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Dino Borie, Dorian Jullien

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Dino Borie† Dorian Jullien‡

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Abstract

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JEL Classification: D80, D81, D01, D03.

Keyword: description invariance, framing effect, description dependence, decision theory.

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†THEMA, Université Cergy-Pontoise and Université Nice Côte d'Azur, CNRS, GREDEG, Nice, France. E-mail:dino.borie@u-cergy.fr.

‡Corresponding author. Université Nice Côte d'Azur, CNRS, GREDEG, Nice, France. E-mail:dorian.jullien@gredeg.cnrs.fr.

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Abstract

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1. Introduction

There are often different ways to present a single decision problem. Four decades of research on "framing effects" in economics and psychology have documented how decision makers tend to behave differently under different presentations of the same decision problem. The paradigmatic example is Tversky and Kahneman's Asian Disease problem (1981, p.543). One presentation of this problem runs as follows:

> Imagine that the U.S. is preparing for the outbreak of an unusual Asian Disease, which is expected to kill 600 people. Two alternative programs to combat the disease have been proposed:
>
> If Program A is adopted, 200 people will be saved.
>
> If Program B is adopted, there is 1/3 probability that 600 people will be saved, and 2/3 probability that no people will be saved.

Another presentation consists in the same decision scenario ("Imagine that the U.S. ...") with different descriptions of the two alternatives :

> If Program C is adopted, 400 people will die.
>
> If Program D is adopted, there is 1/3 probability that nobody will die, and 2/3 probability that 600 people will die.

Both presentations have identical consequences. They only differ in the description of these consequences. However, decision makers tend to choose the sure program A in the first presentation and the risky program D in the second one. This is a framing effect: a preference reversal that violates "description invariance", an axiom of rational choice theory which states that decision makers should not be influenced by the descriptions used in a decision problem. Hence, in the Asian Disease problem, decision makers exhibit *description-dependent preferences*.

In this paper, we provide a relatively simple and intuitive theoretical framework to explain and organize various framing effects that have been found in the literature. In economics, existing discussions of framing effects and description invariance usually highlight five points. Description invariance is (1) an implicit axiom of standard models, and framing effects are (2) descriptively pervasive, (3) mathematically intractable, (4) normatively unjustifiable and (5) explained by prospect theory[1].

These points are usually made by discussing Tversky and Kahneman's work on framing effects in the 1980s. However, since then, other psychologists have greatly contributed to refine our understanding of framing effects. Our goal is to use insights from these psychologists in order to improve on the five points about framing effects and description invariance that are highlighted in economics. More precisely, we characterize description invariance (1) as several axioms, by contrast with its standard interpretation as one monolithic axiom. We do so by taking insights from psychologists (beyond Tversky and Kahneman) who have (2) refined the pervasiveness of violations of description invariance and (3) provided normative justifications for some of them. We then weaken description invariance to formalize description-dependent preferences that are (4) mathematically tractable and (5) that account for some framing effects which are not explained by prospect theory.

The body of evidence that motivates our theoretical framework is presented in section 2. The framework and the results for decisions under certainty are presented in section 3. Section 4 extends the framework and the results for decisions under risk. We then compare our theoretical contributions with other formal accounts of framing effects in section 5. Finally, a set of suggestions for further empirical work on framing effects is discussed in section 6.

2. Motivation

As emphasized in the most cited review and meta-analysis on framing effects in psychology, "the likelihood of obtaining choice reversals [is] directly related to the similarity between features of a given study and features of Tversky and Kahneman's (1981) original "Asian disease problem."" (Levin et al., 1998, p.157). The effect is robust to changes in scenario and type of consequences, e.g., money instead of lives. This motivates our choice to use the empirical structure of the Asian Disease problem to guide the construction of our theoretical framework[2].

Table 1 illustrates a pattern of strict preferences that can emerge from different redescriptions of the consequences in the Asian Disease problem. The results from Kühberger (1995) are the most robust in the literature and will be central in our theoretical account of description-dependent preferences[3].

[1] See Arrow (1982, pp.6-8), Smith (1985), Machina (1987, pp.144-146), Kreps (1988, p.197), Sugden (1991, p.759), Quiggin (1993, §14.5; 2014, p.713), Camerer (1995, p.652; 1999, p.10577; 2004, p.386), Rabin (1996, pp.46-47; 1998, pp.37-38; 2002, p.662), Starmer (2000, pp.338-9, p.352), Camerer2004a, Samuelson (2005, pp.93-95), Varian (2006, pp.549-550), Fudenberg (2006, p.699, p.700, p.708), Fudenberg and Levine (2006, p.1472), Bernheim and Rangel (2007, p.66), Thaler and Sunstein (2008, p.37), Köszegi and Rabin (2008, p.1824), Wakker (2010, p.234, pp.241-2, p.250, p.265, p.350, pp.377-378), Bardsley et al. (2010, pp.130-1).

[2] Other reviews and meta-analyses include Kühberger (1997; 1998; 2017), Kühberger et al. (1999), Piñon and Gambara (2005), Maule and Villejoubert (2007), Keren (2011), Mandel and Vartanian (2012), Takemura (2014, Part. V), Grüne-Yanoff (2016), Seta et al. (2017), Steiger and Kühberger (2018). The results of Kühberger et al. (2002) is of particular interest for economics, as they show in a monetary version of the Asian Disease problem that the framing effect disappears with small stakes but is amplified with large stakes.

[3] Several contributions have indeed successfully replicated the results of Kühberger (1995) (see, e.g., Mandel, 2001; Kühberger and Tanner, 2010; Schulte-Mecklenbeck and Kühberger, 2014; Tombu and Mandel, 2015; for a comprehensive survey, see Broniatowski and Reyna, 2018). Druckman's results seem less robust as replications

Experiment, [Number of subjects]	Choice set		
	Sure program	[% Prefer]	Risky program
Tversky and Kahneman (1981), [$N = 152$]	A: 200 people will be saved	$\succ$ [72%]	B: 1/3 probability that 600 people will be saved, and 2/3 probability that no people will be saved
Tversky and Kahneman (1981), [$N = 155$]	C: 400 people will die	$\prec$ [78%]	D: 1/3 probability that nobody will die, and 2/3 probability that 600 people will die
Kühberger (1995), [$N \approx 48$]	A': 400 people will not be saved	$\prec$ [60%]	B
Kühberger (1995), [$N \approx 48$]	C': 200 people will not die	$\succ$ [57%]	D
Kühberger (1995), [$N \approx 48$]	A'': 200 people will be saved and 400 people will not be saved	$\prec$ [62%]	B
Kühberger (1995), [$N \approx 48$]	C'': 400 people will die and 200 people will not die	$\prec$ [57%]	D
Druckman (2001), [$N \approx 43$]	A''': 200 people will be saved and 400 people will die	$\prec$ [56%]	B''': 1/3 probability that 600 people will be saved, and 2/3 probability that 600 people will die

The symbol $\approx$ is used for the number of subjects in Küberger's experiment because he only mentions that it is "roughly" one third of a group of 145 students in each cases (thus we took $\frac{145}{3}$ and rounded down), and also in Druckman's experiment because he mentions a total of 172 subjects in four groups in which the order of the programs and of the outcome statistics of the mixed presentation varied with no significant difference on the modal preference (thus we took $\frac{172}{4}$).

