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IDENTIFICATION OF THE DAMPING COEFFICIENT OF A NES IN BOTH LINEAR AND NON-LINEAR DOMAINS

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ABSTRACT

In order to have the Targeted Energy Transfer in the acoustic domain, R. Bellet proposed a non-linear vibro-acoustic absorber constituted by a thin viscoelastic circular membrane coupled to a primary system through a coupling box and excited by a loudspeaker. The membrane damping in the Bellet's model is described in term of linear and quadratic viscous-like damping. The damping coefficient were obtained using identification in the linear case. The comparison between experiments and computation revealed that this description was not sufficient to capture all the feature of the very rich dynamics of this nonlinear system. We decided to estimate the damping coefficient in both linear and non-linear domains. For this estimation, a series of experimental tests was realized in a several input levels. Moreover, following these tests we have developed a MATHEMATICA code that allows the nonlinear identification of the damping coefficient of the NES using the harmonic balance method. We present the estimated variation of the damping coefficient as a function of the excitation amplitude, wherein an increase of the dissipation under the nonlinear behavior of the system and a quasi-constant value during the saturation are observed.

1. INTRODUCTION

Many studies are devoted on the energy-pumping phenomenon. First, Gendelman [1] presented this very interesting phenomenon. The energy pumping also called Targeted Energy Transfer, which can be defined as the unidirectional and irreversible transfer of vibrational energy from a main structure (that we want to protect) to the nonlinear absorber called Nonlinear Energy Sink (NES). In order to have the phenomenon of energy pumping in the acoustic field, Romain Bellet proposed a non-linear vibro-acoustic absorber [2] constituted by a thin circular viscoelastic membrane of essentially cubic stiffness coupled to the first mode of resonance of an open tube through a coupling box and excited by an acoustic field supplied by a loudspeaker (see figure 1.a). The membrane damping in the Bellet's model is described in term of linear and cubic viscous-like damping with one damping coefficient. This coefficient was determined in the linear case (a low level of excitement). Therefore, we observe a single value of the damping coefficient independent of the excitation. According to the model of the absorber developed by R. Bellet [2], we have developed an identification method in both linear and nonlinear domains, which allows

estimating the variation of the damping coefficient of the membrane η on term of amplitude of excitation.

2. EQUATIONS

The dimensionless system developed in [2] is described by the following equations:

$$\begin{aligned} \frac{d^2u}{d\tau^2} + \lambda \frac{du}{d\tau} + u + \beta(u - q) &= F \cos\left(\frac{\Omega}{\omega} \tau\right) \\ \gamma \frac{d^2q}{d\tau^2} + c1 \left[(1 + \chi)q + \eta\omega \frac{dq}{d\tau} \right] + c3 \left[q^3 + 2\eta\omega q^2 \frac{dq}{d\tau} \right] &= \beta(u - q) \end{aligned} \quad (1)$$

Where u and v show the dimensionless displacements of air at the end of the tube and in the center of the membrane. The tube damping is introduced by λ , τ is the time normalized by the first natural frequency of the tube, β is the stiffness of coupling between the tube and the membrane. F and Ω present the excitation force and frequency and ω present the resonance frequency of the tube. γ and η show the mass and damping of the membrane. A pre-strain applied to the membrane is presented in this model by the coefficient χ , which is the ratio between this pre-strain and the critical buckling strain. The coefficients $c1$ and $c3$ respectively represent the linear and nonlinear stiffnesses of the membrane are

$$\begin{aligned} \text{defined by} \quad c1 &= \frac{2 * 1.015^4 \pi}{9(1 - \nu^2)} \frac{Eh^3 LS_t}{\rho_a c_0^2 R^6} \text{ and} \\ c3 &= \frac{64}{3\pi^3 (1 - \nu^2)} \frac{Eh^3 LS_t}{\rho_a c_0^2 R^6} \end{aligned} \quad \text{Since } \frac{1.015^4 * \pi}{3} = 1.1 \approx 1.03 = \frac{32}{\pi^3}$$

then we can assume that $c1 \approx c3$. Therefore, the system of equations (1) is written in the following form:

$$\begin{aligned} \frac{d^2u}{d\tau^2} + \lambda \frac{du}{d\tau} + u + \beta(u - q) &= F \cos\left(\frac{\Omega}{\omega} \tau\right) \\ \gamma \frac{d^2q}{d\tau^2} + c1 \left[(1 + \chi)q + q^3 + \eta\omega(1 + 2q^2) \frac{dq}{d\tau} \right] &= \beta(u - q) \end{aligned} \quad (2)$$

3. EXPERIMENTAL SETUP

In this part, a series of experimental tests is realized, in which the amplitude of the sinusoidal voltage applies at the terminal of the loudspeaker changes and the frequency is varied for each input level. For this, we use the NETdB acquisition system, which allows controlling the excitation of the loudspeaker, collecting two measurements: the acoustic pressure is measured in the

middle of the tube by a microphone and the displacement of the membrane in its centre by a displacement sensor, and saving this time data in hdf5 format. (see Figure 2.b)

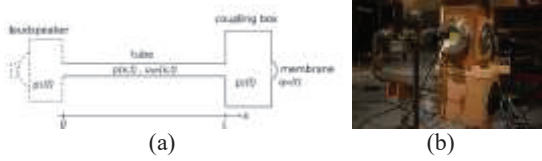


Figure 1. (a) Show scheme Bellet's system and (b) Scheme and picture of the set-up.

