An accelerated spectral approach for predicting the vibroacoustic behavior of a stiffened pipe filled with a heavy fluid

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AN ACCELERATED SPECTRAL APPROACH FOR PREDICTING THE VIBRO-ACOUSTIC BEHAVIOR OF A STIFFENED PIPE FILLED WITH A HEAVY FLUID

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ABSTRACT

This paper proposes a numerical model to predict the dynamic behavior of fluid-loaded cylindrical shell coupled to two rings. The model resolution is based on the circumferential admittance approach (CAA) and a spectral decomposition of the vibratory and pressure fields. For this study, we consider four degrees of freedom in the coupling between the shell and the stiffeners. Furthermore, as we work at a low frequency range, that introduces a significant coupling between the axial/rotational waves and the flexural waves of the stiffened-shell, which contributes consequentially in the dynamic response of the coupled system. The main problem of this coupling is that the calculation of the admittances associated with the radial and the angular rotation leads to convergence issues. Therefore, to tackle this matter, an accelerated approach is proposed. The originality of the approach lies in the use of an intermediate reference calculation at a particular frequency (called reference frequency) in the high wavenumbers domain. The present approach has been proven good results in terms of accuracy and calculation cost.

1. INTRODUCTION

The French Atomic and Alternatives Energies Commission, CEA, is working on the safety of the Sodium-cooled Fast Reactor (SFR). The continuous monitoring of sodium pipes, between the reactor pool and the heat exchanger is for example an important safety related issue. In such a context, liquid sodium leakage detection along pipes is one of the most important safety related issue that claims for new monitoring methodologies. In this study, it is assumed that a significant variation in the properties of the liquid flow modifies the dynamic behavior of the pipes. It results that the analysis of the pipe vibrations, measured from outside the pipe, may be an efficient way for detecting anomalies of the process sodium flow, such as , the presence of leaks in the gas/sodium heat exchanger, mainly used in the SFR. In this context, a numerical vibro-acoustic model is developed to establish the relation between the characteristics of the pipe, and of the contained fluid, and the vibrations properties of the instrumented test section. The model corresponds to an infinite cylindrical shell filled with a heavy fluid (sodium or water) and coupled to two ring stiffeners. In such geometry, it is assumed that the real pipe geometry outside the stiffeners will not importantly affect the vibration behavior of studied section.

The model resolution is based on the circumferential admittance approach (CAA) [1] and a spectral decomposition of the vibratory and pressure fields. The originality of the model, in comparison to previous studies, comes from the fact that we consider four degrees of freedom (3 translations and 1 rotation) in the coupling between the shell and the stiffeners and the use of an accelerated spectral approach to calculate the circumferential admittances. In fact, as we work in a low frequency domain, the flexural waves are coupled to the shear and compressional waves. Therefore, all the degrees of freedom are coupled and must be considered in the resolution of the problem. Moreover, as we work below the ring frequency, that introduces a significant coupling between the radial/rotational displacements of the shell that contributes consequentially in the dynamic response of the entire hull. However, it leads to convergence issues in the calculation of the admittances associated with the radial and the angular rotation. In order to improve the convergence, we propose in this paper an accelerated spectral approach, which consists in modelling the problem by a fluid-loaded shell in the high wave numbers using an intermediate reference calculation at a particular frequency called reference frequency. The vibratory field of the test section can then be analyzed in a wide frequency range in function of the bubble percentage.

2. FORMULATION OF A STIFFENED CYLINDRICAL SHELL MODEL

The model developed in this paper considers an infinite cylindrical shell filled with a heavy fluid (at rest) coupled to two ring stiffeners representing the nearest flanges on both side of the test section. The diagram shown in Fig.1 represents the configuration considered. The connection between the shell and the stiffeners is assumed to be rigid. The ring stiffeners and the shell consist of the same linear elastic homogeneous and isotropic material. The system is excited by a mechanical radial point force applied on the shell. The excitation is supposed harmonic with a time

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dependency $e^{j\omega t}$ in where ω represents the angular frequency.

