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# OBJECTIVE METRICS TO IDENTIFY THE TONEHOLE LATTICE CUTOFF FREQUENCY OF CONICAL WOODWIND INSTRUMENTS

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### ABSTRACT

The tonehole lattice cutoff frequency is a characteristic of many woodwinds that is generally assumed to be linked to the musical personality of a given instrument. For frequencies below the cutoff, the input impedance peaks tend to have large amplitudes and regular spacing in frequency, while the peaks above the cutoff are attenuated and less regularly spaced. An approximate cutoff of an instrument is often identified as a perturbation in the magnitude or phase of the input impedance that separates these two frequency bands. A precise definition of the cutoff exists only for infinite, loss-less lattices, and has been worked out for the cylindrical resonators [1]. Conical resonators are more challenging because there is no possibility of a strict geometric regularity. Here, three objective metrics to identify the cutoff are proposed and compared. The first defines a cutoff frequency band that is defined by thresholds on the modulus of the reflection coefficient. The second metric divides the lattice into  $\Pi$ -shaped cells each of which consists of two toneholes that the section of main bore between them. The resonance of the cells varies across the length of the instrument, and the variation is interpreted as a degree of acoustic irregularity. The third divides the geometry of the lattice into asymmetric unit cells all of which have the same eigenfrequency. The range of eigenfrequencies for which this is possible is interpreted as the range of frequencies for which the lattice can be considered acoustically regular. These three metrics are applied to the measured input impedance and geometries of the saxophone and bassoon.

### 1. ELEMENTS OF CUTOFF FREQUENCY THEORY

The cutoff frequency of a tonehole lattice is a known phenomena due to wave propagation in periodic media that creates pass and stop bands for a wave incident upon the lattice. Below cutoff, a wave propagating into the lattice is evanescent, having mostly reflected back into the resonator and weakly radiated through the first open tonehole. Above the cutoff, the wave propagates into the lattice and may radiate from the first or any of the subsequent toneholes and end of the instrument. A full development of

this theory exists for infinite, loss-less lattices, but is also adapted for finite and lossy systems such as woodwind instruments. It is also possible to define empirical estimations of the cutoff frequency that do not depend on any knowledge of the resonator's geometry. The current section provides a brief description of theoretical and empirical definitions of the cutoff frequency, which are applied to the bassoon in Section 2.

#### 1.1 Standard T- and $\Pi$ -shaped cells

The usual formulation of the cutoff frequency for a periodic lattice is derived from the geometry of the constituent T-shaped cell, consisting of a hole flanked by two sections of the main pipe. In this case, the cutoff frequency corresponds to the eigen frequency of the Helmholtz resonator formed by the T-shaped cell closed on both sides of the main bore section

$$f_c = \frac{1}{2\pi} \frac{1}{\sqrt{m_a C_a}} \quad (1)$$

where  $m_a$  is the acoustic mass of the tonehole chimney and  $C_a$  the acoustic compliance in the main bore for each cell:

$$m_a = \frac{\rho h}{\pi b^2} \quad \text{and} \quad C_a = \frac{2\ell S}{\rho c^2}, \quad (2)$$

where  $\rho$  is the density of the medium,  $h$  the height of the chimney of the tonehole,  $b$  the radius of the tonehole,  $\ell$  the length of the main bore on either side of the cell,  $S$  the main bore section and  $c$  is the speed of sound in the medium. Although this formula was derived initially for a cylinder, it remains valid for a cone [2].

However, the tonehole lattices of real instruments are not periodic, and therefore the definition of T-shaped cells is ambiguous because  $\ell$  is not uniquely defined. One way to avoid this ambiguity is to study  $\Pi$ -shaped cells, defined by two neighboring toneholes and the section of main bore in between. The characteristic frequency of the  $\Pi$ -shaped cell is

$$f_{ch} = \frac{c}{2\pi} \sqrt{\frac{\rho}{2d} \left( \frac{1}{m_n S_n} + \frac{1}{m_{n+1} S_{n+1}} \right)}, \quad (3)$$

where  $d$  is the distance between the center of the two subsequent holes,  $m_n$  is the acoustical mass of the  $n$ -th hole

and  $S_n$  the section of the main bore underneath it. Note that the T- and II-cell definitions are equivalent when the tonehole lattice is perfectly regular.

