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## Beyond-mean-field effects in Rabi-coupled two-component Bose-Einstein condensate

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**Resubmission of the manuscript LE18380 to Phys.  
Rev. Lett.**

Dear Editors,

We thank the referees for their careful reading of our manuscript and for judging our work timely, interesting and suitable for publication in PRL. In our revised version, we address the points raised by referee C. We believe that these changes have indeed improved the quality of the manuscript. In particular, we have now differently fitted our data in order to assess our detection of the three-body term scaling as  $1/\sqrt{\Omega}$  in the beyond-mean-field energy. We would like to resubmit our manuscript to Physical Review Letters.

Please find below our detailed response to the referee's comments and a list of changes.

With best regards,

Lucas Lavoine, Alfred Hammond, Alessio Recati, Dmitry Petrov, and Thomas Bourdel

**Comment (referee C):** *I have only one remark on this part. The new paragraph at the bottom of page 2 is helpful, but I find the note [36] difficult to follow. When going to this note, I was expecting a detailed analysis of the scaling in  $\sqrt{\Omega}g$ , whereas [36] sends the reader to the supplemental material (SM) of a paper where there is no coherent coupling. I suggest that the authors write a new paragraph in the SM explaining in detail why this coefficient in front of  $n^2$  emerges from the Born approximation. In this respect the explanation that they give in their response to the referee is quite helpful (since the authors provide the integral that enters in the calculation) and it would be nice to make it available to the whole community.*

Reply: We have added the exact zero-range two-body calculations of the scattering length in the presence of RF coupling in the supplemental material.

**Comment (referee C):** *(1) The first issue concerns the fraction of theoretical predictions that have been tested experimentally. The modeling developed in the first two pages singles out 3 possible contributions to the energy, depending on the regime of parameters:  $n^{5/2}$  for low coupling and  $n^2 + n^3$  for large coupling. As far as I can tell, only the  $n^2$  variation is observed without ambiguity here. If this is indeed the case, it should be clearly stated in the Introduction in page 1, in the paragraph starting by "Here, we analytically..."*

Reply: We thank the referee for this important comment. We have address these questions in a new paragraph. It has permitted to clarify our findings. First, our experimental data lie in the regime of strong coupling (or large  $\Omega$ ), where Eq. (5) is valid. This is now explicitly discussed. Second, we now discuss in more detail how we can evidence not only the two-body scaling  $\propto \sqrt{\Omega}$  but also the emerging three-body term scaling with  $1/\sqrt{\Omega}$ . This is done through a fit with the prefactor of the three-body term as an additionnal free parameter (in addition to the atom number). We carefully evaluate the uncertainty of this parameter including the one originating from our imperfect knowledge of the magnetic field noise. Finally, we find that this prefactor agrees with our theoretical prediction and is more than 2 sigma away from the value 0, which would correspond to a pure two-body model.

**Comment (referee C):** *(2) What are the readers supposed to understand from Fig. 1? It would be nice to provide a short statement in the text explaining that the energy minimum occurs for the expected value of delta.*

Reply: We follow the referee advice and focus the text on the experimental finding that the minimum size is found at the expected position in  $\delta/\Omega$ ,

where the MF energy is minimal. The rather complex statement about the dependence of the BMF with  $\delta/\Omega$  is moved to a footnote.

**Comment (referee C):** (3) *In the caption of Fig. 2, page 4, it would be useful to give the value of delta that has been used. I assume that it is  $0.23\Omega$  as mentioned in the top of page 3, col.1, but since this value was varied later in Fig. 1, it is important to lift any ambiguity.*

Reply: The value of  $\delta/\Omega$  is now given in the caption of the figure.

**Comment (referee C):** (4) *In Fig. 2, could the authors indicate by a line or a colored zone the region where the crossover between the  $n^{5/2}$  and  $n^2 + n^3$  regime is supposed to occur? This is in relation with my point (1) above: the readers should be able to understand from this figure which part of the predictions have been tested experimentally.*

Reply: We now add theoretical line corresponding the large  $\Omega$  limit of the BMF energy given by Eq. (5). By comparison with the exact calculation it is now possible to point out that our data lie in the large  $\Omega$  limit. A clear deviation would be visible for  $\Omega < 2$  kHz.

**Comment (referee C):** (5) *Still in Fig.2, the green dashed curve gives the BMF description restricted to the two-body term. Does it mean that it varies as  $\Omega^{1/4}$ , assuming  $R_{TF}$  varies as the square root of the energy? It does not seem to be the case... Please clarify.*

Reply: The calculation of the size assuming only the renormalization of the two-body term is actually very close to a  $\Omega^{1/4}$  scaling (green dotted curve in Fig. 1 of the revised manuscript).