Table 1: Experimental Regularities in the Asian Disease problem.

Prospect theory's account of the original Asian Disease problem is that the words "saved" and "die" induce different reference points. The expectation that everybody will die in the save frame $\{A, B\}$ is different from the expectation that everybody can be saved in the die frame $\{C, D\}$. Prospect theory cannot explain straightforwardly that, on the one hand, preferences change across $\{A, B\}$, $\{A', B\}$, $\{A'', B\}$ while the word "saved" remains constant, and, on the other hand, that preferences change across $\{C, D\}$, $\{C', D\}$, $\{C'', D\}$ while the word "die" remains constant[4].

We want our framework to account for two types of explanations of framing effects. On the one hand, framing effects can result from a direct dependence of preferences on descriptions, i.e., as if utility is derived from descriptions. For instance, a given piece of ground beef is judged to taste better when described as "75% lean ground beef" than when described as "25% fat ground beef" (Levin and Gaeth, 1988). That utility is attached to descriptions is exemplified in the Asian Disease problem by the pattern $A \succ B \sim D \succ C$ observed when decision makers evaluate the programs in a joint evaluation mode, i.e., $\{A, B, C, D\}$ simultaneously (Kühberger and Gradle, 2013). This result implies that it is empirically possible to elicit a strict preference for a description of a consequence over another description of that same consequence, i.e., A ("200 people will be saved") $\succ$ C ("400 people will die").

have shown decision makers to be indifferent between the sure and the risky programs (Mandel, 2001) or to strictly prefer the sure one (Mellers and Locke, 2007). Other effects have been obtained with further redescriptions, e.g., of the consequences in the risky program (Küberger and Tanner 2010; Schulte-Mecklenbeck and Kühberger 2014; Tombu and Mandel 2015) or with synonyms of "save" and "die" (Wallin et al., 2016). These effects can also be represented in our framework (trivially), but they are not central in our axiomatic construction of description-dependent preferences.

[4]For detailed discussions of the theoretical problems posed by framing effects to prospect theory, see Kühberger (1997), Takemura (2014, chap.10), Seta el al. (2017), Broniatowski and Reyna (2018).

On the other hand, framing effects can result from tacit information that is "leaked" from the one who describes the decision problem. For instance, in the Asian Disease problem, the very choice of presenting the saved frame $\{A, B\}$ but not the die frame $\{C, D\}$ to a decision maker can leak implicit information. This has been documented in several studies where subjects are asked, under various conditions, to choose the descriptions of the alternatives in the Asian Disease problem that will then be presented to a decision maker. Sher and McKenzie (2008) found that subjects who describe the sure program as "200 people will be saved" tend to have a strong personal preference for the sure program. van Buiten and Keren (2009) found that subjects who are asked to promote the sure program tend to describe it as "200 people will be saved"[5].

These authors argue that if decision makers are able to infer such implicit information (i.e., about the preference of the one who describes the problem or about the implicit recommendations he or she is trying to convey), then the classic preference reversal in the original Asian Disease problem can be interpreted as rational behavior. This interpretation can hold even when the one who describes the problem and the decision maker agree on the fact that the different descriptions of the sure program have equivalent consequences. In that case, a strict preference for a description of a consequence over another description of that same consequence reveals different *informational contents* of the two descriptions from the decision maker's perspective. Our theoretical framework proposed in the next section is designed to account for this kind of case.

3. The framework

We first present the primitives of our framework and then the axiomatic characterization of description invariance and description-dependent preferences.

3.1. Primitives

Our framework is constructed for static decision problems under certainty because violations of description invariance can occur at this basic level, as discussed in the previous section. It has four primitives. The first one is the nonempty choice set X. Its elements are descriptions of objects of choice, interpreted here as *described consequences*. Different described consequences can be about the same consequence. Abstracting from risk, the Asian Disease problem provides a handy illustrative example throughout, which we use with the following notation (see Table 2). Described consequences are denoted by letters with subscripts discriminating different descriptions of a same consequence, e.g., x_1, x_2, consequences are denoted by letters within brackets, e.g., $[x]$, and the symbol $\equiv$ reads 'can be empirically instantiated as'.

The second primitive is a non-trivial binary relation $\approx$ on X. Two described consequences related by this binary relation are considered as two descriptions of the same consequence (and conversly) from the perspective of what we call the decision modeler – henceforth denoted by d_{mo}. The d_{mo} can be identified with a decision theorist considering a decision situation or an experimenter. By definition, $\approx$ is an equivalence relation. If $x \approx y$, we say that x and y are *consequentially equivalent*. In other words, the d_{mo}'s perspective defines equivalence classes of described consequences. We say nothing about the perspective of the decision maker – henceforth

[5]Besides the inference of personal preferences and implicit recommendations, other studies show leakage of information about reference points, trustworthiness and other choice-relevant characteristics of decision problems: see McKenzie and Nelson (2003), Sher and McKenzie (2006; 2008; 2011), McKenzie et al. (2006), Keren (2007; 2011), van Buiten and Keren (2009), Mandel (2012; 2015), Tombu and Mandel (2015), Kühberger and Tanner (2010), Kühberger and Gradl (2013).

Consequences	Described consequences
$[x]$	$x_1 \equiv$ "200 people will be saved"
	$x_2 \equiv$ "400 people will die"
	$x_3 \equiv$ "400 people will not be saved"
	$x_4 \equiv$ "200 people will not die"
$[y]$	$y_1 \equiv$ "600 people will be saved"
	$y_2 \equiv$ "nobody will die"
	$y_3 \equiv$ "nobody will not be saved"
	$y_4 \equiv$ "600 people will not die"
$[z]$	$z_1 \equiv$ "no people will be saved"
	$z_2 \equiv$ "600 people will die"
	$z_3 \equiv$ "600 people will not be saved"
	$z_4 \equiv$ "nobody will not die"

x_1, y_1, z_1, x_2, y_2 and z_2 are used by Kahneman and Tversky (1981), x_3 and x_4 are used by Kühberger (1995), y_4 and z_3 are used by Mandel (2001), and to the best of our knowledge, y_3 and z_4 have not been used in the literature.