## 1.2 The asymmetric division algorithm

Another way to estimate the cutoff frequency of a lattice that is not periodic is by dividing it iteratively into asymmetric T-shaped cells [3]. We start by fixing a test value of cutoff  $f_c$ , as well as the upstream bore length of the first T-shaped cell comprising the first open tonehole. Knowing the bore length upstream of a given hole, the bore length downstream is computed so that the resulting T-shaped cell's cutoff frequency  $f_c$  is equal to the test value. The upstream bore length of the neighboring hole is thus fixed as the remaining part of the distance between the two holes. The algorithm is carried out until one of the two following outcomes.

1. There is no next hole. In this case, the lattice is successfully divided for the value of the test cutoff frequency.
2. One of the computed lengths is negative or imaginary. This means the division was unsuccessful.

The final result of this process is a set of test cutoff frequency values that lead to successful division of the lattice. This set can be interpreted as a range of possible cutoff values for which the lattice can be considered acoustically periodic if losses are ignored.

## 1.3 The reflection coefficient interpretation

The methods presented up to this point rely on precise knowledge of the geometry of the lattice. However, it is common to measure only acoustical characteristics of a given resonator, such as its input impedance, as precise geometrical measurements can be tedious or invasive on intricate bores. The reflection coefficient of the resonator is obtained from the input impedance

$$R(f) = \frac{Z(f)/Z_\theta - 1}{Z(f)/Z_\theta^* + 1}, \quad (4)$$

where

$$Z_\theta = \frac{\rho c}{S} \frac{1}{(1 + 1/jkx)}, \quad (5)$$

is a modification of the characteristic impedance, accounting for wave propagation in a conical bore with an input half-angle  $\theta$  and missing length  $x$ . Two cutoff metrics are derived from two different interpretations of the reflection coefficient.

### 1.3.1 Reflection coefficient modulus drop

Below cutoff, the reflection coefficient modulus is approximately equal to one. At cutoff, the modulus drops quickly because the wave propagates into the lattice and radiates through subsequent toneholes, and is also subject to greater thermoviscous losses due to the longer propagation path

(see for instance [4]). Defining a threshold allows a simple, arbitrary estimation of the cutoff. In the present case, the threshold is set at half of the maximum value of the reflection coefficient modulus, and the estimate of the cutoff  $f_{|R|}$  is the lowest frequency such that

$$\frac{|R(2\pi f_{|R|})|}{|R(2\pi f)|_{\max}} = 0.5. \quad (6)$$

### 1.3.2 Effective length shift

The effective length of the resonator is proportional to the group delay associated to the reflection coefficient, defined as

$$\ell_{\text{eff}}(f) = -\frac{1}{2} \frac{c}{2\pi} \frac{d(\angle R(2\pi f))}{df}. \quad (7)$$

Below cutoff, this effective length corresponds approximately to the distance from the input to the first open tonehole, because waves do not propagate farther into the lattice. Above cutoff, the effective length is subject to large variations, as the wave may propagate into the lattice and undergo multiple reflections from subsequent toneholes. Therefore, the cutoff can be estimated as the first frequency for which the effective length deviates by an arbitrary threshold from its low frequency value.

The average effective length of the bore between 100 Hz and 200 Hz is taken as reference  $\ell_{\text{ref}}$ . The cutoff estimate  $f_\ell$  corresponds to the lowest frequency for which the effective length deviates from this reference by at least the reference value

$$\ell_{\text{eff}}(f_\ell) = \ell_{\text{ref}} \pm \ell_{\text{ref}}. \quad (8)$$

