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Finite element method for vibration investigation of 3D thin-walled braced beams

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Abstract. The flexural-torsional vibration behavior of thin-walled beams with generalized open section is treated using the finite element method. The developed beam element is based on 2 nodes and 7 seven degrees of freedom per node with warping taken as an independent degree of freedom. In the model, effects of rotational inertial kinematic terms are included. The presence of elastic and viscous springs is considered. The model is validated by comparison to original analytical solutions carried out, to test setup in beam vibration and to other benchmark solutions in the literature. Moreover, some tests set up are built. Vibration tests have been investigated on beams with arbitrary cross sections. The flexural torsional vibration modes have been researched in the range 1-800 Hz. Applications in free vibration, forced vibration under constant and variable force excitation are presented. Finally, the effects of elastic springs and dashpot on the dynamic behavior of braced structures and vibration control have been well studied.

Keywords: Thin-walled structure beam; generalized open section; Finite element method; Torsion; Warping; Coupled mode; Vibration; Intermediate bracings; Experimental test.

1. Introduction

Due to their light weight and high efficiency thin-walled open section beams are frequently used in civil structures and mechanical engineering field. Vibration behavior of these beams must be taken in consideration in design procedure due to coupled bending-torsional modes. That, effect causes sudden unexpected structure failure, like what happened in Takoma bridge in 1940. In the literature we can found extensive studies, analytical theories and numerical models employed to investigate the vibration behavior of the thin-walled beams with open sections. It is important to note that torsion exhibit with warping for this types of sections. Vlasov's model [1] is one the first models that included the warping phenomena in the behavior of thin-walled beams, in the 1930's. Mei [2] and Lee [3] offered an analytical analysis of linear vibrations of composite I and Channel open sections. Klausbruckner [4] examined the natural frequencies of Channel beams using theoretically and experimentally methods. The vibration analysis arbitrary open cross-sections have been analyzed by Tanaka [5] and Arpacı [6]. A modified Vlasov's model was used by Ambrosini [7], for linear dynamic analysis. Based on energy method, Prokić [8] derived a semi-analytical method for the free vibration frequencies of simply supported thin-walled beams with arbitrary cross. Petyt [9] formulates the steady state method to solve forced vibration. A numerical solutions and experimental tests to determine natural frequencies for non-symmetric thin walled beam section was presented by Ambrosini [10]. More recently, Pianna [11] compared the experimental with the numerical natural frequencies and buckling loads of aluminum non-symmetric thin-walled beams under axial forces. On the other hand, Nguyen [12] introduced an analytical solution for free vibration of simply supported I beams with intermediate bracings, and validate results using Abaqus.

In what follows, a finite element method for free and forced vibration investigation is detailed in part 2. This developed element is used to study vibration control of beams by adding of intermediate elastic and viscous braces (dashpots). Furthermore, presented element can be used to simulate the response of beams subjected to random signals, like earthquakes. This element is called B3Dw and it is implemented on Matlab software. An analytic method is developed also to solve free vibration problem of thin-walled beams under different boundary conditions. These solutions take in consideration of kinematic rotational terms. So it can be used to compute higher vibration modes with accuracy. These solutions can be found in [13]. In part 3 the model is validated by

experimental test and by comparing to some benchmark results in free and forced vibration. Adding to this, the effect of intermediate braces in vibration control is studied in the last example. Finally, part 4 is the conclusion.

2. Finite element method formulation

2.1 Model basics

A straight element with thin-walled open section depicted in Fig.1 is studied. The rectangular system of coordination (G,x,y,z) is used, with G is the cross section center, the x -axis is parallel to the length of the beam. The shear center was noted C , its co-ordinates in Gyz are (y_c, z_c) . Vlasov's model for non-uniform torsion is used in this model. The displacements components of M can be derived from those of the shear center and centroid points. Then the motion equations are derived using Hamilton's principle. Based on this model and on the Galerkin's approach, dynamic equations of motion are formulated. Solutions for free and forced vibration of thin-walled beams are detailed using finite element approach. In these solutions, the effects of the rotational terms present in the kinetic energy have been considered. The displacements components of point M demonstrated in Mohri [14, 15]:

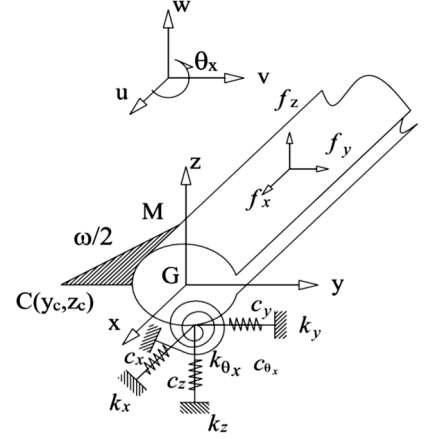


Fig. 1: Thin-walled open section element.

