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NUMERICAL MODELS FOR CLASSICAL GUITARS WITH UPDATED PARAMETERS FROM EXPERIMENTAL DATA

Alexander Brauchler Pascal Ziegler Peter Eberhard

Institute for Engineering and Computational Mechanics, University of Stuttgart, Germany

[alexander.brauchler, pascal.ziegler, peter.eberhard] @itm.uni-stuttgart.de

ABSTRACT

The classical guitar is a popular string instrument in which the sound results from a coupled mechanical process. The oscillation of the plucked strings is transferred through the bridge to the body which acts as an amplifier to radiate the sound of the guitar. In this contribution, a numerical model of a classical guitar is presented. Alongside the numerical modeling, experiments are carried out. The numerical model might be used as virtual prototype to simulate the influence of varying parameters like different bracing patterns or new geometries or to compare different guitars. The numerical finite element model of the guitar consists not only of the guitar body and the neck of the guitar, but also the influence of the air inside the guitar body is taken into account. Furthermore, the struts to reinforce the soundboard and the back of the guitar are included, and orthotropic material properties are considered. The numerical models are compared to experimental measurements and unknown parameters are estimated using experimental data in a model updating scheme. Experimentally, the vibration of the guitar is examined by means of an experimental modal analysis. An experimental setup being capable of determining eigenmodes and eigenfrequencies in a high spatial resolution is presented. For the measurements a scanning laser Doppler vibrometer is used and the guitars are excited via impulses using an electrodynamical shaker.

1. INTRODUCTION

The classical guitar is a popular string instrument in which the sound results from a complex transient process beginning with the oscillation of a plucked string that is then coupled with the guitar body that radiates the sound to the surrounding air. To better understand this transient process, not only measurements shall be carried out, but also a finite element (FE) model shall be created that is able to approximate the vibration of the guitar. The oscillation of a single string is examined in a previous work [1]. The existing literature already includes finite element models of guitars in a high level of detail with well measured material properties of the instrument [2]. Also the influence of the air inside the guitar body is examined [3]. Furthermore, a model updating procedure has been applied to soundboards of string instruments to identify the material parameters [4].

In this contribution the guitar body is examined without any influence from forces of the strings with the goal to

create an FE model that approximates the eigenfrequencies and eigenmodes of a real guitar. For this reason, an experimental modal analysis is carried out to gather knowledge of the eigenfrequencies and the eigenmodes of the particular examined guitar. This particular examined guitar is then modeled in a high level of detail and the eigenmodes and eigenfrequencies of the model are compared with the ones identified in the experiment. A model updating procedure is proposed to identify the only coarsely known material properties of the guitar. This article is divided in an experimental and a numerical part. Section 2 contains the description of an experimental setup to conduct experimental modal analyses on guitars and the identification of modal parameters for one particular guitar model. In Sec. 3 a numerical FE model is developed and a model updating scheme is presented and applied to the particular guitar examined in the experimental modal analysis.

2. EXPERIMENTAL MODAL ANALYSIS

In the following, the experimental setup and the parameter identification procedure to identify the modal parameters of a classical guitar is described. In this investigation a Yamaha GC-12 guitar with cedar top and classical Torres bracing pattern is examined. This model is chosen for its design consisting of all solid woods whose composition can be found on the Yamaha homepage¹. The modal analysis of the guitar body is carried out with the strings unmounted to identify the eigenfrequencies and the eigenmodes as well as the damping and the transmission behavior of the system.

2.1 Experimental Setup

The experimental setup shown in Fig. 1 consists of the guitar mounted with rubber bands and springs on an aluminum frame, as well as an electrodynamical shaker and a laser Doppler vibrometer (LDV). Generally, it is possible to mount any classical guitar on the frame as the attachment of the springs and rubber bands is possible independently of the geometry of a particular guitar. Moreover, the design of the setup makes it easily possible to turn the guitar around and thereby measure the vibration on the front as well as on the back of the guitar. Preliminary experiments and simulations with an FE model showed that determining the oscillation on the soundboard, the fretboard and the

¹ <https://www.yamaha.com/en/>



Figure 1. Setup to conduct an experimental modal analysis of the guitar with approximately free boundary conditions. Front and back of the guitar can be measured.

back of the guitar is necessary to make sure that the identified modes can be clearly distinguished from each other. For this reason the oscillation of the guitar is measured on a total of 245 points, 101 points of which are placed on the back of the guitar and 144 on the soundboard and the fretboard. This results in roughly 3 cm space between two measured points which is considered a good spacial resolution for eigenmodes up to 1000 Hz. The velocity at these predefined points on the guitar is measured with a Polytec PSV-500 scanning LDV which allows the measurement of the different points on the guitar without moving the LDV.

