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# Vibrations in Multilayered Soils: A Simplified and Robust Prediction Equation

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**Abstract.** Wave propagation in soil is controlled by both attenuation and amplification laws. The former is due to geometrical and material damping, while the latter is mainly due to velocity contrasts between the various layers and possibly some geometrical/topographic effects. In the field of seismology and earthquake engineering, several factors are considered to describe the site properties and predict the amplifications features, such as average velocity  $V_{s30}$  and velocity-gradient-based coefficients. The reliability of these factors is questioned in the case of vibration problems, for which surface sources generated frequency components are much higher than in earthquake engineering. With this aim, considering various sites typologies, this paper suggests a set of relevant parameters for predicting vibration away from the vibratory source. Starting from numerical results (FEM), correlations between the proposed factors and the estimated displacements are discussed. These simple prediction equations are shown to be relevant and robust.

**Keywords:** Vibration · Amplification · Gradient · Wave propagation · Prediction equation

## 1 Introduction

While ground motion prediction equations are an important research domain since many decades in earthquake engineering, the necessity of similar approaches for vibration prediction is on the rise. Similarly to the seismic case, attenuation and site effects due to the presence of waves trapped in weaker soils over stiffer soils play an important role in the determination of vibration levels [1].

In the domain of earthquake engineering,  $V_{s30}$  which is the harmonic average shear wave velocities in the first 30 m of soil is usually taken as the main parameter of analysis and classification [2, 3].

The doubts about the capability of the  $V_{s30}$  factor to predict the site amplification, and therefore predict precisely the ground displacement, led to the suggestion of more or alternative factors for the analysis. Such suggestions are, but not limited to, the use of the soil fundamental frequency  $f_0$  along with  $V_{s30}$  [4, 5]. Another suggestion is the use of the gradient of the shear-wave velocity profile factor  $B_{30}$  [6] that describes the variation of soil impedance in the first 30 m.

In the present study, the aim is to suggest prediction equations for vibration from surface sources using adapted parameters capable of describing specific site properties.

## 2 Parameters Suggestion

For the purpose of analyzing the soil behavior under higher frequencies excitation originating from surface sources, the existing parameters used in earthquake engineering should be modified. The modifications aim to consider the peculiar aspects of the wave propagation for these kinds of sources. Higher frequencies are associated with smaller wavelength, since wavelength is inversely proportional to frequency. This will intuitively mean that the parameters in the higher frequencies vibration cases will be associated with shallower depths.

Unlike the seismic cases where the waves come from below the surface of earth, the soil surface response from higher frequencies surface sources is mainly dominated by Rayleigh waves/surface waves (R-waves). The dominance of surface waves in the case of vibration will eliminate the idea of using factors describing the shear wave velocity at higher depths, for instance  $V_{s30}$  and  $V_{s20}$ . The alternative will be the use of a parameter capable of describing the very top layer of a complex soil system where the R-waves will be dominant. In order to find such parameter, an approximation should be adopted that fits the best possible the wide range of soil types encountered by civil engineers. Thus this study is focalized on three soil types representing the range of soils usually encountered in shallow depths (as surface layers). The three chosen soil types are characterized by  $V_s = 100$  m/s, 200 m/s, 400 m/s. The frequency content of pulses used to analyze the soils was chosen to cover the range 0–120 Hz in which most of the vibration from urban surface sources fall within.

The average of these frequencies which is 60 Hz and the average of the Rayleigh wave speed over the chosen range of soils are the main defining parameters. The most suitable average depth which will fulfill the description of the required dominance of Rayleigh waves at the surface is then calculated.

The Rayleigh wave velocities for the soils are calculated assuming a constant value of  $\nu = 0.3$  and using the following Eq. (1):

$$V_R = \frac{0.862 + 1.14\nu}{1 + \nu} \times V_S \quad (1)$$

The associated wavelength using the average Rayleigh wave speed of the three soils and a 60 Hz frequency as mean frequency of the source is then calculated as follows:

$$\lambda_R = \frac{V_R}{f} \simeq 3.5 \text{ m} \quad (2)$$

A wavelength of 3 m was finally adopted and the factor  $V_{s3}$  will be used in order to characterize the effective part of soil that controls the effects of the R-waves.