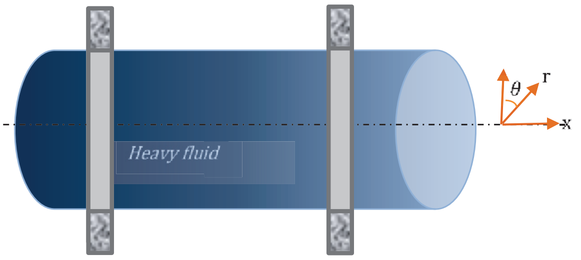

Figure 1. Illustration of fluid-loaded cylindrical shell coupled to two ring stiffeners.

2.1 Principle of the circumferential admittance approach

The system presented in Fig.1 has a complex geometry that makes the analytical resolution of the vibroacoustic problem very difficult. In order to solve this problem easily, we partition the system in two subsystems as shown in Fig.2. The fluid filled cylindrical shell constitutes one subsystem noted "*s*" whereas the ring stiffeners constitutes the second subsystem noted "*p*". We will use the circumferential admittance approach (CAA) for assembling the two subsystems, In the following the thin cylindrical shell and the stiffeners will be respectively described by the Flügge model [2] and a circular plate model (describing in-plane and out-of-plane motions) [3], while the fluid behavior will be described by the Helmholtz equation.

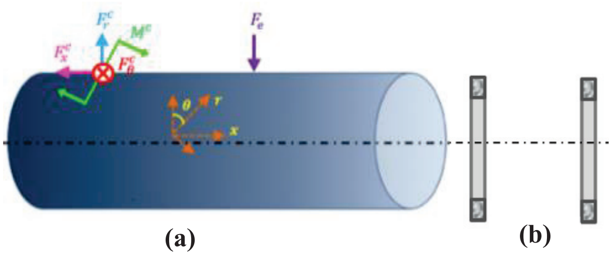

Figure 2. Partitioning and coupling forces: sub-system (a), cylindrical shell filled with heavy fluid; sub-system (b), ring stiffeners.

The cylindrical shell has constant thickness h and radius R_c The reference surface is taken at the middle surface of the shell where cylindrical coordinate system (x, r, θ) is considered. The x coordinate is taken in the axial direction of the shell, where θ and r are, respectively, in the circumferential and radial directions of the shell as shown on Fig.2. We assume that the junction between the shell and the stiffeners is sufficiently rigid, thus, at each junction, the shell and the stiffener have the same linear displacements and tangential rotation. The displacements and forces applied at the junctions between the two subsystems are therefore defined as follows:

- $U^s_{x,i}$, $U^s_{\theta,i}$,$U^s_{r,i}$ et φ^s_i(respectively$U^p_{x,i}$, $U^p_{\theta,i}$,$U^p_{r,i}$and φ^p_i) : the axial, tangential and radial displacements and $\varphi^s_i = \frac{\partial U^s_{r,i}}{\partial x}$ (resp. $\varphi^p_i = \frac{\partial U^p_{x,i}}{\partial r}$) the angular rotation of the shell (respectively the ring-stiffener);

- $F^s_{x,i}$, $F^s_{\theta,i}$,$F^s_{r,i}$ et M^s_i (respectively $F^p_{x,i}$, $F^p_{\theta,i}$,$F^p_{r,i}$ and M^p_i): the axial, tangential, radial forces, and the angular moment acting on the ring stiffener (respectively shell) by the shell (respectively ring stiffener).

As the considered system (excepted the excitation) is axisymmetric (2π-periodic along the circumference), all the physical variables (i.e. forces, shell displacements, pressure, etc.) can be written as Fourier series depending on circumferential orders n:

$$\tilde{f}(n) = \frac{1}{2\pi}\int_0^{2\pi} f(\theta)\, e^{-jn\theta} d\theta, \quad \forall n \in \mathbb{Z} \tag{1}$$

