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► **To cite this version:**

Tong Zhou, Jean-Daniel Chazot, Emmanuel Perrey-Debain, Li Cheng. Partition of Unity Finite Element Method for the Modeling of Structural Vibrations. Forum Acusticum, Dec 2020, Lyon, France. pp.2141-2142, 10.48465/fa.2020.0334 . hal-03235441

HAL Id: hal-03235441

<https://hal.science/hal-03235441>

Submitted on 27 May 2021

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Partition of Unity Finite Element Method for the Modeling of Structural Vibrations

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EXTENDED ABSTRACT

A novel conforming thin plate bending element based on the Partition of Unity Finite Element Method (PUFEM) [1] for solving the forced structural vibrations is presented. The inter-element continuity of displacement and slope is imposed by using Hermite-type Partition of Unity (PU) functions (the components of Hermite functions only associated with the nodal displacement [2]). With the appropriate C^1 PU functions, the PUFEM allows the inclusion of auxiliary enrichment functions into the finite elements space which better capture the plate oscillating behavior. The type of enrichment considered in this work consists of the sum of a power series up to a given order and a finite combination of progressive flexural wave solutions. The applicability and the effectiveness of the PUFEM plate elements are investigated via the frequency response and convergence analysis. It is shown that the method provides very accurate results with better convergence and data reduction, as compared with the classical FEM.

Most existing PUFEM works on acoustics [3] and structural dynamics [4] adopt the piecewise Lagrangian shape functions as PU functions. These functions only ensure C^0 continuity due to the jump of the slope at element interfaces. The Hermite shape functions in classical FEM satisfy the displacement and slope continuity requirement, but these functions do not form a partition of unity [4]. However, the components of Hermite functions associated with the nodal displacement form a partition of unity and we can utilize these PU functions for the development of PUFEM thin plate elements. We may note that from the perspective of the development in classical FEM, there are few works on element shape functions with C^1 continuity compared with the more classical C^0 family.

In the present work, we focus on the PUFEM formulation based on conforming thin plate bending elements. The formulation is first applied for solving a one-dimensional thin plate bending vibration problem, where the wave motion relies only on one spatial coordinate. Figure 1 compares the convergence curves obtained with classical FEM and PUFEM with different enrichment strategies. Results clearly show that enriched plate elements can

provide better computational performance, both in terms of accuracy and data reduction, than classical finite elements.

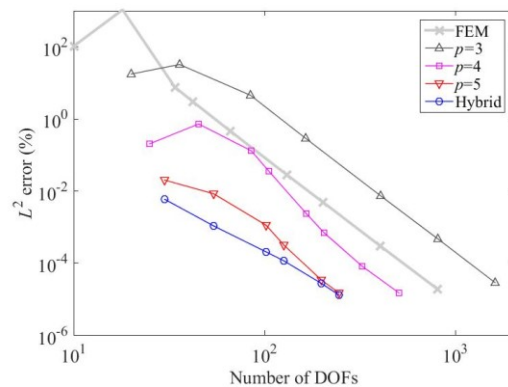


Figure 1. Convergence curves with mesh refinement for classical FEM, the PUFEM with polynomial enrichment of different orders p and hybrid enrichment combining progressive waves and cubic polynomials ($f=1000$ Hz).

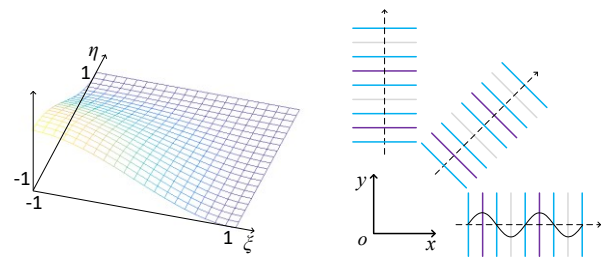


Figure 2. Left: Hermite-type PU function associated with node $(-1,-1)$. Right: plane-progressive flexural wave enrichment functions (over one quarter circle).

Following previous results related to the 1D case, a two-dimensional rectangular PUFEM plate element is developed. We shall investigate a hybrid wave-polynomial enrichment that consists of a finite sum of bending waves propagating in different directions and a power series with respect to spatial coordinates. Increasing the plane wave distributions attached at each node leads to a hierarchic refinement for the 2D wave field. The PU functions and wave enrichment functions

are illustrated in Figure 2. Classical FEM using cubic Hermite shape functions is also considered for comparisons. We use the Lagrange multiplier technique in order to prescribe the boundary conditions at the edges of the plate. The method is tested on simply supported rectangular plates with (i) a singular force acting on the plate and (ii) an uniformly distributed load.

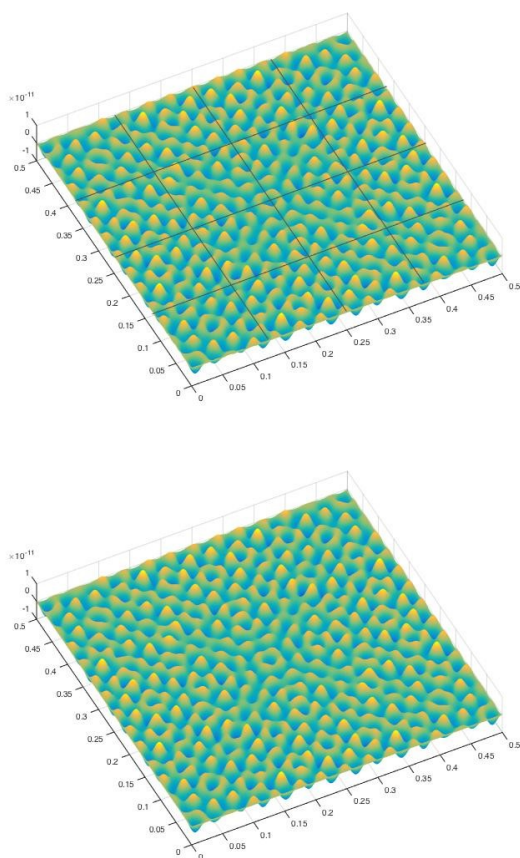


Figure 3. Upper: PUFEM wave-polynomial enrichment using 16 elements. Bottom: reference solution calculated with the modal superposition method.

Numerical results show that the PUFEM plate elements can provide highly accurate predictions over a broad frequency range with a relatively small number of degrees of freedom when compared with classical FEM, and they are also capable to handle non-uniform meshes. The convergence study shows that the PUFEM exhibits a very high rate of convergence in comparison with classical finite elements. It is also found that the polynomial enrichment can provide a good approximation for evanescent or nearly-singular fields in the vicinity of a singular load point and near the borders of the plates. Figure 3 compares structural responses calculated by the PUFEM wave-polynomial enrichment and modal superposition approach when $kh=30$ (k is the wavenumber and h is the element side length). There is a

good agreement between the simulated result and the reference solution and PUFEM can capture many wavelengths by using a low number of DoFs (the number of DoFs per wavelength is about 3 in this example).

The Partition of Unity formulation developed in this work allows constructing a wide family of element shape functions with sufficient regularity for computing the propagation of flexural waves in thin plate structures. Our future works include the development of thin plate elements in order to take into account variable structural thickness and this could be done following previous work by the present authors [6].

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