The excitation of the guitar is realized via a modified electrodynamical shaker that acts like an automatic impulse hammer. During measurement the shaker is triggered by a function generator with a short trigger signal which leads to a single hammering motion of the shaker. A soft rubber tip is used to avoid possible damage to the guitar under test and the contact force is measured by a ICP sensor. Throughout the experiment the position of the shaker remains at a constant position on the guitar. Preliminary experiments revealed that the impact on the guitar is very reproducible and excites the guitar body well up to frequencies around 1000 Hz [5].

2.2 Parameter Identification

In general, many different approaches for the experimental modal analysis exist and have been applied to musical instruments. Here, the complex mode indicator function (CMIF) in combination with the enhanced frequency response function (EFRF) is used for the modal analysis due to its advantage of making use of the fine spacial resolution of the measurements. In the following, the chosen method for the parameter identification shall be briefly explained. A thorough description of the method can be found in [6].

In the performed experiment the force input to the system is measured at one point and the velocity output of the system is measured at 245 points. Accordingly, the matrix of mobilities $\mathbf{Y}(\omega)$ is used to realize the parameter

identification with the modal analysis. From the matrix of mobilities the CMIF is calculated through a singular value decomposition at each considered frequency which results in

$$\text{Re}(\mathbf{Y}(\omega)) = \mathbf{U}(\omega) \mathbf{\Sigma}(\omega) \mathbf{V}(\omega)^H, \quad (1)$$

where $\mathbf{U}(\omega)$ is the matrix of left singular vectors, $\mathbf{V}(\omega)$ is the matrix of right singular vectors and $\mathbf{\Sigma}(\omega)$ is the matrix of singular values. The superscript H denotes the Hermitian of a matrix. For each reference (input position) k one singular value can be calculated and these singular values at the considered frequencies then form the CMIF

$$\text{CMIF}_k(\omega) = \Sigma_k(\omega) \quad \text{with } k = 1, 2, \dots, K, \quad (2)$$

where K is the total number of references. For the conducted experiment this simplifies to one curve as $K = 1$ for only one reference. The resulting CMIF which contains peaks for all modes existing in the data is displayed in Fig. 2 and is used to identify the eigenfrequencies of the system. In the low frequency range up to 18 Hz the eigenfrequencies related to the rigid body modes due to the soft support of the guitar can be found. This is appropriately far away from the first eigenfrequency of the guitar at 89 Hz so that the influence of the support can be neglected.

After identifying the eigenfrequencies ω_r in the CMIF the left singular vectors $\mathbf{u}_r(\omega_r)$ and the right singular vectors $\mathbf{v}_r(\omega_r)$ at these eigenfrequencies are used to transform the matrix of mobilities into a modal domain and form an enhanced frequency response function (EFRF)

$$\bar{\mathbf{Y}}_r(\omega) = \mathbf{u}_r^T \mathbf{Y}(\omega) \mathbf{v}_r \quad (3)$$

for each mode. Note that the left singular vectors $\mathbf{u}_r(\omega_r)$ are already quite good approximations for the mode shapes of the system. The EFRFs for each mode have the advantage that, with a sufficient spacial resolution of the measurements, a curve for each mode is created without any influence of the neighboring modes. Therefore, basic single degree of freedom parameter identification techniques

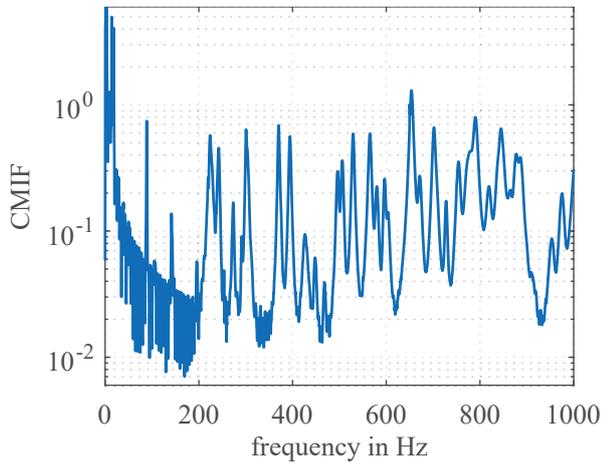


Figure 2. Complex mode indicator function calculated from the experimentally measured mobilities.

are applicable to identify the modal parameters for each mode separately with the EFRF. In the presented case the multi-point peak fitting method is applied on the EFRFs to identify the modal parameters [7, 8].