Where $n \in \mathbb{Z}$, because the system is 2π-periodic along the circumference, and with:

$$f(\theta) = \sum_{n\in\mathbb{Z}} \tilde{f}(n) e^{jn\theta} \tag{2}$$

In the following, the calculation will be carried out for each circumferential order n. We omit it of the notation. For each subsystem α ($\alpha = s, p$), we define the circumferential admittance between the junction i ($i = 1{,}2$), and the junction$j$ ($j = 1{,}2$) as the ratio of the displacement at the junction i to the applied force at the junction j:

$$\tilde{Y}^{\alpha}_{\xi_i \zeta_i} = \frac{\tilde{\xi}^{\alpha}_i}{\tilde{\zeta}^{\alpha}_j} \tag{3}$$

where,

$\xi^{\alpha}_i \in \{U^{\alpha}_{x,i},\ U^{\alpha}_{\theta,i}, U^{\alpha}_{r,i},\ \varphi^{\alpha}_i\}$ and $\zeta^{\alpha}_j \in \{F^{\alpha}_{x,j},\ F^{\alpha}_{\theta,i}, F^{\alpha}_{r,j},\ M^{\alpha}_j\}$

By making use of the superposition principle for linear passive systems, displacements continuity and equilibrium conditions at the junctions between the shell and the ring stiffeners, we can obtain the following linear equation system (see [1]):

$$[\tilde{Y}^s + \tilde{Y}^p][\tilde{F}^p] = -\tilde{\overline{\overline{W}}}^s \tag{4}$$

where,

$\tilde{\overline{\overline{W}}}^s$ is the free displacement vector of the cylindrical shell when it is uncoupled from the ring stiffeners and only excited by an external radial point force.

$$\tilde{F}^p_j = \begin{bmatrix} \tilde{F}^r_{xj} \\ \tilde{F}^r_{\theta j} \\ \tilde{F}^r_{rj} \\ \tilde{M}^r_j \end{bmatrix} \qquad \tilde{\overline{\overline{W}}}^s = \begin{bmatrix} \tilde{\overline{\overline{U}}}^s_{xj} \\ \tilde{\overline{\overline{U}}}^s_{\theta j} \\ \tilde{\overline{\overline{U}}}^s_{rj} \\ \tilde{\overline{\overline{\varphi}}}^s_j \end{bmatrix} \tag{5}$$

And $\tilde{Y}^s$ $(resp. \tilde{Y}^p)$ are the 4x4 circumferential admittance matrices of the shell (respectively the ring stiffener) given by:

$$\tilde{Y}^s = \begin{bmatrix} \tilde{Y}^s_{Ux_iFx_j} & \tilde{Y}^s_{Ux_iF\theta_j} & \tilde{Y}^s_{Ux_iFr_j} & \tilde{Y}^s_{Ux_iM_j} \\ \tilde{Y}^s_{U\theta_iFx_j} & \tilde{Y}^s_{U\theta_iF\theta_j} & \tilde{Y}^s_{U\theta_iFr_j} & \tilde{Y}^s_{U\theta_iM_j} \\ \tilde{Y}^s_{Ur_iFx_j} & \tilde{Y}^s_{Ur_iF\theta_j} & \tilde{Y}^s_{Ur_iFr_j} & \tilde{Y}^s_{Ur_iM_j} \\ \tilde{Y}^s_{\varphi_iFx_j} & \tilde{Y}^s_{\varphi_iF\theta_j} & \tilde{Y}^s_{\varphi_iFr_j} & \tilde{Y}^s_{\varphi_iM_j} \end{bmatrix}$$
$$\tilde{Y}^p = \begin{bmatrix} \tilde{Y}^p_{Ux_iFx_j} & \tilde{Y}^p_{Ux_iF\theta_j} & \tilde{Y}^p_{Ux_iFr_j} & \tilde{Y}^p_{Ux_iM_j} \\ \tilde{Y}^p_{U\theta_iFx_j} & \tilde{Y}^p_{U\theta_iF\theta_j} & \tilde{Y}^p_{U\theta_iFr_j} & \tilde{Y}^p_{U\theta_iM_j} \\ \tilde{Y}^p_{Ur_iFx_j} & \tilde{Y}^p_{Ur_iF\theta_j} & \tilde{Y}^p_{Ur_iFr_j} & \tilde{Y}^p_{Ur_iM_j} \\ \tilde{Y}^p_{\varphi_iFx_j} & \tilde{Y}^p_{\varphi_iF\theta_j} & \tilde{Y}^p_{\varphi_iFr_j} & \tilde{Y}^p_{\varphi_iM_j} \end{bmatrix} \quad (6)$$

