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# COMPRESSION AND EXPANSION NONLINEAR EFFECTS IN AN ELECTRODYNAMIC LOUDSPEAKER: EXPERIMENTS VS. MODEL FAILURE

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## ABSTRACT

An electrodynamic loudspeaker is known to be a nonlinear device due to many physical phenomena resulting in position and current dependent loudspeaker parameters. The most common position-dependent nonlinear sources are the force factor  $Bl(x)$  and stiffness  $K_{ms}(x)$ . These nonlinear effects have many consequences, the most commonly used in measurements being harmonic distortion and intermodulation distortion. Another consequence is the compression effect of the dynamics. Due to a lower  $Bl(x)$  and a higher  $K_{ms}(x)$  for larger displacements, one would expect a compression behavior, i. e. a decreasing ratio of  $X/U$  or  $X/I$  with increasing excitation level ( $X$ ,  $I$ , and  $U$  are the displacement, current, and voltage, respectively). Nevertheless, measurements in the low frequencies of many loudspeakers show an expansion behavior, i.e., an increase in the  $X/U$  and  $X/I$  ratio with increasing excitation level. This non-intuitive behavior, which runs counter to the fundamental theory of simple models of  $Bl(x)$  and  $K_{ms}(x)$ , is studied and discussed in this paper.

## 1. INTRODUCTION

The electrodynamic loudspeaker is the most frequently used type of loudspeaker in the audio industry. Its properties have been studied since its invention nearly a century ago and have given rise to several physical models enabling its behavior to be predicted and its properties to be defined in terms of several parameters.

The simplest and most common model is the Thiele / Small model, which consists of a set of electromechanical parameters describing the performance of the loudspeaker in the low frequencies (frequency range of the diaphragm's piston movement) [1, 2]. These parameters are often used for the prediction of the loudspeaker's behavior by modeling the equivalent electrical circuit.

However, it is known that these parameters only correspond to reality for low signal levels, i.e. for the linear regime. This is because at higher signal levels the loudspeaker behaves in a nonlinear way, leading to distortion. The simple Thiele / Small model is no longer sufficient for these levels, and much work has been done to describe the mechanisms of these nonlinearities and their consequences [3, 4]. There are several models that extend the

Thiele/Small model to include these nonlinearities [5–7].

The vast majority of nonlinear loudspeaker models use the traditional Thiele / Small model and allow some of the parameters to be dependent on displacement and current. There are many nonlinear phenomena in the loudspeaker, such as displacement-dependent force factor  $Bl(x)$ , displacement-dependent stiffness  $K_{ms}(x)$ , current- and displacement-dependent voice coil inductance  $L_e(x, i)$  [4], force factor modulation [8], nonlinearity due to eddy currents [9], temperature-dependent parameters [10], hysteresis in magnetic and suspension materials, and others [11].

The two most apparent displacement dependent parameters are the force factor  $Bl(x)$  and the stiffness  $K_{ms}(x)$ . The force factor  $Bl$  depends on the density of the magnetic field within the air gap where the voice coil moves. Its maximum is usually in the middle of the air gap between the pole pieces of the magnetic circuit, close to the rest position of the voice-coil. Throughout sound reproduction, as the voice moves, it emerges from its ideal position of maximum magnetic field density, and the  $Bl$  force factor decreases. Usually, the  $Bl(x)$  curve is represented by a concave polynomial whose maximum is close to the rest position. Fig. 1 shows an example of a typical force factor  $Bl(x)$  dependent on the displacement.

Stiffness  $K_{ms}$  represents the rigidity of the suspension parts (surround and spider). Unlike the force factor, the displacement-dependent stiffness  $K_{ms}(x)$  is represented by a convex polynomial, the minimum of which is located near the rest position. In fact, the suspension parts are stretched more for a larger excursion, resulting in higher values of  $K_{ms}$  (stiffer suspension). Fig. 2 shows an example of a typical displacement-dependent stiffness  $K_{ms}(x)$ .

These nonlinear phenomena lead to symptoms such as harmonic distortion for sinusoidal excitation, intermodulation distortion for two- and multi-tone excitations, and other effects such as dynamic compression. This paper shows that dynamic compression, a symptom resulting from nonlinear effects, is very different in simulation (described in Sec. 2) and in real measurement (Sec. 3). Moreover, the nonlinear behavior shown experimentally in Sec. 3 is not coherent with the harmonic distortion shown in Sec. 4. All these questionable results are explained by the measurement of nonlinear stiffness in Sec. 5.