With the identified modal parameters it is then possible to reconstruct the EFRFs as single degree of freedom systems

$$\bar{Y}_{\text{rec}}(\omega) = \frac{{}_r A(\omega)}{\omega_r^2 - \omega^2 + 2i\omega_r\omega\zeta_r}, \quad (4)$$

where ${}_r A(\omega)$ are the modal constants and ζ_r are the modal damping coefficients. These reconstructed EFRFs then are transformed back into the physical domain and summed up to reconstruct the matrix of mobilities

$$\mathbf{Y}_{\text{rec}}(\omega) = \sum_{r=1}^R \mathbf{u}_r \bar{Y}_{\text{rec}}(\omega) \mathbf{v}_r^T, \quad (5)$$

where R is the number of identified modes in the considered frequency range. The reconstructed mobilities can then be directly compared to the measured mobilities and reveal immediately whether the approximation of the modal parameters is sufficiently good.

In Fig. 3 the reconstructed mobility is compared to the measured mobility in three different points. The first two positions are on the soundboard of the guitar, the first one on the bottom and the second one above the bridge around the middle of the soundboard. The third compared node is approximately in the middle on the back of the guitar. All three reconstructed mobilities approximate their measured counterparts very well and hence, the parameter identification can be rated as successful. Nevertheless, there are three possible modes visible in at least one of the plots which are not identified as such in the CMIF at around 105 Hz, around 700 Hz and around 770 Hz. These modes might be either very close to another more dominant mode or are not well excited at the chosen position for the excitation. Both problems could be solved by including multiple excitation points to the parameter estimation [6].

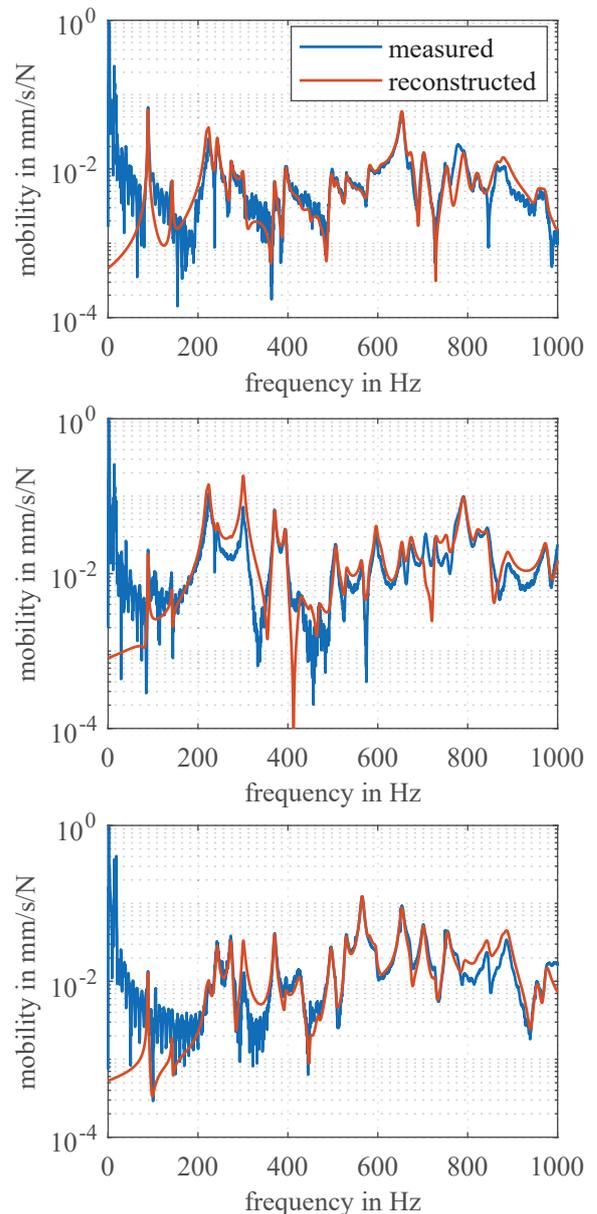


Figure 3. Three measured mobilities in comparison to the related reconstructed mobilities from the experimental modal analysis (top: soundboard bottom, middle: above the bridge, bottom: middle of the back).