By inverting the system given in the Eq.4, we deduce the coupling forces $\tilde{F}^p$ exerted by the two ring stiffeners on the shell filled with water. These forces and moments are finally reinjected in the Flügge spectral model and a 2D inverse Fourier transform is used in order to estimate the vibration field value of the shell when coupled to the ring stiffeners. In the following, we discuss how to calculate the circumferential admittances of the shell.

2.2 Estimation of Shell Circumferential Admittances and Free Displacements

2.2.1 Principle

To estimate the admittances of the shell coupled to the fluid, it is considered that it is excited by a radial loading F_e positioned at $x = 0$. The amplitude of this loading is unitary on each circumferential order. We can deduce the admittance between two junctions by translating the displacements of the shell $\tilde{\xi}$ a long x:

$$\tilde{Y}^c_{\xi_i\zeta_i} = \tilde{\xi}\left(x_i - x_j, k_\theta\right), \quad \forall(i,j)\epsilon([1,2])^2 \quad (7)$$

2.2.2 Mathematical Formulation of Fluid-loaded shell model

To determine the dynamic response of the cylindrical shell coupled to the fluid, various equations of motion have been derived and are summarized by Leissa [2]. In this study, the cylindrical shell coupled to the fluid is modelled using Flügge equations of motion. The external forces and the parietal pressure appear in the second member of these equations as follow:

$$\left[R_c^2\frac{\partial^2}{\partial x^2} + \frac{1-\nu}{2}\frac{\partial^2}{\partial\theta^2} - \rho R_c^2\frac{1-\nu^2}{E}\frac{\partial^2}{\partial t^2} + \beta^2\frac{1-\nu}{2}\frac{\partial^2}{\partial\theta^2}\right]U_x^s + \left[R_c\frac{1+\nu}{2}\frac{\partial^2}{\partial x\partial\theta}\right]U_\theta^s + \left[R_c\nu\frac{\partial}{\partial x} - \beta^2R_c^3\frac{\partial^3}{\partial x^3} + \beta^2R_c\frac{1-\nu}{2}\frac{\partial^3}{\partial x\partial\theta^2}\right]U_r^c = -\frac{(1-\nu^2)R_c^2}{Eh}F_x(\theta)\delta(x) \quad (8)$$

$$\left[R_c^2\frac{\partial^2}{\partial x^2} + \frac{1-\nu}{2}\frac{\partial^2}{\partial\theta^2} - \rho R_c^2\frac{1-\nu^2}{E}\frac{\partial^2}{\partial t^2} + \beta^2\frac{1-\nu}{2}\frac{\partial^2}{\partial\theta^2}\right]U_x^s + \left[R_c\frac{1+\nu}{2}\frac{\partial^2}{\partial x\partial\theta}\right]U_\theta^s + \left[R_c\nu\frac{\partial}{\partial x} - \beta^2R_c^3\frac{\partial^3}{\partial x^3} + \beta^2R_c\frac{1-\nu}{2}\frac{\partial^3}{\partial x\partial\theta^2}\right]U_r^s = -\frac{(1-\nu^2)R_c^2}{Eh}F_\theta(\theta)\delta(x) \quad (9)$$