4. ANALYTICAL STUDY

In order to linearize the system of nonlinear differential equations (2) we use the harmonic balance technique [3] where the coordinates of the problem are expressed in Fourier series. In our case we are limited to the first harmonic where the displacements of the two oscillators are then expressed in the form $u(t) = U \cos(\omega t - \theta_t)$ and $q(t) = Q \cos(\omega t - \theta_m)$. By introducing these expressions into the system (2) and neglecting the higher harmonics, we obtain the following algebraic system for the amplitudes U and Q :

$$\begin{aligned} & \left[-\gamma Q \left(\frac{\Omega}{\omega} \right)^2 + c_1 \xi Q + \frac{3}{4} c_1 Q^3 + \beta Q \right] \cos \left(\frac{\Omega}{\omega} \tau - \theta_m \right) \\ & - c_1 \eta \Omega Q \left(1 + \frac{1}{2} Q^2 \right) \sin \left(\frac{\Omega}{\omega} \tau - \theta_m \right) = U \beta \cos \left(\frac{\Omega}{\omega} \tau - \theta_t \right) \\ & \left[\left(\frac{\Omega}{\omega} \right)^2 + 1 + \beta \right] U \cos \left(\frac{\Omega}{\omega} \tau - \theta_t \right) - \lambda \frac{\Omega}{\omega} U \sin \left(\frac{\Omega}{\omega} \tau - \theta_t \right) \\ & - Q \beta \cos \left(\frac{\Omega}{\omega} \tau - \theta_m \right) = F \cos \left(\frac{\Omega}{\omega} \tau \right) \end{aligned} \quad (3)$$

The coefficient ξ defined by $\xi = 1 + \chi = \frac{f_{m1}^2}{f_{m0}^2}$, Where f_{m1}

is the first resonance frequency of the membrane with pre-stress and f_{m0} the resonance frequency of the membrane without pre-stress. The application of the harmonic balancing of the first equation of the algebraic system (4) permits the following equations:

$$\begin{aligned} & \left[-\gamma Q \left(\frac{\Omega}{\omega} \right)^2 + c_1 \xi Q + \frac{3}{4} c_1 Q^3 + \beta Q \right] \cos(\theta_m) \\ & - c_1 \eta \Omega Q \left(1 + \frac{1}{2} Q^2 \right) \sin(\theta_m) = U \beta \cos(\theta_t) \\ & \left[-\gamma Q \left(\frac{\Omega}{\omega} \right)^2 + c_1 \xi Q + \frac{3}{4} c_1 Q^3 + \beta Q \right] \sin(\theta_m) \\ & - c_1 \eta \Omega Q \left(1 + \frac{1}{2} Q^2 \right) \cos(\theta_m) = U \beta \sin(\theta_t) \end{aligned} \quad (4)$$

Consequently, we can determine this expression:

$$\begin{aligned} & \left[-\gamma Q \left(\frac{\Omega}{\omega} \right)^2 + c_1 \xi Q + \frac{3}{4} c_1 Q^3 + \beta Q \right]^2 + \left[c_1 \eta \Omega Q \left(1 + \frac{1}{2} Q^2 \right) \right]^2 \\ & = (U \beta)^2 \end{aligned} \quad (5)$$

Combining this expression with the experimental results via a developed MATHEMATICA code permits the estimation of the damping coefficient variation of the NES as a function of the amplitude of excitation, which shows a the more the forcing the more the damping, with the maximum of damping during pumping and quasi-constant value during the saturation (fig 3). We observe the same shape of this curve for two different experiments.

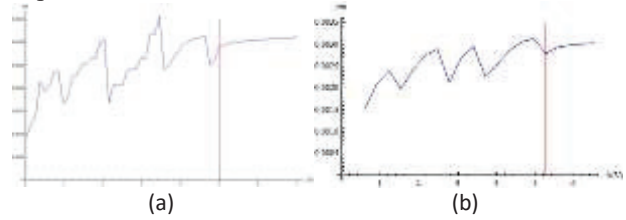


Figure 2. The estimation of the damping coefficient variation of the NES as a function of the amplitude of excitation for two different experiments (for (a) the membrane thickness $h = 0.21\text{mm}$ and the natural frequency of the membrane $f_{m1} = 29\text{Hz}$, for (b) $h = 0.2\text{mm}$ and $f_{m1} = 33.5\text{Hz}$). The red solid lines indicate the saturation threshold.

5. CONCLUSION

This work presents the variation of the damping coefficient according to the amplitude of excitation. A clear increase of the dissipation is observed under nonlinear motion of the membrane. This clearly indicates that some dissipative phenomenon is neglected in our model must be accounted for a better description of dissipation over its complete functioning range.

6. REFERENCES

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