In total 32 eigenmodes are identified in the frequency range between 0 and 1000 Hz with modal damping ratios between 0.5 % and 1.5 %. Six exemplary modes are shown in Fig. 4 which are in good agreement with existing literature, see for example [9]. For each mode the front and the back of the guitar are displayed and the sign of the displacement is chosen so that the oscillation is visible as seen from the front. The visible mesh is the mesh of points that are measured during the experiment. These modes are used in the following as reference to compare against numerically calculated ones.

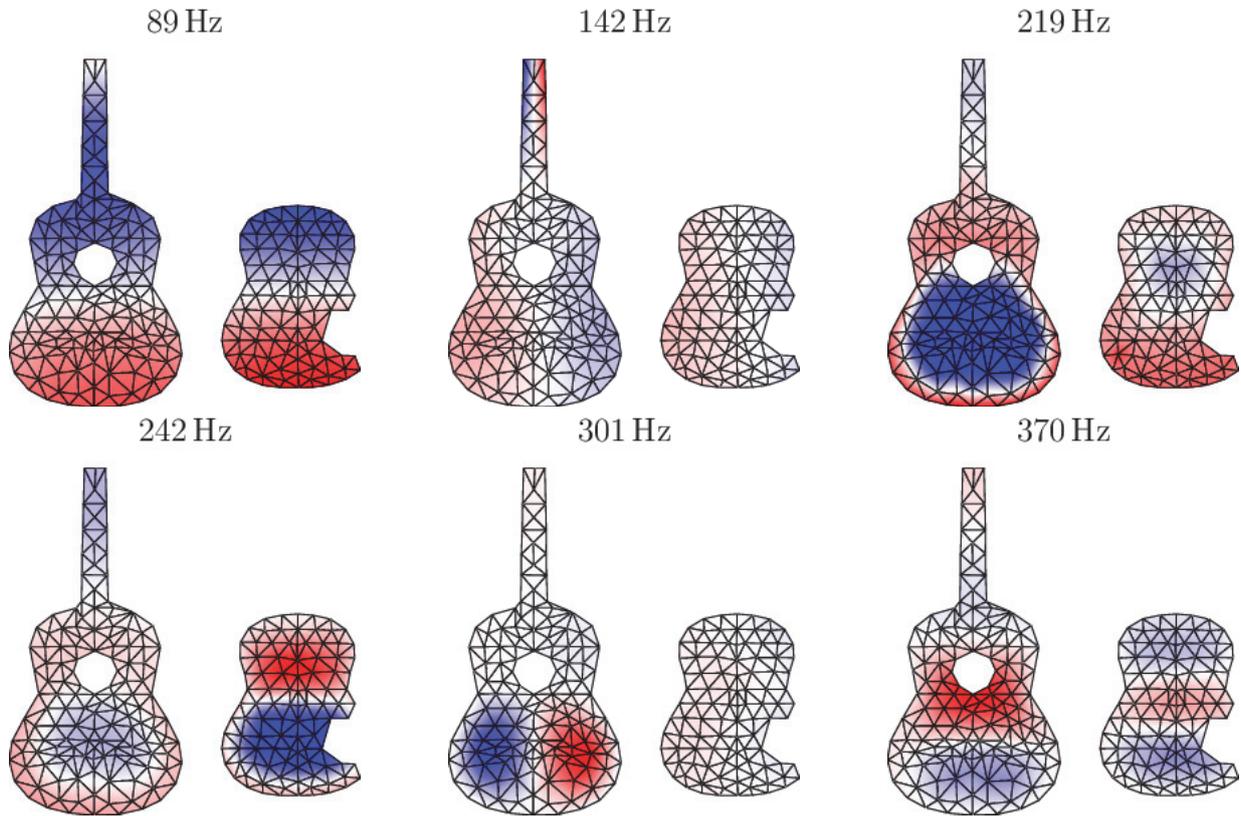


Figure 4. Six modes of the experimental guitar calculated with the modal analysis.

