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# Semi-lagrangian formulation of an extended GSOM Model for Multimodal Transportation Systems

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**Abstract:** The object of the paper is to introduce a macroscopic multimodal transportation model, based on the GSOM (generic second order modelling) approach. In multimodal transportation systems, there are two flows; the flow of vehicles, and the flow of passengers. These two flows are not independent, since vehicles carry passengers. Hence the idea of the GSOM approach: to describe first the vehicle flow, and to consider the passenger load of vehicles as an attribute of vehicles. The resulting model is treated in a semidiscretized lagrangian way: vehicles are discretized, and passengers are modelled by continuous quantities (passenger load). Nodes in the model recapture the main complexity of the transportation system. They can represent such distinct features intersections (cars), stations (buses, trains), or intermodal poles. The advantage of the model is that it provides a unifying macroscopic view of multimodal transportation systems, can accomodate various vehicle and passenger attributes, and thus should provide a useful tool for the management of such systems.

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**Keywords:** Multimodal, traffic, transportation, particle discretization, intersection model, conservation equation, passenger flow, passenger density

## 1. INTRODUCTION

The new technologies (communication, information, automation and autonomization) will impact the transportation system in a massive way. The system should become progressively fully multimodal. The object of the paper is to propose a macroscopic modelling framework for such a fully multi-modal transportation system, including guided transportation (trains, trams) or new modes of transportation Sossoe and Lebacque (2016). Macroscopic models are particularity suitable for the simulation but also management and planning of large transportation systems. Two flows must be taken into account: passengers and vehicles. The passenger flow is strongly linked to the vehicle flow, since passengers are transported by vehicles. Further, the passenger flow exhibits a high innate complexity, because it is the result of passenger activity plans, information and interactions.

This complexity has been addressed in various ways. One natural approach is multi-agent modelling as exemplified by generic models such as Urbansim Waddell (2002), and more recently Matsim (Horni et al. (2016)), following ideas from statistical physics. More specific models have been developed, such as (Monteiro et al. (2014)), (Holmgren (2012)), (Ma and Lebacque (2013)), (Xu et al. (2016)). The references (Moccia et al. (2011)), (Dib et al. (IEEE, 2016)), (Yang et al. (2017)) provide examples of application in the area of optimization and management. Recently, in order to address specific problems in multimodal networks, such as dynamic assignment, new macroscopic originally

tools, originally devised for traffic on very large networks, have been adapted to multimodal flow. Examples are MFD (macroscopic fundamental diagram) modelling, refer to (Zheng et al. (2013), Zheng and Geroliminis (2016)), (Loder et al. (2017)), and bidimensional modeling, refer to (Sossoe (2017)). The resolution of these models does not go to the link/node level.

The aim of the paper is to develop a macroscopic network-based multimodal model. Our starting point is constituted by GSOM models (Lebacque et al. (2007); Lebacque and Khoshyaran (2013)) which were initially designed for vehicular multimodality. They combine a kinematical wave model of the LWR (Lighthill-Whitham-Richards) type with vehicle/driver/passenger attribute dynamics. The attributes include: OD information, path/mode choice, driver behaviour, vehicle type (either physical: electric, diesel, etc., or service: taxi, bus, on demand service, internet service, etc.), driver information, vehicle dynamic characteristics (battery charge, engine temperature f.i.), V2S, V2I and V2V communication, and passenger load. The passenger flow dynamics can be modelled in this framework via the passenger load attribute. This attribute can be disaggregated per destination. Services can be represented as vehicle sub-type attributes. The model will be discretized into particles: vehicles or groups of vehicles with similar characteristics. Guided transportation systems can be described in the same way; stations being nodes, and the passenger load attribute of vehicles (trains, trams...) accounts for passenger dynamics. Intermodal platforms can be treated as special nodes. The GSOM

framework provides a systematic way of modeling nodes Khoshyaran and Lebacque (2008); Costeseque et al. (2015) which will be adapted to this new context.

The outline of the paper is the following. First we present the GSOM model for multimodal (vehicle + passenger) flow on a link (section 2). Then we address the semi-lagrangian discretization of the model, which yields the particle semi-discretization (section 3). The proposed node model is derived from the internal state node model concept (Khoshyaran and Lebacque (2008), Khoshyaran and Lebacque (2009), Costeseque et al. (2015)) and described in section 4 with its semidiscretization.

