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Finite elements in Fourier transformed domain for the simulation of trains in tunnels

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ABSTRACT

For the simulation of railway tunnels in an anisotropic horizontally layered medium a combined approach with boundary and finite elements is presented. To reduce the numerical efforts it assumed that the structure is straight and infinitely long. In this case, a Fourier transformation about the longitudinal coordinate reduces the dimensionality. In the 2.5D approach, a sequence of 2D simulations is needed instead of one large simulation in 3D [1]. The soil is simulated by the boundary element method. The structure is divided into finite elements in 2.5D. In first attempt an analytic solution for a thin plane shell element was used to simulate the tunnel shell [2-5]. The derivation of a plane triangular element for an isotropic medium for the super structure and the tunnel wall, and a point element for the rail in the 2.5D domain are derived. The derivation is based on a modification of Plancherel's theorem. This theorem leads to a coupling of the positive and negative wave numbers. A derivation with respect to the negative wave number side leads to a set of linear equations that depend on the unknowns at the positive wave number side.

1. INTRODUCTION

The idea of the project is to allow the simulation of vibrations from railway tunnels in a layered anisotropic medium. The problem is split into two parts. In the first part is a boundary element method that simulates the vibrations in a horizontally layered anisotropic medium in 2.5D. In this part, the tunnel is simulated as a hole in the layered medium.

The second part is the simulation of the tunnel structure and the superstructure from the base to the rail using a finite element approach in 2.5D.

The final step is the coupling of the finite element part with the boundary element part. This done by coupling the nodal displacements and the stresses at the interface.

2. DERIVATION OF A FEM ELEMENT

2.1 Potential for an isotropic medium

Eqn. (1) presents the potential Π in the frequency domain for an isotropic medium in 3D. The displacement in the orthogonal directions x, y, z are u_x, u_y, u_z . λ and μ are the Lamé's material parameter ρ is the mass density and ω is the angular frequency. A is the area of the element in the y - z plane. The load on the element is given by q_x, q_y, q_z .

$$\begin{aligned} \Pi = \int \int_{(t) x=-\infty(A)} \int \left\{ \frac{1}{2} \left[(\lambda + 2\mu) (u_{x,x}^2 + u_{y,y}^2 + u_{z,z}^2) + \right. \right. \\ \left. \left. + 2\lambda (u_{x,x} u_{y,y} + u_{x,x} u_{z,z} + u_{y,y} u_{z,z}) + \right. \right. \\ \left. \left. + \mu \left((u_{x,y} + u_{y,x})^2 + (u_{x,z} + u_{z,x})^2 + (u_{y,z} + u_{z,y})^2 \right) \right] \right\} \\ \left. + \frac{1}{2} \omega^2 \rho (u_x^2 + u_y^2 + u_z^2) - (q_x u_x + q_y u_y + q_z u_z) \right\} \\ dA dx d\omega \end{aligned} \quad (1)$$

Eqn. (2) presents a modification of Plancherel's theorem.

$$\int_{-\infty}^{\infty} f(x) h(x) dx = \frac{1}{4\pi^2} \int_{-\infty}^{\infty} \hat{f}(k_x) \hat{h}(-k_x) dk_x \quad (2)$$

With this theorem, it is easy to transform the potential to the frequency wavenumber domain. The coordinate x is transformed to the wavenumber k_x . A symmetric version is given in Eqn. (3).