## 2. DYNAMIC COMPRESSION - SIMULATIONS

As discussed in the Introduction section, there are many nonlinear phenomena in the loudspeaker that cause many symptoms. One of the symptoms is a theoretical dynamic compression, which is illustrated in Fig. 3. The FRF (Frequency Response Function)  $X/U$  between displacement and voltage is compressed for higher voltage levels. The results of a simulation presented in Fig. 3<sup>1</sup> show that for an arbitrarily selected loudspeaker at frequencies below the resonant frequency (e.g., 20 Hz), the displacement is e.g., 0.6 mm for 1 V, but only 1 mm for 2 V. By doubling the input voltage (an increase of 100%), the displacement increases by only 67%. In other words, the displacement would increase by less than  $n\%$ , for the same increase ( $n\%$ ) in the input voltage due to the dynamic compression.

This theoretical dynamic compression symptom is due to both  $Bl(x)$  (Fig. 1) and  $K_{ms}(x)$  (Fig. 2). Indeed, at low frequencies, the force applied to the membrane is proportional to the displacement and stiffness  $K_{ms}$ . The value of the nonlinear stiffness increases with the amplitude of the displacement. Therefore, an increase in applied force of  $n\%$  results in an increase in displacement of less than  $n\%$ . Similarly, since the force is proportional to the force factor  $Bl$  and the current, and  $Bl(x)$  is lower for larger displacement amplitudes, more current is required to obtain a higher force. The displacement voltage ratio is therefore, from this theoretical point of view, lower at higher voltage levels.

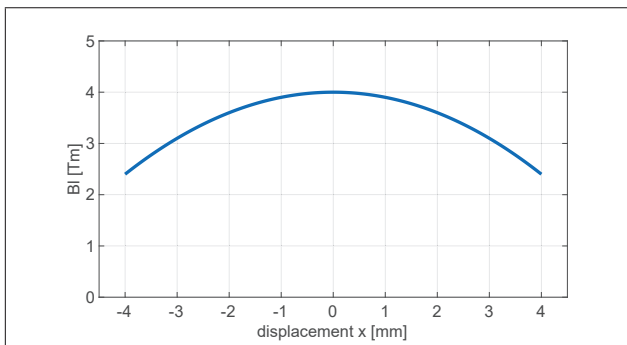


Figure 1. Simulated  $Bl(x)$ .

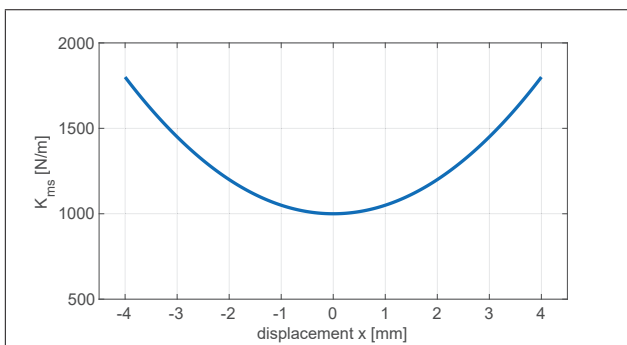


Figure 2. Simulated  $K_{ms}(x)$ .

<sup>1</sup> parameters  $R_e = 6 \Omega$ ,  $L_e = 0.1 \text{ mH}$ ,  $Bl(x) = 4 - 10^5 x^2 \text{ Tm}$ ,  $M_{ms} = 2 \text{ g}$ ,  $R_{ms} = 1 \text{ Ns/m}$ ,  $K_{ms}(x) = 1000 + 10^8 x^2 \text{ N/m}$