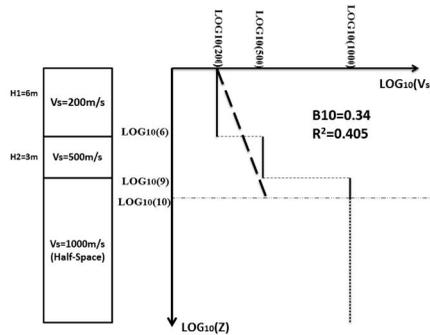
For the purpose of measuring the heterogeneity of the soil (the change of impedance with depth) the most pertinent factor used and previously suggested in measuring

soil properties in the seismic domain is the gradient of the shear-wave velocity profile. The effect of soil heterogeneity will be manifested through the amplification caused from reflected and refracted waves.

Measuring the heterogeneity (the change of impedance with depth) of different sites will be done using gradients for different depths. Gradients  $B_Z$  are found through a linear regression using the following equation:

$$\log_{10} V_{s(Z)} = B_Z \times \log_{10}(z) + A_Z \quad (3)$$

where  $Z$  is the depth of the soil section taken,  $A_Z$  is the origin ordinate of the regression and  $V_{s(Z)}$  the shear wave velocity at the given depth  $Z$ . Figure 1 shows an example of a soil column with a simplified representation of the gradient physical meaning.



**Fig. 1.** A simplified representation of the definition of a gradient ( $B_{10}$  in this case)

Working with a wide range of high frequencies, and therefore a wide range of wavelengths associated with them, makes it theoretically challenging to find the most relevant depth which influences the vibration prediction at the surface. Moreover in different sites, the soil types are changing and different soils are vulnerable to different frequencies. Thus three types of gradients are suggested for the study describing different depths:  $B_{20}$ ,  $B_{15}$  and  $B_{10}$ .

Gradients were chosen to describe values smaller than 30 m and the three gradients will be studied to examine which one correlates better with the numerical predictions of displacement.

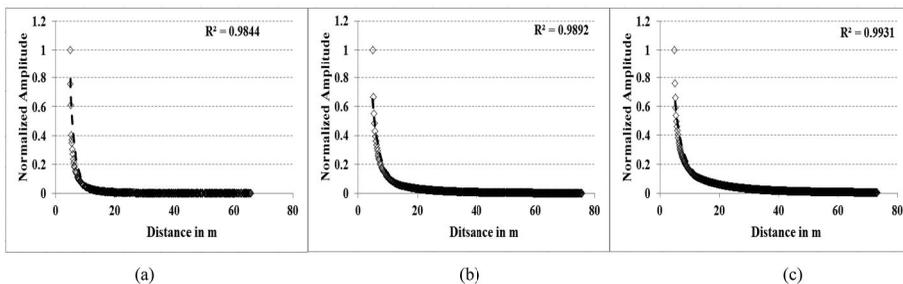
### 3 Computation and Data Selection

For the purpose of the present study, finite elements models were executed using CESAR-LCPC software. A surface displacement was applied as the source where a pulse is injected. Absorbing layers were applied at the boundaries to avoid spurious waves reflected at the model artificial boundaries.

Varying simulations were done taking into account four factors: a varying  $V_{s3}$ , a varying depth of the top layer, a varying depth of the bedrock and a wide variation of gradient values in the range between 0 and 1 (the minimum and the maximum).

### 4 Results

In order to quantify the efficiency of the suggested parameters, a comparison between the multilayered cases and the homogeneous case was done. The comparison considered normalized peak displacement values as a function of distance from source. For the homogeneous cases (half-space) normalized attenuation laws were established for the three types of soils as shown in Fig. 2.



**Fig. 2.** Attenuation laws for homogeneous soils (a)  $V_s = 100$  m/s, (b)  $V_s = 200$  m/s, (c)  $V_s = 400$  m/s

The attenuation equations for the homogenous cases are summarized as following:

$$\text{Attenuation Level } AL = 10.53 \times (d + 5)^{-1.75} \text{ for } V_s = 400 \text{ m/s} \quad (4a)$$

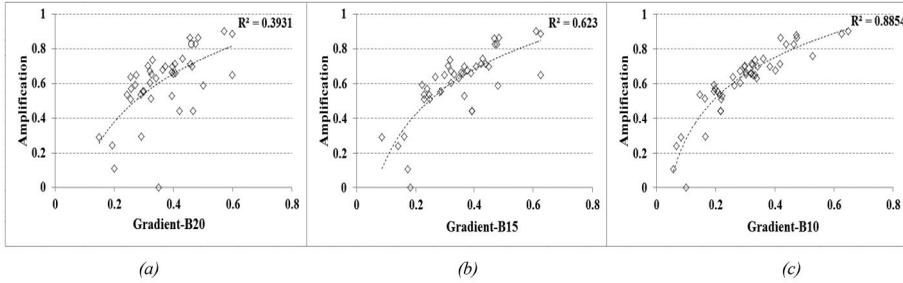
$$\text{Attenuation Level } AL = 19.15 \times (d + 5)^{-2.16} \text{ for } V_s = 200 \text{ m/s} \quad (4b)$$