$$\begin{aligned} \Pi = \frac{1}{2\pi} \int \int_{(\omega) k_x=-\infty(A)} \int \left\{ (\lambda + 2\mu) \left[\begin{aligned} &k_x^2 \hat{u}_x(k_x) \hat{u}_x(-k_x) \\ &+ \hat{u}_{y,y}(k_x) \hat{u}_{y,y}(-k_x) \\ &+ \hat{u}_{z,z}(k_x) \hat{u}_{z,z}(-k_x) \end{aligned} \right] + \right. \\ \left. + \lambda \left[\begin{aligned} &jk_x \hat{u}_x(k_x) \hat{u}_{y,y}(-k_x) - jk_x \hat{u}_x(-k_x) \hat{u}_{y,y}(k_x) \\ &+ jk_x \hat{u}_x(k_x) \hat{u}_{z,z}(-k_x) - jk_x \hat{u}_x(-k_x) \hat{u}_{z,z}(k_x) \\ &+ \hat{u}_{y,y}(k_x) \hat{u}_{z,z}(-k_x) + \hat{u}_{y,y}(-k_x) \hat{u}_{z,z}(k_x) \end{aligned} \right] + \right. \\ \left. + \mu \left[\begin{aligned} &(\hat{u}_{x,y}(k_x) + jk_x \hat{u}_y(k_x)) (\hat{u}_{x,y}(-k_x) - jk_x \hat{u}_y(-k_x)) \\ &+ (\hat{u}_{x,z}(k_x) + jk_x \hat{u}_z(k_x)) (\hat{u}_{x,z}(-k_x) - jk_x \hat{u}_z(-k_x)) \\ &+ (\hat{u}_{y,z}(k_x) + \hat{u}_{z,y}(k_x)) (\hat{u}_{y,z}(-k_x) + \hat{u}_{z,y}(-k_x)) \end{aligned} \right] \right. \\ \left. + \rho \omega^2 (\hat{u}_x(k_x) \hat{u}_x(-k_x) + \hat{u}_y(k_x) \hat{u}_y(-k_x) + \hat{u}_z(k_x) \hat{u}_z(-k_x)) - \right. \\ \left. - (q_x(-k_x) \hat{u}_x(k_x) + q_y(-k_x) \hat{u}_y(k_x) + q_z(-k_x) \hat{u}_z(k_x)) \right\} \\ dA dk_x d\omega \end{aligned} \quad (3)$$

Variation is done with respect to the nodal displacement depending on the negative wavenumber $-k_x$. The result are linear equations that depend on the unknown nodal displacements depending on the positive wavenumber $+k_x$.

2.2 Derivation of finite elements in 2.5D

A triangular element is developed. Fig. 1 gives the definition of the nodes of the element. A standard element is defined in Fig. 2 to generalize the equations. The shape functions are linear and identical for the displacements u_x, u_y, u_z . Eqn. (4) defines the shape functions N_i and the nodal displacements u

$$\begin{aligned}\hat{u}_x(\pm k) &= N_1 u_{x1}(\pm k) + N_2 u_{x2}(\pm k) + N_3 u_{x3}(\pm k) \\ \hat{u}_y(\pm k) &= N_1 u_{y1}(\pm k) + N_2 u_{y2}(\pm k) + N_3 u_{y3}(\pm k) \\ \hat{u}_z(\pm k) &= N_1 u_{z1}(\pm k) + N_2 u_{z2}(\pm k) + N_3 u_{z3}(\pm k)\end{aligned}\quad (4)$$

The shape functions are linear and scaled to one at one node and to zero at the other two nodes. For the standardized geometry, the shape functions are defined in Eqn. (5).

$$N_1 = \zeta_1, \quad N_2 = \zeta_2, \quad N_3 = \zeta_3 = 1 - \zeta_1 - \zeta_2 \quad (5)$$

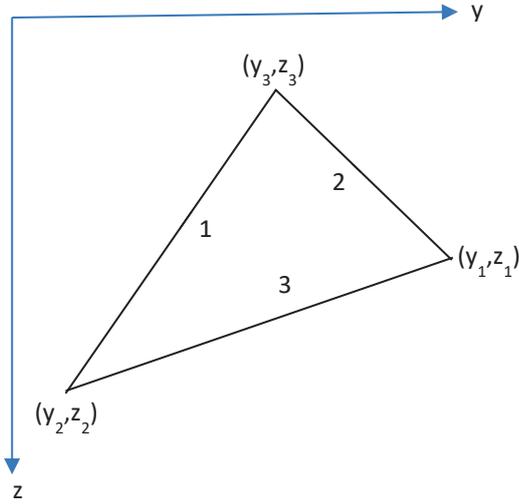


Figure 1. Geometry of the triangular element.

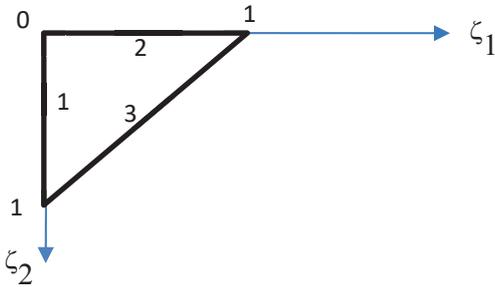


Figure 2. Standardized geometry of the triangular element.