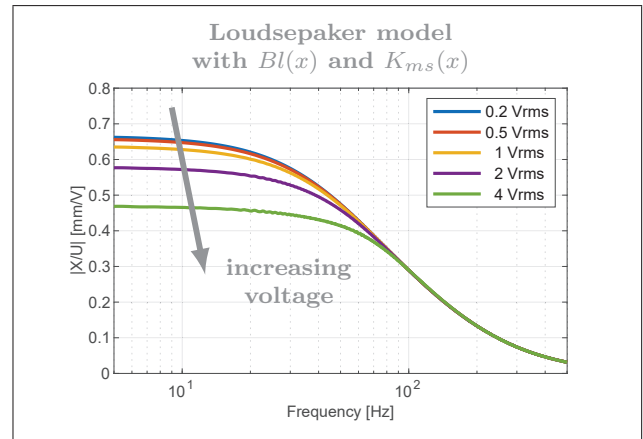


Figure 3. Simulated FRF between the displacement and voltage (first harmonic ratio) of a loudspeaker with a typical  $Bl(x)$  and  $K_{ms}(x)$  dependence.

## 3. EXPERIMENTAL CONTROVERSY

To verify or refute the simulation results presented in Fig. 3, two commercially available woofers (size 6 and 5 inches) are selected and measured. They have a resonant frequency of 40 Hz and 55 Hz respectively. The picture of the two woofers is shown in Fig. 4.

The measurement of the FRF between displacement and voltage is carried out using a simple setup in which the displacement sensor (Panasonic HG-C1030-P) is used to measure the membrane excursion. The Synchronous Swept-Sine signal [12] generated by Matlab, the RME Fireface 400 sound card, and an arbitrary audio amplifier are used to stimulate the loudspeakers in a frequency range between 10 Hz and 500 Hz. The FRF is then calculated as a ratio between the first harmonic of the displacement and the voltage.

Fig. 5 shows the measured FRF between displacement and voltage for the two loudspeakers and for several rms (root mean square) voltages ranging from 0.04 to 4 V. While loudspeaker n.1 has highly level-dependent FRF values, loudspeaker n.2 is much less level dependent. What is very surprising, however, is the increase in the value of the FRF with the voltage level. It is in contradiction with the simulation and expectations set out in the previous section and indicates an expansion rather than a compression character. Similar behavior can be observed in Fig. 6 which shows the FRF (displacement vs. current). Both the increase in the FRF value and the decrease in the resonance frequency with the voltage level can be observed.

These measurement results lead to two important conclusions, to be addressed in the next section:

- Loudspeaker n.1 exhibit much stronger nonlinear behavior than loudspeaker n.2.
- The values of  $X/U$  increase with the voltage level (an expansion character) contrary to the prediction of the simulation.



Figure 4. x.

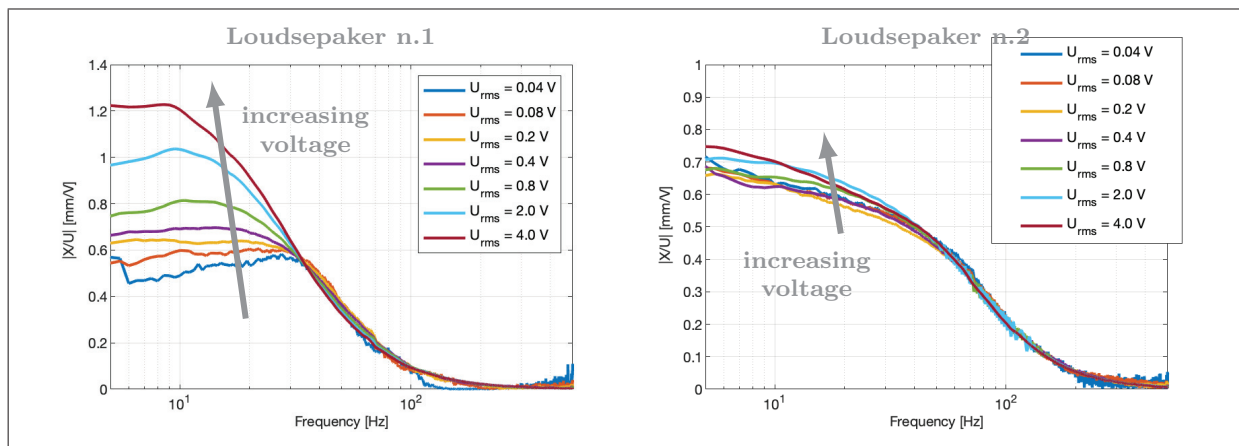


Figure 5. Measured FRF between the displacement and voltage (first harmonic ratio), for several levels of input voltage. Loudspeaker n.1 (left), loudspeaker n.2 (right).