## 2. LINK MODEL.

### 2.1 Notations

- $a \in \mathcal{A}$ : the arcs;
- $d \in \mathcal{D}$ : the destinations;
- $x$  the position, which can be instantiated on a link  $a$  as:  $x = \xi_a$  with  $\xi_a \in [0, \ell_a]$ ;
- $t$  the time;
- $\rho(x, t)$  the density,  $v(x, t)$  the speed and  $q(x, t)$  the flow of vehicles. Note that  $q \stackrel{def}{=} \rho v$ ;
- $I(x, t)$ : the vector of attributes. It comprises the following:
  - $\chi(x, t) \stackrel{def}{=} (\chi^d(x, t))_{d \in \mathcal{D}}$ : the vector of fractions of vehicles with destination  $d \in \mathcal{D}$  ( $\chi^d(x, t)$  denotes the fraction of vehicles at time  $t$  and location  $x$  with destination  $d$ );
  - $\varpi(x, t)$ : the load of passengers;
  - $\mu(x, t) \stackrel{def}{=} (\mu^d(x, t))_{d \in \mathcal{D}}$ : the vector of fractions of passengers with destination  $d \in \mathcal{D}$  ( $\mu^d(x, t)$  denotes the fraction of passengers at time  $t$  and location  $x$  with destination  $d$ );
  - $\kappa(x, t)$  denotes a vector of supplementary attributes pertaining to passengers or vehicles.

$$I = (\chi, \varpi, \mu, \kappa) \quad (1)$$

- Note that  $\varpi$  has the unit of number of passengers per vehicles, thus  $\rho\varpi$  denotes the density of passengers per unit length. The speed of passengers is  $v(x, t)$  thus the flow of passengers is given by:

$$p \stackrel{def}{=} \rho\varpi v \quad (2)$$

- Velocity and density are connected through the FD (fundamental diagram):

$$v = V_e(\rho) \quad (3)$$

We make a standing simplifying hypothesis: all attributes are neutral (i.e. they do not impact the FD).

- The vehicles have an attribute which is their capacity with respect transport passengers,  $\varpi_{max}$ . This attribute is connected to vehicles, thus must satisfy an advection equation

$$\partial_t \varpi_{max} + v \partial_x \varpi_{max} = 0 \quad (4)$$

The passenger load is bounded as follows:

$$0 \leq \varpi(x, t) \leq \varpi_{max}(x, t) \quad \forall x, t \quad (5)$$

This attribute must be included into  $I$ :

$$I = (\chi, \varpi_{max}, \varpi, \mu, \kappa) \quad (6)$$

### 2.2 The model equations.

The GSOM model equations in eulerian coordinates  $(x, t)$  are the following:

$$\begin{cases} \partial_t \rho + \partial_x (\rho v) = 0 & (7.1) \\ \partial_t (\rho I) + \partial_x (\rho I v) = -\rho \Phi(I, \rho) & (7.2) \\ v = V_e(\rho) & (7.3) \\ I = (\chi, \varpi_{max}, \varpi, \mu, \kappa) & (7.4) \\ 0 \leq \varpi \leq \varpi_{max} & (7.5) \end{cases} \quad (7)$$

The source term  $\Phi$  concerns only  $\kappa$ . Thus passengers can board or unboard only at nodes, at which locations passengers and vehicles can change their path. The lagrangian coordinates are  $(n, t)$  with  $n$  the vehicle index and  $t$  the time. The relationship between eulerian and lagrangian coordinates results from the definition of the vehicle index:

$$n(x, t) \stackrel{def}{=} \int_{-\infty}^t q(x, \tau) d\tau \quad (8)$$

Let us introduce the spacing  $r \stackrel{def}{=} 1/\rho$  and define the FD in lagrangian coordinates:  $\mathcal{V}_n(r) \approx V_e(\rho)$ . Let us also define  $\phi_n(r, I) \approx \Phi(\rho, I)$ . Note that in lagrangian coordinates we include an explicit dependency of the FD and the source term on the vehicle (in eulerian coordinates such a dependency would be expressed via some attribute). Following for instance (Lebacque and Khoshyaran, 2013), equation (7) can be expressed as

$$\begin{cases} \partial_t r + \partial_n v = 0 & (9.1) \\ \partial_t I = -\phi_n(I, r) & (9.2) \\ v = \mathcal{V}_n(r) & (9.3) \\ 0 \leq \varpi \leq \varpi_{max} & (9.4) \end{cases} \quad (9)$$

