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# EXPERIMENTAL METHODS FOR CHARACTERIZATION OF ONE PORT NONLINEAR ACOUSTIC SYSTEMS: APPLICATION TO SOUND ABSORBERS

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## ABSTRACT

Numerous sound absorbers dedicated to noise reduction at low frequencies are based on nonlinear properties. Impedance is classically used to describe the behaviour of a sound passive linear absorber, considered as a one-port acoustic system, and measurement methods are available. The objective of this work is to develop identification techniques to characterize nonlinear elements at low frequencies and high levels. We used a specific set-up of impedance tube named "Short Kundt's Tube", with a swept-sine as excitation and one pressure measurement inside the tube. Different approaches are discussed. The first one is defined as a linearisation method and gives access to the acoustic impedance quantities. The second one is based on a nonlinear model which is able to characterize energy transfer to higher harmonics. This approach is defined as a scattering matrix linking the harmonic components of the incident and reflected pressure wave amplitudes. These two approaches are defined in frequency domain. Finally the first step of a third approach is considered. It is based on an Hammerstein model, and permits to obtain a nonlinear relation between the incident and the reflected pressure waves in time domain reducing in the favourable case the dependency with respect the excitation level. Finally, these techniques are illustrated characterizing nonlinear vibroacoustic absorbers.

## 1. INTRODUCTION

Sound absorbers like nonlinear Helmholtz resonators or nonlinear vibroacoustic absorbers, based on nonlinear properties, permit to reduce noise at low frequencies. These absorbers, also known as NES for Nonlinear Energy Sink, consist in a thin structure submitted to large deformations and present nonlinear resonances which permit to absorb sound energy [1, 2]. To improve the knowledge of a NES as an acoustic device with absorbing properties we need to develop specific tools.

Two approaches to measure the acoustic characteristics, dependent of the excitation level and developed in frequency domain, are first discussed in this work. The first one, named "linearisation method", gives access to the acoustic impedance and/or the reflection coefficient. The second one, named "nonlinear model", permits to charac-

terize energy transfer between harmonics. It is defined as a scattering matrix [3] relating the harmonic components of the incident pressure wave amplitudes with the harmonic components of the reflected pressure wave amplitudes.

A third approach, developed in time domain, permits to have a direct relation between the incident and the reflected pressure waves. Many methods of identification are available in time domain (as MISO, Volterra...) and we have chosen here a Hammerstein model for its simplicity of implementation.

Data needed in the proposed approaches were obtained from a non standard equipment named "Short Kundt's Tube" able to reach very high levels at low frequencies. The absorbers are excited by a synchronized swept-sine exhibiting an exponential instantaneous frequency. We measure the acoustic pressure inside the tube, at one point only, in front of the device under study. This set-up needs also a prior calibration of the source from measurements of known acoustic loads (*i.e.* characteristics are known analytically).

This paper is organized as follows : Section 2 presents the measurement set-up. In Section 3, the linear impedance model which permits to have a first linearized characterization of nonlinear absorbers is described. The source calibration is also presented. Section 4 is dedicated to the nonlinear impedance model which takes into account the nonlinearities present in the system. These techniques are applied for the characterization of a nonlinear acoustic absorber, made of a loudspeaker membrane.

## 2. MEASUREMENT SET-UP : SHORT KUNDT'S TUBE (SKT)

We use the measurement set-up developed in [4] (see Fig. 1). It is composed of a sound source, a short tubular section and a device under study (DUT), not shown on the figure. The source and the DUT are connected through the tubular section having a rear length of  $x_{rear} = 0.13$  m, a front length of  $x_{front} = 0.35$  m and a diameter  $D_t = 0.175$  m. The rear side is closed by an air-tight disc placed inside the section. The acoustical scheme of this set-up is represented Fig. 2.

The source is composed of four loudspeakers (type Beyma 10LW30/N, diam 0.25 m) controlled by voltage  $u(t)$  (or  $U(f)$  in frequency domain) connected perpendic-

ularly at the tubular section by smooth 3D-printed horns with initial diameter 0.25 m and final diameter 0.1 m. The rear face of each loudspeaker is enclosed in a 37 ℓ air-tight box. The four loudspeakers are connected at the same abscissa and contribute equally to the volume velocity of the source. A microphone (G.R.A.S 40BH 1/4") is localized inside the right section at 0.035 m away from the DUT ( $x_m = 0.035$  m).