#### 4. HARMONIC DISTORTION

The measured FRFs of displacement versus voltage (Fig. 5) and displacement versus current (Fig. 6) of the two loudspeakers indicate that loudspeaker n.1 seems to have a much stronger nonlinear behavior than loudspeaker n.2. This section examines whether this hypothesis is consistent with the measurement of harmonic distortion.

A sinusoidal voltage signal with a frequency of 20 Hz and an amplitude of 4 V rms is applied to the terminals of each loudspeaker, and the membrane velocity is measured. The velocity spectra, calculated using the FFT (Fast Fourier Transform) algorithm, and the measured velocity signals are shown in Fig. 7 for the two speakers.

Surprisingly, the loudspeaker n.1's harmonic distortion is considerably lower than the n.2 loudspeaker one. This results in a contradiction with the results in Figs. 5 and 6, where the nonlinear behavior of the loudspeaker n.1 was much more significant. The stronger distortion of loudspeaker n.2 (Fig. 7) is apparent both in the time domain, where the waveform is noticeably distorted, and in the frequency domain, where each of the higher harmonics is at least 10 dB higher than that of loudspeaker n.1. This following section clarifies all these results that do not fit the

nonlinear model and do not appear to form a logical piece.

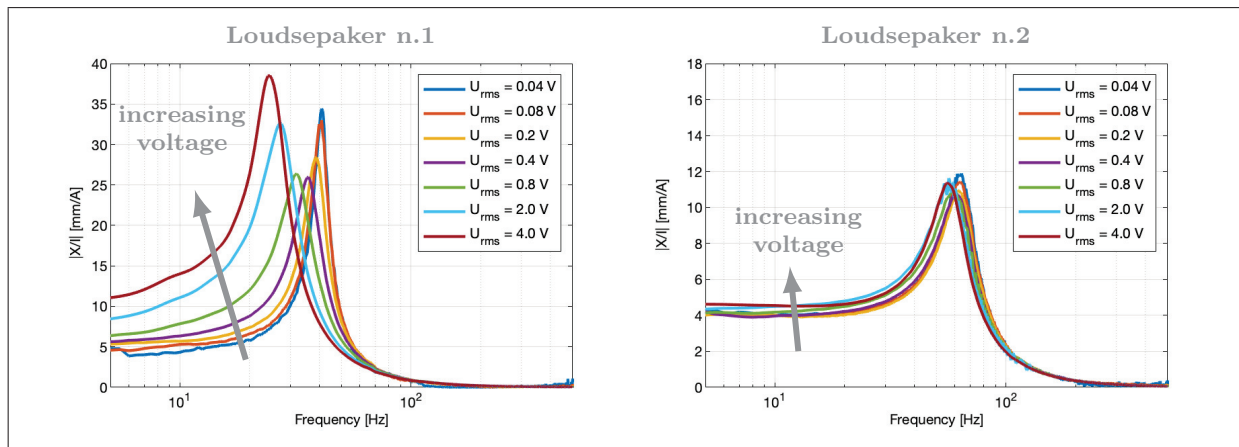
#### 5. LEVEL DEPENDENT STIFFNESS

The resonant frequency shift (see displacement vs. current FRF in Fig. 6) is an important indicator of stiffness behavior. The resonance frequency  $f_{res}$  depends on the mass  $M_{ms}$  and the stiffness  $K_{ms}$  as