For the original geometry, the definition in Eqn. (6) holds.

$$\begin{aligned}N_i &= a_i + b_i y + c_i z, \quad \frac{\partial N_i}{\partial y} = N_{i,y} = b_i, \quad \frac{\partial N_i}{\partial z} = N_{i,z} = c_i \\ a_1 &= \frac{y_2 z_3 - y_3 z_2}{2A}, \quad a_2 = \frac{y_3 z_1 - y_1 z_3}{2A}, \quad a_3 = \frac{y_1 z_2 - y_2 z_1}{2A} \\ b_1 &= \frac{z_2 - z_3}{2A}, \quad b_2 = \frac{z_3 - z_1}{2A}, \quad b_3 = \frac{z_1 - z_2}{2A} \\ c_1 &= \frac{-y_2 + y_3}{2A}, \quad c_2 = \frac{-y_3 + y_1}{2A}, \quad c_3 = \frac{-y_1 + y_2}{2A}\end{aligned}\quad (6)$$

Two times the area of the triangular element is given in Eqn. (7).

$$2A = |y_1 z_2 - y_2 z_1 + y_3 z_1 - y_1 z_3 + y_2 z_3 - y_3 z_2| \quad (7)$$

In the 2.5D case, we need the integrals about two shape functions and one shape function with a derivative and

about two derivatives. The derivatives of the linear shape function leads to constant expressions. The integrals are defined in Eqn. (8) using the standardized geometry.

$$\begin{aligned}\iint_A f(y, z) dy dz &= 2A \int_0^{1-\zeta_2} \int_0^{\zeta_1} f(\zeta_1, \zeta_2) d\zeta_1 d\zeta_2 \\ 2A \iint_A \zeta_1^p \zeta_2^q \zeta_3^r d\zeta_1 d\zeta_2 &= 2A \frac{p!q!r!}{(p+q+r+2)!} \\ 2A \int_0^{1-\zeta_2} \int_0^{\zeta_1} 1 d\zeta_1 d\zeta_2 &= \frac{2A}{2} \\ 2A \int_0^{1-\zeta_2} \int_0^{\zeta_1} \zeta_1 d\zeta_1 d\zeta_2 &= \frac{2A}{6} \\ 2A \int_0^{1-\zeta_2} \int_0^{\zeta_1} \zeta_i \zeta_j d\zeta_1 d\zeta_2 &= \begin{cases} \frac{2A}{12}, & i = j \\ \frac{2A}{24}, & i \neq j \end{cases}\end{aligned}\quad (8)$$

For the coupling of the stresses with the boundary pressures on the element, the following lengths in Eqn. (9) and integrals in Eqn. (10) are needed.

$$\begin{aligned}l_1 &= \sqrt{(y_2 - y_3)^2 + (z_2 - z_3)^2} \\ l_2 &= \sqrt{(y_3 - y_1)^2 + (z_3 - z_1)^2} \\ l_3 &= \sqrt{(y_1 - y_2)^2 + (z_1 - z_2)^2}\end{aligned}\quad (9)$$

$$\begin{aligned}l_1 \int_0^1 \zeta_2^q \zeta_3^r d\zeta_1 &= l_1 \frac{q!r!}{(q+r+1)!} \\ l_2 \int_0^1 \zeta_1^q \zeta_3^r d\zeta_2 &= l_2 \frac{q!r!}{(q+r+1)!} \\ l_3 \int_0^1 \zeta_1^q \zeta_2^r d\zeta_3 &= l_3 \frac{q!r!}{(q+r+1)!} \\ l_i \int_0^1 \zeta_j^q \zeta_k^r d\zeta_i &= \begin{cases} \frac{l_i}{3}, & j = k \\ \frac{l_i}{6}, & j, k \neq i \end{cases}\end{aligned}\quad (10)$$

These equations allow deriving the elemental forces in dependency of the unknown displacements.

3. COUPLING FEM WITH BEM

The coupling of FEM and BEM is two sided. The pressure loads from BEM a used as a load along the side of the element that couples with the boundary element. The nodal displacements of the BEM and the coupled nodes FEM element are the same and therefore both share the same degrees of freedom (DOFs).

4. CONCLUSIONS

A method is presented to simulate the vibrations in a layered soil caused by railway traffic in a tunnel. Especially the interfaces lead to reflections that can increase the vibration level far away from the tunnel. In the conventional models, these interfaces are not included,

because a classical BEM does not allow for a layered medium.

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