Modelling of a closed acoustical space by boundary reconstruction approach

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ABSTRACT

A common way to model the sound field in a closed space relies on the assumption of a diffuse sound field. In cases of pronounced modal behavior, typical of small spaces / low frequencies, the simplified diffuse model fails. Analytical modal solutions exist for spaces of primitive geometry and fully reflective boundaries, such as rectangular or spherical cavities. The spaces of complex geometry can be modelled by FEM, mirror image, tracing, diffusion and similar approaches. The present work uses a semi-analytical technique where the acoustical field is represented as a sum of two fields: 1) the primary field created by the original sound source(s) in an infinite space and 2) the secondary field created by the reflections from boundaries. The secondary field is composed of a large number of elementary solutions, each corresponding to a simple source radiating in an infinite space. The secondary sources are located outside the region occupied by the acoustical volume concerned. The field within this volume is obtained by matching the strengths of secondary sources to the field of primary source in order to achieve the desired boundary conditions at the contour surface of the modelled acoustical volume. The theoretical formulations of this technique are accompanied by several examples involving variations in the shape and boundary conditions of the acoustical space.

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ABSTRACT

A sound field synthesis technique using auxiliary sources for the modelling of closed cavity of arbitrary shape and boundary conditions has been investigated already. The technique suffers from two drawbacks which cannot be overcome by applying known physical principles: neither the type nor the positions of auxiliary sources can be made known in advance. In the present paper an adaptive multipole is used as the auxiliary source. When coupled to an auto-search algorithm convenient auxiliary source positions could be identified.

The theoretical basis of this technique, named the Boundary Reconstruction Approach (BRA), is illustrated by an example involving variations in the degree and number of multipoles of the auxiliary sources.

1. INTRODUCTION

The simplest way to model the sound field in a closed space relies on the assumption of a diffuse sound field, [1-2]. In cases of pronounced modal behavior, typical of small spaces and/or low frequencies, the simplified diffuse model fails. Simple analytical solutions exist for spaces of primitive geometry and fully reflecting walls, such as rectangular cavities. Absorbing boundaries can be modelled analytically by somewhat complex approaches, such as described e.g. in [3-4]. The spaces of complex geometry can be modelled by FEM, mirror image, tracing, diffusion and similar approaches.

In [5] Cremer has investigated the modelling of sound radiation from a vibrating body by using a superposition of spherical waves. The basic idea was to mimic the vibrating surface by a superposition of sound waves from simple sources located within the body contour. While this initial work concerned a cylindrical shape, Heckl has later demonstrated that the method itself can be used for bodies of arbitrary shape, [6].

Using the same basic idea the modelling of radiation into a free space was replaced by the modelling of sound generation within a closed cavity of arbitrary shape, [7]. This paper could serve a reference text on the subject.

2. THE BR APPROACH

The present work uses the same base modelling as used in [7]. The acoustical field in a cavity is represented as a sum of two fields: 1) the primary field created by the original sound source(s) in an infinite space and 2) the secondary field created by the reflections from cavity boundaries. The secondary field is composed of a number of elementary fields, each corresponding to a simple source radiating into an infinite space. The secondary sources are located outside the region occupied by the cavity. The field within this volume is obtained by matching the strengths of secondary sources to the field of primary source(s) in order to achieve the desired boundary conditions at the contour surface of the cavity.

The secondary field which accounts for the reflections from the boundaries could theoretically be obtained by a superposition of a continuum of elementary sources, i.e. by an infinite number of these, each of an infinitesimally small strength. A numerical computation of the secondary sources can be done by replacing the source continuum with a finite number S of discrete sources, each of finite strength. The unknown strengths of these sources can be arranged in a $S\times 1$ strength vector $\boldsymbol{Q}$. The acoustical state at the contour surface of an otherwise infinite medium has to be defined in a (large) number of discrete control points C. This leads to the following matrix formulation expressed in frequency domain:

$$(\mathbf{T}_{cs} - \mathbf{Z}\mathbf{H}_{cs})\boldsymbol{Q} = \mathbf{Z}\boldsymbol{V}_0 - \boldsymbol{P}_0 \qquad (1)$$

Here $\mathbf{T}_{cs}$ stands for the transfer matrix between the vector $\boldsymbol{Q}$ and the sound pressure at the control points C while $\mathbf{H}_{cs}$ stands for the transfer matrix between the vector $\boldsymbol{Q}$ and the normal components of particle velocities at the points C. $\mathbf{Z}$ represents the impedance matrix relating the pressure and normal velocities at C while $\boldsymbol{P}_0$ and $\boldsymbol{V}_0$ stand for pressure and normal velocities at C.

The unknown source strengths vector $\boldsymbol{Q}$ can be easily obtained from (1) by inversion. If the matrices involved in Eq. (1) are regular the sources in $\boldsymbol{Q}$ should fully satisfy the boundary conditions defined by the impedance $\mathbf{Z}$. However these conditions apply only to the control points C; in between these points the boundary conditions may be exceedingly violated. The known remedy is to use much more control points than auxiliary sources, $C \gg S$. In such a case the matrices become non-square and the computation is done by a convenient pseudo-inversion.