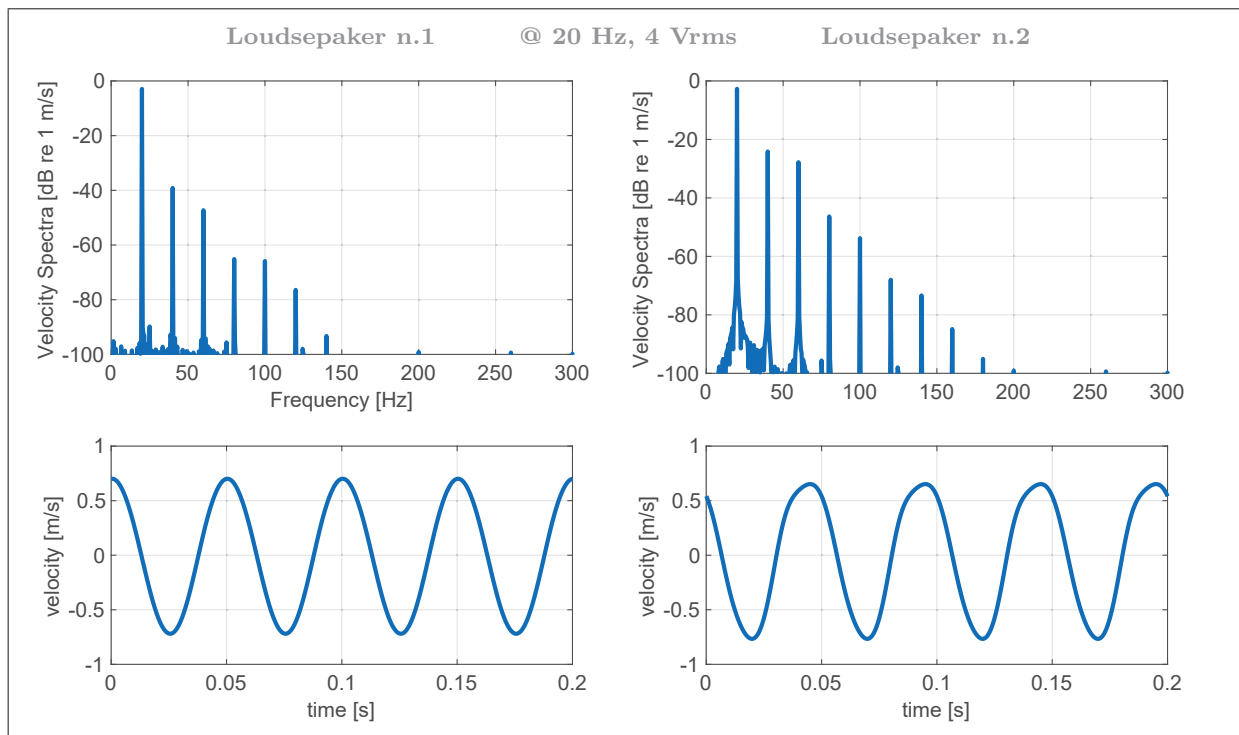
$$f_{res} = \frac{1}{2\pi} \sqrt{\frac{K_{ms}}{M_{ms}}}. \quad (1)$$

While the moving mass  $M_{ms}$  is almost independent of the input voltage level, the stiffness is not. In the case of loudspeaker n.1, the resonance frequency drops from 40 Hz at low voltage level to 25 Hz at high voltage level. According to Eq. (1), and considering a constant mass  $M_{ms}$ , this would mean a decrease in stiffness  $K_{ms}$  of 60 % between the low and high voltage level. Moreover, according to the polynomial representation of the nonlinear stiffness  $K_{ms}$  (Fig. 2), the stiffness should increase with increasing level while in our case it decreases.

This type of behavior, i.e., the decrease in stiffness with increasing excitation amplitude, is well known in the field



**Figure 6.** Measured FRF between the displacement and current (first harmonic ratio), for several levels of input voltage. Loudspeaker n.1 (left), loudspeaker n.2 (right).



**Figure 7.** Measured velocity (waveform and spectra) at 20 Hz and 4 Vrms of loudspeaker n.1 (left) and loudspeaker n.2 (right).

of material sciences. For example, filled rubber insulators follow the well-known Payne effect where the stiffness is high for small excitation amplitudes and low for large amplitudes [13]. Nevertheless, these well-known properties of rubber materials are often neglected in loudspeaker suspension modeling.

