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Convolutional Neural Network for Material Decomposition in Spectral CT scans

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Abstract—Spectral computed tomography acquires energy-resolved data for the recovery of the density of the different material constituents. Spectral computed tomography reconstruction requires the decomposition of the spectral measurements into material projections and the reconstruction of the volumes from the decomposed projections. Material decomposition is a nonlinear inverse problem that has been traditionally solved using model-based material decomposition algorithms. However, the model is needed and difficult to estimate in real prototypes, and it is also difficult to design relevant regularizers.

In this study, we propose a deep-learning method for material decomposition in the projection domain. We validate our methodology with numerical phantoms of human knees that are created from synchrotron CT scans. We consider four different scans for the training and the one scan for the validation. The measurements are corrupted by Poisson noise, with the assumption that no more than 10^5 photons hit the detector.

We compare these results to the decomposition obtained using a regularized Gauss-Newton algorithm. We observe that this method depends particularly on the regularization parameter,

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which enforces a trade-off between noise and resolution, while the proposed deep-learning approach provides a compromise between noise and resolution, which is accompanied by a 100-fold reduction in computation time.

Index Terms—Spectral computed tomography, Convolutional neural network, Material decomposition, Knee phantom

I. INTRODUCTION

Spectral computed tomography (SCT) acquires energy-resolved data based on photon-counting detectors, which allows decomposition of a sample into maps of different materials [1], [2]. Among other applications [3], SCT has been proposed for detection of atherosclerosis in cardiovascular disease [4], [5], and more recently for osteoarthritis, using a contrast agent [6]. With SCT, the contrast between the different types of soft tissues is improved.

Material decomposition is a nonlinear, nonconvex, ill-posed inverse problem [7]. Regularized variational methods are state-of-the-art for material decomposition [7], [8]. The regularized Gauss-Newton (RGN) method was shown to outperform the commonly used maximum likelihood estimation. In our previous study, we investigated the use of the RGN approach for application to osteoarthritis [9]. However, such model-based material decomposition methods require the knowledge of the

detector response function of the scanner, the source spectrum, and the attenuation of each material with the energy, which cannot be accurately known in practice. Moreover, the detector response function suffers from pixel-to-pixel deviations, and can change over time [1], [8].

Deep-learning methods have been proposed to solve inverse problems, including image denoising and image reconstruction, with limited or noisy data [10]–[12]. Recently, deep-learning methods were proposed to solve the material decomposition problem in the image domain [13]. As decomposition in the image domain can lead to artifacts, decomposition in the projection domain is to be preferred [3]. However, decomposition in the projections has not been addressed using deep-learning approaches. Moreover, in previous studies, deep-learning methods were not compared to variational approaches.

In this study, we propose a deep-learning approach for solving the material decomposition problem in SCT in the projection domain, and we compare this to the RGN method [8], [9]. Our deep-learning approach is based on a U-net convolutional neural network (CNN) [14], to decompose projections at different energy bins into projections for different materials. Our method is assessed for application to osteoarthritis using unique human knee numerical phantoms created from synchrotron CT acquisitions presented in [9]. The methods are then compared in terms of mean square error (MSE), structural similarity (SSIM), and noise, computed as the standard deviation within a homogeneous region.

II. SPECTRAL COMPUTED TOMOGRAPHY

A. Forward model

We assume an object that is composed of M materials with V voxels scanned with a sensor that acquires I energy bins over P pixels and Θ projections. Let

$$\mathbf{s} = (s_{1,1}^1, \dots, s_{i,p}^\theta, \dots, s_{I,P}^\Theta)^\top$$

be the spectral measurement vector, where $s_{i,p}^\theta$ is the data acquired in the i -th energy bin, at the p -th pixel, and for the θ -th projection. Similarly, let

$$\boldsymbol{\rho} = (\rho_{1,1}, \dots, \rho_{m,v}, \dots, \rho_{M,V})^\top$$

be the unknown mass densities vector, where $\rho_{m,v}$ is the mass density of the m -th material at the v -th voxel of the sample. We note

$$\mathbf{s} = G(\boldsymbol{\rho}) \quad (1)$$

where G is the full forward model that maps $\boldsymbol{\rho}$ onto \mathbf{s} . The goal of SCT is to invert (1).

