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# GEOMETRICAL THEORY OF DIFFRACTION FOR SOUND AND VIBRATION

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## ABSTRACT

We present a geometrical theory of diffraction (GTD) for sound radiation and structural response. Six types of hybrid rays are identified travelling either from structure-to-fluid (sound radiation) or from fluid-to-structure (structural response). Three rays correspond to sound radiation by the surface, edge, or corner of a structure while the three others are the reciprocal paths corresponding to sound-to-vibration conversion by the surface, edge, or corner. We present the calculation of geometrical properties of wavefronts (principal directions and curvatures) and their laws of transformation during an interaction process. Furthermore, some simple explicit relationships for diffraction coefficients are given under the light fluid assumption. Finally, two examples are discussed to illustrate the concepts. The first one is a pure radiation problem while the second one involves transmission through walls, structural response and sound radiation.

## 1. INTRODUCTION

The geometrical theory of diffraction (GTD) founded by Keller [1] more than sixty years ago, is today a well-accepted and widespread theory in the fields of electromagnetism and acoustics.

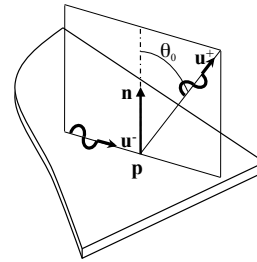
The main idea of the GTD is that waves propagate as rays. Although this idea is ancient and goes back at least to Greek antiquity, the novelty of GTD leads on the discovery that it exists diffracted rays in addition to reflected and refracted rays.

Nowadays, the GTD has not been extended to describe interaction between sound and vibration. Such a theory has been recently proposed in Ref. [2]. This constitutes an important step to model high frequency vibrations beyond the classical diffuse field assumption adopted in statistical energy analysis [3].

In this paper, we present a short review of the content of Ref. [2]. We show that sound structure interaction may be described with only six types of hybrid rays i.e. structural and acoustical. We first present these rays and illustrate their interest with two examples.

## 2. REVIEW OF HYBRID RAYS

Interaction between structure and sound appears in the phenomena of sound radiation and structural response. In



**Figure 1.** Radiation of a supersonic structural ray, the acoustical ray starts with an emission angle  $\theta_0$  defined in Eq. (1).

both cases, a vibrational energy is exchanged between the structure and the surrounding fluid. The description of these interactions in the framework of geometrical acoustics implies that a ray propagating in the structure is converted in a ray propagating in the fluid, or reciprocally. This is what we call hybrid rays.

Their existence is predicted by Fermat's principle. We know that the radiation process may be decomposed in three modes: radiation by surface, edges or corners of the structure.

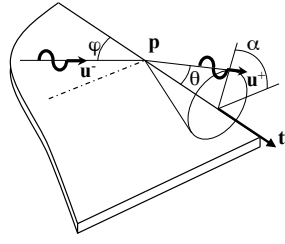
Consider a structural ray travelling inside the plate in direction  $\mathbf{u}^-$ . This ray is converted into a sound ray at point  $\mathbf{p}$  in direction  $\mathbf{u}^+$ . For radiation by surface, the point  $\mathbf{p}$  is anywhere on the structure as shown in Fig. 1. Then, an acoustical ray starts from  $\mathbf{p}$  in the same direction but with an emission angle  $\theta_0$ . Snell's law then reads

$$\sin \theta_0 = \frac{c_f}{c_s} \quad (1)$$

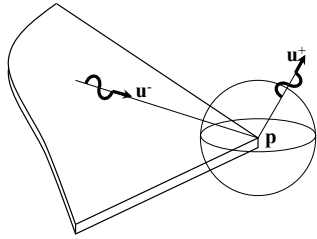
where  $c_f$  is the speed of sound and  $c_s$  that of flexural wave. It follows that a sound ray exists only if  $c_s > c_f$ , that is the structural ray is supersonic.