Note that (9) is slightly more general than (7). Since the source term  $\phi$  concerns only  $\kappa$ , it follows that (9.2) can be expressed as:

$$\begin{cases} \partial_t \chi = 0 ; \partial_t \varpi_{max} = 0 ; \quad \partial_t \varpi = 0 ; \quad \partial_t \mu = 0 \\ \partial_t \kappa = -\varphi_n(I, r) \end{cases}$$

### 2.3 Boundary conditions

The equilibrium supply and demand functions  $\Sigma_e$  and  $\Delta_e$  result from the FD. The flow-density FD in eulerian coordinates is given by:  $Q_e(\rho, I; x) \stackrel{def}{=} \rho V_e(\rho, I; x)$ .

$$\begin{cases} \Sigma_e(\rho, I; x) = \max_{\beta \geq \rho} Q_e(\beta, I; x+) \\ \Delta_e(\rho, I; x) = \min_{\beta \geq \rho} Q_e(\beta, I; x-) \end{cases} \quad (10)$$

As a consequence of the standing hypothesis, that the attributes are neutral, the equilibrium functions do not depend on  $I$ . This simplifies the expression of the local traffic supplies  $\sigma$  and demands  $\delta$  considerably (Lebacque et al. (2005), Mammam et al. (2009)).

$$\begin{cases} \sigma(x, t) = \Sigma_e(\rho(x+, t); x) \\ \delta(x, t) = \Delta_e(\rho(x-, t); x) \end{cases} \quad (11)$$

The vehicle and attribute flows at any location  $x$  and time  $t$  are given by the **min principle**:

$$\begin{cases} q(x, t) = \min[\delta(x, t), \sigma(x, t)] \\ (qI)(x, t) = q(x, t) I(x-, t) \end{cases} \quad (12)$$

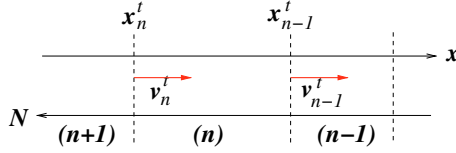


Fig. 1. Lagrangian discretization

Note that the passenger flow is given by:  $(q\varpi)(x, t) = q(x, t) \varpi(x-, t)$ .

### 3. SEMI-DISCRETIZED LINK MODEL.

This model is obtained by applying the Godunov scheme (Lebacque (1995), (Lebacque and Khoshyaran, 2013), Costeseque and Lebacque (2014)) to (9). Time is discretized in time-steps  $(t) \stackrel{def}{=} [t \Delta t, (t+1) \Delta t]$  of duration  $\Delta t$ . Vehicular traffic is divided into particles  $(n)$  of size  $\Delta v_{h_n}$ , which are the cells of the discretization. Each particle  $(n)$  has a capacity for transporting passengers which is  $K_{n,max}$ . The passenger traffic is not discretized explicitly, but only subsidiarily to the discretization of vehicular traffic, which is why the scheme is called semi-discretized. The position of particle  $(n)$  at time  $t \Delta t$  is  $x_n^t$ . The interdistance between particles  $(n)$  and  $(n-1)$  at time  $t \Delta t$  is denoted  $r_n^t$ . The resulting scheme is straightforward and described by:

$$\begin{cases} r_n^t \stackrel{def}{=} (x_{n-1}^t - x_n^t) / \Delta v_{h_n} \\ v_n^t = V_n(r_n^t) \\ r_n^{t+1} = r_n^t + (\Delta t / \Delta v_{h_n}) [v_{n-1}^t - v_n^t] \\ I_n^{t+1} = \Psi_n(r_n^{t+1}, r_n^t, I_n^t, \Delta t) \end{cases} \quad (13)$$

Here  $\Psi_n(r_n^{t+1}, r_n^t, I_n^t, \Delta t)$  denotes the solution of

$$\begin{cases} \dot{I}_n(\tau) = -\phi_n(I_n(\tau), r_n(\tau)) \\ I_n(\tau)|_{\tau=t \Delta t} = I_n^t \\ r_n(\tau) = r_n^t + (\tau - t \Delta t) [r_n^{t+1} - r_n^t] \end{cases} \quad (14)$$

$\dot{I}_n$  denotes the time-derivative with respect to the time  $\tau$ . The notations are illustrated by figure 1.