In the case of rigid boundaries, $\mathbf{Z} \to \infty$, the expression (1) simplifies into:

$$\mathbf{H}_{cs}\boldsymbol{Q} + \boldsymbol{V}_0 = 0 \qquad (2)$$

The objective is to get the strength vector $\boldsymbol{Q}$. Once this has been accomplished the sound pressure at an arbitrary point within the cavity contour can be obtained simply by adding the free-field pressure from the original source at this point and that created by the secondary sources.

Any possible sharp edges of the modelled cavity may represent conflicting requirements on the secondary sound field created by the (remote) auxiliary sources. To address this issue the authors have applied in [7] the mirror-image technique to the modelled rectangular box in order to find the positions of some of these sources.

Such a source positioning cannot apply to cavities of irregular shape. Thus the approach described suffers from a key drawback, the one of finding suitable positions of secondary sources. For cavities of oval shape such positions are on a surface similar but larger of that of the cavity. Figure 1 shows the frequency response of sound pressure inside an air-filled hard-walled spherical cavity of 1m radius driven eccentrically at 0.3m by a point source. The corresponding FEM result is shown for comparison. The matching of the two results is very good. Slight discrepancies rising with frequency originate from the "stiffening" effect of the FE method.

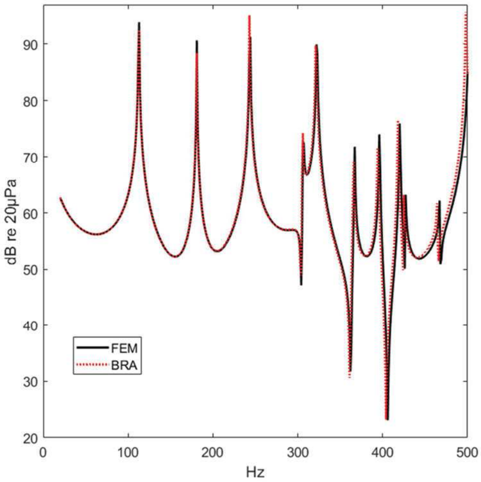

Figure 1. Sound pressure level in a spherical cavity.

3. THE ADAPTIVE MULTIPOLE

Each secondary source should satisfy two conditions: 1) it should be acoustically transparent, i.e. it should not scatter the sound field and 2) an analytical form of its free field radiation law should be known. The simplest source fitting these requirements is the monopole, as used in [7]. The BR approach does not prescribe the type of secondary source providing it remains acoustically transparent. The monopole can be thus substituted by a more complex source, a higher order multipole. One example is a 4 Degree-of-Freedom (DoF) multipole: a monopole added by three orthogonal dipoles. Taking that a dipole can be approximated by two closely spaced monopoles of equal strengths but opposite sign, a finite-spacing multipole can be conceived as a cluster of M closely spaced monopoles. The task of finding the multipole M-DoF strengths becomes then equivalent to finding the strength of each monopole in the cluster using Eqs. (1,2). One possible technique of identifying the multipole positions consists of using a greedy search scan of candidate positions as demonstrated in [8].

Fig. 2 shows the error of the search procedure in dependence of the number of secondary sources. Three types of multi-pole sources, 4-, 6- and 8-poles, were used with the maximum number of DoF equal to 720. The search was done over 1270 candidate positions around the contour modelling a 1.8m×1.42m×1.26m rectangular cavity of 2-2.5j normalised local-reacting impedance walls. The primary source was a group of 11 monopoles at 200 Hz having the positions, amplitudes and phases selected at random. The error is given as the normalised difference of the attained and prescribed impedances averaged over the 6858 control points. The final error is seen to be less than 2% using the 8-pole sources. An equivalent but uniform distribution of 720 ordinary monopoles has produced an error of only -19.6dB. This shows the benefits of the multipole search method.

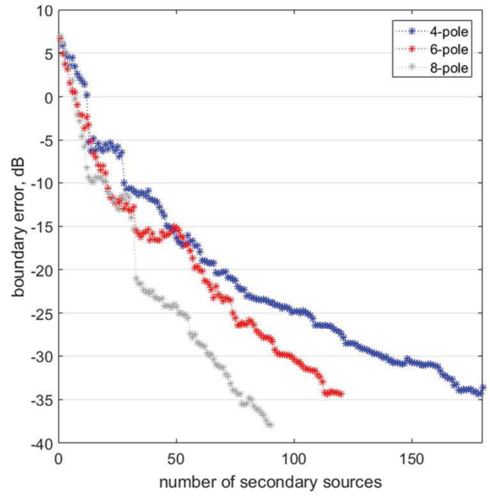

Figure 2. Computation error of the search procedure.

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