Radiation by edges is rather a problem of diffraction. The point  $\mathbf{p}$  is located on the edge of tangent vector  $\mathbf{t}$  and the incoming ray has incidence  $\varphi$  as shown in Fig. 2. The possible directions of sound radiation all belongs to Keller's cone of axis  $\mathbf{t}$  and angle  $\theta$  given by

$$\cos \theta = \frac{c_f}{c_s} \cos \varphi \quad (2)$$



**Figure 2.** Diffraction of a structural ray by edge, the acoustical ray lies in a Keller's cone which angle  $\theta$  is given in Eq. (2).



**Figure 3.** Diffraction of a structural ray a corner, the acoustical ray may follow any direction.

while the azimuthal angle of emission  $\alpha$  can take any value. Note that for supersonic structural ray ( $c_s > c_f$ ), there is always a diffracted ray. But in the subsonic case, no diffraction occurs for incidence  $\varphi < \arccos c_s/c_f$ .

Finally, consider a structural ray impinging on a plate corner as shown in Fig. 3. The point  $\mathbf{p}$  is fixed at corner. A corner diffracts structural rays in all directions in the fluid. More generally this is the case for any singular point such as force point, attachment point of additional mass or stiffness, or small hole.

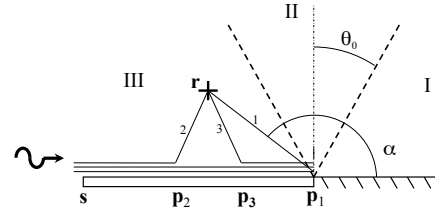
By virtue of reciprocity, all previously determined paths can be inverted. It follows that acoustical rays hitting the plate with the particular incidence  $\theta_0$  are absorbed and transformed into structural rays. In a similar way, acoustical rays reaching a plate edge are also absorbed for any incidence in the subsonic case and incidence  $\theta > \arccos c_f/c_s$  in the supersonic case. Finally, absorption by corner or any singular points occurs for any incidence.

Thus, there exists six types of hybrid rays *i.e.* rays travelling in both fluid and structure. They appear in vibroacoustics each time a structure radiates or absorbs acoustical energy. But all other classical rays may be required in practice.

### 3. RADIATION OF SOUND

In this section, we detail a simple application of GTD to a sound radiation problem.

We consider a structural plane wave incident on the edge of a semi infinite baffled plate as shown in Fig. 4.



**Figure 4.** Plane wave hitting the edge of a plate. Three rays are radiated towards the fluid and three zones numbered in roman characters are defined depending on the number of rays reaching the zone.

Let  $\mathbf{r}$  denote a point above the plate. To determine the acoustical pressure at this point, we need to make an inventory of all rays passing through that point. First, the incident ray is diffracted by the edge leading to the field  $p_1$ . Since any point in the fluid is located on a unique Keller's cone, the point  $\mathbf{r}$  is always reached by such a diffracted ray. Secondly, in the supersonic frequency range, the incident structural ray radiates continuously acoustical rays at angle  $\theta_0$  which may reach or not the reception point  $\mathbf{r}$ . The attached field is noted  $p_2$ . Finally, the incident ray is reflected by the edge and the resulting reflected plane wave also radiates acoustical rays at angle  $-\theta_0$  whose field is noted  $p_3$ . Three zones are thus clearly defined depending on the number of rays reaching  $\mathbf{r}$ . They are summarized as follows.

- zone I:  $p(\mathbf{r}) = p_1(\mathbf{r})$
- zone II:  $p(\mathbf{r}) = p_1(\mathbf{r}) + p_2(\mathbf{r})$
- zone III:  $p(\mathbf{r}) = p_1(\mathbf{r}) + p_2(\mathbf{r}) + p_3(\mathbf{r})$

These zones and the three types of acoustical rays are shown in Fig. 4.

Now, let us calculate the field attached to each ray. The incident structural ray is assumed to be a plane wave with a unit magnitude.