The scheme (13) may be expressed in a more accessible way in terms of the trajectories of particles, that is in terms of the positions  $x_n^t$ . In this context we can also approximate (14) by a simple explicit euler scheme. Thus (13), (14) yield

$$\begin{cases} r_n^t \stackrel{def}{=} (x_{n-1}^t - x_n^t) / \Delta v_{h_n} & (15.1) \\ v_n^t = V_n(r_n^t) & (15.2) \\ x_n^{t+1} = x_n^t + \Delta t v_n^t & (15.3) \\ I_n^{t+1} = I_n^t - \Delta t \phi_n(r_n^t, I_n^t) & (15.4) \\ 0 \leq \varpi_n^t \leq \varpi_{n,max}^t & (15.5) \end{cases} \quad (15)$$

Considering that  $I = (\chi, \varpi_{max}, \varpi, \mu, \kappa)$  we can express (15.4) as

$$\begin{cases} \chi_n^{t+1} = \chi_n^t, & \varpi_{n,max}^{t+1} = \varpi_{n,max}^t, & \varpi_n^{t+1} = \varpi_n^t \\ \mu_n^{t+1} = \mu_n^t, & \kappa_n^{t+1} = \kappa_n^t - \Delta t \varphi_n(I_n^t, r_n^t) \end{cases} \quad (15.5)$$

(15.5) is trivial except at network nodes.

### 4. NODE MODELLING

Nodes represent those elements which connect links. The exact nature of nodes is context-dependent; they can

be intersections (connecting links dedicated to vehicular traffic), stations (connecting links dedicated to public transportation, guided modes etc) and intermodal poles (connecting links pertaining to all modes).

#### 4.1 Notations

- Nodes  $z \in \mathcal{Z}$ , Arcs  $a \in \mathcal{A}$ ;
- OD (origin-destination) couples  $w \in \mathcal{W}$ ;
- Entry points  $(i), i \in \mathcal{I}(z)$  of a node  $(z)$ ;
- Exit points  $(j), j \in \mathcal{O}(z)$  of a node  $(z)$ ;
- Entry link  $(a)$  ( $a \in \Gamma^-(z)$ ) corresponding to an entry point  $(i)$  of node  $(z)$ :  $a \stackrel{def}{=} I(z, j)$ ;
- Exit link  $(b)$  ( $b \in \Gamma^+(z)$ ) corresponding to an exit point  $(j)$  of node  $(z)$ :  $b \stackrel{def}{=} O(z, j)$ ;
- Nodes may serve as entry points of passengers, hence the definition:  $\Pi_{z,j}^d =$  arrival rate of passengers at node  $(z)$  with destination  $(d)$ , exiting the node through exit point  $(j) \in \mathcal{O}(z)$  towards the link  $(b)$ ,  $b = 0z, j$ );
- Nodes may serve as entry points for vehicles, hence the definition:  $\Xi_{z,j}^d =$  arrival rate of vehicles at node  $(z)$  with destination  $(d)$ , exiting the node through exit point  $(j) \in \mathcal{O}(z)$  towards the link  $(b)$ ,  $b = 0z, j$ );
- $\theta_{z,ij}^d$  denotes the generic assignment coefficient for passengers in the node  $(z)$ , that is to say the fraction of passengers with final destination  $d \in \mathcal{D}$ , having entered the node through entry point  $i \in \mathcal{I}(z)$  which exit the node through exit point  $j \in \mathcal{O}(z)$ .
- $\gamma_{z,ij}^d$  denotes the generic assignment coefficient for vehicles in the node  $(z)$ , that is to say the fraction of vehicles with destination  $d \in \mathcal{D}$ , having entered the node through entry point  $i \in \mathcal{I}(z)$  which exit the node through exit point  $j \in \mathcal{O}(z)$ . Vehicles carry passengers, but, especially in the case of public transportation, they share only part of their passengers' paths.
- $N_{z,j}$ : the passenger queue in node  $(z)$  about to exit the node through exit point  $j \in \mathcal{O}(z)$ . This queue is managed following a FIFO rule, and passengers inside are described as a continuum (semi-discretized approximation).
- $M_{z,j}$ : the vehicle queue in node  $(z)$  about to exit the node through exit point  $j \in \mathcal{O}(z)$ . This queue is managed following a FIFO rule, and vehicles inside are described as a queue of particle.
- $M_z$  denotes the total number of vehicles in the node  $(z)$ . At any time  $t$ :  $N_z^t = \sum_{j \in \mathcal{O}(z)} \#M_{z,j}^t$  (with  $\#S$  being the cardinal of a set  $S$ ).
- $\beta_{z,i}$  denotes the split coefficient for the node supply relative to the node entry point  $i \in \mathcal{I}(z)$ .

