydrology,  
200 since hydrologic time series are often very short; see, for example, Schertzer et al. (2002)  
201 and Sivakumar et al. (2002a) for a discussion. It is our hope that analysis of a very long time  
202 series will help to have more confidence in the search for the presence of chaos in  
203 hydrological time series.

## 204 **Data, analysis, and results**

205 We focus here our attention on the analysis of long-term high-resolution streamflow records  
206 from karstic systems in order to highlight possible chaotic dynamics in their temporal  
207 variations. We study 30-min discharge data observed over a period of about 20 years from  
208 two karstic springs of the Pyrénées Mountains (Ariège): the Aliou and the Baget springs  
209 (Figure 1). Owing to the geographic proximity of these mid-altitude basins, both basins are  
210 under the influence of the same rainfall input function. Both watersheds are characterized  
211 by similar hydrological indexes with a mean daily discharge around  $0.5 \text{ m}^3/\text{s}$ , an area around  
212  $13 \text{ km}^2$  and a median altitude around 900m a.s.l.

213 The Aliou watershed, which is known as extremely karstified with a very short time response  
214 (less than a day), exhibits a discharge increase from  $0.1 \text{ m}^3/\text{s}$  to nearly  $30 \text{ m}^3/\text{s}$  in as little as  
215 8 hours followed by a decrease that is equally quick (Figure 1, top). **The 30-min sampling rate**  
216 **was adopted in the first time by analogy with other karstic springs. However, we recently put**  
217 **in evidence that for these two karstic springs, it is the best sampling rate in term of**  
218 **information. Then, in order to verify that it is the best sampling rate in terms of information**  
219 **content in hydrologic processes, we consider here 3-min sampling rate for one year as**  
220 **demonstrated in Labat et al. (2011). The spectral density of such a discharge series shows**  
221 **that below the 30-min timescale, the power spectrum density is flat meaning that there is**  
222 **only noise in the signal. It also shows a power-law behavior of the spectral density up to a 1-**  
223 **hour period. Therefore, the 30-min sampling time that has been adopted since the beginning**  
224 **of the 1990s, appears as the best compromise in terms of hydrological information. The**  
225 Baget spring also exhibits a very short response time in flow dynamics (Figure 1, bottom),  
226 although the discharge is not as high as that for the Aliou spring. Figure 1 displays a

227 visualization of the 30-min sampling rate for Aliou and Baget springs discharge time series  
228 (left) and compares it to the daily mean classical value (right). One can see that the 30-min  
229 data allow a far better resolution of the flood event than the classic daily mean sampling  
230 rate. It reveals the complete dynamics of the hydrological system response, especially the  
231 sharp rising part of flow (i.e. flood) within a short period for the two watersheds.

232 For analysis, we consider two different periods of data for each watershed: 1995–2000 and  
233 2005–2015. Data from the intervening period 2000–2005 are not considered in this study,  
234 because continuous runoff measurements at high sampling rates could not be made **due to**  
235 **several technical issues that were encountered during the measurement. With this, the**  
236 **longest series corresponds to more than 160,000 consecutive streamflow data. To our**  
237 **knowledge, this dataset constitutes the longest available series at this sampling rate on**  
238 **karstic system worldwide.** The analysis of the power spectrum densities for the two longest  
239 discharge time series (Figure 2) highlight a power law behavior with a break-in-scale around  
240 10 hours for Aliou and 16 hours for Baget. **This break-in-scale, indicated in Figure 2 by a**  
241 **vertical line, corresponds to the frequency where the two power law behaviors cross. This is**  
242 **an indication that two stochastic processes can be deciphered in the discharge time series,**  
243 **corresponding to high-frequency and low-frequency processes. While the high-frequency**  
244 **process corresponds roughly to the response of the system to rainfall impulses, the low-**  
245 **frequency process corresponds roughly to the response of the system to the synoptic**  
246 **climate variations.**

247 We apply the correlation dimension method to identify the presence of chaotic dynamics in  
248 these two streamflow time series. **The delay time is estimated based on the mutual**  
249 **information function, as it is a much better representation of nonlinear dynamics when**

