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# INFLUENCE OF CRANDALL LINES ON ERGODIC BILLIARD IN STATISTICAL ENERGY ANALYSIS

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## ABSTRACT

The main relationship of statistical energy analysis is the so-called coupling power proportionality (CPP) which states that the mean power exchanged between two subsystems is proportional to the difference of their modal energies. A very important condition for the validity of CPP is that the vibrational field must be perfectly diffuse in all subsystems. This is a special state of vibration for which the vibrational energy is homogeneously and isotropically distributed. There are two ways to obtain a diffuse field. Firstly, if the excitation is composed of many uncorrelated forces on the whole plate (like rain-on-the-roof), then the field is forced to be diffuse. Secondly, if the excitation is a single point force and the geometry of subsystem is an ergodic billiard, then homogeneity and isotropy of the vibrational field are a priori enforced. However, even when diffuseness of the field is enforced by ergodicity, certain special lines called Crandall lines present a higher level of energy. On these lines, rays interfere constructively, leading to an enhancement of the vibrational energy. We show that if the coupling between the two plates is fixed on one of these lines of high energy, the CPP slightly differs from the theoretical value predicted by SEA. Numerical simulations are provided to support these results.

## 1. INTRODUCTION

Statistical energy analysis is a statistical theory of sound and vibration developed by Lyon and co-workers in the sixties to estimate the vibroacoustic response of structures in the high frequency range [1, 2, 3]. Equations of statistical energy analysis are based on an energy balance in each subsystem. The principle is to analyse the exchanges of vibrational energies between each subsystems [4, 5, 6, 7, 8]. The main result of SEA is the coupling power proportionality (CPP) which states that the mean power exchanged between two subsystems is proportional to the difference of modal energies.

However there exists some assumptions to apply successfully SEA [9]. One of the most restrictive assumption is the diffusivity of the vibrational field. A vibrational field is diffuse if it is ergodic (homogeneous and isotropic) [10]. However, for certain simple structures there exists some special regions (called Crandall lines) where rays inter-

fere constructively, leading to an enhancement of vibrational level of energy [11].

In this study, we highlight the importance of diffuse field assumption in SEA even if subsystems are geometrically ergodic. For this purpose, we calculate the coupling power proportionality (CPP) for different configurations. Then, these results are compared to a reference calculation obtained with numerical simulation.

## 2. STATISTICAL ENERGY ANALYSIS

Statistical energy analysis consists in subdividing complex systems into simple subsystems (like plates or beams) and studying the exchange of vibrational energies. We consider a system composed of two coupled subsystems coupled by a spring. The vibrational source apply on the first plate is random and have a flat spectrum in the band  $\Delta\omega$  centred on  $\omega$ .

The modal density of plate  $i$  defined as the number of modes per unit circular frequency  $\omega$  (rad/s) is

$$n_i = \frac{A_i}{4\pi} \sqrt{\frac{m_i}{D_i}} \quad (1)$$

where  $m_i$  is the mass per unit area,  $A_i$  the plate area, and  $D_i$  the bending stiffness. Then, the number of resonant modes in the frequency bands  $\Delta\omega$  is  $N_i = n_i \Delta\omega$ .

The coupling power proportionality states that the exchanged power  $\langle P_{ij} \rangle$  is proportional to the difference of vibrational energies  $\langle E_i \rangle$  and  $\langle E_j \rangle$ .

$$P_{ij} = \beta_{ij} \left( \frac{E_i}{n_i} - \frac{E_j}{n_j} \right) \quad (2)$$