$$u_M = u - y(v' \cos \theta_x + w' \sin \theta_x) - z(w' \cos \theta_x + v' \sin \theta_x) - \omega \theta_x' \quad (1)$$

$$v_M = v - (z - z_c) \sin \theta_x - (y - y_c)(1 - \cos \theta_x) \quad (2)$$

$$w_M = w + (y - y_c) \sin \theta_x - (z - z_c)(1 - \cos \theta_x) \quad (3)$$

The flexural torsional motion equations are derived from Hamilton's principle, based on the weak form of the strain energy, kinetic energy and the applied external loads variations. For this aim, let us introduce the following trial vectors needed for the motion equations.

$$\{q\}^t = \{u \ v \ w \ \theta_x\}; \quad \{\varphi\}^t = \{u' \ v' \ w' \ \theta_x' \ v'' \ w'' \ \theta_x''\} \quad (4a-d)$$

$$\{\gamma\}^t = \{u' v'' w'' \theta_x \theta_x''\}; \quad \{f\}^t = \{f_x \ f_y \ f_z \ m_x\}$$

In these equations and what follows, $()'$ and $()''$ define the successive x derivatives. The beam strain energy variation is based on the beam part and the contribution of the elastic 3D springs given by:

$$\begin{aligned} \delta U_b &= \int_L (EAu' \delta u' + EI_z v'' \delta v'' - EI_z w'' \delta w'' + EI_\omega \theta_x'' \delta \theta_x'' + GI_t \theta_x' \delta \theta_x') dx = \int_L \{\delta \gamma\}^t [D] \{\gamma\} dx \\ \delta U_s &= \int_L k_u u \delta u + k_v v \delta v + k_w w \delta w + k_\theta \theta_x \delta \theta_x = \int_L \{q\}^t [k] \{q\} dx \end{aligned} \quad (5a-c)$$

$$\delta U = \delta U_b + \delta U_s$$

In this, EA , EI_y , EI_z are the axial and bending beam stiffness. The torsion stiffness is based on the classical St-Venant term (GI_t) and warping part (EI_ω) . $[D]$ is the constitutive matrix defined in [15]. $[k]$ is the spring stiffness matrix.

The kinetic energy T of a straight thin-walled element with a mass density ρ and where $m = \rho A$ is given by:

$$\begin{aligned} \delta T &= \int_L m(\dot{u} \delta \dot{u} + \dot{v} \delta \dot{v} + \dot{w} \delta \dot{w} + I_0 \dot{\theta}_x \delta \dot{\theta}_x) dx + \int_L m(z_c(\dot{\theta}_x \delta v + \dot{v} \delta \theta_x) - y_c(\dot{\theta}_x \delta w + \dot{w} \delta \theta_x)) dx + \\ &\quad \int_L \left(m \frac{I_z}{A} (\dot{v}' \delta \dot{v}') + m \frac{I_y}{A} (\dot{w}' \delta \dot{w}') + m \frac{I_\omega}{A} (\dot{\theta}_x' \delta \dot{\theta}_x') \right) dx \\ \delta T &= \int_L m \{\delta \dot{q}\}^t [M_1] \{\dot{q}\} dx + \int_L \frac{m}{A} \{\delta \dot{\varphi}\}^t [M_2] \{\dot{\varphi}\} dx \end{aligned} \quad (6a,b)$$

Where ($\dot{\quad}$) define the time derivative. The first term is classical in beam theory; the second term is the contribution of the flexural torsional coupling of open section beams. $[M_1]$ and $[M_2]$ are mass matrices defined in [13]. The last is the related to rotational inertial effects. For the equilibrium, the variation expression of external load work δW is needed,

$$\delta W = \int_L (f_x \delta u + f_y \delta v + f_z \delta w + m_x \delta \theta_x) dx = \int_L \{\delta q\}^t \{f\} dx \quad (7)$$

Taking account for the presence of viscous damping in the beam and due to dashpots, the damping parts variation are:

$$\delta \Gamma_b = \int_L (C_u \dot{u} \delta u + C_v \dot{v} \delta v + C_w \dot{w} \delta w + C_\theta \dot{\theta}_x \delta \theta_x) dx = \int_L \{\delta q\}^t [C] \{\dot{q}\} dx \quad (8a-c)$$

$$\delta \Gamma_d = \int_L (c_u \dot{u} \delta u + c_v \dot{v} \delta v + c_w \dot{w} \delta w + c_\theta \dot{\theta}_x \delta \theta_x) dx = \int_L \{\delta q\}^t [C_d] \{\dot{q}\} dx$$