**Comment (referee C):** (6) *In the conclusion I again face issue (1). The authors summarize the various possible regimes found theoretically and then write "We quantitatively verify our theoretical findings...". For this to be correct, one would need a figure where the  $n^{5/2}$  variation on one hand and the  $n^2 + n^3$  variation on the other hand show up in a convincing way.*

Reply: In the conclusion we now state clearly that our measurements are in agreement with the theory for large  $\Omega$ . We also argue that we detect both the two-body term scaling with  $\sqrt{\Omega}$  and the three-body term scaling with  $1/\sqrt{\Omega}$ .

### List of changes

- Abstract: We have change the sentence about the experiment in order to clarify our findings.
- The paragraph describing our work at the end of the introduction has been slightly modified to clarify our finding: "In the large  $\Omega$  limit, we experimentally evidence these two contributions by preparing a condensed  $^{39}\text{K}$  spin-mixture at the point of vanishing mean field and by quantitatively measuring its expansion in a waveguide as a function of  $\Omega$ ."
- The paragraph following equation 5 explaining the correspondence to few body calculations as well as the limitations of the our Bogoliubov approach has been shorten and clarified.
- Figure 1 is now inserted as an inset of figure 2. Its description in the text has been modified in order to focus more on the experimental finding and on the zeroing of the mean-field energy: "For each value of the Rabi frequency  $\Omega$ , we first measure the condensate density profile after expansion and extract the condensate sizes from fits with 1D Thomas-Fermi density profiles  $\propto 1 - (z/R_{\text{TF}})^2$  (see inset of Fig. ??). As a function of the detuning  $\delta$ , a clear minimum in size appears at  $\delta/\Omega \approx 0.23$ , i.e., a position where we expect  $a_{--}$ , and thus also the MF energy to be minimized. As explained above, under the conditions (3), the MF term vanishes at its minimum ( $a_{--} \approx 0$ ) and the expansion of the cloud is governed by the BMF term. " The fact that the BMF term may vary with  $\delta/\Omega$  is now explain in footnote 35.
- A full paragraph is now dedicated to explaining how much of the theoretical model has been tested experimentally: "For our explored range  $\Omega/2\pi > 3$  kHz, Eq. (5) (red dash-dotted curve in Fig. ??) is a good approximation to Eq. (4) (black dashed curve), meaning that we only probe the large coupling behavior of the BMF energy. The dominant two-body energy contribution (green dotted curve) scaling as  $\sqrt{\Omega}$  is clearly evidenced in our data for  $\Omega/2\pi > 6$  kHz. The three-body term ( $\propto 1/\sqrt{\Omega}$ ) is smaller. To give concrete numbers, for  $\Omega/2\pi = 10$  kHz,  $\tilde{\Omega}/(-2g_{\uparrow\downarrow}n) \approx 1$  and the initial two-body and three-body energies per particle are calculated to be 20 Hz and 3.2 Hz, respectively. In order to better understand how much of the model is experimentally tested, we fit our data with two free parameters: the total atom number and a prefactor  $\eta$  in front of the three-body term (in Eq. 5). We then repeat this fitting procedure for different value of the magnetic field noise

$\Delta B$ . We find  $\eta = 0.85(35)$ , where the error bar is dominated by the uncertainty in  $\Delta B$ . This value, in agreement with the expected value  $\eta = 1$ , shows that our measurements are sufficiently accurate to detect the emerging weak three-body contribution to the BMF equation of state from its different scaling with  $\Omega$ . "

- In the conclusion, the sentence "We quantitatively verify our theoretical findings in an experiment where the BMF energy governs the condensate expansion and can thus be accurately measured as a function of  $\Omega$ . Our results open the path to the creation of coherently-coupled quantum droplets in which the two-body interaction (MF+BMF) is compensated by BMF three-body effects [?]." is now modified to clarify our findings: "The experimentally measured condensate expansions are governed by the BMF energy and are in good agreement with the theory at large  $\Omega$ . We detect not only the dominant two-body BMF term  $\propto \sqrt{\Omega}$ , but also evidence the smaller three-body term  $\propto 1/\sqrt{\Omega}$ , opening the path to the creation of coherently-coupled quantum droplets in which the two-body interaction (MF+BMF) is compensated by BMF three-body effects. Exploration of the small  $\Omega$  regime will require a further reduction of the magnetic field fluctuations."
- Two references were added: 29 and 42.
- Some typos were corrected and minor changes also done.