Table 2: Consequences and Described Consequences in the Asian Disease problem.

denoted by d_{ma} – on the equivalence of the consequences of different descriptions because this has not been investigated in the literature on framing effects.

In the literature on framing effects, this equivalence relation corresponds to the judgment of decision theorists and experimenters who consider that the logical equivalence of descriptions implies their consequential equivalence, which should imply the indifference of a rational d_{ma}. The relation of consequential equivalence is therefore not an empirical element in an experiment, because the d_{ma} is not explicitly informed about it. Expressions of this equivalence relation can however be empirically observed in the scientific literature on framing effects, where decision theorists and experimenters explicitly state that some descriptions are about the same consequence. That consequential equivalence of descriptions from the d_{mo}'s perspective should imply indifference from the d_{ma}'s perspective is the axiom of description invariance that we seek to make explicit and then to weaken.

The third primitive is a partial binary operation $\circ$ on X. If $x \approx y$, then x and y can be concatenated together to form a single described consequence denoted $x \circ y$. The *concatenation* $x \circ y$ is consequentially equivalent to x and to y. To illustrate with the Asian Disease problem, if the d_{mo} considers that "200 people will be saved" (x_1) and "400 people will not be saved" (x_3) are consequentially equivalent ($x_1 \approx x_3$), then they can be concatenated to form "200 people will be saved and 400 people will not be saved" ($x_1 \circ x_3$), which is consequentially equivalent to either one of the two initial described consequences. In other words, we have three different descriptions of a same consequence: x_1, x_3 and $x_1 \circ x_3$.

The three primitives introduced so far can be used to formalize the *descriptive structure* of a decision problem from the d_{mo}'s perspective, independently of the d_{ma}'s perspective.

Definition 1. Let X be a nonempty set, $\approx$ a non-trivial binary relation on X, and $\circ$ a partial binary operation on X. The triple $\langle X, \approx, \circ \rangle$ is a *descriptive structure* if two conditions hold:

(i) $\approx$ is an equivalence relation,

(ii) for all $x, y \in X$, $x \approx y$ if and only if $x \circ y \in X$.

Therefore, in a descriptive structure, $x \circ y \approx x$.

The last primitive of our framework is the d_{ma}'s preference relation $\succsim$ on X. As usual, $x \succsim y$ means that x is at least as desirable as y. The strict preference $\succ$ and indifference $\sim$ relations are respectively defined as the asymmetric and symmetric parts of $\succsim$. We emphasize that $\succsim$ is defined over described consequences, i.e., over X. This contrasts with standard models, where d_{ma}'s preferences are defined over consequences. That is, in standard models, the d_{ma}'s preferences are

implicitly defined over the d_{mo}'s partition of X, i.e., over $X/\approx$, the set of equivalence classes of X under $\approx$. Furthermore, in standard models, the d_{ma}'s indifference relation implicitly holds among all the elements of a given equivalence class, that is, among all different descriptions of a same consequence. This is indeed the standard interpretation of description invariance.

Within the present framework, it is possible to characterize three relevant cases of preferences with different degrees of dependence to descriptions. The first case is when there is no dependence to descriptions at all. It corresponds to the standard interpretation of description invariance. In standard models, description invariance is implicit because preferences are only defined over consequences, i.e., over the d_{mo}'s equivalence classes of consequences $[x]_{\approx}$, $[y]_{\approx}$, and so on. They are not defined over described consequences such as x_1, y_1, x_2, y_2, and so on. In other words, the standard models force decision theorists to work within an *unframed descriptive structure*. By contrast, we have constructed our choice set so that we always work within a *framed descriptive structure*. Hence description invariance can be formally explicated. The standard models' case where it is impossible to discriminate different descriptions of a given consequence corresponds here to the case where the d_{ma}'s indifference relation holds among all the elements of a given equivalence class:

$$\forall x, y \in X, \quad x \approx y \Rightarrow x \sim y \ [6].$$

The second case is when the d_{ma} can have strict preferences between different descriptions of the same consequence, with however an ordered preference relation over the different consequences. This is what we call *tidy description dependence*. In other words, tidy description dependence requires that there are no preference reversals by the the d_{ma} across the d_{mo}'s equivalence classes. Formally,

$$\forall x_1, x_2, y_1, y_2 \in X, \text{ if } x_1 \approx x_2,\ y_1 \approx y_2 \text{ and } x_1 \not\approx y_1 \text{ then } x_1 \succsim y_1 \Rightarrow x_2 \succsim y_2.$$

In words, if the d_{ma} prefers the description (x_1) of a first consequence over the description (y_1) of a different consequence (i.e., $x_1 \succsim y_1$ and $x_1 \not\approx y_1$), then he or she also prefers an equivalent description (x_2) of the first consequence to an equivalent description (y_2) of the second consequence (i.e., $x_2 \succsim y_2$ if $x_1 \approx x_2$ and $y_1 \approx y_2$). With tidy description dependent preferences, the d_{ma} is not necessarily indifferent between all equivalent descriptions of a single consequence, and may therefore violate description invariance. However, the d_{ma}'s preference relation over described consequences is consistent with a unique weak order over the d_{mo}'s equivalence classes on the described consequences.

This is not possible anymore in a third case that we call *untidy description-dependent preferences*: when preferences over described consequences do not induce a unique weak order over the consequences. We can formalize the distinction as follows. Given a descriptive structure $\langle X, \approx, \circ \rangle$ for the d_{mo}, and a weak order $\succsim$ describing the d_{ma}'s preference relation over described consequences, define a binary relation $\succsim^*$ over the equivalence classes in $X/\approx$ by $[x] \succsim^* [y]$ if and only if

$$x_i \succsim y_i \quad \forall x_i \in [x], \forall y_i \in [y],$$

[6] We are not the first one to state description invariance in this way, see, e.g., Bacharach (2003, p.65), Blume et al. (2013, p.9), Lerner (2014, p.40) and esp. Giraud (2004a, p.52), who are discussed in section 5. It can be argued that description invariance in standard models is in fact characterized by $x \approx y \Rightarrow [x] \sim [y]$. If we would have defined preferences over consequences, this would be the natural way of characterizing description invariance. However, with preference defined over described consequences, our characterization is more natural.

which can be interpreted as the d_{ma}'s induced preference relation over the consequences. Description-dependent preferences are then tidy if the induced preference relation over consequences $\succsim^*$ is complete and transitive, and untidy otherwise.