3. FINITE ELEMENT MODEL

In the following the numerical model of the experimentally examined guitar is described. Modeling and simulation are carried out in the commercial software Abaqus and all numerical details can be found in the Abaqus reference [10]. The FE model is created through a reverse engineering process measuring the existing guitar by hand and with the help of X-Ray pictures. The material parameters are taken from literature and then updated in a model updating scheme.

3.1 Modeling Procedure

The geometry of the classical guitar is measured by means of X-Ray pictures and mechanical tools and then modeled in Abaqus in a very high level of detail. Screenshots showing the geometric model can be seen in Fig. 5. All parts of the guitar have been modeled including not only the guitar body with the neck and the fretboard but also all parts and struts inside the guitar body. For the structural parts either volume elements (C3D8) or shell elements (S4) are used. The shell elements are a reasonable choice for the soundboard, the back and the sides of the guitar body as these parts satisfy the condition that these parts' length and width are much greater than their thickness. Besides the parts of the structure, the air inside the guitar body is modeled with acoustic elements (AC3D8) and hence, the coupling between the air and the structure is taken into account. Furthermore, a layer of acoustic infinite elements (ACIN3D4)

is applied to the soundhole surface to model a sufficiently accurate boundary condition at the soundhole. In Fig. 5 the mesh is not visible as it is too fine to be displayed accordingly. However, the model consists of 332604 elements and a convergence analysis has been carried out to make sure that it is meshed fine enough to expect converged results.

The use of different material properties is highlighted in Fig. 5 by the use of distinct colors for each material. The soundboard is made of solid western redcedar while the back, sides as well as the neck and the head are made of solid mahogany. Moreover, the fretboard material is ebony and the bridge is made of rosewood. The struts and other parts inside the guitar body are assumed to be spruce. For all materials the material properties are taken from existing literature in a first step [11]. As stated in previous works, it is necessary to model the full anisotropic material behavior of all the different kinds of wood and the grain direction has to be determined carefully [2, 12].

Because wood is a natural material it is expected that the material parameters vary by a significant amount between different instruments. Thus, the numerical model is not expected to behave precisely like the experimentally examined guitar right away with the material parameters taken from the literature. Nonetheless, these parameters are a good starting point for a model updating procedure in which the material parameters are optimized with respect to a predefined objective function. In total, there are 50 material parameters to identify as there are five different

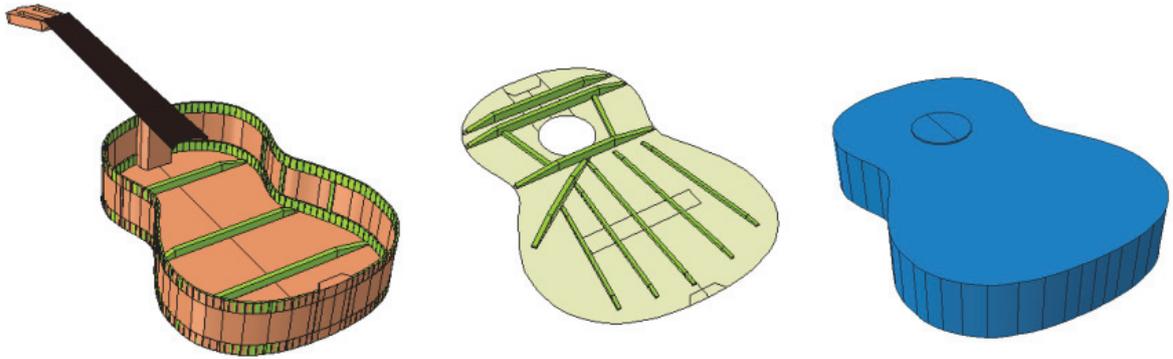


Figure 5. Finite element model of the guitar. The colors visualize the different materials of the guitar and the most right picture displays the simplified air volume inside the guitar.

kinds of wood used in the guitar and each material has got a density as well as three Young's moduli, three Poisson Ratios and three shear moduli. As identifying 50 parameters via numerical optimization is a very demanding task computationally, it makes sense to carry out a sensitivity analysis to find the parameters with the largest influence on the mode shapes and the eigenfrequencies. Such a sensitivity analysis has been carried out in an existing work for a violin [12]. In this publication it is stated that for a constant moisture content in the wood the Young's moduli and the shear moduli are the most influential parameters regarding their influence on the eigenfrequencies.