$$\left[R_c\nu\frac{\partial}{\partial x} - \beta^2R_c^3\frac{\partial^3}{\partial x^3} + \beta^2R_c\frac{1-\nu}{2}\frac{\partial^3}{\partial x\partial\theta^2}\right]U_x^s + \left[\frac{\partial}{\partial\theta} - \beta^2R_c^2\frac{3-\nu}{2}\frac{\partial^3}{\partial x^2\partial\theta}\right]U_\theta^s + \left[1 + \beta^2\left(R_c^4\frac{\partial^4}{\partial x^4} + 2R_c^2\frac{\partial^4}{\partial x^2\partial\theta^2} + \frac{\partial^4}{\partial\theta^4}\right) + \rho R_c^2\frac{1-\nu^2}{E}\frac{\partial^2}{\partial t^2} + \beta^2\left(1 + 2\frac{\partial^2}{\partial\theta^2}\right)\right]U_r^s = \frac{(1-\nu^2)R_c^2}{Eh}\left(F_r(\theta)\delta(x) - M(\theta)\delta'(x) - p\right) \quad (10)$$

Where $\beta = \frac{h}{R_c\sqrt{12}}$ is the non-dimensional thickness parameter, $\delta(x)$ and $\delta'(x)$ are the Dirac delta distribution and its derivative, and p is the parietal pressure exerted by the fluid on the shell.

For a fluid loaded cylinder, the pressure satisfies the Helmholtz equation:

$$\nabla^2 p(x,r,\theta) + (k_0)^2 p(x,r,\theta) = 0 \qquad (11)$$

With $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2}$ the Laplacian operator and $k_0 = \left(\frac{\omega}{C_0}\right)^2$ the acoustic wavenumber.

We assume that the pressure also satisfies the Somerfield radiation condition and Euler's relation at the interface with the shell. The problem of Eqs. (8)-(9) can be solved in the wave numbers domain. This consists in using the spatial Fourier transform defined by:

$$f(x,\theta) \xrightarrow{SFT} \tilde{\tilde{f}}(k_x,n) = \frac{1}{2\pi}\int_{-\infty}^{+\infty}\int_{0}^{2\pi} f(x,\theta)e^{-jk_x x}\, e^{-jn\theta}\, d\theta dx \qquad (12)$$

Where $\tilde{\tilde{f}}(k_x,n)$ denotes the space Fourier transform of $f(x,\theta)$ and k_x, n are respectively the axial wavenumber and the circumferential wavenumber.

The Flügge equations can be written in the wavenumber space as:

$$\begin{bmatrix} \tilde{\tilde{Z}}_{11} & \tilde{\tilde{Z}}_{12} & \tilde{\tilde{Z}}_{13} \\ \tilde{\tilde{Z}}_{21} & \tilde{\tilde{Z}}_{22} & \tilde{\tilde{Z}}_{23} \\ \tilde{\tilde{Z}}_{31} & \tilde{\tilde{Z}}_{32} & \tilde{\tilde{Z}}_{33} \end{bmatrix} \begin{bmatrix} \tilde{\tilde{U}}_x^s \\ \tilde{\tilde{U}}_\theta^s \\ \tilde{\tilde{U}}_r^s \end{bmatrix} = \begin{bmatrix} \tilde{\tilde{F}}_{xx} \\ \tilde{\tilde{F}}_{\theta\theta} \\ \tilde{\tilde{F}}_{rr} \end{bmatrix} - \frac{1-\nu^2}{Eh_c} {R_c}^2 \begin{bmatrix} 0 \\ 0 \\ \tilde{\tilde{p}} \end{bmatrix} \qquad (13)$$

Where $[\tilde{\tilde{Z}}_{3*3}]$ is a square matrix (3x3) which depends on the Flûgge operator and the system parameters. Analytical expressions of $\tilde{\tilde{Z}}$ are given in [1].

The spectral pressure at the interface is related to the spectral radial displacement by:

$$\tilde{\tilde{p}}\,(k_x,r,n) = -j\omega\tilde{\tilde{Z}}_{fluid}\tilde{\tilde{U}}_r^s(k_x,n) \qquad (14)$$

With $\tilde{\tilde{Z}}_{fluide} = j\rho_0\omega\frac{J_n(k_r r)}{k_r J_n'(k_r R_c)}$ and $k_r = \sqrt{k_0^2 - k_x^2}$. Introducing Eq. (14) in (13) and solving the resulting equation system gives us the spectral displacements.