B. Two-step inverse problem

The forward model G can be seen as the composition of the X-ray transform X and a spectral mixing operator F . The X-ray transform X applies to each material independently. If we let $\mathbf{a}_m = (a_{m,1}^1, \dots, a_{m,p}^\theta, \dots, a_{m,P}^\Theta)^\top$ be the projected mass density of the m -th material, we have

$$\mathbf{a}_m = X(\boldsymbol{\rho}_m), \quad 1 \leq m \leq M, \quad (2)$$

where $\boldsymbol{\rho}_m = (\rho_{m,1}, \dots, \rho_{m,v}, \dots, \rho_{m,V})^\top$ is the mass density of the m -th material.

Spectral mixing applies to each projection angle independently; i.e.,

$$\mathbf{s}^\theta = F(\mathbf{a}^\theta), \quad 1 \leq \theta \leq \Theta, \quad (3)$$

where $\mathbf{s}^\theta = (s_{1,1}^\theta, \dots, s_{i,p}^\theta, \dots, s_{I,P}^\theta)^\top$ is the spectral measurement for the θ -th projection angle, and $\mathbf{a}^\theta = (a_{1,1}^\theta, \dots, a_{m,p}^\theta, \dots, a_{M,P}^\theta)^\top$ are the mass densities projected at the θ -th projection angle. We consider the standard nonlinear mixing used in [1], [8]; i.e.,

$$s_{i,p}^\theta = \int_{\mathcal{E}} n_0(E) d_i(E) \exp \left[- \sum_{m=1}^M a_{m,p}^\theta \tau_m(E) \right] dE \quad (4)$$

where \mathcal{E} is the energy range of the source, n_0 is the source energy spectrum, d_i is the detector response function of the i -th bin, and τ_m is the m -th material mass attenuation coefficient.

Equations (2) and (3) suggest a two-step approach where the spectral projections are first decomposed into material projections, before using a standard CT algorithm to reconstruct each material density volume from the decomposed material projections. The first step, which is referred to as material decomposition, is the main focus of this work.

C. Model-based material decomposition

Material decomposition aims to invert (3). Assuming the sample is composed of $M = 2$ materials, as bone and soft tissue here, we seek the solution by minimizing the cost function

$$C(\mathbf{a}^\theta) = \frac{1}{2} \|\mathbf{F}(\mathbf{a}^\theta) - \mathbf{s}^\theta\|_{\mathbf{W}}^2 + \alpha \mathcal{R}(\mathbf{a}_{\text{soft}}^\theta, \mathbf{a}_{\text{bone}}^\theta), \quad (5)$$

where α is a regularization parameter, \mathcal{R} is a regularization functional, and $\mathbf{a}_{\text{soft}}^\theta$ and $\mathbf{a}_{\text{bone}}^\theta$ are the projected mass densities of soft tissues and bone, respectively. The first term of the cost function is a weighted least squares data fidelity term, with $\mathbf{W} = \mathbf{Diag}(1/\mathbf{s}^\theta)$, which is chosen to handle data corrupted by Poisson noise. The second term is the regularization term that conveys prior knowledge about the solution. As in [8], regularization consists of first-order and second-order Tikhonov regularization for bone and soft tissue, respectively, which has a denoising effect.

To minimize (5), we use the Gauss-Newton algorithm implemented in the Matlab toolbox SPRAY [8], [15].

III. DEEP LEARNING FOR MATERIAL DECOMPOSITION

A. Proposed approach

In this study, we propose to learn mapping from the measured spectral projection to the projected material maps; i.e.,

$$h_\beta: \hat{\mathbf{s}}^\theta \mapsto \mathbf{a}^\theta \quad (6)$$

where $\hat{\mathbf{s}}$ is the normalized data with $\hat{\mathbf{s}}^\theta = \ln(\bar{\mathbf{s}}^\theta / \mathbf{s}^\theta)$, where $\bar{\mathbf{s}}^\theta$ represents the measurement in the absence of the object, i.e., $\bar{\mathbf{s}}^\theta = F(0)$, and β represents the parameters of the