Following this advice, in the first step of the model updating approach the mode shapes shall be optimized with respect to the Young's moduli along the grain E_L and the shear moduli G_{LT} and G_{LR} to match the mode shapes calculated in the experimental modal analysis. The parameters are varied through factors which can vary between 0.5 and 1.5 for each parameter, hence the material parameters should stay in a realistic interval. Furthermore, the shear moduli for each material are varied through the same factor and, thus, there are in total 10 optimization variables. Considering that in a first step the mode shapes shall be matched, the objective function

$$\max_{\forall \mathbf{p} \in \mathbb{P}} \left(\min_{i=1 \dots N} \max_{j=1 \dots M} \text{MAC}_{ij}(\mathbf{p}) \right) \quad (6)$$

is proposed, where $\mathbb{P} = [0.5, 1.5]^{10}$, N is the number of modes identified in the experiment and M is the number of modes calculated in the numerical model. The term $\text{MAC}_{ij}(\mathbf{p})$ is the modal assurance criterion (MAC) value of the mode pair consisting of experimental mode i and numerical mode j . The MAC values are calculated as

$$\text{MAC}_{ij} = \frac{|\varphi_i^H \varphi_j|^2}{\varphi_i^H \varphi_i \varphi_j^H \varphi_j} \quad (7)$$

where φ_i is the mode shape of the experimental mode i and φ_j is the mode shape of the numerical mode j [6]. The parameter identification through numerical optimization is performed with the standard Matlab function `patternsearch` where in each iteration firstly, the numerical eigenmodes are calculated in Abaqus. Then, the MAC

Table 1. Initial values of the in the model updating scheme updated parameters in GPa and the corresponding update factors p_{E_L} and p_G .

	E_L	G_{LR}	G_{LR}	p_{E_L}	p_G
Spruce	11.9	1.47	1.43	1.13	0.5
Cedar	8.47	0.74	0.73	1.5	1.5
Mahogany	10.7	0.94	0.63	1	0.75
Ebony	17.0	0.95	0.36	0.5	1.19
Rosewood	13.5	0.11	0.10	1.5	1.5

matrix between numerical and experimental modes is computed and the objective function stated in Eq. (6) is evaluated.

3.2 Simulation Results

In this section the results of the model updating scheme are discussed and compared to the results of the experimental modal analysis. In Tab. 1 the initial values of the varied parameters from the literature and the corresponding factors calculated in the model updating scheme are displayed. First and foremost all parameters besides the Young's modulus of mahogany are varied by a significant amount. Another striking fact is that six out of ten factors reach one of the predefined limits.

Six modes calculated with the numerical model are displayed in Fig. 6. These modes correspond to the experimentally identified modes in Fig. 4 and they are shown in the same way as they are compared to the experimental modes. That means, that the numerical modes are evaluated at the nearest nodes to the in the experiment measured nodes. Furthermore, the MAC matrix comparing these six numerical modes with the experimental modes is shown in Fig. 7. First and foremost, each of the six numerical modes can be correlated clearly to its experimental counterpart and moreover, the correlated modes are satisfyingly similar. This is not only visible by comparing Fig. 6 with Fig. 4 but also proven by high MAC values in Fig. 7. Four out of the six modes yield MAC values around 0.9 which is a very good result especially when taking into account