$$\delta \Gamma = \delta \Gamma_b + \delta \Gamma_d$$

Based on the strain energy defined in (5), the kinetic energy (6), the load contribution (7) and the damping (8), the motion equations of the beam fulfil the extended Hamilton's principle given by [9]:

$$\delta \int_{t_1}^{t_2} (U + \Gamma - T - W) dt = 0 \quad (9)$$

2.2 Finite element formulation of the motion equations

First step is discretization of structure, mesh process, elements are then defined with the connectivity nodes. In a second step, the variables are written in terms of the displacements of the degree of freedom of the basic elements. For this aim, shape function matrix and its gradient are needed. The basic element stiffness and the mass matrices are then computed by numerical integration using Gauss method. The matrices and vectors of the structure are obtained by the assembling process over the elements. In the present work, the warping is considered as an independent displacement with regard to classical 3D beams. Based on this assumption, each node has 7 degrees of freedom: $\{d\}_e = (u_i, v_i, w_i, \theta_{xi} v'_i, w'_i, \theta'_{xi}, i = 1, 2)$ [13]. The continuous displacement vector $\{q\}$ and its gradient $\{\varphi\}$ defined previously in Eq. (5) are written as in terms of the $\{d\}_e$:

$$\{q\} = [N(\xi)] \{d\}_e \text{ and } \{\varphi\} = [G(\xi)] \{d\}_e; \{\gamma\} = [B(\xi)] \{d\}_e \quad (10a,b)$$

ξ is the local element coordinate, $[N(\xi)]$ and $[G(\xi)]$ are respectively the shape functions and the gradient matrices. $[B(\xi)]$ is the linear matrix. They are defined in [14, 15]. We remind that the mesh process, linear shape functions are adopted for the displacement u and polynomial Hermite's functions are used in discretization of the other components (i.e: v, w, θ_x). Using relationships (10) in (5-9), the motion equations system is the following:

$$[K] \{d\} + [C] \{\dot{d}\} + [M] \{\ddot{d}\} = \{F(t)\} \quad (11)$$

$[K]$, $[C]$ and $[M]$ are respectively the stiffness, damping and mass matrices, they are given by:

$$[K_b] = \sum_e \frac{l}{2} \int_{-1}^1 [B]^t [D] [B] d\xi; [K_s] = \sum_e \frac{l}{2} \int_{-1}^1 [N]^t [k_s] [N] d\xi; [K] = [K_b] + [K_s] \quad (12a-e)$$

$$[M] = \sum_e \frac{l}{2} \int_{-1}^1 m [N]^t [M_1] [N] + \frac{m}{A} [G]^t [M_2] [G] d\xi$$

$$\{F(t)\} = \sum_e \frac{l}{2} \int_{-1}^1 [N]^t \{f(t)\}_e d\xi$$

The structure damping is defined as $[C] = [C_b] + [C_d]$. (13)

The beam damping used here is Rayleigh type and it is given by: $[C_b] = \alpha [M] + \beta [K_b]$ where α and β are the mass stiffness proportional coefficients. Otherwise, the dashpot damping matrix is given by:

$$[C_d] = \sum_e \frac{l}{2} \int_{-1}^1 [N]^t [c_d] [N] d\xi \quad (14)$$

2.3 Free and forced vibration

For the free and undamped vibration, the load forces and damping vanish ($\{F\} = \{0\}$, $[C] = [0]$). The vibration modes are carried out from the classical solutions of the eigenvalue problem given by:

$$(-\Omega^2[M] + [K])\{\phi\} = \{0\} \quad (15)$$

Specific and efficient solver is used to get the vibration modes (Eigen frequencies Ω_j and the eigenvectors $\{\phi_j\}$). In the present study, 3D flexural torsional vibration modes happen. In case of forced vibration under an imposed frequency ω ($\{F\} = \{F_0\}e^{i\omega t}$, with $i^2=-1$), the steady state method is applied in the solution [9]. Thus, the dynamic response (Nodal displacements $=f(\omega)$) is investigated. According to this method, solution is gotten by following two steps: the first one is proceeding with free vibration analysis demonstrated in the previously to obtain n specific vibration modes. In the second step, motion equations (11) are projected on the mode basis $\{d\} = [\phi]\{X\}e^{i\omega t}$, $[\phi]$ is the mode matrix, one gets the following system:

$$(-\omega^2[\phi]^T[M][\phi] + i\omega[\phi]^T[C][\phi] + [\phi]^T[K][\phi])\{X\} = [\phi]^T\{F_0\} \quad (16)$$