The conductivity factor  $\beta_{ij}$  verifies reciprocity  $\beta_{ij} = \beta_{ji}$ . For two plates coupled by a spring the conductivity factor is given by Mace [12]

$$\beta_{ij} = \frac{K^2}{32\pi\omega^2 \sqrt{m_i D_i m_j D_j}} \quad (3)$$

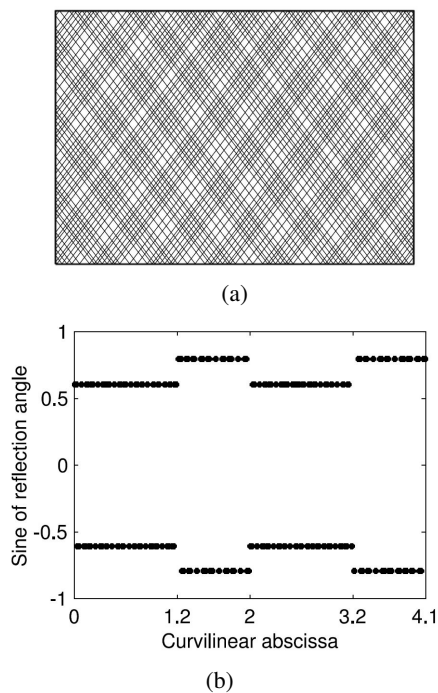
with  $K$  the stiffness of the coupling spring.

## 3. DIFFUSE FIELD AND CRANDALL LINES

### 3.1 Ray dynamics

To ensure the validity of SEA, vibrational field has to be diffuse in subsystems. In other words, each rays has to ex-

plore each point of the plate in each directions during its travel. Then, rays passes through the entire phase space. Figure 1a presents a ray path in a rectangular plate excited by a single point with high directivity. On each edge, the incidence can only take two values of opposite signs and so the angular momentum can take four values as can be observed from figure 1b. About the position space, a ray explore the entire domain and the probability density function is uniform. Therefore, the vibrational field in a rectangular plate is homogeneous but not isotropic.



**Figure 1:** (a) Ray propagation in rectangular plate; (b) Poincare's section

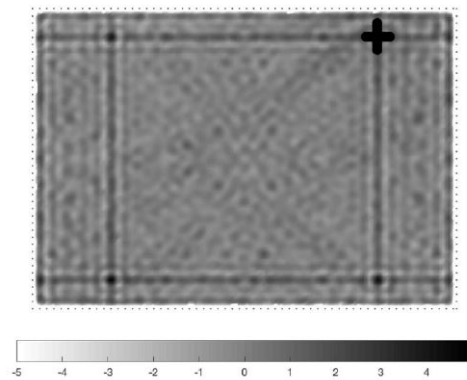
However, a point force sends rays in all directions with a uniform density probability so isotropy is enforced into the vibrational field. Figure 2 shows maps of energy level (dB, ref = mean value) and probability density distribution of energy level in the rectangular plate excited with a random noise centred on the octave 4 kHz. The vibrational field of a rectangular plate excited with a point force is homogeneous and isotropic and so the field is diffused in the plate.

However, even is the field look diffuse, there is a enhancement of the level of energy on four 'special' lines called Crandall lines.

### 3.2 Crandall lines

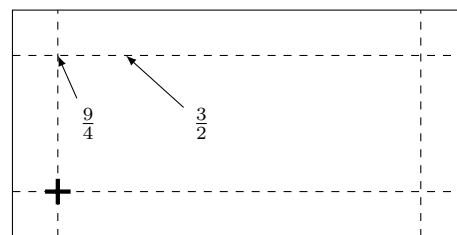
For certain simple configurations, like a rectangular plate, there exist lines with an enhancement of the vibration levels. Crandall lines has been reported for the first time by Crandall in 1978. These lines can be highlighted using sand spread on the plate.

Let us consider a rectangular plate excited with random noise. At a general point in the plate, rays interfere incoherently so the response is uniform. Along these specials



**Figure 2:** Distribution of energy density level for the rectangular plate. The cross indicates the force position.

lines, rays interfere constructively and the vibration level is  $3/2$  times the background level. At the intersection of the lines, the energy density is  $9/4$  times the background level as shown on figure 3. It is easy to predict the presence of these lines of enhance response using a reverberation field method of analysis.



**Figure 3:** Relative mean-square velocity distribution for rectangular plate. The cross indicates the force position.

## 4. NUMERICAL SIMULATION

We now examine the validity of the coupling power proportionality (CPP) for different cases to check if Crandall lines affect the CPP.