The spectral rotation $\tilde{\tilde{\varphi}}^s$ is calculated from $\varphi^s = \frac{\partial U_r^s}{\partial x}$ by:

$$\tilde{\tilde{\varphi}}^s = jk_x\tilde{\tilde{U}}_r^s \qquad (15)$$

2.2.3 Numerical resolution with an accelerated spectral approach

Applying a discrete inverse Fourier transform to these spectral displacements, we can deduce the circumferential displacements and finally, the shell admittances with Eq. (14). However, for applying this discrete inverse Fourier transform, it is required to truncate the wavenumber space with a cut-off wavenumber, k_{xmax}.This number is defined considering the natural wavenumbers associated to the shell and the acoustic fluid: $k_{mxax} = A_{xmax} max[k_f, k_0, k_l]$, $A_{xmax}$is a margin coefficient, $k_f = \sqrt{\frac{\sqrt{12}}{h}k_l}$ the natural wavenumber for the flexural motions and $k_l = \omega\sqrt{\frac{\rho(1-\nu^2)}{E^*}}$ the natural wavenumber for the longitudinal motions.

In the past [4], it has been shown that accurate results were obtained with a margin coefficient of 1.5. However, these studies considered only the coupling along the radial direction. In the present work, we take into account the four degrees of freedom (axial/tangential/radial displacements and angular rotation) for the coupling between the shell and the stiffeners. It results that one should in particular estimate the admittance, $\tilde{Y}_{\varphi_i M_j}^s$. The estimation of this one by the inverse Fourier transform can lead to numerical errors due to a slow convergence of the spectral rotation in the wavenumber domain. This can be highlighted by replacing the Flügge model by the Love-Kircchoff model. In this case, the spectral radial displacement of the shell excited by a radial force is given by:

$$\tilde{\tilde{U}}_r^s(k_x,\mathrm{n}) = \frac{1}{\mathrm{D}\left[k_x^2 + (\frac{n}{R_c})^2\right]^2 - \rho\omega^2 h + \tilde{\tilde{Z}}_{fluide}} \qquad (16)$$

Whereas the spectral angular rotation of the shell excited by a moment is given by:

$$\tilde{\tilde{\varphi}}^s(k_x,\mathrm{n}) = \frac{k_x^2}{\mathrm{D}\left[k_x^2 + (\frac{n}{R_c})^2\right]^2 - \rho\omega^2 h + \tilde{\tilde{Z}}_{fluide}} \qquad (17)$$

One can observe that the decrease of $\tilde{\tilde{U}}_r^c$ for large wavenumber *kx* is about $1/_{k_x^4}$ whereas the decrease of $\tilde{\tilde{\varphi}}^s$ is about $1/_{k_x^2}$.The decrease is slower for the spectral rotation for a moment excitation, $\tilde{\tilde{\varphi}}^s$, than for the spectral radial displacement for a radial force excitation, $\tilde{\tilde{U}}_r^s$. Obviously, in order to enhance the convergence about k_x, we should truncate the wavenumber space by using a higher cut-off wavenumber before applying the discrete inverse Fourier transform. This can however lead to significant computing time. In order to overcome this issue, we propose an accelerated spectral approach. This one is based on the fact that the spectral rotation in the high wavenumber domain are not strongly dependent on the frequency.

The spectral accelerated approach is performed in three steps:

(1) The first step is based on an accurate estimation of the circumferential rotation $\tilde{\varphi}_M^{reference}$ at a frequency, called the reference frequency. A high margin coefficient is used for this calculation (typically, $A_{xmax} = 100$) to ensure a good convergence.