These definitions are illustrated by figure 2.

#### 4.2 Node modeling principles

The node model is based on the internal state model which is based on the buffer concept, which is described in Khoshyaran and Lebacque (2008), Khoshyaran and Lebacque (2009), see also Delle Monache et al. (2014).

This model allows us to calculate the through-flow of vehicles, i.e. the flow of vehicles entering node  $(z)$  through

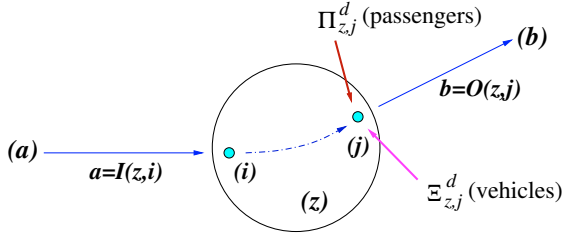


Fig. 2. Node modeling principles

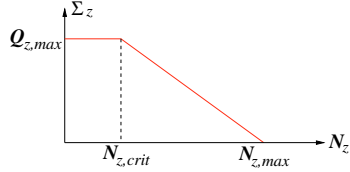


Fig. 3. Global equilibrium supply function of a node.

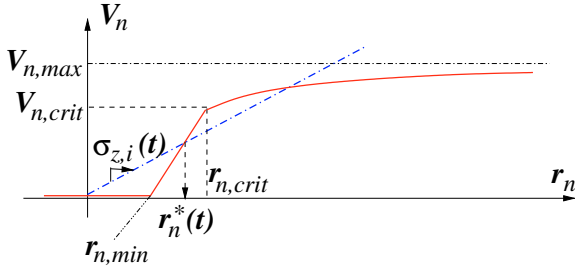


Fig. 4. headway applying to a vehicular particle exiting a link, and entering a node.

the entry point  $i \in \mathcal{I}(z)$ , and exiting the node through exit point  $j \in \mathcal{O}(z)$ . This through-flow is obtained by classical supply/demand analysis.

While crossing the node, passengers stay in the vehicle if they have the same exit link as their vehicle. Otherwise the passengers exit the vehicle and join the passenger queue say  $N_{z,j}$  pertaining to their exit link  $b = O(z, j)$  and board vehicles exiting the node towards link  $(b)$ . Thus passengers have their own through-flows.

#### 4.3 Node inflows

##### Node supply.

Any node  $(z)$  is assumed to have a global supply which depends on the total number of vehicles  $N_z^t$ :  $\Sigma_z(N_z^t)$ . Typically the function  $\Sigma_z(N_z)$  has the aspect depicted by figure 3. This node fundamental diagram expresses the physical qualities of the node: the maximum global through-flow  $Q_{z,max}$ , the storage capacity  $N_{z,max}$ , the critical capacity  $N_{z,crit}$ . The supply for link  $(a) \in \Gamma^-(z)$  with  $a = I(z, j)$  is a fraction  $\beta_{z,i}$  of the total node supply:

$$\sigma_{z,i}^t \stackrel{def}{=} \beta_{z,i} \Sigma_z(N_z^t)$$

where  $t$  denotes either time or time-step.