Defining the modal matrix terms:

$$\begin{aligned} \text{Generalized mass: } m_j &= \{\phi_j\}^t[M]\{\phi_j\}; & \text{Generalized stiffness: } k_j &= \{\phi_j\}^t[K]\{\phi_j\}; \\ \text{Generalized force: } f_j &= \{\phi_j\}^t\{F_0\}; & \text{Generalized damping: } c_j &= \{\phi_j\}^t[C]\{\phi_j\}; \end{aligned} \quad (17a-d)$$

The solution of the problem sought in a real and imaginary parts $\{X\} = \{A\} + i\{B\}$, where the components are given by:

$$A_j(\omega) = \frac{f_j(k_j - \omega^2 m_j)}{(k_j - \omega^2 m_j)^2 + \omega^2 c_j^2} \quad B_j(\omega) = \frac{-f_j \omega c_j}{(k_j - \omega^2 m_j)^2 + \omega^2 c_j^2} \quad (18a,b)$$

$$\text{Real displacement vector: } \{x_{Re}(\omega)\} = \sum_{j=0}^n A_j \{\phi_j\} \quad (19)$$

$$\text{Imaginary displacement vector: } \{x_{Im}(\omega)\} = \sum_{j=0}^n B_j \{\phi_j\} \quad (20)$$

$$\text{Displacement amplitude can be calculated by mean the following equation: } \{d\} = \sqrt{\{x_{Re}\}^2 + \{x_{Im}\}^2} \quad (21)$$

The present model is validated below on the basis of some benchmark solutions and original test results.

3 Validation examples

3.2 A simply supported beam with arbitrary cross section

In this section, a simply supported beam with arbitrary non-symmetric cross section treated in Prokić [8] was considered for model validation. Beam properties and dimensions are presented in Fig.2.

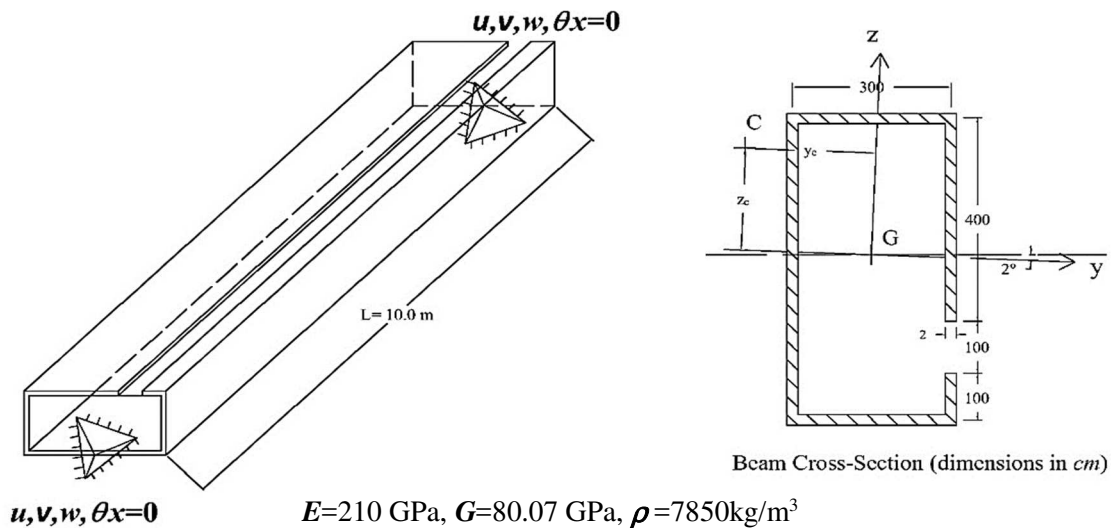


Fig. 2. A simply supported beam with arbitrary cross section: geometry and boundary conditions, and Section dimensions and material properties.

The eigen values of the present work and Prokić's results are reported in (Table 1). The natural frequency solutions are obtained analytically by taking into account or omitting the rotational terms. The present analytic eigen values, finite element solutions, Abaqus and ADINA results are compared to Prokić's benchmark solutions.

Table 1: Comparison between the present models resonance frequencies (Hz) and references values.

	Present Analytic (1)	Present Analytic (2)	Present (FE, B3Dw) (1)	Present (FE, B3Dw) (2)	ADINA (Beam)	Abaqus (B31OS)	Prokić [8] (1)	Prokić [8] (2)
$n=1$	8.77	8.780	8.78	8.78	8.78	8.77	8.68	8.69
	13.15	13.17	13.16	13.17	13.16	13.12	13.02	13.04
	34.19	34.3	34.12	34.30	34.12	33.36	33.83	34.12
$n=2$	34.28	34.41	34.23	34.41	34.23	34.07	33.86	33.93
	51.81	52.10	51.84	52.10	51.84	51.23	51.32	51.58
	133.79	135.88	131.45	135.88	131.50	<u>121.32</u>	130.35	134.74
$n=3$	76.27	76.80	76.43	76.80	76.43	75.67	75.61	76.00
	115.53	116.98	115.68	116.98	115.70	<u>112.73</u>	114.52	115.79
	283.70	304.99	283.92	304.99	283.90	<u>244.30</u>	281.51	302.45
$n=4$	134.64	136.29	135.13	136.29	135.10	<u>132.78</u>	-	-
	203.3	207.82	203.75	207.82	203.80	<u>195.01</u>	-	-
	510.25	541.74	480.03	541.74	480.00	<u>386.03</u>	-	-

(1): with rotational terms and (2): rotational terms neglected, (-): not studied.