(2) For each frequency of the frequency band of interest, one defines the spectral rotation discrepancy by:

$$\delta\tilde{\tilde{\varphi}}_M^s(k_x, \mathrm{n}) = \tilde{\tilde{\varphi}}_M(k_x, \mathrm{n}) - \tilde{\tilde{\varphi}}_M^{reference}(k_x, \mathrm{n}) \quad (18)$$

This quantity decreases quickly in the wavenumber domain. One can then easily estimate the circumferential rotation discrepancy $\delta\tilde{\varphi}_M^s(\mathrm{x}, \mathrm{n})$ by the inverse Fourier transform after truncating the wavenumber space by the standard cut-off wavenumber

(3) The circumferential rotation is then finally deduced with :

$$\tilde{\varphi}_M(\mathrm{x}, \mathrm{n}) = \delta\tilde{\varphi}_M^s(\mathrm{x}, \mathrm{n}) + \tilde{\varphi}_M^{reference}(\mathrm{x}, \mathrm{n}) \quad (19)$$

The Fig.3 shows the amplitude of the admittance of a shell, $\tilde{Y}_{\varphi\,M}^{s}$, for the circumferential mode orders n=2, (a) and n=3, (b). The admittance is calculated with and without acceleration for different values of the margin coefficient.

As shown in the figure, in the case of a calculation without acceleration of convergence, the results converge very slowly when A_{xmax}= 3 increases. A high margin coefficient, A_{xmax}= 100, permits to have accurate results but it led to prohibitive computation time, *t*= 0.2177 s. In contrary, the calculation with the acceleration converges correctly with only A_{xmax}= 1.5. Indeed, it gives the same results as the calculation without acceleration using a high margin coefficient. In conclusion, the acceleration approach has proven very good results in terms of accuracy and calculation cost, *t*= 0.0340 s.

	$n = 2$	$n = 3$
Without acceleration, $A_{xmax} = 3$	0.0237 s	0.0211s
Without acceleration, $A_{xmax} = 10$	0.0481 s	0.0489 s
Without acceleration, $A_{xmax} = 100$	0.2177 s	0.2262 s
Reference solution	0.2028 s	0.4368 s
With acceleration, $A_{xmax} = 1.5$	0.0340 s	0.0685 s

Table 1. Time computation of the calculation of the input admittance of a shell, $\tilde{Y}_{\varphi\,M}^{s}$ with/without acceleration.

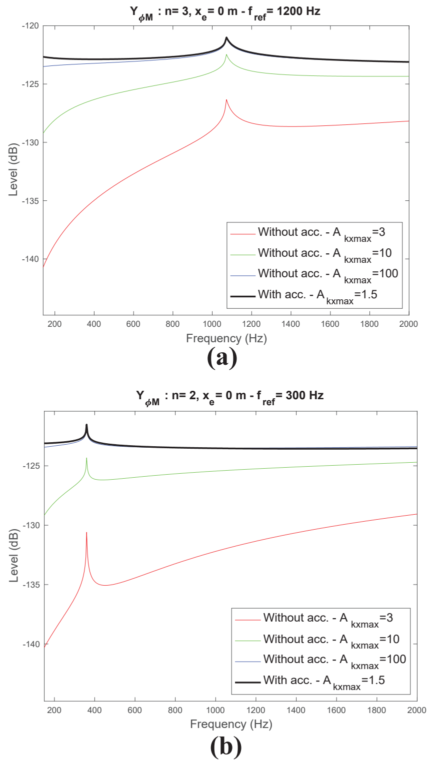

Figure 3. Comparison of the amplitude of the input admittance $\tilde{Y}_{\varphi\,M}^{s}$ for different values of cut-off wavenumber K_{xmax} and with/without acceleration for the circumferential orders: n=*2*, (a) and n=*3*, (b).

For comparison purposes, a reference calculation is made for the circumferential order n=*2* at the reference frequencies 100 Hz and 1000 Hz, which correspond respectively to the cases where the reference frequency is lower and higher than the frequency of appearance of this circumferential order. As shown in the Fig.4, the decrease of delta function about k_x is greater than the decrease of the spectral rotation. Therefore, that makes the rotation in the physical space easily retrieved by applying a *2D*-inverse Fourier transform. On the other hand, at 1000 Hz the reference frequency resolution ensures to describe properly the resonance peaks of the system. One can deduce that to apply correctly the accelerated approach, the spectral reference rotation should be calculated above the frequency of appearance of the higher circumferential order considered.