##### First particle dynamics.

The first particle in link  $(a)$ , let us say  $(n)$ , has the position  $\xi_a^{n,t}$ . The dynamics of the particle  $(n)$  are described by the system (15). In order to apply (15.2) it is necessary to estimate  $r_n^t$  from the supply applying to  $(a)$ , since  $(n)$  has no predecessor on  $(a)$ . Refer to figure 4.

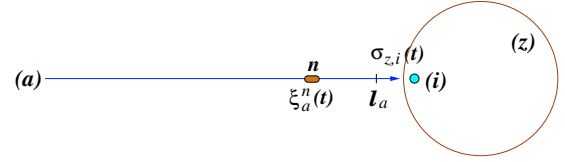


Fig. 5. Vehicular particle exiting a link, entering a node.

A lower bound for  $r_n^t$  is given by the distance to the head of the link,  $\ell_a - \xi_a^{n,t}$ . This bound applies if this distance is large enough and the node supply  $\sigma_{z,i}^t$  does not impose any further constraints on the dynamics of particle  $(n)$ . If the supply is low we must also consider the headway  $r_n^*(t)$  which is the smaller of the two solutions of the equation:

$$r \sigma_{z,i}^t = V_n(r)$$

i.e. the solution in the interval  $[r_{n,min}, r_{n,crit}]$  (refer to figure 4). Thus  $r_n^t$  is obtained as follows

$$r_n^t = \max [r_n^*(t), \ell_a - \xi_a^{n,t}] \quad (16)$$

The position of the particle  $(n)$  at time  $(t+1)\Delta t$  results from (16), (15.2), and (15.3):

$$\xi_a^{n,t+1} = \xi_a^{n,t} + \Delta t V_n(r_n^t) \quad (17)$$

with  $r_n^t$  given by (16). The attribute dynamics is still given by (15.4).

##### Particle entry into a node.

Please refer to Figure 5 for a description of the setting. If  $\xi_a^{n,t+1} \geq \ell_a$  then the particle  $(n)$  enters the node  $(z)$  during time step  $(t) = [t\Delta t, (t+1)\Delta t]$ , at time  $(t + \tau_n^t)\Delta t$  with

$$\tau_n^t = (\ell_a - \xi_a^{n,t}) / (\xi_a^{n,t+1} - \xi_a^{n,t})$$

If the destination  $(d)$  of  $(n)$  is actually the node  $(z)$  then the particle  $(n)$  exits the network at time  $(t + \tau_n^t)\Delta t$ .

Otherwise the particle  $(n)$ , given its destination  $(d)$ , chooses the exit point  $j \in \mathcal{O}(z)$  with probability  $\gamma_{z,ij}^d$ :

$$\mathbb{P}(\text{choice} = j \mid \text{entry} = i, \text{destination} = d) = \gamma_{z,ij}^d(t)$$

If the particle  $(n)$  chooses the exit point  $(j)$  it joins the queue  $M_{z,j}^t$  at time  $(t + \tau)\Delta t$ . This information is important since the queue is managed following a FIFO rule.

##### Passenger dynamics at a node entry.

While entering the node the passenger load of the particle is  $\varpi_n^t$ , with a composition in terms of passenger destination  $\mu_n^{d,t}$ . The assignment coefficient for passengers is  $\theta_{z,ij}^d(t)$ . Thus for any  $k \in \mathcal{O}(z)$ ,  $\varpi_n^t \mu_n^{d,t} \theta_{z,ik}^d(t)$  passengers choose this link exit point  $(k)$ . If  $k = j$  the passengers stay in vehicle  $(n)$  and contribute to  $\varpi_n^{t+1}$ . Otherwise they join the passenger queue  $N_{z,j}^t$ . We summarize this as follows, in the case  $(j)$  is the exit point of particle  $(n)$ :