250 compared to the autocorrelation function, which is essentially a linear tool. Nevertheless, for  
251 the two time series considered here, the autocorrelation function method also offers  
252 somewhat similar values of delay time obtained using the mutual information. The  
253 estimated delay time value using the mutual information function method is 40 hours for  
254 the Aliou watershed and 60 hours for the Baget watershed (Figure 3), which is about 1.5 and  
255 2.5 days, respectively. Both values are in good accordance with previous studies based on  
256 daily data on these watersheds over the 1970–2000 time period (Labat et al., 2000). By  
257 comparison, for example, a delay time of 7 days was selected in the study by Islam and  
258 Sivakumar (2002), 10 days by Elshorbagy et al. (2002), 14 days in Ng et al. (2007), and up to  
259 146 days in Pasternack (1999).

260 Based on these delay time values, the data are reconstructed and the correlation integrals  
261 are estimated. Figure 4 shows, for five selected embedding dimensions, the correlation  
262 integral plots for the longer time series from Aliou and Baget. The correlation integral  
263 exhibits a power law behavior in the log–log plot, characterized by a linear behavior across  
264 low values of radius. Some authors have used the Takens-Theiler estimates of correlation  
265 dimension to identify the scaling regions more clearly (e.g. Wang et al., 2006), but here we  
266 prefer to identify the region where correlation between linear scaling and correlation  
267 integral presents the highest values.

268 The linear scaling regimes in these plots allow reliable determination of the slope, i.e.  
269 correlation exponent ( $\nu$ ). Figure 5(a) to (d) show the relationship between the correlation  
270 exponent and embedding dimension for the four time series. For all the four time series, the  
271 correlation exponent  $\nu$  first increases with the embedding dimension and then reaches a  
272 saturation value. The saturation value of the correlation exponent is 1.17 and 1.95 for the

273 Aliou streamflow time series and 2.75 and 1.55 for the Baget streamflow time series. The  
274 low and fractal values of the correlation exponents suggest the presence of low-dimensional  
275 deterministic dynamic nature of the four streamflow time series characterized by a certain  
276 number of dominant variables. The correlation exponent values also indicate that there are  
277 at least 2 or 3 independent variables dominantly governing the temporal dynamics of  
278 streamflow in the Aliou and Baget watersheds. More precisely, if we use  $m > d$  (Takens,  
279 1981) and  $m \geq 2d+1$  (Abarbanel et al., 1990) for the lower and upper limits of number of  
280 dominant variables, it can be concluded that different number of dominant variables are  
281 acting in streamflow dynamics for the different periods for the same basins. Both basins are  
282 located in the same zone and continuous and multiresolution wavelet analyses (not provided  
283 here) do not show any significant changes in the rainfall regimes during the two intervals.  
284 One may suspect, therefore, that the changes in the number of dominant variables can be  
285 related to the influence of snow on some winters or to some changes in the internal  
286 structure of karst in relationship with floods. For example, some hydraulic connections could  
287 have been changed due to the large amount of sediments and materials carried out by water  
288 in the drainage network.

289 The correlation dimension results for the Aliou and Baget watersheds provide further  
290 support to the results reported by most of the chaos studies in hydrology, at least from two  
291 perspectives:

292 (1) The dynamics of streamflow (and other hydrological processes), including in karstic  
293 watersheds, often exhibit low-dimensional chaotic behavior. This is particularly the  
294 case for high-resolution time series, such as those finer than daily, where the  
295 smoothing effects (which are often observed at daily scale due to averaging, as

296 shown in Figure 1) are not significant; see also Regonda et al. (2004) and Salas et al.  
297 (2005).

298 (2) Data size (in terms of the sheer number of values in time series) is not necessarily a  
299 serious limiting factor in the estimation of correlation dimension (or any other chaos  
300 indicator), as long as the time series is long enough to represent the essential  
301 features of the system dynamics; see also Sivakumar et al. (2002a, b) and Sivakumar  
302 (2005).