Firstly, the diffracted field is

$$p_1(\mathbf{r}) = \exp(ik_s d_1) D_f^{\text{edge}}(\alpha) \frac{\exp(ik_f r_1)}{\sqrt{r_1}} \quad (3)$$

where  $d_1 = |\mathbf{p}_1 - \mathbf{s}|$ ,  $r_1 = |\mathbf{p}_1 - \mathbf{r}|$  and  $k_s = \omega/c_s$ ,  $k_f = \omega/c_f$  are respectively the structural and acoustical wavenumbers. The term  $\exp(ik_s d_1)$  is attached to the structural part of the ray while  $D_f^{\text{edge}} \exp(ik_f r_1) / \sqrt{r_1}$  is the cylindrical acoustical wave from the edge to the receiver point.  $D_f^{\text{edge}}(\alpha)$  is a diffraction coefficient whose explicit expression is given in Ref. [2].

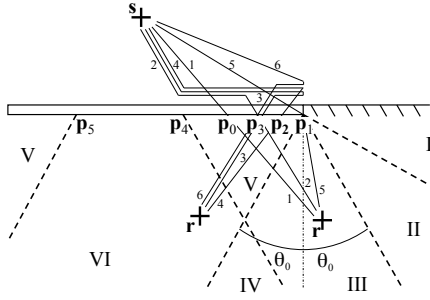
Secondly, the field radiated by the incident ray is

$$p_2(\mathbf{r}) = \exp(ik_s d_2) L_f \exp(ik_f r_2) \quad (4)$$

where  $d_2 = |\mathbf{p}_2 - \mathbf{s}|$  and  $r_2 = |\mathbf{p}_2 - \mathbf{r}|$ .  $L_f$  is the factor for radiation and  $\exp(ik_f r_2)$  the field attached to the acoustical plane wave.

Thirdly, the field radiated by the reflected ray is

$$p_3(\mathbf{r}) = \exp(ik_s d_3) R_s \exp(ik_s d_3) L_f \exp(ik_f r_3) \quad (5)$$



**Figure 5.** Cylindrical wave hitting the plate. Six rays are transmitted through the plate and six zones numbered in roman characters are defined depending on the number of rays reaching the zone.

where  $d_3 = |\mathbf{p}_3 - \mathbf{p}_1|$  and  $r_3 = |\mathbf{p}_3 - \mathbf{r}|$ . The term  $R_s \exp(ik_s d_3)$  is due to the reflected field  $R_s$  being the reflection factor.

This simple example illustrates the ease with which a ray-tracing approach applies to sound radiation problems. Of course, the semi-infinite nature of the plate leads a small number of rays. In the case of a finite plate, successive reflections occur on plates edges and the fields are series of structural rays radiating sound rays. An energetic approach to this problem is presented in Refs. [4, 5].

#### 4. TRANSMISSION OF SOUND

The second application is concerned with a transmission problem. Consider a cylindrical acoustical wave emanating from the source point  $\mathbf{s}$  and impinging on a semi-infinite baffled plate as shown in Fig. 5. One searches the acoustical field in the lower half-plane. From the source  $\mathbf{s}$  to the receiver point  $\mathbf{r}$ , six rays can propagate.

The first one is the acoustical ray that reaches the receiver point in straight line directly through the plate. The field is noted  $p_1$  with

$$p_1(\mathbf{r}) = \frac{\exp(ik_f s_0)}{\sqrt{s_0}} T_f \frac{\sqrt{s_0}}{\sqrt{s_0 + r_0}} \exp(ik_f r_0) \quad (6)$$

where  $s_0 = |\mathbf{p}_0 - \mathbf{s}|$ ,  $r_0 = |\mathbf{p}_0 - \mathbf{r}|$  and  $T_f$  is the transmission coefficient.

The second ray is absorbed by the structure at incidence  $\theta_0$ , travels in the plate, and is radiated into the lower fluid with emission angle  $\theta_0$ . The corresponding field is

$$p_2(\mathbf{r}) = \frac{\exp(ik_f s_4)}{\sqrt{s_4}} L_s \exp(ik_s d_1) L_f \exp(ik_f r_3) \quad (7)$$

where  $s_4 = |\mathbf{p}_4 - \mathbf{s}|$ ,  $r_3 = |\mathbf{p}_3 - \mathbf{r}|$ ,  $d_1 = |\mathbf{p}_4 - \mathbf{p}_3|$ , and  $L_s = -1/L_f$  is the attachment coefficient.