$$\text{Attenuation Level } AL = 54.53 \times (d + 5)^{-3.02} \text{ for } V_s = 100 \text{ m/s} \quad (4c)$$

where the *Attenuation Level* is the normalized level of vibration after attenuation in respect to the value at the source (smaller than 1) and  $d$  is the distance from source.

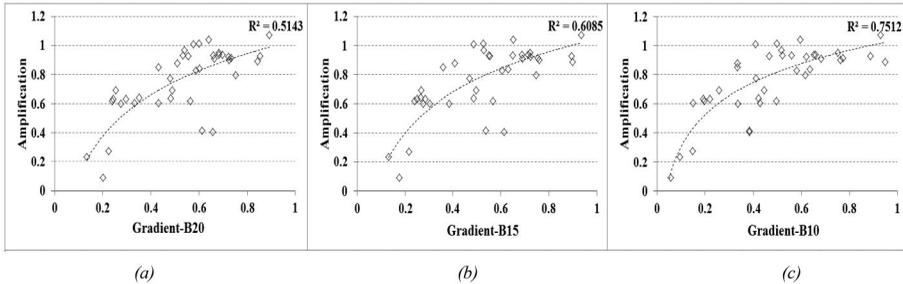
For each case of  $V_{s3}$ , the amplification factor will be calculated as shown in Eq. (5). The amplification factor describes the difference between the total maximum displacement in the multilayered case and the homogeneous case normalized with the latter.

$$\text{Amplification factor } AF = \frac{PGD(\text{Multilayered}) - PGD(\text{Homogeneous})}{PGD(\text{Homogeneous})} \quad (5)$$

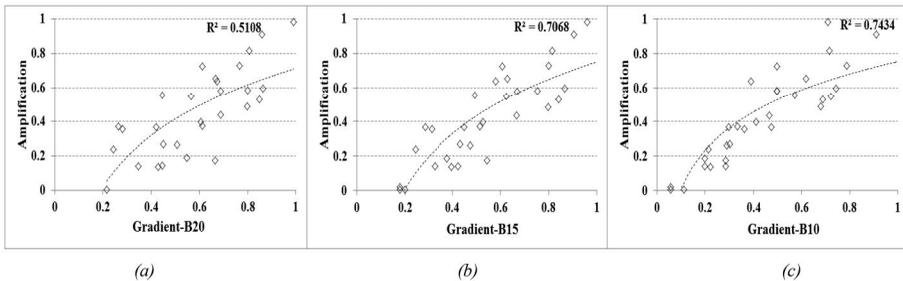


**Fig. 3.** Amplification factors for  $V_s = 400$  m/s versus gradient (a)  $B_{20}$  (b)  $B_{15}$  (c)  $B_{10}$ .

The amplifications are then averaged for different segments of distance from the source and tested with different gradients. The correlation of the calculated values of each group plotted against the varying gradient indicates the efficiency of the chosen gradient in detecting the amplification. Figure 3 illustrates the correlation variability of peak displacement values averaged for the first 20 m next to the source for different gradients ( $B_{20}$ ,  $B_{15}$  and  $B_{10}$ ) for the case of  $V_{s3} = 400$  m/s. The  $B_{10}$  case shows a very coherent distribution of amplification factor versus the ascending gradient. Similarly for the case of  $V_{s3} = 200$  m/s and 100 m/s (Figs. 4 and 5), the best correlation is seen for the  $B_{10}$  gradient.



**Fig. 4.** Amplification factors for  $V_s = 200$  m/s versus gradient (a)  $B_{20}$  (b)  $B_{15}$  (c)  $B_{10}$ .



**Fig. 5.** Amplification factors for  $V_s = 100$  m/s versus gradient (a)  $B_{20}$  (b)  $B_{15}$  (c)  $B_{10}$ .

The amplification equations are summarized as following (using  $B_{10}$  for the first 20 m from source):

$$\text{Amplification factor } AF = 0.34 \times \ln(B_{10}) + 1.06 \text{ for } V_s = 400 \text{ m/s} \quad (6a)$$

$$\text{Amplification factor } AF = 0.314 \times \ln(B_{10}) + 1.05 \text{ for } V_s = 200 \text{ m/s} \quad (6b)$$