By analyzing depicted values in the table1, we can conclude that the present results are a good agreement with the Prokić's values when the effect of rotational terms is neglected or not. It is confirmed that the rotational terms present in the mass matrix are important effect in the case of higher frequencies. Moreover, in Abaqus solutions, some eigenvalues are very singular and must be considered outlined in the Table 1. To verify this singularity, another FE code is used (software) with 3D beam elements with open section. ADINA's Results matched well with the present FEM and analytical results.

3.2 Experimental test setup procedure and comparison

Experimental tests are performed to get natural frequencies. Vibration response of the beams extracted using shaker with harmonic impulse. Test procedure and specimen properties are presented with details in Fig. 3 and 4. After that, finite element calculated results were compared to experimental values.

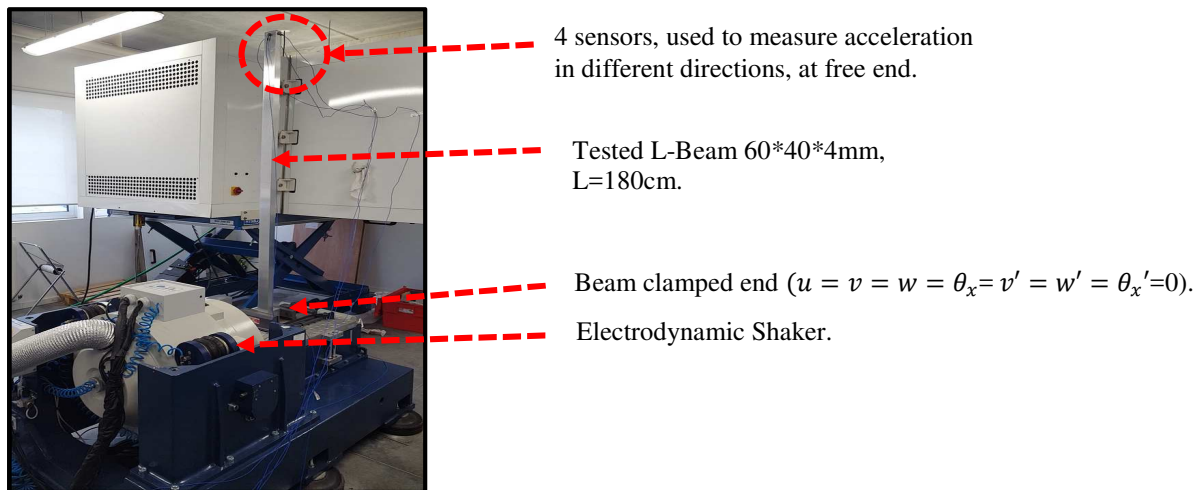


Fig. 3. Experiment setup to determine the resonance frequencies of the cantilever L-beam.