The axioms we present next provide a formal characterization of these three degrees of dependence to description.

3.2. Axioms and results

We first provide an explicit characterization of description invariance and then show how to weaken it to represent tidy and untidy description-dependent preferences.

3.2.1. Basic axioms

The unframed descriptive structure of standard models has forced economists and psychologists to conceive the axiom of description invariance as one monolithic axiom, i.e., as an axiom that is either satisfied or not satisfied. However, within the framed descriptive structure of our framework, it is easy to see that description invariance is in fact jointly implied by several axioms. All these axioms are therefore implicit in standard models. Furthermore, the first five axioms that we present below will be satisfied in our characterization of description invariance and the first three will also be satisfied in our characterizations of description-dependent preferences[7].

Axiom 1 (Descriptive structure)**.** $\langle X, \approx, \circ \rangle$ *satisfies Definition 1.*

Axiom 2 (Weak order)**.** $\succsim$ *is complete and transitive.*

Axiom 3 (Weak commutativity)**.** *For all* $x, y \in X$*, if* $x \circ y$ *and* $y \circ x \in X$*, then* $x \circ y \sim y \circ x$*.*

Axiom 4 (Weak associativity)**.** *For all* $x, y, z \in X$*, if* $(x \circ y) \circ z$ *and* $x \circ (y \circ z) \in X$*, then* $(x \circ y) \circ z \sim x \circ (y \circ z)$*.*

Axiom 5 (Weak idempotency)**.** *For all* $x \in X$*, if* $x \circ x \in X$*, then* $x \circ x \sim x$*.*

Axiom 1 corresponds to the structure of the objects of choice from the d_{mo}'s point of view. Its empirical interpretation has already been discussed. Axiom 2 is a standard requirement about preferences, that is, the d_{ma}'s preference relation over the objects of choice is a weak order.

Axioms 3 and 4 state that whenever the concatenation is defined, the d_{ma} is indifferent to the order in which the described consequences are concatenated. The following example illustrates the empirical implications of both axioms.

Example 1. From Table 2, the following concatenations can be defined by the d_{mo}:

$$x_1 \circ x_3 \equiv \text{ “200 people will be saved and 400 people will not be saved”,}$$
$$x_3 \circ x_1 \equiv \text{ “400 people will not be saved and 200 people will be saved”.}$$

and

$$(x_1 \circ x_3) \circ x_2 \equiv \text{ “200 people will be saved and 400 people will not be saved, i.e., 400 people will die”,}$$
$$x_1 \circ (x_3 \circ x_2) \equiv \text{ “200 people will be saved, i.e., 400 people will not be saved and 400 people will die”.}$$

All the basic descriptions (x_1, x_2, x_3) and the concatenations are consequentially equivalent. Therefore weak commutativity (Axiom 3) implies that the d_{ma} is indifferent between the order in

[7] Our axioms are inspired by Krantz et al. (1971)'s characterization of an “extensive structure” for measurement. The main distinctions are that we add the idempotency axiom, that our concatenation operation is not total and that we do not use any Archimedean axioms.

which the described consequences are concatenated: $x_1 \circ x_3 \sim x_3 \circ x_1$. And weak associativity (Axiom 4) implies that the d_{ma} is indifferent between the order in which the concatenations of the described consequences are presented: $(x_1 \circ x_3) \circ x_2 \sim x_1 \circ (x_3 \circ x_2)$. ♦

Weak commutativity and weak associativity jointly imply that the order of presentation of different basic descriptions within a described consequence can be reversed without altering its informational content, hence without altering the d_{ma}'s indifference. This is explicit in our framework and implicit in standard models and informal discussions of description invariance by economists and psychologists. These two axioms are clearly not violated in the set of empirical evidence on framing effects reviewed in the previous section.

Axiom 5 states that the d_{ma} is indifferent between a described consequence and the concatenation of this described consequence with itself. Again, this is implicit in standard models as well as in informal discussions of description invariance by economists and psychologists. This axiom is also not violated in the set of empirical evidence from the previous section.

To sum up, the implications of Axioms 3-5 are not at stake in the framing effects we are interested in, by contrast with the next axiom.

3.2.2. Framed descriptive structure with description invariance

The following axiom is also implicit in standard models and informal discussions of description invariance by economists and psychologists.

Axiom 6 (Independence of common description)**.** *For all $x, y, z \in X$, if $x \circ z$ and $y \circ z \in X$, then $x \sim y$ iff $x \circ z \sim y \circ z$.*

Independence of common description (ICD) states that the d_{ma}'s indifference between two described consequences is not affected by their concatenation with a common described consequence. And conversely, the d_{ma}'s indifference between two concatenations sharing a common described consequence is not affected by the cancellation of the common described consequence. This axiom can be naturally interpreted as a separability condition on the descriptions of a given consequence. The following example illustrates its empirical implications.

Example 2. From Table 2, $x_1 \circ x_2$ and $x_3 \circ x_2$ can be defined by the d_{mo} to correspond to:

$$x_1 \circ x_2 \equiv \text{ “200 people will be saved and 400 people will die”,}$$
$$x_3 \circ x_2 \equiv \text{ “400 people will not be saved and 400 people will die”.}$$

ICD implies that a d_{ma} indifferent between these two concatenations ($x_1 \circ x_2 \sim x_3 \circ x_2$) should be indifferent between the two distinct basic descriptions “200 people will be saved” (x_1) and “400 people will not be saved” (x_3), i.e., $x_1 \circ x_2 \sim x_3 \circ x_2 \Rightarrow x_1 \sim x_3$. And conversly, a d_{ma} indifferent between the two basic descriptions ($x_1 \sim x_3$) should be indifferent between the two concatenations that are constructed by adding a common basic description ($x_1 \circ x_2 \sim x_3 \circ x_2$), i.e., $x_1 \sim x_3 \Rightarrow x_1 \circ x_2 \sim x_3 \circ x_2$. ♦

If ICD and the axioms presented above hold, then our framework is equivalent to the standard models. That is, the d_{ma} remains indifferent between all equivalent descriptions of a given consequence. In other words, when Axioms 1-6 hold, we are in a *framed descriptive structure* with *description invariance*, which is equivalent to the standard models' *unframed descriptive structure.*

Proposition 1. *Let X be a nonempty set, $\approx$ and $\succsim$ nontrivial binary relations on X, and $\circ$ a partial binary operation on X. Suppose that the quadruple $\langle X, \approx, \succsim, \circ \rangle$ satisfies axioms 1-5. Then the following two statements are equivalent:*

(i) $\langle X, \approx, \succsim, \circ \rangle$ satisfies Independence of common description.