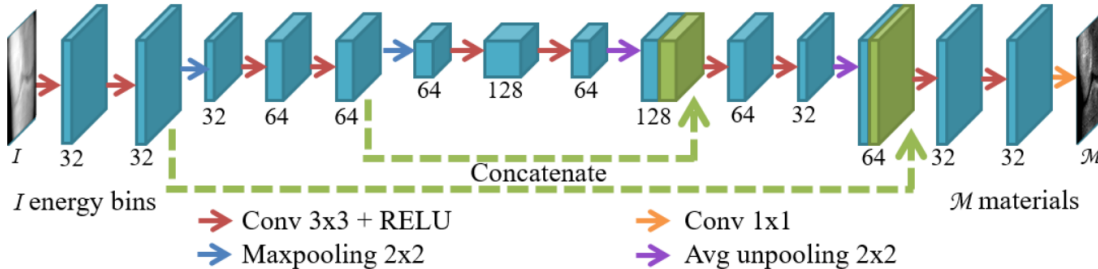


Fig. 1: U-Net architecture used to recover M material images from I spectral images at a given projection angle.

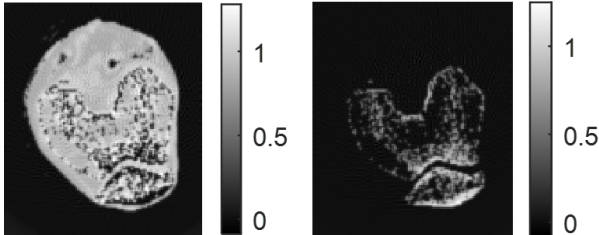


Fig. 2: Representative numerical phantom of material mass densities obtained from synchrotron CT scans. Single slices are shown for soft tissue (left) and bone tissue (right).

mapping. As depicted in Fig. 1, we consider a convolutional neural network with a U-net architecture.

Given N training input-output pairs $\{\hat{s}^{(n)}, \mathbf{a}^{(n)}\}$, $1 \leq n \leq N$, where N is the number of samples multiplied by the number of projections Θ , we consider the following loss function

$$L(\beta) = \sum_{n=1}^N \|\mathbf{h}_{\beta}(\hat{s}^{(n)}) - \mathbf{a}^{(n)}\|^2. \quad (7)$$

The inputs of the networks are $I = 4$ images, each of size 51×161 , and the outputs are $M = 2$ images, each of size 51×161 . We minimize (7) using the adaptive moment estimation, or Adam, algorithm [16] with PyTorch [17] running on a GeForce NVIDIA RTX 2080 Ti graphics card. The learning rate is set to 10^{-3} , and the batch size to 15. We use an early stopping criterion of 200 epochs.

B. Data

1) *Knee acquisitions*: Scans of eight knee samples at different stages of osteoarthritis were provided by the Institut d’Anatomie Paris Descartes. All of the knees were scanned by synchrotron CT at the European Synchrotron Radiation Facility (ESRF, Grenoble, France) [18]. The synchrotron CT scans were performed with a voxel size of $50 \mu\text{m}$ and at 55 keV, an energy that has shown improved contrast for cartilage visualization. The study was approved by the Ethics Committee of Descartes University, Paris.

2) *Generation of material mass density phantoms*: We create height realistic numerical phantoms from the synchrotron CT scans. All of the synchrotron volumes are subsampled by a

factor of 5 to reach a resolution of $250 \mu\text{m}$, which corresponds to the resolution of current SCT prototypes. For each knee, we automatically segment soft tissue and bone using a K-means algorithm [19]. After segmentation, the density of each material is taken from the synchrotron volume, assuming that only one material is present in each voxel. Fig. 2 shows a representative slice of the numerical phantoms obtained.

3) *Training data*: The material mass density phantoms are used to simulate SCT data according to (2) and (3). First, we again subsample the volumes, here by a factor of 4 to obtain a final resolution of 1 mm. We implement data augmentation by rotating the volumes around an axis perpendicular to the detector plane (-10 to 10°). We also perform two-dimensional scaling of the volumes using a factor ranging from 0.55 to 1.45. Then, the material density volumes are projected according to (2). We use the Matlab function `radon` and consider $\Theta = 720$ projections over 180° . Each volume is augmented to obtain 156 volumes, leading to $N = 156 \times 720 = 112,320$ projections of size 51×161 . Finally, spectral measurements are simulated according to (4) by means of the Matlab toolbox SPRAY [15]. We consider the same detector response function as in [1] ($I = 4$ energy bins) and a conventional 120 kVp X-ray source. Spectral measurements are corrupted by Poisson noise; the maximum number of photons is set to 10^5 .