- If  $k \neq j$ ,  $\varpi_k^{t+1} \stackrel{def}{=} \sum_{d \in \mathcal{D}} \varpi_n^t \mu_n^{d,t} \theta_{z,ik}^d(t)$  passengers join the queue  $N_{z,k}^t$  at time  $(t + \tau_n^t)\Delta t$ . Their composition with respect to destinations  $d \in \mathcal{D}$  is given by:  $\mu_k^{d,t+1} \stackrel{def}{=} \varpi_n^t \mu_n^{d,t} \theta_{z,ik}^d(t) / \varpi_k^{t+1}$ .
- If  $k = j$  no passengers from particle  $(n)$  join queue  $N_j^t$ . All passengers with exit-point  $(j)$  stay in vehicle, thus



we can define  $\varpi_j^{t+1} \stackrel{def}{=} \sum_{d \in \mathcal{D}} \varpi_n^t \mu_n^{d,t} \theta_{z,ij}^d(t)$  the load of passengers in particle ( $n$ ) while this particle is queued in the particle queue  $M_{z,j}^t$ . The composition of this passenger load per destinations is given by:  $\mu_j^{d,t+1} \stackrel{def}{=} \varpi_n^t \mu_n^{d,t} \theta_{z,ik}^d(t) / \varpi_j^{t+1}$ . The residual capacity of the particle is given by  $\varpi_{n,max} - \varpi_j^{t+1}$ .

#### 4.4 Node queue dynamics for vehicles.

##### *Vehicular queue dynamics in a node: arrival process.*

These queues are denoted  $M_{z,j}^t$  (vehicular queue pertaining to exit point ( $j$ ) of node ( $z$ ) at the beginning of time-step ( $t$ )). They are constituted of car particles managed following a FIFO dynamics. A queue can be fed according to two processes.

1. A particle enters the node ( $z$ ) through an entry point ( $i$ ) and joins the queue  $M_{z,j}^t$ . This process is described in subsection 4.3 (subsubsection "Particle entry into a node").

2. A vehicular particle enters the node from outside. The arrival rate of vehicular particles during time-step ( $t$ ) is given, and denoted  $\Xi_z^{d,t}$ . Thus the inflow rate of vehicular particles with destination ( $d$ ) which chose the exit point ( $j$ ) is given by:

$$\Xi_{z,j}^{d,t} = \Xi_z^{d,t} \gamma_{z,j}^{d,t} \quad (18)$$

All arrival processes are assumed Poisson. Thus during the time-step ( $t$ ) of duration  $\Delta t$ , the number  $h_{z,j}^{d,t}$  of vehicular particles generated with destination ( $d$ ) and exit point ( $j$ ) follows the law

$$\mathbb{P} [h_{z,j}^{d,t} = h] = \left\{ \left( \lambda_{z,j}^{d,t} \right)^h / h! \right\} \exp \left( -\lambda_{z,j}^{d,t} \right) \quad (19)$$

with  $\lambda_{z,j}^{d,t} \stackrel{def}{=} \Xi_{z,j}^{d,t} / \Delta t v_h$ . If needful and relevant, i.e. if there exists a passenger inflow at node ( $z$ ), passengers could be loaded into the generated vehicular particles. In this paper we mainly consider passengers boarding vehicles as they leave the node to enter a link.

##### *Vehicular queue dynamics in a node: exit process.*

Now we analyze the exit of a particle into link ( $b$ ), with  $b = O(z,j)$ . It is necessary to introduce a specific time-step for the link:

$$\Delta t_b \stackrel{def}{=} \Delta t v_h / Q_{b,max} \quad (20)$$

with  $Q_{b,max}$  the maximum flow in link ( $b$ ). During each such time-step of duration  $\Delta t_b$ , we check link ( $b$ ) and the vehicular queue  $M_{z,j}^t$ . If  $M_{z,j}^t$  is not empty, the first particle of the queue may leave and enter link ( $b$ ), provided that the supply of ( $b$ ) is sufficient. Let us denote by ( $m$ ) the last particle on link ( $b$ ); its position is  $\xi_b^{m,t}$ . The setting is described in figure 6.