(ii) For all $x, y \in X$, $\quad x \approx y \Rightarrow x \sim y$.

All proofs are in the appendix.

We next show how to weaken this structure in order to formalize description-dependent preferences.

3.2.3. Framed descriptive structure with description dependence

We propose two ways of characterizing a *framed descriptive structure* with *description dependence* in order to suitably represent the framing effects we are interested in. We first present what is common to the two characterizations. In both cases, we keep the basic axioms and propose to weaken the ICD axiom. The ICD axiom can be decomposed in two parts:

Axiom 6.1 (Weak substituability)**.** *If* $x \circ z$ *and* $y \circ z \in X$*, then* $x \sim y$ *implies* $x \circ z \sim y \circ z$.

Axiom 6.2 (Weak simplifiability)**.** *If* $x \circ z$ *and* $y \circ z \in X$*, then* $x \circ z \sim y \circ z$ *implies* $x \sim y$.

Example 2 above already illustrates the empirical interpretation of these two axioms (the first part of the example illustrates weak simplifiability and the second part illustrates weak substituability). Weak substituability is the "if" direction of ICD: if the d_{ma} is indifferent between two informational contents, then adding a common informational content to both preserves the indifference of the d_{ma}. Weak simplifiability is the "only if" direction of ICD: if the d_{ma} is indifferent between two informational contents, then the retrieval of a common informational content from both preserves the indifference of the d_{ma}. To account for framing effects, we keep weak substituability and drop weak simplifiability. The following example provides a justification for this.

Example 3. From Table 2, $x_1 \circ (x_1 \circ x_2)$ and $x_2 \circ (x_1 \circ x_2)$ can be defined by the d_{mo} to correspond to:

$x_1 \circ (x_1 \circ x_2) \equiv$ "200 people will be saved, i.e., 200 people will be saved and 400 people will die",
$x_2 \circ (x_1 \circ x_2) \equiv$ "400 people will die, i.e., 200 people will be saved and 400 people will die".

Firstly, the basic axioms together with weak substituability imply that the d_{ma} is indifferent between these two described consequences, i.e., that $x_1 \circ (x_1 \circ x_2) \sim x_2 \circ (x_1 \circ x_2)$[8]. This indifference is plausible as the concatenation $x_1 \circ x_2$ can be seen as clarifying that the informational content implied by either x_1 or x_2 is indeed identical.

Secondly, if a d_{ma} is indeed plausibly indifferent between these two concatenations, i.e., if $x_1 \circ (x_1 \circ x_2) \sim x_2 \circ (x_1 \circ x_2)$, then weak simplifiability implies that he or she is also indifferent between x_1 and x_2, i.e., between "200 people will be saved" and "400 people will die". The results discussed in the previous section shows that this implication is clearly not plausible. That most d_{ma}s are not indifferent between "200 people will be saved" and "400 people will die" is at the heart of the classical results from Tversky and Kahneman (1981) and their many replications. Furthermore, this indifference has also been intuitively and normatively questioned from the results of Sher and McKenzie (2008) and others on informational leakage. ♦

We have therefore good empirical, intuitive and normative reasons to drop weak simplifiability, but not weak substituability. Without weak simplifiability, Proposition 1 does not hold anymore: the d_{ma} is no longer necessarily indifferent between different descriptions of a same consequence.

[8] This is so because by weak idempotence, $x_1 \circ x_1 \sim x_1$ and $x_2 \circ x_2 \sim x_2$. Weak substituability implies that $(x_1 \circ x_1) \circ x_2 \sim x_1 \circ x_2$ and $x_1 \circ (x_2 \circ x_2) \sim x_1 \circ x_2$. Rearranging by weak associativity, weak commutativity and transitivity of $\sim$ implies that $x_1 \circ (x_1 \circ x_2) \sim x_2 \circ (x_1 \circ x_2)$.

Such cases of *description dependence* are representable when Axiom 6.2 is dropped, that is, when Axioms 1-3 and 6.1 characterize a framed descriptive structure with description dependence. In this structure, it is thus possible that different descriptions of a same consequence convey different information.

We now wish to discriminate between the cases of tidy and untidy description dependence already presented above. Recall that tidy description-dependent preferences are necessarily consistent with an induced weak order over the consequences, as with the preference pattern $x_1 \succ x_1 \circ x_2 \succ x_2 \succ y_1 \succ y_4 \succ y_2 \circ y_3$. By contrast, untidy description-dependent preferences are not consistent with an induced weak order over the consequences, as with the pattern $x_1 \succ x_1 \circ x_2 \succ z_3 \succ x_2$ for $x_1 \approx x_2 \not\approx z_3$.

When Axioms 1-3 and 6.1 hold together with the following axiom 7, then the d_{ma} has necessarily tidy description-dependent preferences.

Axiom 7 (Independence of common description across consequences)**.** *For all $x, y, z, t \in X$, if $x \not\approx y$, $x \circ z$ and $y \circ t \in X$, then $x \succsim y$ iff $x \circ z \succsim y \circ t$.*

Independence of common description across consequences (ICDAC) requires that, for each pair of different consequences, the d_{ma} has a preference between them that does not change when their descriptions are concatenated with another one or when such a concatenation is removed. Note that if ICD holds, ICDAC holds trivially, but not the converse. ICDAC prevents that an indifference between two consequences implies an indifference among all descriptions of both consequences. The following example illustrate the empirical implications of this axiom.

Example 4. From Table 2, $x_1 \equiv$ "200 people will be saved" $\not\approx y_1 \equiv$ "600 people will be saved" and the following concatenations can be defined:
$x_1 \circ x_2 \equiv$ "200 people will be saved and 400 people will die"
$y_1 \circ y_2 \equiv$ "600 people will be saved and nobody will die"
ICDAC requires that the d_{ma} strictly prefers the description "600 people will be saved" over the description "200 people will be saved" if and only if he or she strictly prefers any description involving a concatenation with the former, such as "600 people will be saved and nobody will die", over any description involving a concatenation with the latter, such as "200 people will be saved and 400 people will die". ♦

Proposition 2. *Let X be a nonempty set, $\approx$ and $\succsim$ non-trivial binary relations on X, and $\circ$ a partial binary operation on X. Suppose that the quadruple $\langle X, \approx, \succsim, \circ \rangle$ satisfies axioms 1-3. Then the following two statements are equivalent:*

(i) $\langle X, \approx, \succsim, \circ \rangle$ satisfies Independence of common description across consequences.

