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To cite this version:


HAL Id: hal-03235393
https://hal.science/hal-03235393
Submitted on 27 May 2021

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CONTINUATION OF A BRASS INSTRUMENT MODEL WITH CONSTRAINTS: TOWARDS A FIT OF NUMERICAL AND EXPERIMENTAL RESULTS

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ABSTRACT

Numerical continuation using the Asymptotic Numerical Method (ANM) has been used to analyze and compare brass instruments numerically in previous work. By combining the ANM and Harmonic Balance Method (HBM), branches of periodic solutions can be calculated. Several descriptors could be extracted from the bifurcation diagrams obtained using this approach. The question of experimental validation of the obtained results then becomes an important issue. In this study, measurements were performed on musicians for comparison with the numerical results obtained. A new approach of continuation, with constraints, allows us to fit numerical on experimental bifurcation diagrams by releasing some parameters of the model. The results obtained show the capability of the simple 1D lip model to perform very good fit on experimental results, while providing new information (lip parameter trajectories) likely to bring new elements of comparison of the different musical instruments studied.

1. INTRODUCTION

The system formed by the couple {player - trumpet} falls into the category of non-linear dynamical systems likely to be studied using different numerical tools such as numerical continuation methods. Continuation of periodic solutions using the ANM (Asymptotic Numerical Method) has already been applied to brass instruments [1, 2]. In this study we propose a novel approach of continuation with constraints, which allows to calculate bifurcation diagrams sharing important features of experimental data collected on musicians. In order to perform continuation on this augmented system, one or more parameters of the system must be relaxed so that they become unknowns with respect to the continuation parameter. This approach can therefore be seen as a method of inversion by continuation, allowing to retrieve the evolution of particular parameters of the models, taking into account some constraints to be respected by the outputs.

2. PHYSICAL MODEL OF THE \{PLAYER-TRUMPET\} SYSTEM AND CONTINUATION

We consider a one-dimensional lip model, coupled to the resonator impedance described by a series of complex modes similar to what is proposed in [2]. The coupling between the mechanical oscillator and the acoustic resonator is achieved by a stationary Bernoulli flow equation, considering turbulent mixing in the mouthpiece with no pressure recovery. The mechanical and acoustic equations are given in system 1, where \( y \) is the vertical lip position (\( y_0 \) is the lip position at rest), \( \omega_1, Q_1, \mu \) and \( b \) the mechanical lip parameters, \( s_k \) and \( C_k \) with \( k \in [1, N] \) the modal parameters of the N resonances of the acoustic impedance of the instrument, \( Z \), the characteristic impedance, \( u \) the volume flow, \( p \) the downstream air pressure at the input of the instrument (in the mouthpiece), and \( p_0 \) the upstream (mouth) static pressure.

\[
\begin{aligned}
\dot{y}(t) + \frac{\omega_1^2}{\mu} \dot{y}(t) + \omega_1^2 (y(t) - y_0) = \frac{1}{\mu_1} (p_0 - p(t)) \\
\dot{p}_k(t) = Z_k C_k u(t) + s_k p_k(t), \forall k \in [1, N]
\end{aligned}
\]

with \( p(t) = 2 \sum_{k=1}^{N} \Re(p_k(t)) \) and \( u = \sqrt{2|p_0 - p|/\rho} \cdot \text{sign}(p_0 - p) \cdot \theta(y) \), where \( \theta(y) = \frac{|y + 1/2|}{b} \), \( b \) is the lip width and \( \rho \) the air density.

The case of a negative opening of the lips is managed by introducing the Heaviside function \( \theta(y) \). The modal parameters of the N modes of the impedance are extracted from the measured input impedance using the high resolution method ESPRIT [3].

In this work, we choose to work with the Asymptotic Numerical Method (ANM) implemented in the software MANLAB [4]. This method is based on the expansion of the solutions under the form of truncated Taylor series, providing analytical formulations of the branch of solution. Recently, this method has been associated to the Harmonic Balance Method (HBM) for the search of periodic solutions of oscillating systems [5].

One requirement of MANLAB relies on the recast of...
nonlinearities of the model into, at most, quadratic nonlinearities. The complete quadratic dimensionless model can be found in [2].

3. CONTINUATION WITH CONSTRAINTS

Two constraints are introduced as follows:

\[ \| \tilde{p} \|_{L^2} = S \gamma + I, \]  
\[ \text{with } \gamma = p_0 / P_M \text{ is the dimensionless mouth pressure with } P_M = \mu_0 \omega_0^2 y_0, \text{ and } S \text{ and } I \text{ are constant values, and} \]
\[ \| \tilde{p} \|_{L^2} = 2 \left\| \sum_{k=1}^N \Re(\tilde{p}_k) \right\|_{L^2}. \]

The second constraint simply writes as follows:

\[ f_0 = F, \]

where \( F \) is a constant value.

Adding two equations to the system requires two parameters of the model to be relaxed, that is two unknowns to be introduced. We choose to relax \( Q_L \) and \( \zeta = Z_0 b y_0 \sqrt{\frac{2}{\mu_0 P_M}} \) \( \zeta \) can be seen as an “embouchure” parameter. This requires to recast the system of equations in order to preserve the quadratic property of the model.

4. RESULTS AND CONCLUSIONS

Figure 1 shows the result of continuation with constraints applied to the physical model described in previous section. In the bottom plot, the evolution of \[ \| \tilde{p} \|_{L^2} \] with respect to \( \gamma \) measured on a trumpet player during a slow crescendo-decrescendo maneuver is represented in red. A linear fit of the red curve is applied, which gives \( S \) and \( I \) (Eq. 2) and defines the constraint on \[ \| \tilde{p} \|_{L^2} \]. The constraint on \( f_0 \) is such as it remains constant and equal to the value at the initial calculation point (about the playing frequency of a Bb4).

It can be seen that the two constraints are well respected: \( f_0 \) is constant with respect to \( \gamma \), and \[ \| \tilde{p} \|_{L^2} \] evolves linearly with respect to \( \gamma \) (the solution branches, in blue, are superimposed with the target constraint on Fig. 1 bottom plots). The stability of the branch was computed and the branch was found stable across the whole range of \( \gamma \). The variations of \( \zeta \) and \( Q_L \) are represented on the top plots. Significant variations of these two variables are observed, showing the importance of adapting these parameters in order to match the constraints.

These results highlight the ability of the ANM to calculate the evolution of some parameters of the model while applying some mathematical constraints to the output of the continuation calculation. By defining these constraints from experimental data, this approach can be seen as an inversion method, allowing to retrieve the parameter values of the model necessary to achieve a given performance (playing a crescendo-decrescendo at completely constant playing frequency).

This method then shows great perspectives for the parametrisation of physical models of brass instruments, as well as for objective comparison of brass instruments.

Figure 1. Results from continuation with constraints. Evolution of \( \zeta, Q_L, f_0 \) and \[ \| \tilde{p} \|_{L^2} \] with respect to the dimensionless mouth pressure \( \gamma \). In red is the evolution of \[ \| \tilde{p} \|_{L^2} \] with respect to \( \gamma \) measured on trumpet player during a slow crescendo-decrescendo maneuver.

5. REFERENCES