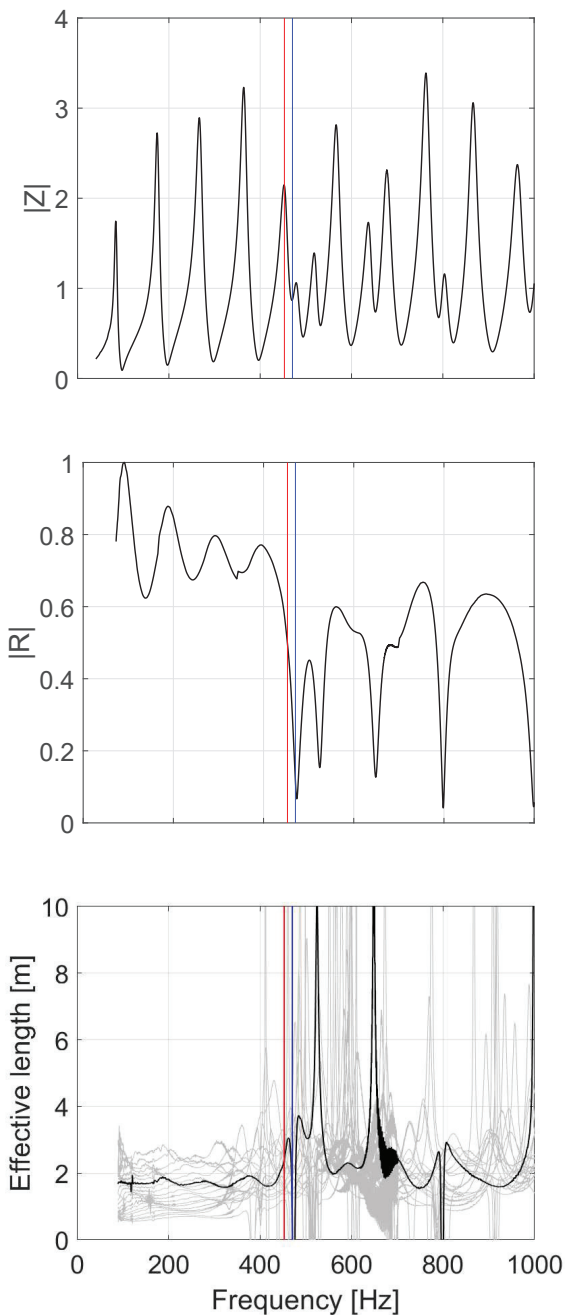
As for the reflection coefficient modulus drop, the deviation from low frequency acoustic length must be defined by an arbitrary threshold, this case  $\ell_{\text{ref}}$  provides satisfactory results.

## 2. APPLICATION TO THE BASSOON

The complete geometry of the bore of a modern German bassoon has been measured in order to compute the characteristic frequencies and apply the asymmetric division algorithm. The input impedance of the first register fingerings of the bassoon are measured using an impedance sensor adapted for woodwind measurements [5].

Figure 1 shows the impedance, reflection coefficient, and effective length for the E fingering of the bassoon. On this particular fingering the metrics defined in Section 1.3 are very similar.

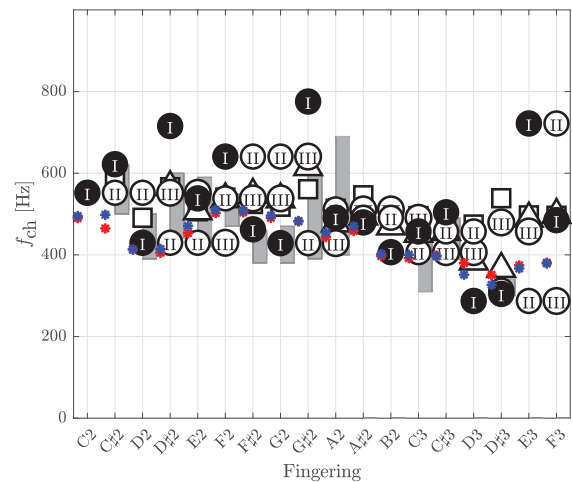
All four metrics are computed for the lowest 18 fingerings of the bassoon excluding the two lowest, Bb1 and B1, because they have respectively 0 and 1 open toneholes. Figure 2 shows that at least 3 of the 4 cutoff metrics overlap for most fingerings. The characteristic frequency  $f_{\text{ch}}$  was computed for the first 3 cells, starting from the first open tonehole. This way, they provide a first interval for the cutoff, while neglecting the downstream holes that do not have as strong a role. Both metrics due to the reflection coefficient are very close together in comparison to



**Figure 1.** Impedance modulus and reflection coefficient (modulus and effective length) for the E fingering of the bassoon, with overlaid metrics calculated from the reflection coefficient. The vertical red and blue lines are defined by Equations 6 and 8 and correspond to the red and blue asterisks in Figure 2. The grey curves in the bottom panel show the effective length of all the fingerings in the first register, where longer lengths correspond to lower notes.