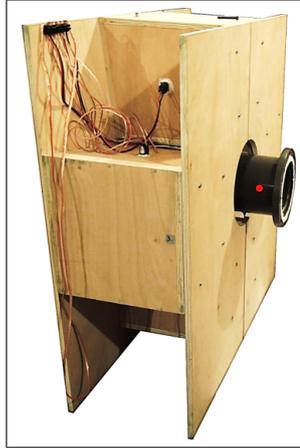


Figure 1. Picture of the source

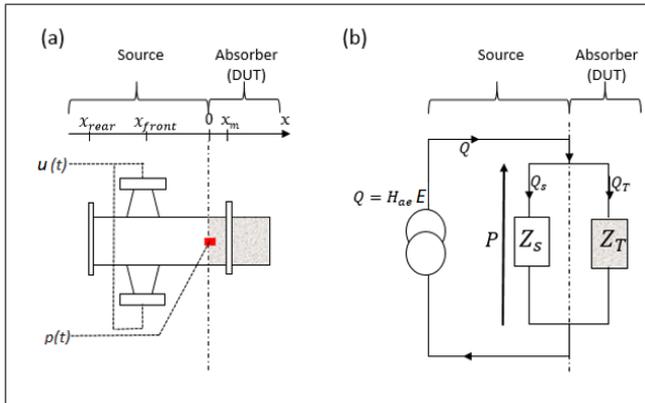


Figure 2. Scheme of (a) the experimental set-up and (b) the equivalent electroacoustic circuit of the one-microphone identification method

Assuming plane wave mode, the source was designed [4] for a low frequency range, between 10 to 700 Hz, in order to expose the DUT to high pressure levels ( $> 300$  Pa).

The air-tightness of the source is critical and has been checked.

The source can be used with different types of excitation signals (white noise, sine, swept-sine). In a first time, we have made the choice to use a stepped-sine allowing us to have informations frequency by frequency. A second type of signal has been used, the swept-sine, in order to reduce the experimentation time and obtain a characterization of a DUT for all the frequencies from one run.

### 3. LINEAR IMPEDANCE MODEL

This approach permits to obtain a "linear" characterization of nonlinear absorbers in terms of apparent impedance and apparent reflection coefficient. The dependency of these quantities with respect, for example, to the excitation level gives interesting information on the nonlinear behaviour of the DUT (softening or hardening effects, energy dissipation).

#### 3.1 Principle of the method

Classically (*i.e.* under linear assumption), the impedance permits to describe the behaviour of an acoustic load and measurement methods are available. The impedance is defined, for any cross-section of the tube, as the ratio between the mean acoustic pressure and the volume velocity. It is also possible to equivalently characterize the acoustic load using the reflection coefficient.

We consider a DUT connected to the SKT, in plane waves hypothesis. We assume that the DUT is characterized by its impedance  $Z_T(f)$ . If the DUT is nonlinear, the impedance  $Z_T$  corresponds to a linear approximation of the relationship between the fundamental components of the spectra of the pressure  $p(t)$  measured by the microphone and the associated volume velocity. We will show that  $Z_T$  can be obtained through a calibration set from the measured pressure  $p(t)$ .

Assuming the source and the DUT linear and time-invariant, the coupled system SKT+DUT can be represented by the electro-acoustic scheme shown in Fig. 2(b). The source is characterized by its impedance  $Z_S(f)$  and the transfer function  $H_{ac}(f)$  between the acoustic volume velocity  $Q(f)$  and the control voltage  $U(f)$  of the loudspeakers as  $Q(f) = H_{ac}(f) U(f)$ . The equivalent impedance  $Z(f)$  loading the volume velocity source is defined by

$$Z(f) = \frac{Z_T(f)Z_s(f)}{Z_T(f) + Z_s(f)} = \frac{H_m(f)}{H_{ac}(f)} \quad (1)$$

where  $H_m(f)$  is the transfer function between the measured control voltage  $U(f)$  of the source and the measured acoustic pressure  $P(f)$  at the microphone.

It follows that the impedance  $Z_T(f)$  of the DUT can be expressed as

$$Z_T(f) = \frac{H_m(f)Z_s(f)}{H_{ac}(f)Z_s(f) - H_m(f)} \quad (2)$$

and the reflection coefficient  $R_T(f)$  as

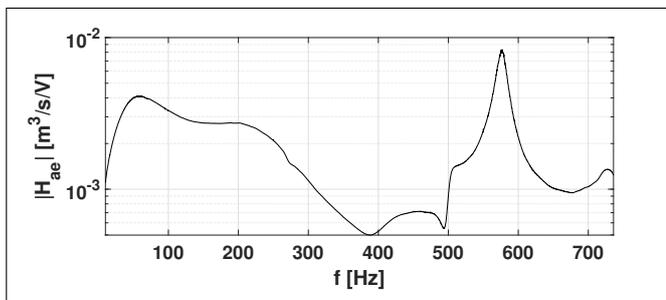
$$R_T(f) = \frac{H_m(f)Z_s(f) - Z_c(f)(Z_s(f)H_{ac}(f) - H_m(f))}{H_m(f)Z_s(f) + Z_c(f)(Z_s(f)H_{ac}(f) - H_m(f))} \quad (3)$$

where  $Z_c = \rho c/S$  is the specific acoustic impedance of air,  $\rho$  is the density of air,  $c$  the sound wave velocity and  $S$  the area of the waveguide section of the SKT.