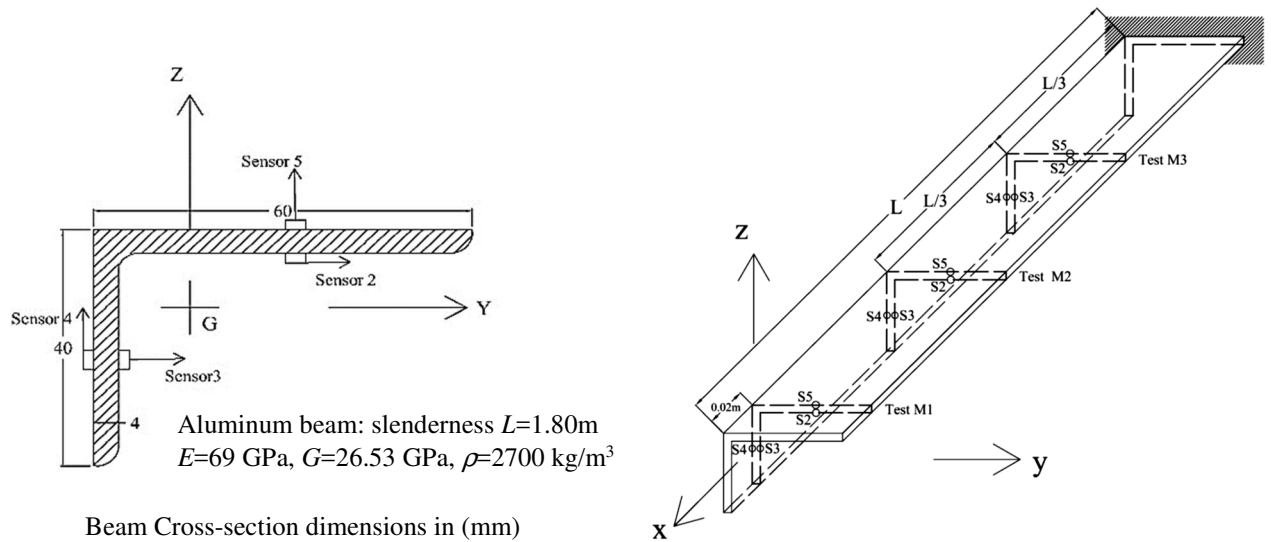


Fig. 4. Tested beam properties with the locations of 4 sensors, for the 3 set of measurements.

Table 2. Comparison between test and numerical natural frequencies (Hz).

Mode	Test	B3Dw	Abaqus (B31OS)	Error % (Test/B3Dw)
1	8.03	7.61	7.53	5.2
2	16.7	17.49	17.47	4.7
3	43	42.57	42.28	1.0
4	50	49.88	49.87	0.2
5	93	94.61	94.72	1.7
6	123	119.53	118.75	2.8
7	161	160.67	160.52	0.2

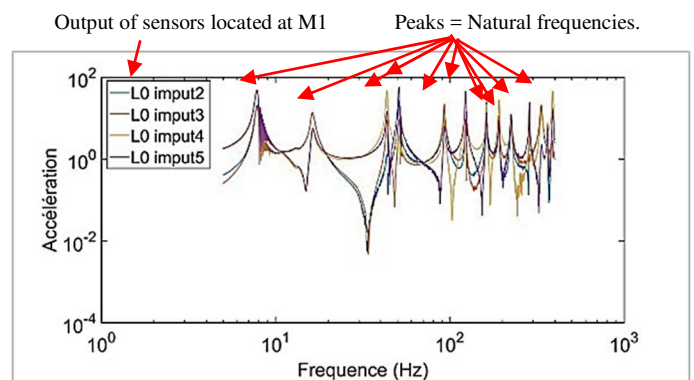


Fig.5. Test results: frequency acceleration response

By analyzing results presented in Table 2 and Fig.5, we can see that test results match well with B3Dw values, so the efficiency of finite element method was proved.

3.3 Vibration control using intermediate bracings (elastic springs and dashpots)

A simply supported beam of 6 m length, with doubly symmetric I cross section ($h=300$, $b=150$, $t_f=10.7$, $t_w=7.1$ mm) is considered. The material properties are: $E = 210$ GPa, $G = 80.77$ GPa, $\rho = 7800$ kg/m³.