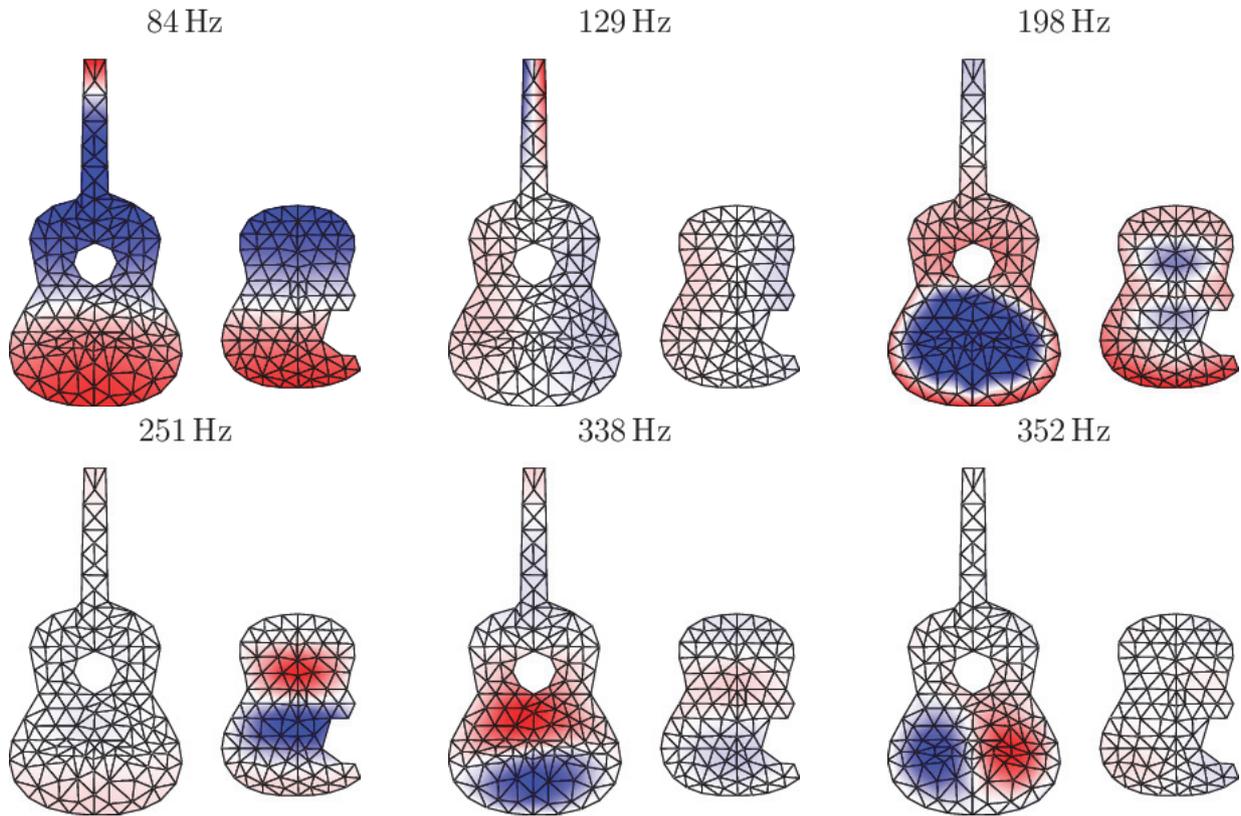


Figure 6. Six modes of the finite element model evaluated at the nearest nodes to the experimentally measured points on the real guitar.

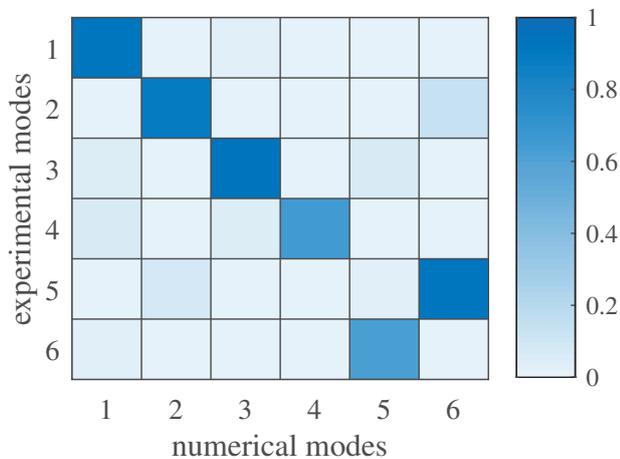


Figure 7. Modal assurance criterion matrix as comparison between the experimentally and numerically calculated modes displayed in Fig. 4 and Fig. 6.

that there is some amount of inaccuracy in finding the exact same nodes in simulation and experiment. The other two pairs of modes, (4, 4) and (6, 5), still yield MAC values around 0.7 and, therefore, are still well classifiable as a match.