The third one is similar to the second one, although it is reflected by the edge before to be radiated.

$$p_3(\mathbf{r}) = \frac{\exp(ik_f s_4)}{\sqrt{s_4}} L_s \exp(ik_s (d_2 + d_3)) R_s L_f \exp(ik_f r_2) \quad (8)$$

where  $r_2 = |\mathbf{p}_2 - \mathbf{r}|$ ,  $d_2 = |\mathbf{p}_1 - \mathbf{p}_4|$  and  $d_3 = |\mathbf{p}_1 - \mathbf{p}_2|$ .

The fourth ray is successively absorbed at incidence  $\theta_0$  and diffracted by the edge.

$$p_4(\mathbf{r}) = \frac{\exp(ik_f s_4)}{\sqrt{s_4}} L_s \exp(ik_s d_2) D_f^{\text{edge}} \frac{\exp(ik_f r_1)}{\sqrt{r_1}} \quad (9)$$

where  $r_1 = |\mathbf{p}_1 - \mathbf{r}|$ .

The fifth one is the acoustical ray impinging on the edge and directly diffracted toward the receiver point.

$$p_5(\mathbf{r}) = \frac{\exp(ik_f s_1)}{\sqrt{s_1}} D_f^{\text{edge}} \frac{\exp(ik_f r_1)}{\sqrt{r_1}} \quad (10)$$

with  $s_1 = |\mathbf{p}_1 - \mathbf{s}|$ .

Finally, the sixth ray is diffracted into the structure by the edge, travels into the structure and is radiated with emission angle  $\theta_0$  toward the receiver point. Its field is

$$p_6(\mathbf{r}) = \frac{\exp(ik_f s_1)}{\sqrt{s_1}} D_s^{\text{edge}} \exp(ik_s d_3) L_f \exp(ik_f r_2) \quad (11)$$

These rays do not reach every point of the lower half-plane and the pattern. They are summarized as follows.

- zone I:  $p(\mathbf{r}) = p_4 + p_5$
- zone II:  $p(\mathbf{r}) = p_1 + p_4 + p_5$
- zone III:  $p(\mathbf{r}) = p_1 + p_2 + p_4 + p_5$
- zone IV:  $p(\mathbf{r}) = p_1 + p_4 + p_5$
- zone V:  $p(\mathbf{r}) = p_1 + p_2 + p_3 + p_4 + p_5 + p_6$
- zone VI:  $p(\mathbf{r}) = p_1 + p_3 + p_4 + p_5 + p_6$

These zones and the three types of acoustical rays are shown in Fig. 5.

#### 5. CONCLUSION

In conclusion, the geometrical theory of diffraction has been applied to sound radiation and structural response of plane structures. It has been shown that fluid-structure interaction gives rise to six structural-acoustical rays. These are radiation of supersonic structural rays, diffraction by edges of structural rays, diffraction by corners of structural rays, absorption of acoustical rays at incidence  $\theta_0$ , absorption of acoustical rays by edges, absorption of acoustical rays by corners.

#### 6. REFERENCES

- [1] J.B. Keller. Geometrical theory of diffraction. *J. Opt. Soc. Am.*, 52:116–130, 1952.
- [2] A. Le Bot. Geometrical theory of diffraction for sound radiation and structural response. *Wave Motion*, 87:179–192, 2019.
- [3] A. Le Bot. *Foundation of statistical energy analysis in vibroacoustics*. Oxford University Press, Oxford, UK, 2015.

- [4] V. Cotoni, A. Le Bot. Radiation of plane structures at high frequency using an energy method. *Int. J. of Acoustics and Vibration*, 6: 209–214, 2001.
- [5] V. Cotoni, A. Le Bot, L. Jezequel. High frequency radiation of L-shaped plates by a local energy flow approach. *J. Sound Vib.*, 250: 431–444, 2002.