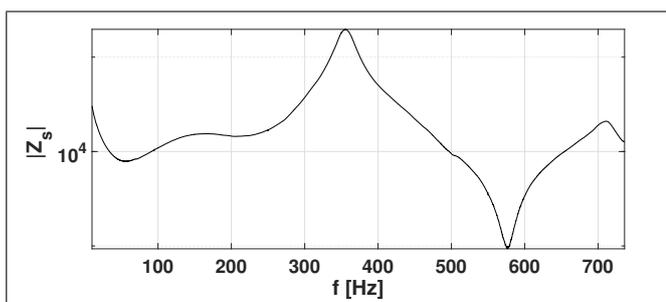
To obtain  $Z_T(f)$  and  $R_T(f)$ , one measurement of the transfer function  $H_m(f)$  is required if the source characteristics  $H_{ac}(f)$  and  $Z_s(f)$  are known. These quantities

can be estimated from  $H_m(f)$  measured for several known acoustic loads. This step is named set-up calibration of the source. From Eqn. (2) and (3), a set-up calibration based on two different known loads is theoretically sufficient to determine  $H_{ae}(f)$  and  $Z_s(f)$ . An over-determination of the calibration data is wisest to have a better estimation of this coefficients [5].

The modulus (respectively argument) of  $H_{ae}(f)$  and  $Z_s(f)$  are reported Fig. 3 and Fig. 4 (respectively Fig. 5 and Fig. 6). These quantities have been obtained from a set-up calibration involving a logarithmic swept-sine excitation of duration 16 s from  $f_1 = 10$  Hz to  $f_2 = 730$  Hz and for an excitation level of  $a = 6.5$  V (see Eqn. (19)). We can observe a resonance pic (Fig. 3) on  $H_{ae}(f)$  at  $f = 577$  Hz corresponding to a zero of  $Z_s(f)$  (associated to rotation phases on  $arg(H_{ae}(f))$  and  $arg(Z_s(f))$ ) at the same frequency. This frequency corresponds to a zero pressure inside the tube. Attention was paid to shift this resonance towards high frequencies by decreasing the length of the rear tube. In the same way, the zero of  $H_{ae}(f)$  at  $f = 495$  Hz corresponds to a maximum of  $Z_s(f)$ . We have checked that  $H_{ae}(f)$  and  $Z_s(f)$  do not depend on the excitation level.



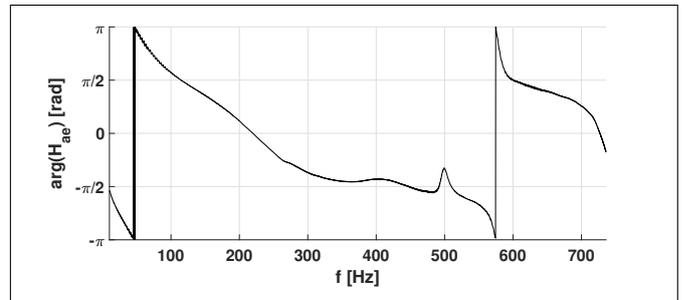
**Figure 3.**  $H_{ae}$  estimated from six known acoustic loads



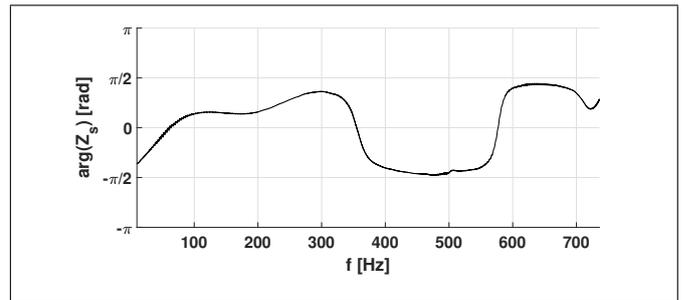
**Figure 4.**  $Z_s$  estimated from six known acoustic loads