(ii) For all $x_1, x_2, y_1, y_2 \in X$, if $x_1 \approx x_2$, $y_1 \approx y_2$ and $x_1 \not\approx y_1$, then $x_1 \succsim y_1 \Rightarrow x_2 \succsim y_2$.

Proposition 2 shows that it is the ICDAC axiom that is crucial in the distinction between tidy description-dependent preferences and untidy description-dependent preferences. Indeed, proposition 2 shows that when ICDAC holds in our framework, preferences cannot exhibit untidy description dependence. In other words, when Axioms 1-3 and 6.1 hold without Axiom 7, then the d_{ma} can have untidy description-dependent preferences.

The simplicity of our framework, and formal proximity with standard models, make the classical equivalence results between utility, preference and choice easy to obtain. If the preference relation $\succsim$ is order dense, then standard results about the representability of a weak order apply. Hence utility representations of the different degrees of description-dependent preferences presented in this section are straightforward corollaries. On the other hand, choice functions that can represent

description-dependent preferences require a translation of our preference axioms in choice correspondence rules. In the appendix, we propose a set of correspondence rules that allows to represent description-dependent choices[9].

4. Framing under risk

The main framing effect we wish to account for is obviously the one observed in the Asian Disease problem. To be consistent with our motivations exposited in section 2, we need to account for this framing effect by showing how the observed preferences are influenced by the descriptions of the consequences under risk and not by the descriptions of the probabilities. This can be done in two simple steps. Firstly, we need to extend our framework under risk, i.e., to conceptualize objects of choice as lotteries with described consequences. Secondly, we need to find the conditions under which description-dependent preferences are representable (as well-defined utility functions and choice functions) within this extension.

A risky alternative can be described in several ways. For instance, $(x, p+q; y, 1-p-q)$ and $(x, p; x, q; y, 1-p-q)$ are obviously two different descriptions of the same gamble. That a d_{ma} should be indifferent between these two described gambles is a requirement of description invariance. This requirement corresponds to the axiom of "coalescing" that is implicit in expected utility theory, rank-dependent utility theory and cumulative prospect theory: if a gamble has two (probability-consequence) branches yielding identical consequences, those branches can be combined by adding their probabilities without affecting the utility (see Birnbaum, 2005). As the framing effects we are interested in do not involve different descriptions of the probabilities, we need to extend our choice set under risk (i.e., to define our space of gambles) in a way that leaves aside the descriptions of the probabilities[10].

The extension of our framework under risk is as follows. Let $\Delta(X)$ be the set of all simple distributions on the nonempty set of described consequences X from the previous section. We denote the elements of $\Delta(X)$ by p, q, etc., for each $p, q \in \Delta(X)$ and $\alpha \in [0,1]$, define $\alpha p+(1-\alpha)q \in \Delta(X)$ pointwise by $(\alpha p + (1-\alpha)q)(x) = \alpha p(x) + (1-\alpha)q(x)$ for all $x \in X$ [11].

The descriptive structure is now $\langle \Delta(X), \approx, \circ \rangle$. By identifying X with the set of degenerate lotteries, the restrictions of $\approx$ and $\circ$ to X (still denoted $\approx$ and $\circ$ by a slight abuse of notation) induce the descriptive structure $\langle X, \approx, \circ \rangle$ on described consequences from the previous section.

We show that expected utility theory can be extended in our framework under risk in order to represent description-dependent preferences. The most fundamental axioms of expected utility theory are of course the weak order, the independence and the Archimedean axioms. These axioms can be stated in our framework as follows.

Weak Order. *$\succsim$ is a weak order on $\Delta(X)$.*

Independence. *For all $p, q, r \in \Delta(X)$ and $\alpha \in (0,1)$, $p \succsim q$ if and only if $\alpha p + (1-\alpha)r \succsim \alpha q + (1-\alpha)r$.*

[9] The results in the appendix can be seen as a complement to the general framework for choice functions with frames proposed by Salant and Rubinstein (2008), which cannot account for framing effects violating description invariance (i.e., when different frames are different descriptions).

[10] Defining our choice set under risk in a way that allows to make explicit the axiom of coalescing for described gambles is possible. It however involves a slightly more complicated extension than the one we provide below, which is sufficient to represent the framing effects we are interested in.

[11] Formally, $\Delta(X) = \{l : X \to [0,1] \mid \#\{x \mid l(x) > 0\} < \infty, \ \sum_{x \in X} l(x) = 1\}$. It is readily seen that the gamble $(x, p; x, q; y, 1-p-q) \notin \Delta(X)$: it is not a function from X to $[0,1]$.

Archimedean. *For all* $p, q, r \in \Delta(X)$*, if* $p \succ q \succ r$*, then there exist* $\alpha, \beta \in (0,1)$ *such that* $\alpha p + (1-\alpha)r \succ q$ *and* $q \succ \beta p + (1-\beta)r$*.*

While the weak order and archimedean axioms in our framework are straightforward translations of their standard form, the independence axiom imposes a stronger requirement than its standard form. Indeed, here the independence axiom applies to concatenations of consequentially equivalent described lotteries, which do not exist in standard models. Such concatenations are however not involved in the framing effects we are interested in. The following proposition generalizes proposition 2.

Proposition 3. *Let* X *be a nonempty set,* $\approx$ *and* $\succsim$ *non-trivial binary relations on* $\Delta(X)$*, and* $\circ$ *a partial binary operation on* $\Delta(X)$*. Suppose that the quadruple* $\langle X, \approx, \succsim, \circ \rangle$ *satisfies axioms 1-3, 7. Then the following two statements are equivalent:*

(i) $\succsim$ *is an Archimedean weak order on* $\Delta(X)$ *which satisfies the independence axiom.*

(ii) $\succsim$ *on* $\Delta(X)$ *admits an expected utility representation.*

Moreover this representation is unique up to positive affine transformation.