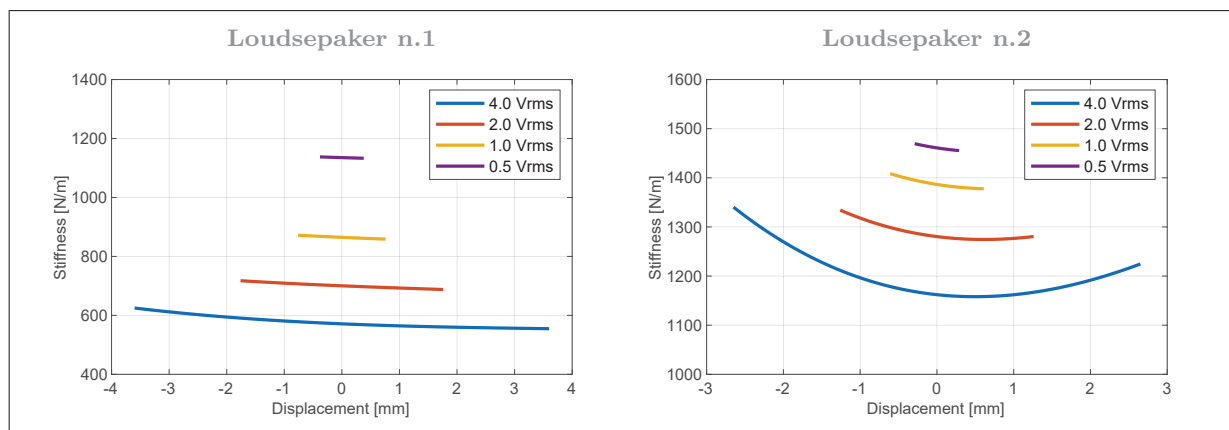
Only a few studies on loudspeakers have reported this behavior [14–16]. In [17], the authors described a method that can be used to measure this effect on loudspeaker suspensions directly. Fig. 8 shows the nonlinear values of  $K_{ms}$  measured using the technique presented in [17]. These results may explain the surprising behavior presented in this paper.

For loudspeaker n.1 (Fig. 8 on the left), the  $K_{ms}(x)$

curves are almost flat for each voltage level, but the value of  $K_{ms}$  depends very much on the voltage level. This is consistent with the expansion-like behavior of the FRF (displacement vs. voltage, Fig. 5) and with the low harmonic distortion (Fig. 7). On the other hand, for loudspeaker n.2 (Fig. 8 on the right), the  $K_{ms}$  curves are very curved (high displacement dependent), but their values depend much less on the voltage level compared to loudspeaker n.1.

Considering a single excitation level (for example, 4 Vrms), the loudspeaker n.1 has a much flatter  $K_{ms}(x)$  curve compared to the loudspeaker n.2. Therefore, for a single harmonically excitation, loudspeaker n.1 distorts less than loudspeaker n.2, as confirmed by the measure-





**Figure 8.** Measured  $K_{ms}$  as a function of displacement, for several levels of input voltage. Loudspeaker n.1 (left), loudspeaker n.2 (right).

ment of harmonic distortion (Fig. 7).

For loudspeaker n.1, the mean value of  $K_{ms}(x)$  ranges from 1150 N / m for 0.5 Vrms to 600 N / m for 4 Vrms, while for loudspeaker n.2, it ranges from 1450 N / m to 1200 N / m only. Consequently, the variation of the mean  $K_{ms}$  with the excitation level is much smaller for loudspeaker n.2 than for loudspeaker n.1. Therefore, the resonance frequency shift for the loudspeaker n.2 is much smaller. Two facts support this behavior. Firstly, in the measurement of FRF displacement vs. current (Fig. 6), the resonant frequency shifts accordingly. Second, the FRF displacement vs. current (Fig. 6) and the displacement vs. voltage (Fig. 5) show that below the resonant frequency where the displacement depends mainly on stiffness, relatively less effort is required to obtain larger displacements.

## 6. CONCLUSION

This paper shows that basic nonlinear loudspeaker models based on nonlinear static functions (polynomials)  $Bl(x)$  and  $K_{ms}(x)$  fail to explain some of the symptoms observed experimentally. The first, which occurs below the resonant frequency, is the increasing value of the Frequency Response Functions (displacement vs. voltage and displacement vs. current) with increasing voltage level. This symptom can be seen as an expansion effect, whereas the model predicts a compression effect instead. The second is the decreasing resonance frequency with increasing voltage level; the model predicts the opposite behavior. Finally, experience with two speakers shows that two different sets of measures can give a contradictory indication from a nonlinear point of view. All these results can be explained by a more complex nonlinear stiffness behavior that is generally neglected in loudspeaker modeling.

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