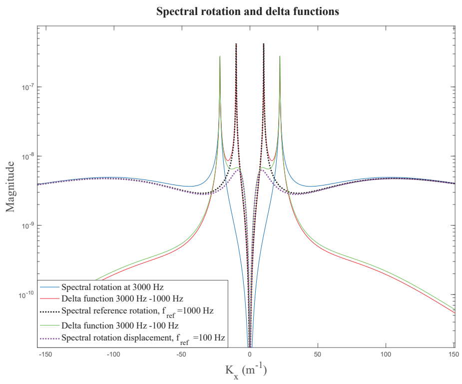

Figure 4. Illustration of spectral rotation, $\tilde{\tilde{\varphi}}_M$ at 3000 Hz, delta functions, $\delta\tilde{\tilde{\varphi}}_M^s$ and spectral reference rotation, $\tilde{\tilde{\varphi}}_M^{reference}$ at $f_{reference}$= 100 Hz and $f_{reference}$= 1000 Hz.

2.3 Example of stiffened shell coupled to fluid

A Finite elements model was developed to check the validity and accuracy of the accelerated approach. A comparison of the results of present approach (b) and the ones obtained by the FEM (c) is shown in Fig.5. It can be observed that the results of present method are in well agreement with the results calculated by FEM. Which indicates the present is a more accurate method.

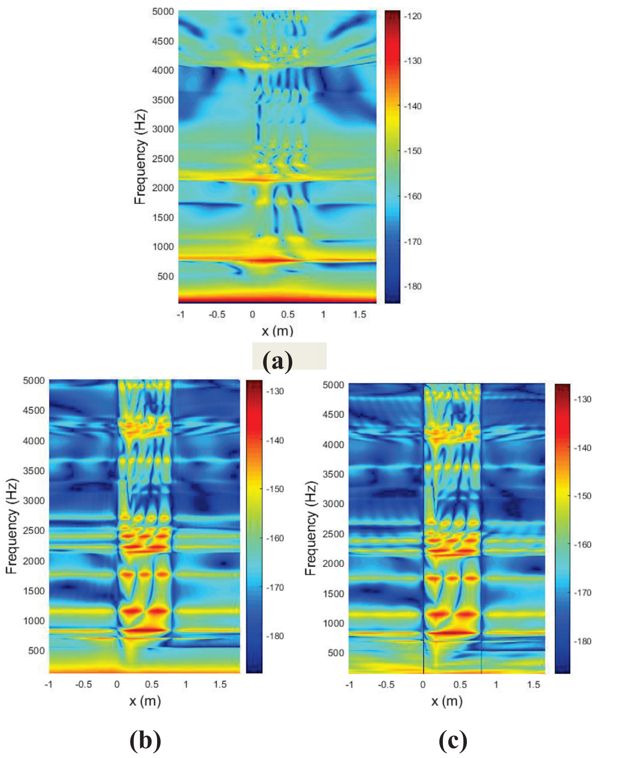

Figure 5. Radial displacement of a shell in vacuo coupled to two stiffeners and excited by a radial force point: model developed without acceleration (a) / Model developed with acceleration (b) / Finite Element Model (c).

3. CONCLUSION

A low frequency analytical model to predict the dynamic behavior of fluid-loaded pipe has been presented. The problem was modelled as a fluid loaded infinitely long shell coupled to two ring stiffeners. The CAA approach has been used for partitioning the problem in two sub-systems so the fluid loaded shell constitutes one subsystem and the ring stiffeners constitute another subsystem. The two subsystems are coupled together by assembling the circumferential admittance calculated for each of the two uncoupled subsystems considering the four degrees of freedom between the fluid-loaded shell and the stiffeners. The cylindrical shell is then described by the Flügge model whereas the ring stiffeners is described by the circular plate model. A convergence issue for estimating the shell admittances associated with the angular rotation was highlighted. In order to tackle this problem, an accelerated convergence approach is proposed based on a reference calculation describing the high wavenumber components. To validate the proposed approach, a comparison with a finite elements model has been presented. The proposed approach has proven very good results in terms of accuracy and calculation cost.

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