On the other hand some drawbacks still remain. Although the eigenfrequencies of the numerical modes are already close to the experimental ones, there still is around

10 % deviation between experimental and numerical eigenfrequencies. In addition, the fifth and sixth mode in the simulation appear in the wrong order and especially the deviation of 51 Hz between the sixth numerical eigenfrequency and its experimental counterpart is quite high. Yet, the differences of the frequencies are not surprising because the objective function for the updating process only takes the mode shapes into account but not the eigenfrequencies.

All in all the first step of model updating can be rated a success as there is a very clear correlation between the numerical model and the real guitar.

4. CONCLUSION

The goal of the paper was to create a numerical FE model that approximates the modal behavior of a real guitar well although the material parameters are only coarsly known due to the natural variation of wood. For this reason an experimental setup has been designed with which the oscillation of the guitar can be measured from both sides and an experimental modal analysis has been performed for a Yamaha GC-12 classical guitar. The reconstructed mobilities by means of the experimental modal analysis showed very good agreement with the measured mobilities and, hence, the modal parameter identification yields very good results.

Furthermore, the experimentally examined guitar has been reverse engineered to create a finite element model

with very high geometrical accuracy. The numerical model includes all necessary parts including the bracing pattern and all other parts inside the guitar body as well as the air inside the guitar. The parts of the guitar are built with five different kinds of wood and all the orthotropic material properties of all of them have been included in the model.

Due to the variation of the material properties of wood the material properties found in existing literature are only starting values for a model updating scheme. A model updating scheme has been applied to approximate the material properties by performing a numerical optimization with an objective function in which the mode shapes of the numerical model are compared with the experimentally identified ones. This procedure already resulted in a good agreement of the mode shapes and eigenfrequencies between numerical model and experiment. Nevertheless further work has to be put into the model updating to improve the results of the numerical model.

5. REFERENCES

- [1] A. Brauchler, P. Ziegler, and P. Eberhard, "Experimentelle und numerische Untersuchung des Einflusses realistischer Randbedingungen auf Saitenschwingungen (in German)," in *Fortschritte der Akustik - DAGA 2019*, (Rostock), March 18-21 2019.
- [2] H. Tahvanainen, H. Matsuda, and R. Shinoda, "Numerical simulation of the acoustic guitar in virtual prototyping," in *Proceedings of ISMA 2019*, (Detmold), September 13-17 2019.
- [3] M. Elejabarrieta, A. Ezcurra, and C. Santamaria, "Coupled modes of the resonance box of the guitar," *The Journal of the Acoustical Society of America*, vol. 111, no. 5, pp. 2283–2292, 2002.
- [4] R. Viala, V. Placet, and S. Cogan, "Identification of the anisotropic elastic and damping properties of complex shape composite parts using an inverse method based on finite element model updating and 3d velocity fields measurements (femu-3dvf): Application to bio-based composite violin soundboards," *Composites Part A: Applied Science and Manufacturing*, vol. 106, pp. 91–103, 2018.
- [5] A. Brauchler, P. Ziegler, S. Benatti Camargo, and P. Eberhard, "Experimental and simulative examination of the string-soundboard coupling of an acoustic guitar," in *Proceedings of ISMA 2019*, (Detmold), September 13-17 2019.
- [6] R. Allemang and D. Brown, "A complete review of the complex mode indicator function (cmif) with applications," in *Proceedings of ISMA International Conference on Noise and Vibration Engineering*, September 18-20 2006.
- [7] H. Irretier, *Experimentelle Modalanalyse*. Lecture Notes, Kassel: Universität Kassel, 2004.
- [8] D. Ewins, *Modal Testing, Theory, Practice, and Application*. Baldock: Research Studies Press, 2nd ed., 2000.
- [9] R. French, *Engineering the guitar: theory and practice*. New York: Springer Science & Business Media, 2008.
- [10] Abaqus, *Analysis User's Guide*. Simulia, Providence, 2014.
- [11] D. Kretschmann, "Mechanical properties of wood," in *Wood Handbook: Wood as an Engineering Material*, Madison: Forest Products Laboratory, 2010.
- [12] R. Viala, V. Placet, S. Cogan, and E. Foltête, "Model-based effects screening of stringed instruments," in *Model Validation and Uncertainty Quantification, Volume 3*, pp. 151–157, Springer, 2016.