We can now account for framing effects by representing that utility is attached directly to described consequences. Recall that the original Asian Disease problem involves three consequences under different descriptions : "200 people will be saved" (x_1) or "400 people will die" (x_2) *versus* 1/3 probability that "600 people will be saved" (y_1) or "nobody will die" (y_2), and 2/3 probability that "no people will be saved" (z_1) or "600 people will die" (z_2). The original preference reversal observed in this problem can now be represented straightforwardly in expected utility theory extended within our framework as:

$$u(x_1) > \frac{1}{3}u(y_1) + \frac{2}{3}u(z_1)$$
$$\frac{1}{3}u(y_2) + \frac{2}{3}u(z_2) > u(x_2).$$

Moreover, the choice patterns observed in Druckman (2001) can also be rationalized if the function u satisfies the following restriction on the concatenation of the described consequences x_1 and x_2:

$$\frac{1}{3}u(y_1) + \frac{2}{3}u(z_2) > u(x_1 \circ x_2).$$

A complete study of the preferences in the Asian Disease problem would allow to estimate more precisely what restrictions the function u must satisfy on the concatenation between the set of all described consequences. The discussion and examples in the previous section established that this weak order is compatible with tidy-description dependent preferences. In other words, the preferences in the Asian Disease problem are compatible with both a well-behaved utility function over the consequences under certainty and with a straightforward extension of expected utility theory within our framed descriptive structure under risk.

Furthermore, under expected utility theory extended within our framework, the implied risk attitudes are dependent on the descriptions of the consequences, not on the consequences *per se*. This is the main point of the Asian Disease problem. Then, in a sense, our result here reconciles

the insights of psychologists with economic analysis. The same reasoning applies to other framing effects besides the Asian Disease problem.

Consider for example the problem used by DeMartino et al. (2006), which has nearly the same empirical structure as the Asian Disease problem. The decision scenario is simply that the d_{ma} is told "You Receive £50". Then he or she has to choose between a sure consequence and a lottery, under either one of the two following descriptions:

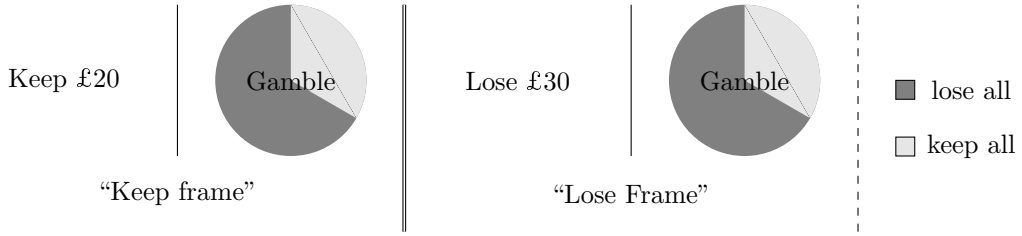

Figure 1: DeMartino et al. (2016)'s decision problem.

Most d_{ma}s prefer to keep the £20 in the first frame and to take the lottery in the second frame. As the lottery is described identically in the two frames, i.e., the diagram is exactly the same, this result furnishes further evidence that it is the description of the sure consequence that is crucial in the framing effects we are interested in. This effect can indeed be represented straightforwardly in our framework. From the d_{mo}'s perspective (in line with the judgments of DeMartino et al. 2006):

$x_1 \equiv$ "Keep £20", $x_2 \equiv$ "lose £30" $\in [x]_{\approx}$
$y_1 \equiv$ "keep all" $\in [y]_{\approx}$
$z_1 \equiv$ "lose all" $\in [z]_{\approx}$

The tidy description-dependent preferences of a d_{ma} who strictly prefers "Keep £20" over "lose £30" ($x_1 \succ x_2$) can be represented as compatible with expected utility theory such that:

$$u(x_1) > \frac{1}{3}u(y_1) + \frac{2}{3}u(z_1)$$
$$\frac{1}{3}u(y_1) + \frac{2}{3}u(z_1) > u(x_2).$$

5. Comparison with other formal accounts of framing effects

The main difference of our framework with other formal accounts of framing effects is that we consider a richer set of behavioral data from psychology to guide our axiomatic constructions. Among these accounts, the closest from our framework is Giraud's (2004b;2004a;2005). We are indeed inspired from what he calls a "normative" equivalence relation on the choice set, which is close to the perspective of the d_{mo} in our framework. However, we do not necessarily identify the d_{mo}'s perspective with a normative one that the d_{ma} would like to take. Furthermore, while he formalizes description invariance as one monolithic axiom, we formalize it as a set of axioms. This makes it possible to weaken description invariance without dropping it altogether. That is, we do not impose the axiom in a way that forces a binary choice between taking it to be satisfied or not.

Another difference is that Giraud cannot formalize a direct dependence of preferences on descriptions without interpreting it as a failure of the d_{ma} to recognize that different descriptions

are about the same consequences. To bypass this shortcoming, he gives an account of the original Asian Disease problem which is more complex than ours. In his account, the preference reversal is explained through a nonadditive decision weight function that makes the d_{ma}'s beliefs directly dependent on descriptions and preferences indirectly so when revealed (see Giraud 2004a;b).

The few other formal accounts of framing effects in the literature have all been conducted in frameworks that are less close to the standard one than Giraud's and ours. Nevertheless, the same differences between our framework and Giraud's occur between our framework and these other accounts.

Blume, Easley, and Halpern (2013) account for framing effects within their "constructive decision theory" framework in which the objects of choice are syntactic descriptions of events in a simple programming language. Description invariance is not derived and then weakened but imposed by a subset of the syntactic descriptions that describe the same event according to the d_{ma}'s beliefs. This allows to represent both the d_{ma}s revealing a framing effect (by putting two syntactic descriptions of two frames in different subsets) and those who do not (by putting two syntactic descriptions of two frames in the same subset).

Gold and List (2004) use the resources of predicate logic to construct a choice set in which the elements are either "target" propositions about the d_{ma}'s preferences or various "background" propositions about the decision situation. Framing effects are formalized as arising from a sequential process whereby beliefs about two different sets of background propositions can influence contradictory beliefs about the same target proposition. For instance, the d_{ma} believes that it is true that she or he strictly prefers the sure program over the risky one and the d_{ma} believes that it is false that she or he strictly prefers the sure program over the risky one (see pp.267-268). They make description invariance formally explicit but not as straightforwardly as in our framework[12].