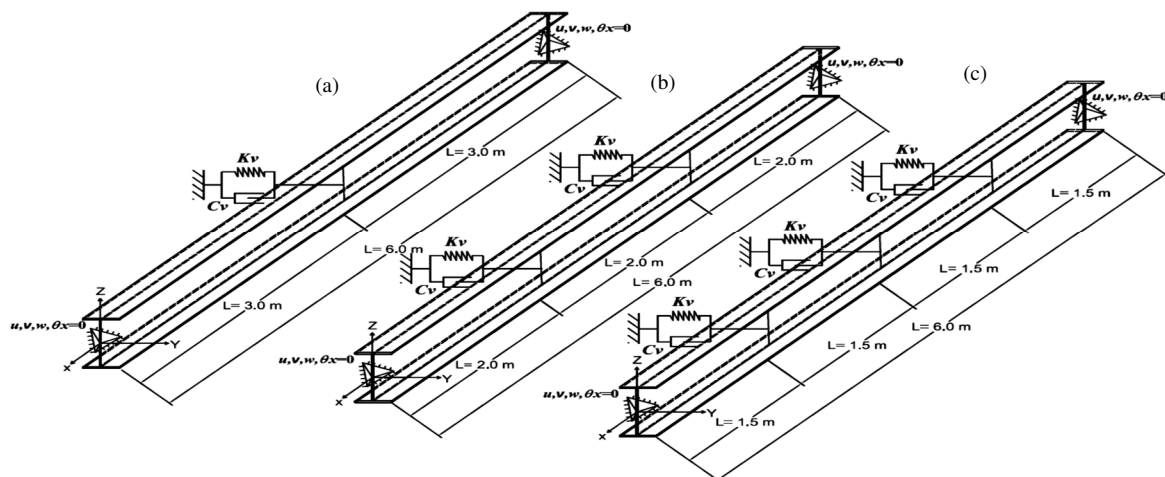
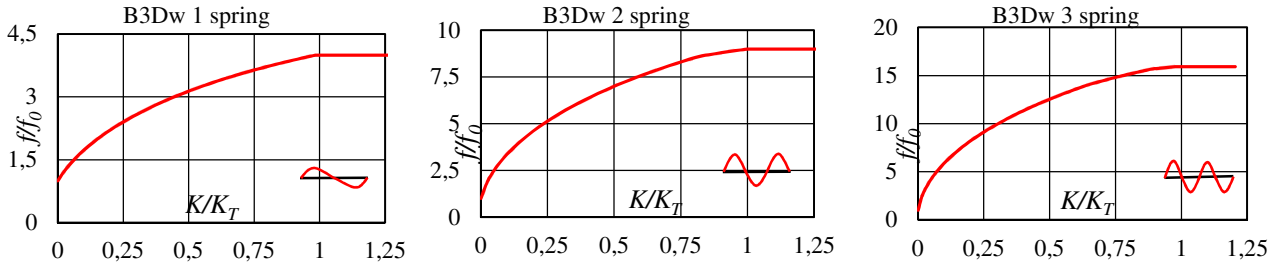
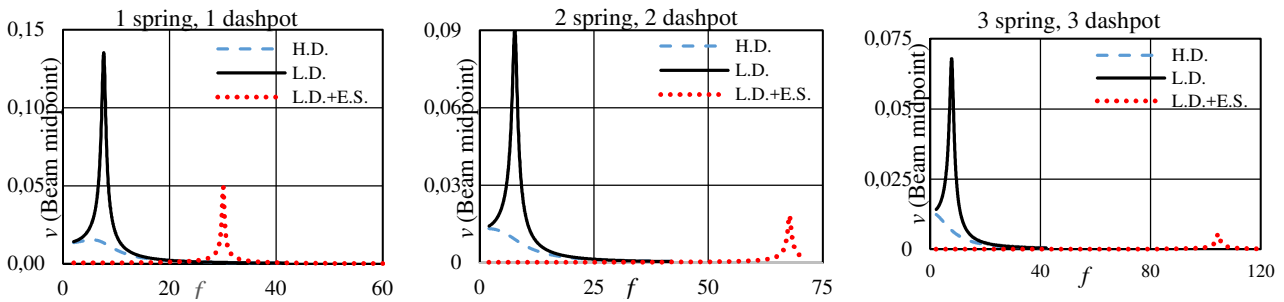


Fig. 6. Simply supported IPE300 beam, with intermediate bracings. (a) one brace at $L/2$, (b) 2 braces at $L/3$ and $2L/3$, (c) 3 braces at $L/4$, $L/2$ and $3L/4$, $k_v = 0-3.0 \times 10^7$ (Elastic spring) [N/m] and $C_v = 582$ (Low Damping) - 5820 (High Damping) [N.S/m]

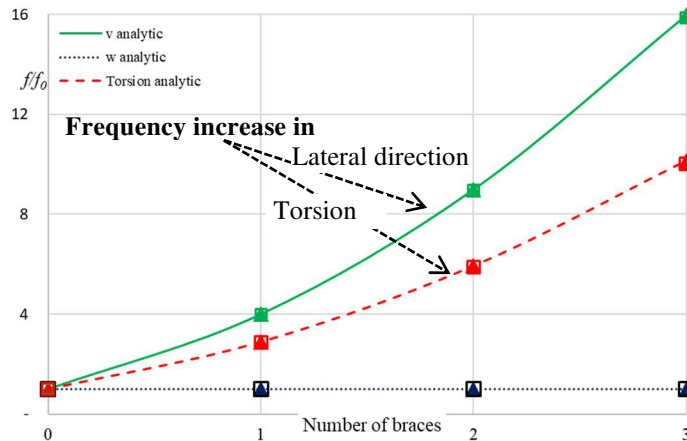
In 3D behavior, the vibration of slender and unrestrained main beams are predominated by lateral bending and torsion modes (modes v , θ). This problem is familiar in bridge applications. In order to improve the vibration behavior of 3D structures against torsion modes, recourse to braces in design are commonly adopted. The natural frequency for torsional and lateral vibration is considerably increased with increasing the number of bracing points along the beam. Also beam displacements decrease in the same time by adding dashpots parallel to elastic springs in lateral direction. In this example we will study the effect of intermediate $nb=1,2,3$ braces on vibration behavior. For this reason, the 3 beams presented in the Fig. 6 are studied. First case in free vibration elastic lateral springs. Then, forced vibration by applying a harmonic distributed load of amplitude 1000 N/m in y direction, in presence of some lateral elastic springs and dashpots (varied from 1 to 3). Results are depicted in Fig. 7, where f_0 denote the fundamental natural frequency of unbraced beam, K_T is the necessary stiffness required to provide full bracing.