303

#### 304 **Conclusion**

305 Considering the heterogeneous karstic systems, chaotic approach appears as a valuable  
306 method for determining the level of complexity of such systems. The chaos analysis gives  
307 relevant and useful information on the system not only for designing or exploitation of the  
308 water resources but also for flood prevention. Since previous studies on chaos applications  
309 in streamflow series have focused in a large majority on mean daily data, they could not  
310 account for the short-term variations in the streamflow process. Therefore, past results  
311 regarding the presence of chaotic behavior in streamflow may not be as strong as they can  
312 be. This study provides the first observation of chaotic behavior in long, continuous, and  
313 high-resolution (30-min) streamflow series from karstic systems.

314 It is important to note that streamflow process in karstic systems (and any hydrologic system  
315 for that matter) exhibits at least two different kinds of responses at different scales. The  
316 short-term response of the karstic systems (almost at daily scale) corresponds to the  
317 hydrologic response to a given rainfall event and highlights a large amount of the

318 nonlinearity of the rainfall-runoff relationship. On the other hand, the medium- to long-term  
319 response of the karstic system corresponds to its response to seasonality of rainfall.  
320 Considering these two different kinds of system responses and developing models to  
321 integrate them will likely offer a better way to study the streamflow dynamics. We strongly  
322 encourage hydrologists to apply nonlinear dynamic and chaos methods towards  
323 development of such an integrated framework in order to provide a new vision of the  
324 hydrological systems.

### 325 **Acknowledgments**

326 The database benefits from CRITEX Program RBV-SOERE from INSU-CNRS and Aliou and  
327 Baget watersheds are part of the French national observatory SO KARST ([www.sokarst.org](http://www.sokarst.org)).  
328 B. Sivakumar acknowledges the financial support from the Australian Research Council (ARC)  
329 through the Future Fellowship grant (FT110100328).

330

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434 Figures

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437 Figure 1: Visualization of the 30-min discharge time series for Aliou and Baget watersheds. Figure 1-a  
438 represents the two 30-min discharge times series for Aliou watershed and Figure 1-b compares 30-  
439 min high resolution sampling rate and daily sampling rate for the flood corresponding to the  
440 restricted interval 2/12/1995–7/12/1995. Figure 1-c represents the two 30-min discharge time series  
441 for Baget watershed and Figure 1-d compares 30-min high resolution sampling rate and daily  
442 sampling rate for the flood corresponding to the restricted interval 9/6/2000–14/6/2000.

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444 Figure 2: Power spectrum density function of the normalized frequency (in hours<sup>-1</sup>) for the longest  
445 discharge time series available for the Aliou (a) and Baget (b) karstic systems. The vertical line  
446 corresponds to the frequency where the two power law behaviors cross. It indicates that two stochastic  
447 processes can be deciphered in the discharge time series, corresponding to high-frequency and low-frequency  
448 process processes. The high-frequency process corresponds roughly to the response of the system to rainfall  
449 impulses, whereas the low-frequency process corresponds roughly to the response of the system to the  
450 synoptic climate variations

451

452 Figure 3: Average mutual information function of the time delay obtained for the longest discharge  
453 time series available on Aliou (a) and Baget (b) karstic systems. The first minimum roughly  
454 corresponds to 40 hours for Aliou watershed and 60 hours for Baget watershed.

455

456 Figure 4: Log-log plot of the correlation integral function for five selected embedding dimensions  $m$   
457 obtained for the longest 30-min discharge time series on the Aliou (a) and Baget (b) watershed. The  
458  $C(r)$  function exhibits a clear power law behavior highlighted by the straight line.

459

460 Figure 5: Correlation exponent versus embedding dimension for discharge time series from Aliou and  
461 Baget watersheds. The top plots correspond, respectively, to the first (a) and second discharge time  
462 (b) series of Aliou watershed and the bottom plots correspond, respectively, to the first (c) and  
463 second (d) discharge time series of Baget watershed. The correlation exponent  $\nu$  first increases with  
464 the embedding dimension and then reaches a saturation value. The saturation value  $\nu$  of the  
465 correlation exponent is 1.17 and 1.95 for the Aliou streamflow time series and 2.75 and 1.55 for the  
466 Baget streamflow time series.

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