4) *Training procedure*: The total of eight knee phantoms available are we split into four knees for training, one for validation, and three for testing. We use the training dataset to update the weights of the network, the validation dataset to control for overfitting during training, and the test dataset to determine whether our algorithm can be generalized to unseen samples.

After material decomposition, we reconstruct the 3D density volumes using filtered back projection (Matlab function `iradon`).

IV. RESULTS

Fig. 3 shows the material projections decomposed using the model-based RGN algorithm and the CNN methods, together with the decomposition error. For the RGN method, we display the results for two regularization parameters ($\alpha = 0.03$, $\alpha = 3$). Table I gives the MSE, SSIM, and noise values for both of the decomposed material projections at 90° , as it is consistent along all projections, and the reconstructed

material maps. We evaluate noise as the standard deviation in a (homogeneous) region of interest. The SSIM is also computed in a homogeneous region of interest, and the MSE is computed on the whole projection. The RGN algorithm with the lowest regularization parameter has higher MSE and SSIM, but also higher noise, compared to the higher regularization parameters. U-Net leads to low-noise decompositions with good compromise for the MSE and SSIM. In Fig. 3, it can be seen that for the RGN method, a low regularization parameter gives noisy decomposition and a high regularization parameter gives blurry images. Visually, U-net gives better results, as the images have both low noise and sharp details. Moreover, U-net decomposes a whole knee, i.e., 720 projections, in 40 s, while the RGN method needs 1 h for central processing units. Using graphical processing units, the U-net decomposition of the whole knee is reduced to only 2 s.

V. DISCUSSION

Here, we propose a CNN approach based on the U-net architecture for learning material decomposition for SCT, and compare this to a RGN method. The algorithms were assessed on numerical phantoms created from human knee synchrotron CT data [9]. A similar approach has already been applied for material decomposition in the image domain [13]. Here, we applied the projection domain, and we show that this gives comparable results to those for the model-based RGN method. Moreover, the RGN method decomposes materials on the assumption of ideal energy responses of the scanner, which is not realistic in practice. Thus, we expect model-based approaches to underperform learning approaches in nonideal situations.

The RGN method with low regularization led to the best results in terms of the MSE and SSIM, but the images were highly corrupted by noise. The RGN with high regularization was better in terms of noise, but the images were very blurred. U-net gives a good compromise between noise and recovery of detail. This can be explained because U-net implicitly learns the prior probability density of the decomposed materials, while this is assumed for the RGN method. Moreover, deep learning

TABLE I: Quantitative data for the decomposed projections and the reconstructed materials.

		Projected soft tissue	Projected bone	Soft tissue	Bone
MSE	GN	0.020	0.009	0.0013	0.0006
	$\alpha = 0.03$				
	GN $\alpha = 3$	0.021	0.010	0.0038	0.0018
	U-net	0.038	0.015	0.0032	0.0015
SSIM	GN	0.89	0.84	0.96	0.96
	$\alpha = 0.03$				
	GN $\alpha = 3$	0.91	0.67	0.51	0.52
	U-net	0.91	0.72	0.69	0.69
Noise	GN	0.76	0.43	0.32	0.26
	$\alpha = 0.03$				
	GN $\alpha = 3$	0.74	0.39	0.18	0.18
	U-net	0.71	0.46	0.21	0.19

allows the computation time to be reduced after the training, by a factor of 100 compared to the RGN method.

Our methodology has some limitations. The low resolution of the phantom used does not allow us to visualize all of the detail in the cartilage, and thus to assess possible improvements in the cartilage characterization. The level of noise used can also be considered to be low in comparison to realistic acquisition conditions. In addition, the simulations used are phantoms with only two materials.

For further studies, we will assess the potential application of the proposed approach for early detection of cartilage deterioration in osteoarthritis. For this, we will use high-resolution images and lower the maximum number of photons to 10^3 , which corresponds to more realistic conditions. We expect that the CNN approach will be more robust to noise while the RGN method will be highly compromised. To visualize the finer structures, such as the cartilage in the knee joint, we will need to work at the same resolution as the SCT (i.e., $250\mu\text{m}$). In addition, this method should be validated on experimental data. Finally, a possible improvement of the proposed method will be to decompose the images into three materials within the cartilage, which would lead to automatic segmentation of the cartilage.