Bourgeois-Gironde and Giraud (2009) analyze framing effects through the framework of Bolker-Jeffrey expected utility in which preferences are over propositions. They make explicit the implicit axiom of description invariance in this framework by showing how it is hidden both in the mathematical structure of the Boolean algebra with which the choice set is constructed and in its interpretation as a set of propositions. Once made explicit, they do not weaken it. Rather, they propose to "bypass" it (p.390) by refining the d_{ma}'s beliefs through a new set of information ("good news" and "bad news") associated with each event. Following McKenzie and co-authors (see section 2), framing effects are thus rationalized by refining description invariance to the informational (and not only logical) structure of decision problems.

Among these three contributions, the closest to ours in spirit is Bourgeois-Gironde and Giraud (2009) because of their inspiration from McKenzie and co-authors' results to motivate an account of framing effects as not necessarily irrational. They however do not consider the other results from psychology that motivated our formalization of a direct dependence of strict preferences on descriptions. Bacharach (2003) provides an analysis of framing effects in this spirit, but it is quite informal and less motivated by behavioral data (concerning the direct dependence of strict preferences on descriptions)[13].

[12] Dietrich and List (2013, 2016, 2013, esp. pp.628-631, 2016, esp. pp.200-201) provide a general framework to deal with choice reversals that is close to ours in the sense that they distinguish two perspectives on a given decision situation: one ("objective") from the modeler and the other ("subjective") from the decision maker. The implicit relaxation of description invariance in their work is however different from ours: for them, if the decision maker perceives two descriptions as being about the same object, then he or she is necessarily indifferent between the two descriptions.

[13] Both Ryan (2005) and Lanzi (2011) claim to provide a framework to formalize framing effects. Their accounts

6. Suggestions for empirical work on variations of the Asian Disease problem

To conclude the paper, we discuss some suggestions for empirical work on the Asian Disease problem that seem relevant in the light of our theoretical contribution. Each of the axioms constitutive of description invariance that we unpacked in this paper are empirically testable. Indeed, the examples that we have proposed to illustrate the empirical implications of these axioms suggest empirical tests that could reveal new framing effects. We emphasize that all these experiments would not test some new requirements of economic rationality that we invented. Rather, they would test some requirements of economic rationality that are implicit in standard models. We propose to use the Asian Disease problem to illustrate two practical issues concerning the empirical tests of these axioms.

One issue concerns the choice set that the d_{ma} would actually face. A traditional experimental design would consist in binary choices between one description of the sure program and one description of the risky program. For instance, to test the axiom of weak commutativity (Axiom 3), the d_{ma} should choose between the following two alternatives:

> If the sure program is adopted, 200 people will be saved and 400 people will not be saved.
>
> If the risky program is adopted, there is 1/3 probability that 600 people will be saved, and 2/3 probability that no people will be saved.

And he or she should also choose between the following two alternatives:

> If the sure program is adopted, 400 people will not be saved and 200 people will be saved.
>
> If the risky program is adopted, there is 1/3 probability that 600 people will be saved, and 2/3 probability that no people will be saved.

The observation of a preference reversal would then reveal a new kind of framing effect that could be interpreted as an empirical refutation of weak commutativity.

A less traditional experimental design would consist in following Kühberger and Gradl (2013, experiments 3 and 4) who presented four described alternatives simultaneously to the d_{ma}: the two original descriptions of the sure program and the two original descriptions of the risky program. Then the d_{ma} had to rate his or her likeliness to choose each of these four descriptions, on a scale between -3 ("would not choose this option") and +3 ("would choose this option") (ibid, p.113). Therefore, a convenient way of testing our axioms would be to add further descriptions in this kind of experimental design, e.g., the descriptions that we used to illustrate the empirical implications of our axioms in section 3.

Adding further descriptions in this kind of experimental design has two advantages over traditional experimental designs. Firstly, it provides a more natural way of eliciting indifference (through identical scores of likeliness to choose) between a plurality of different descriptions of a same consequence, which is required by most of our axioms. Secondly, it provides a more natural

do not seem very tractable. In any case they do not provide concrete applications to framing effects showing how their formalization should work (they do so for other phenomena). Lerner (2014) provides an account of framing effects through intensional logic. Though she illustrates how her framework formalizes the Asian Disease, it is again not very tractable.

way of eliciting preferences over different descriptions simultaneously within and across equivalence classes of consequences under certainty, which is a key requirement of the ICDAC axiom (Axiom 7) to make the distinction between tidy and untidy description-dependent preferences observable. For instance, it does not seem problematic to include in the design proposed by Kühberger and Gradl the following four descriptions: "600 people will be saved and nobody will die", "600 people will be saved", "200 people will be saved", and "200 people will be saved and 400 people will die". Any d_{ma} who gives different scores of likeliness to choose between the first two descriptions and/or between the last two descriptions exhibits description-dependent preferences. Furthermore, these preferences are tidily dependent as long as the lowest score between the first two is higher than the highest score between the last two. This is a clear requirement of the ICDAC axiom and any empirical violation of it can be interpreted as the observation of untidy description-dependent preferences.

The other issue concerns the empirical translation of the concatenation operation. In the example for weak commutativity exposed above, the word "and" in "400 people will not be saved and 200 people will be saved" was the empirical translation of the concatenation operation. However, other empirical translations are possible, such as "i.e.", "therefore", or even just a comma. If d_{ma}s are not indifferent to different empirical translations of the concatenation operation, then new framing effects could be observed. We are not arguing that such framing effects would be theoretically interesting. We simply want to point out that the empirical translation of the concatenation operation is not a trivial matter.

Indeed, in a set of preliminary classroom experiments meant to check the empirical plausibility of our axioms, we used two different empirical translations of the concatenation operation: just a coma or "Hence if this alternative is implemented,". Using the traditional design of a binary choice between a description of the sure program and a description of the risky program (in a within-subjects design), weak commutativity (Axiom 3) and weak associativity (Axiom 4) tended to be satisfied regardless of the empirical translation of the concatenation operation. However, weak idempotency (Axiom 5) tended to be satisfied with "Hence if this alternative is implemented," but it tended to be violated with a simple coma. That is, we took the original Asian Disease problem and replaced the descriptions of the sure program with, on the one hand, either "200 people will be saved, 200 people will be saved" or "200 people will be saved. Hence if this alternative is implemented, 200 people will be saved", and, on the other hand, with either "400 people will die, 400 people will die" or "400 people will die. Hence if this alternative is implemented, 400 people will die".

In the condition with just the coma, we observed the original framing effect (i.e., most d_{ma}s chose the sure program in the save frame and the risky one in the die frame). In the condition with "Hence if this alternative is implemented,", we observed no framing effect (i.e., roughly half of the d_{ma}s had a consistent strict preference for the sure program and the other half had a consistent strict preference for the risky one).

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