the metrics derived from the geometry. A full study would be needed to know whether this is the case for other instruments, and how it depends on the chosen thresholds. The asymmetric division algorithm provides a finer cutoff interval than the characteristic frequencies for most fingerings, but fails to successfully divide the lattice for some of the highest notes. This may be due to the so-called

“partial holes” specific to the bassoon, that open automatically with certain holes and complicate the interpretation of tonehole pairs. Overall, the cutoff estimates show an almost constant trend, between 400 and 600 Hz, for all of the fingerings. This is coherent with Benade’s findings [6]. However, he estimated the cutoffs by human observation of the input impedance, where as here the estimates are performed by multiple objective definitions. Note that the trend is markedly different on another conical woodwind, the saxophone, for which the cutoff increases progressively for higher fingerings [7].



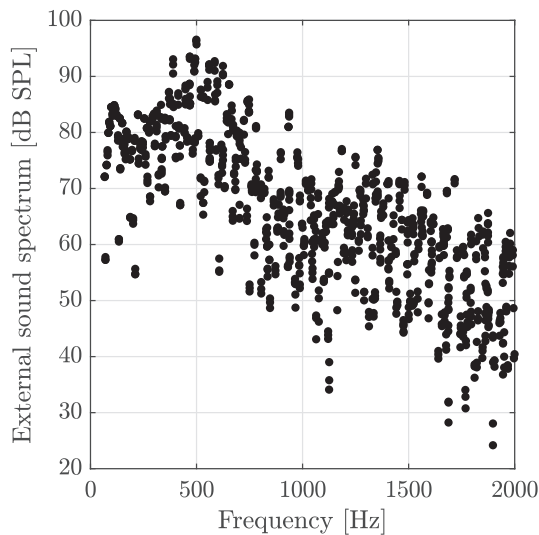
**Figure 2.** Comparison of the four cutoff metrics applied to the bassoon. Circled I, II, III correspond respectively to the characteristic frequency  $f_{ch}$  of Eq. (3) of the first, second and third  $\Pi$ -shaped cells. The triangles mark the average of these three values, and the squares mark the average of all the cells in the lattice. The grey bars show the intervals of successful test cutoff values in the asymmetric division algorithm (Section 1.2). The red and blue stars mark  $f_{|R|}$  and  $f_l$ , defined by Eqs. (6) and (8), respectively. See also Figure 1.

### 3. IMPACT OF THE PRODUCED SOUND

This section examines the radiated sound of the bassoon for which the cutoff estimations are shown in Figures 1 and 2.

The measurements were performed while the bassoon was “played” by an artificial mouth, using a “Légère” synthetic double reed, with the same fingerings used for the impedance measurements. The air pressure was 4.3 kPa, sufficient to produce musical notes through the entirety of the instrument’s register. The recording devices comprised a Piezotronics microphone: PCB 130E20, and a National Instruments sound card: USB-4431. The microphone was placed approximately 30 cm in front of the instrument at the height of the junction of the boot joint. This position was suggested by sound engineers as an efficient placement in order to record an optimal sound with a single microphone [8]. However, the authors are aware of the limitations inherent in this type of microphone placement [9, 10].

Each note was recorded three times with the same setup. The amplitude of the harmonic components of all the notes are superimposed on Figure 3, demonstrating a formant ranging from approximately 400 to 700 Hz. These frequencies match the cutoff frequencies determined by the metrics in Section 2. Other authors have also observed this formant and attributed it to the tonehole lattice cutoff, although their methodology for determining the cutoffs is likely the same as Benade [11]. Further study is required in order to determine the perceptual difference in spectral characteristics due to this formant.



**Figure 3.** Amplitude of the harmonics of the radiated sound, produced with an artificial mouth, on the fingerings C2 to F3 (see Figure 2). The formant around 500 Hz corresponds to the cutoff region.

#### 4. CONCLUSION

Although the tonehole lattice of the bassoon is not geometrically periodic, acoustical descriptors show that it is notably regular, in that the cutoff is relatively constant throughout all the fingerings of the first register. This sets the bassoon apart from another reed conical instrument, the saxophone, for which the cutoff frequency varies greatly. This acoustical regularity influences the produced sound by introducing a formant around the cutoff for all fingerings. Other implications of the cutoff, for instance on directivity or other timbre features of the instrument, remain to be explored. The set of cutoff descriptors provided in this paper can be simply applied to compare the tonehole lattices of other instruments, based on their geometry or their input impedance.

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