VI. CONCLUSION

In this study, we propose a U-net approach for material decomposition in the projection domain for application to knee osteoarthritis. We compare it to a RGN method. Our neural network provides a compromise between image noise and resolution, which can be essential for detection of cartilage deterioration in osteoarthritis. In addition, our approach is rapid and does not require knowledge of the energy response of the scanner, which makes it a good candidate for future scanners.

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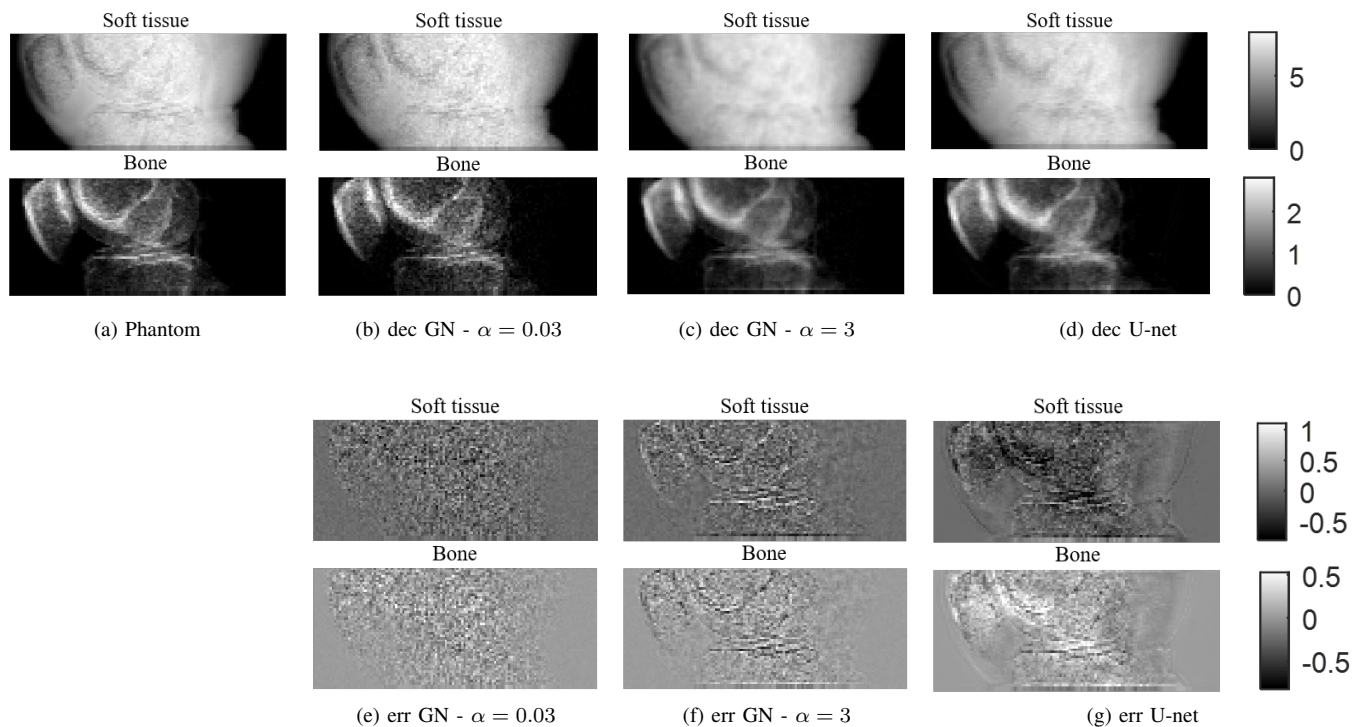


Fig. 3: Decomposed projections for an angle of 90° and a maximum number of photons of 10^5 . (a) The ground truth for the decomposition. (b, c) The decomposed materials with the Gauss-Newton method with regularization parameters of $\alpha = 0.03$ (b) and $\alpha = 3$ (c). (d) The decomposed materials with U-net. (e, f) The errors between the decomposed materials with the Gauss-Newton method and the ground truth, with the regularization parameters of $\alpha = 0.03$ (e) and $\alpha = 3$ (f). (g) The error between the decomposed materials with U-net and the ground truth.

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