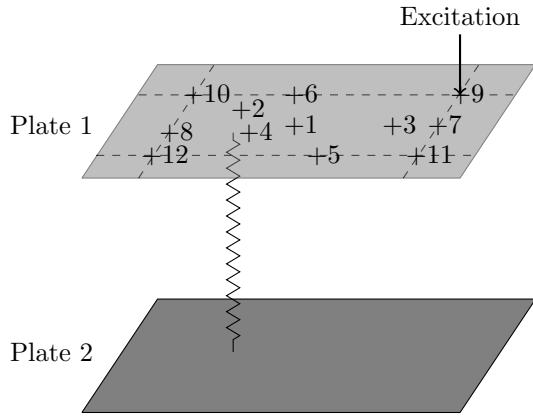
### 4.1 Presentation of the different cases

The system studied is composed of two rectangular plates coupled with a spring as shown in figure 4.

Three different simulations are carried out, with three different attachment point of the spring on the first plate, to check whether or not the CPP is verified. In the first case, the spring is fixed on 'general' point, out of Crandall lines. In the second case, the spring is fixed on a Crandall line and finally, in the third case the spring is fixed at the intersection of two Crandall lines. The other side is fixed to the centre on plate 2.

### 4.2 Parameters of simulation

Both of the plates are made of steel ( $E_0=210$  Gpa,  $\rho=7800$  kg.m<sup>-3</sup>,  $\mu=0.3$  and  $\eta=0.001$ ) The spring stiff-



**Figure 4:** Two plates connected by a spring. Plate 1 is excited with a random excitation. Crandall lines are represented with dashed lines.

ness is  $K=1.10^{-5}\text{N.m}^{-1}$ . Plate 1 is excited by a point force with a band of frequency of an octave centred on  $\omega_c=2\pi\times 4000\text{ rad.s}^{-1}$ . Both plates are 0.841 m wide and a length of 1.189 m. The thickness of the plate 1 is 2mm and 2.5mm for the plate 2.

## 5. RESULTS AND COMMENTS

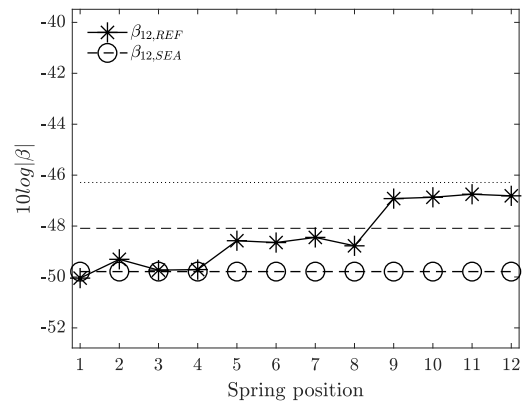
Figure 5 presents the ratio  $\beta$  of exchanged power calculated respectively from Eq.3 and by the reference calculation in the case of two coupled rectangular plates for different attachment positions of the spring on plate 1. The first four points represent case A, the following four for case B and the last four points for case C. Let's analyse the results for these three cases.

### (A) Spring attached near the center of the plate 1.

In this case, there is no enhancement of vibrational energy and so  $\beta_{1,2,SEA}$  is found to be in good agreement with  $\beta_{1,2,REF}$ .

**(B) Spring attached on a Crandall line.** In this case, the vibration level is  $3/2$  times the background level and so the expected value is  $10\log(\frac{3}{2}) \approx 1.76$  superior to the case (A). Dashed line of figure 5 represents this expected value. The results (spring position 5 to 8) shown are in agreement with this prediction.

**(C) Spring attached at the intersection of two Crandall lines.** In this case, the vibration level is  $9/4$  times the background level and so the expected value is  $10\log(\frac{9}{4}) \approx 3.52$  superior to the case (A). Dotted line of figure 5 represents this expected value. Again, results (spring position 9 to 12) of the calculation are in fine agreement with the prediction.



**Figure 5:** Comparison of energy transfer predicted by statistical energy analysis and reference calculation for different positions of spring attachment on plate 1.

## 6. CONCLUSION

In statistical energy analysis, diffuse field is an important assumption. However, we have shown that even for an ergodic system, there is an enhancement of the vibrational energy on Crandall lines and so the field is not perfectly diffused. Then, CPP slightly differs from the theoretical value predicted by SEA.

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