This particle is associated to the headway  $r_b^t \stackrel{def}{=} \xi_b^{m,t} / \Delta t v_h$ . If  $r_b^t < r_{b,min}$  then no particle can leave the queue  $M_{z,j}^t$ . If  $r_b^t \geq r_{b,min}$  then a particle say ( $m+1$ ) leaves the queue  $M_{z,j}^t$  and enters ( $b$ ). In order to calculate its position, we

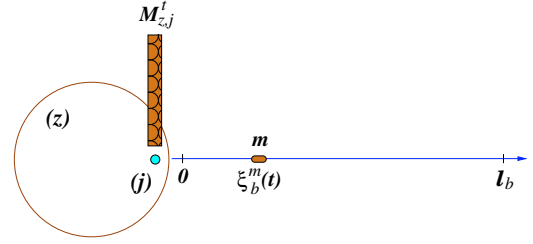


Fig. 6. Vehicular queue at an exit point of a node.

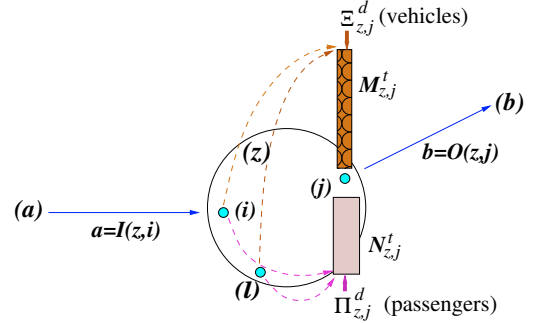


Fig. 7. Internal node dynamics: summary.

evaluate  $v_b^t = V_b(r_b^t)$  which is  $\geq 0$ . The position of ( $m+1$ ) at  $t+1$  follows:

$$\xi_b^{m+1,t+1} = \Delta t v_b^t \quad (21)$$

Passengers in the passenger queue  $N_{z,j}^t$  may board the particle ( $m+1$ ) as it leaves the node.

#### 4.5 Passenger queue dynamics in a node.

The queues  $N_{z,j}^t$  are managed in a FIFO way but they are continuous. The queue inflow results from passengers entering the node inside vehicles (subsubsection "passengers dynamics at a node entry" of subsection 4.3) and of the passenger inflow at the node  $\Pi_{z,j}^{d,t}$ . The queue is parameterized by the passenger index  $\varpi$ , which can also be interpreted as the number of passengers to the exit of the queue. Thus  $\varpi$  ranges from 0 to  $\#N_{z,j}^t$ . Another descriptor of the queue is the composition with respect to destination  $\mu_{z,j}^{d,t}(\varpi)$ .  $\mu_{z,j}^{d,t}(\varpi) d\varpi$  denotes the number of passengers with destination ( $d$ ) whose index is comprised between  $\varpi$  and  $\varpi + d\varpi$ . This composition is propagated according to an advection process since it is an attribute attached to passengers. The "propagation velocity" inside the queue is exactly equal to the outflow of the queue, the rate of which we denote  $\psi_{z,t}^t$ . Thus it follows:

$$\partial_t \mu_{z,j}^{d,t}(\varpi) + \psi_{z,t}^t \partial_{\varpi} \mu_{z,j}^{d,t}(\varpi) = 0 \quad (22)$$

Passengers exit the queue by boarding vehicle particles of the vehicle queue  $M_{z,j}^t$  within the constraints imposed by the residual capacity of these vehicles.

## 5. CONCLUSION

The macroscopic multimodal model introduced in this paper is based on the GSOM approach and has been expressed in terms of a set of conservation equations. This model has been discretized in a semi-discretized lagrangian

form: discrete vehicles and continuous passenger load. The description of node dynamics based on a buffer approach has been carried out. The resulting model allows a synthetic and macroscopic description of large transportation systems. It has been applied to the management of metro lines in incident conditions (Lebacque and Khoshyaran (2018)). The macroscopic approach with discretization schemes allows us to choose the resolution of the model and to adapt it to the application.

Further investigations will concern alternative discretization schemes: fully discretized eulerian and lagrangian schemes, and the inclusion of specific attributes such as communication and information. Specific infrastructures (parkings) and more precise description of nodes (internal node travel time for instance will account for pedestrian links or for boarding/unboarding time) should also be considered, in order to treat multimodal transportation system management. Other investigations will concern the control and management of the system. Indeed management measures can be applied at many stages of the model, macroscopic but also local (passengers boarding vehicles, dynamics of vehicles and headways etc).

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