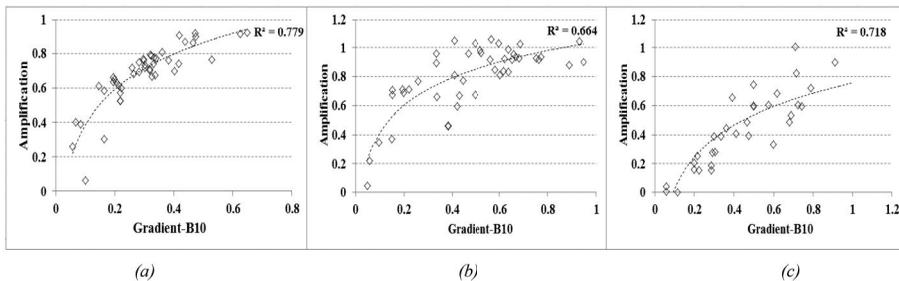
$$\text{Amplification factor } AF = 0.323 \times \ln(B_{10}) + 0.75 \text{ for } V_s = 100 \text{ m/s} \quad (6c)$$

In order to summarize, using Eqs. (4a)–(4c) and (6a)–(6c) the final prediction equation will be in the following form:

$$\text{Final Vibration Amplitude} = AL + AL \times AF \quad (7)$$

where  $AL$  and  $AF$  are respectively the attenuation level and the amplification factor as previously defined.

Changing the segment of distance from the first 20 m to the first 35 m for example, shows that the consistency continues to be present as shown in Fig. 6 but with a small reduction in the correlation fitting value. The rather quick drop of vibration amplitudes away from source as previously seen in Fig. 2 imposes the necessity of being prudent when choosing the size of window in which the values are chosen to be analyzed.



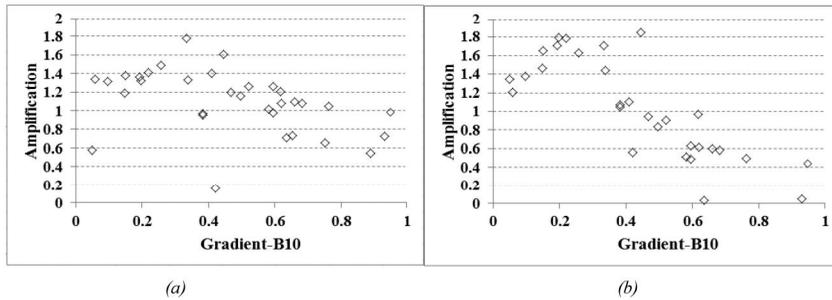
**Fig. 6.** Amplification factors averaging the first 35 m using  $B_{10}$  for (a)  $V_s = 400$  m/s (b)  $V_s = 200$  m/s, (c)  $V_s = 100$  m/s.

Enlarging excessively the window of distance from source will increase the imprecision due to averaging values from a very different range of amplitudes. In other words, enlarging excessively the window will start underestimating the values near the source where the most influential amplification are present in terms of ground displacement, while clearly conserving the same pattern of amplification.

#### 4.1 Mid and Far-Field

While the near field displacement was shown to be well correlated with the variation of the gradient, it is still of clear importance to check for mid-field (20–40 m from source)

and far-field responses (40–60 m from source). No precise correlation could be established when taking soil segment **disrupted** from the source. For the mid-field most of the amplification seems rather constant independently from the gradient. In the far-field, similarly to the mid-field, a clear correlation could not be established but it is noted that the amplifications started to move toward the lower gradients. Hence the lower gradients affect more the far field than the high gradients. An example showing the amplification in a  $V_{s3} = 200$  m/s soil for 20–40 m and 40–60 m from source segments (representing mid-field and far-field) are respectively shown in the Fig. 7.