(a) Effect of lateral elastic bracings in y direction on lateral bending frequency ratios.



(b) Effect of viscoelastic lateral bracings on vibration behaviour. (H.D.: High damping, L.D.: Low damping, ES: Elastic spring)



(c) Braced beams : improvement of vibration behaviour in presence of elastic springs

Fig. 7. Effect of lateral elastic bracings and dashpots on vibration frequency and beam displacements.

Depicted curves in Fig. 7a, c show that, the natural frequency for torsional and lateral vibration is considerably increased with increasing the number of bracing points along the beam. So the first frequency becomes the $(n+1)^{th}$ by adding n intermediate braces. On the other hand, from Fig. 7b we note that beam displacement decreases significantly in conjunction of frequency increasing, by adding dashpots in parallel with elastic springs. Also by analyzing Fig. 7b we observe that, by increasing number of braces beam displacement decrease linearly. By this way, the impact of intermediate viscoelastic springs in vibration control is more efficient.

4. Conclusion

A finite element method to investigate vibrations behavior of thin-walled beams with generalized open cross sections was developed. This element is used to solve: forced vibration, continuous beams, beams with intermediate dashpot and elastic braces. Also, this finite element method can be used to investigate vibration behavior of beam it is excited by random signals as earth quake. This element has been checked by comparing results to commercial codes and references results and by experimental test. According to this, our model gives accurate results with minimum calculation. The effect of introducing intermediate bracings to control bending and torsional vibration was treated also. A new concept was deduced from the last example that by adding n intermediate viscoelastic braces to an unrestrained beam the first n modes can be removed and beam displacements are reduced near to resonance. Thus, the importance of adding viscoelastic braces in vibration controlled is proven. Finally, experimental tests show very good agreement between the results obtained with the theory demonstrated in this paper and those obtained in the experiences.

References

- [1] Vlasov, V.Z., Thin-walled Elastic Beams, Moscow, 1959, 2nd edition, translated from Russian and published for the national Science foundation, Washington, DC, (1961).
- [2] Mei C., Coupled vibrations of thin walled beams of open section using the finite element method, International Journal of Mechanical Science (1970)12, 883-891.
- [3] Lee J and Kim S. E., Flexural-torsional coupled vibration of thin-walled composite beams with channel sections, Computers and Structures (2002) 80, 133-144.
- [4] Klausbruckner M. J. and Pryputniewicz R. J, Theoretical and experimental study of coupled vibrations of channel beams, Journal of Sound and Vibration (1995) 183 (2), 239-252.
- [5] Tanaka M. and Bercin A. N., Free vibration solution of uniform beams of nonsymmetrical cross section using Mathematica, Computers and Structures (1999) 71, 1-8.
- [6] Arpaci A. and Bozdog E., on free vibration analysis of thin-walled beams with nonsymmetrical open cross-sections, Computers and Structures (2002) 80, 691-695..
- [7] Ambrosini R. D., Riera J. D. and Danesi R. F., A modified Vlasov theory for dynamic analysis of thin-walled and variable open section beams, Engineering Structures (2000) 22, 890-900.
- [8] Prokić A., On triply coupled vibrations of thin-walled beams with arbitrary cross-section. Journal of Sound and Vibration, (2005) 279, 723–737.
- [9] Petyt M., Introduction to Finite Element Vibration Analysis, Cambridge University Press, Second Edition 2010.
- [10] Ambrosini D., On free vibration of nonsymmetrical thin-walled beams, Thin-Walled Structures, 47 (2009) 629–636
- [11] Piana G., Lofrano E., Manuello A., Rutta G., Natural frequencies and buckling of compressed non-symmetric thin-walled beams, Thin-Walled Structures (2017) 111189-196.
- [12] Nguyen C.T., Moon J, Le VN, Lee H-E, Natural frequency for torsional steel I-girders with intermediate bracings. Thin-Walled Structures 49 (2011) 534-542.
- [13] Jrad W., Mohri F., Robin G., Daya E. M., Al hajjar J., Analytical and finite element of the free and forced vibration of unrestrained and braced thin-walled beams, Submitted to Journal of vibration and control, (2019).
- [14] Mohri F., Azrar L., Potier-Ferry M., Vibration analysis of buckled thin-walled beams with open sections, Journal of sound and vibration (2004) 275 (1-2), 434-446.
- [15] Mohri F., Damil N., Potier Ferry M., Large torsion finite element model for thin-walled beams, Computers and Structures, (2008) 86 671–683.