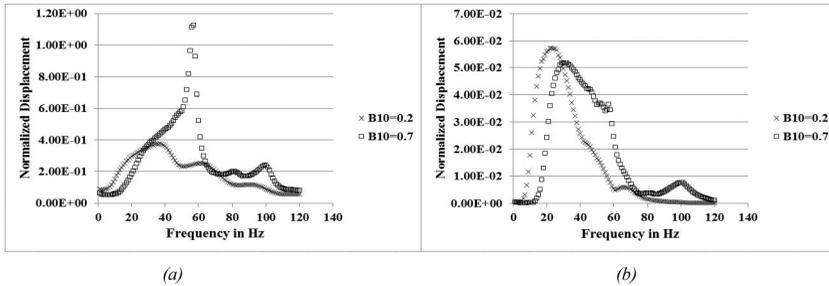
**Fig. 7.** Amplification factors for  $V_{s3} = 200$  m/s soils using  $B_{10}$  for (a) 20–40 m segment (b) 40–60 m segment

It is very important to indicate that the chosen windows to define the mid and far field response are chosen independently of the properties of the site. In other words changing the soil properties whether it is the  $V_{s3}$  or the gradient will influence the frequency response and therefore the wavelengths. Wavelengths will play a pivotal role in defining the response of soil in relation with distance. For instance, weaker soils and smaller gradients will be associated with bigger wavelengths/lower frequencies and therefore the distance segments choice does not represent the wavelength dependency.

The response of a site where  $V_{s3} = 200$  m/s with two different gradients in the frequency domain are shown in Fig. 8. The comparison shows that for the case of near field (0–20 m segment), the vibration is quite dominated by the higher frequencies and the effect of higher gradients is strongly manifested as shown in Fig. 8(a). For the far field (40–60 m segment) window the peak displacement caused in the lower gradient case is higher than that of the higher gradient with a dominance of lower frequencies/larger wavelengths as shown in Fig. 8(b).

However the aim is to get a formulation capable of predicting peak vibration levels not only for the near field, without neglecting the fact that away from source, levels of vibration become unimportant as shown in Fig. 2. Moreover that the most important amplifications are present near the source and the amplification pattern is clearly dominated by it (as clearly shown previously in the comparison between results from the first 20 and 35 m).

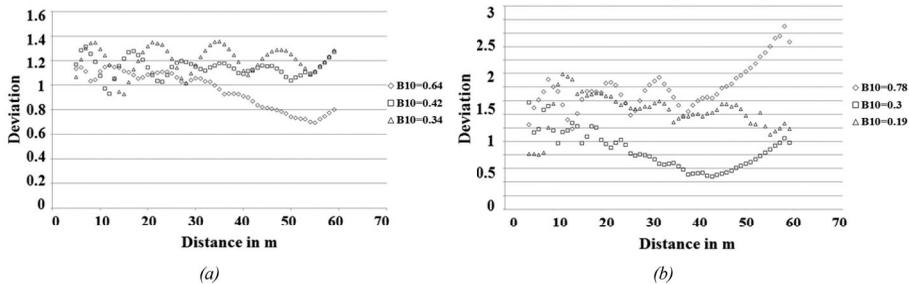
In order to check the validity in the far field of the equations which are derived from the first 20 m, few tests were carried out using the formulation in Eqs. (4a)–(4c),



**Fig. 8.** Normalized displacement for  $V_{s3} = 200$  m/s-in frequency domain for (a) 0–20 m segment (b) 40–60 m segment

(6a)–(6c) and (7). Results from equations were then compared with FEM results between 5 m and 60 m from the source.

Figure 9(a) shows the results of the standard deviation ratio between the results of FEM models and those found by the prediction equations for three different sites with different gradients where  $V_{s3}$  is equal to 400 m/s. The results from the prediction equation shows a good agreement with the FEM results since most of the predicted values are within 20% precision even at far distances (beyond 40 m). Figure 9(b) shows the same for  $V_{s3} = 100$  m/s. The deviation shows a less coherent agreement than the  $V_{s3} = 400$  m/s case but the deviation is still within an acceptable range for varying distances. As a consequence, the formulas are reliable also for distances larger than 20 m (i.e. mid-field and far-field).



**Fig. 9.** Deviation ratio between FEM and prediction equations results for different sites (gradients) where (a)  $V_{s3} = 400$  m/s (b)  $V_{s3} = 100$  m/s

## 5 Conclusions

The scope of this study is to establish a set of formulation that serves as a prediction method for vibration levels in the civil engineering area. For this purpose results obtained from FEM models were analyzed in relation to the relevantly introduced parameters. The proposed parameters ( $V_{s3}$ ,  $B_{10}$ ) showed a very good efficacy in

describing the attenuation and the amplification for a given site. A well correlated formula was developed for the prediction of vibration near the source which also showed a good reliability far from the source. The  $B_{10}$  factor, particularly, showed to be very reliable especially that it is calculated via a linear regression that does not correlate well constantly. A further and more advanced analysis, in the frequency domain, is still needed but was beyond the scope of this paper. Such analysis will help to refine the results and establish a more efficient prediction method for vibration levels induced by surface sources.

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