### 3.2 Application to a nonlinear vibroacoustic absorber

We illustrated here the estimation of  $Z_T(f)$  considering a nonlinear device. The nonlinear device consists in a loud-speaker membrane used as a NES, as described in [1]. Fig. 7. shows the modulus of impedance  $Z_T$  and Fig. 8 the reflection coefficient  $R_T$ . These two quantities (obtained from Eqns. (2) and (3) knowing  $H_{ae}(f)$  and  $Z_s(f)$ ) have been measured for six excitation levels using a logarithmic sweep excitation source from  $f_1 = 10$  to  $f_2 = 100$  Hz. We can first remark that the estimated functions  $Z_T(f)$  are



**Figure 5.** Angle of  $H_{ae}$  estimated from six known acoustic loads



**Figure 6.** Angle of  $Z_s$  estimated from six known acoustic loads

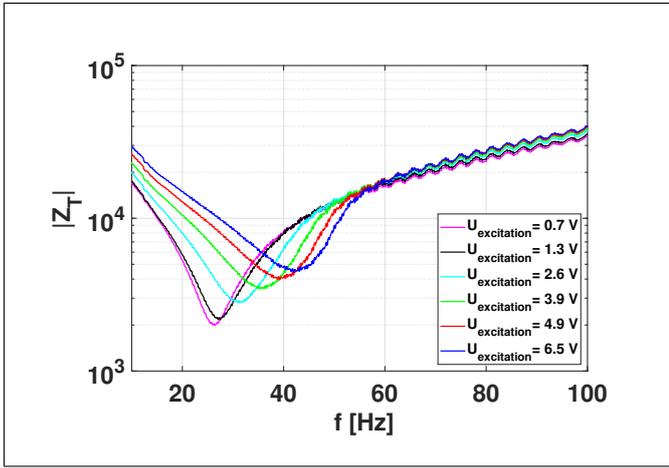
not so smooth as in the linear case. The curves are smooth from  $f = f_1$  to  $f = 2f_1$ . For  $f$  greater than  $2f_1$ , oscillations appear and increase at each harmonic of  $f_1$  (i.e  $2f_1, 3f_1, \dots$ ). This phenomenon is due to the nonlinear behaviour of the DUT and the global analysis in time of the Fourier transform. However, the hardening nature of the DUT can be observed Fig. 7 where the resonance frequency clearly increases with the excitation level.

Although same comments can be made on the reflection coefficient Fig. 8 (and also explain the reflection coefficient higher than 1 from  $f = 50$  Hz), the reflection coefficient allows to compare the amount of energy removed out of the system by the absorber to the case of a total energy reflection  $|R_T|^2 = 1$ . We can see on Fig. 8 that the energy is extracted by the absorber over a frequency range widening with the increase of the excitation level. Note that only a part of extracted energy is absorbed by the NES while another part is radiated by the outward face of the membrane or converted to other frequencies.

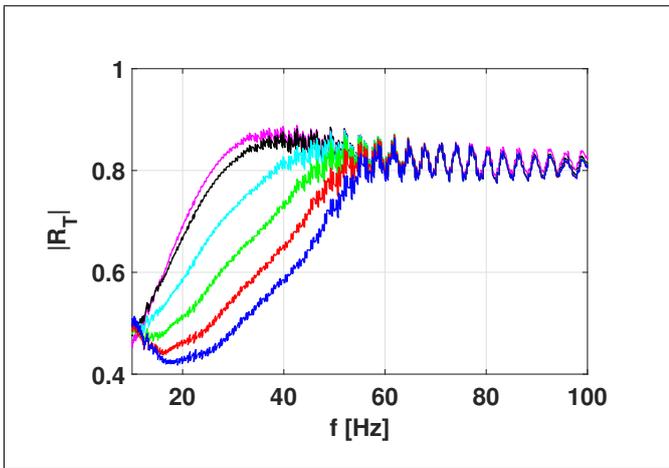
An other linear estimation would be possible taking into account only the fundamental harmonic of  $P(f)$  and  $U(f)$ . Results are not shown here.

## 4. NONLINEAR IMPEDANCE MODEL

The linear impedance model is not sufficient to obtain a complete characterisation of nonlinear absorbers. To take into account nonlinearities, we develop a model taking into account the energy transfer between harmonics.



**Figure 7.** Estimation of impedance for a loudspeaker membrane



**Figure 8.** Estimation of reflection coefficient for a loudspeaker membrane

## 4.1 Scattering matrix modelling

### 4.1.1 Nonlinear model

We assume here that the acoustic source generates an excitation at only one frequency  $f$  (named excitation frequency or fundamental frequency). A multi-port model proposed in [3] consists of, in terms of impedance formulation, characterizing the relationship between the harmonic components  $P_n(f)$  of the acoustic pressure at the microphone position and the harmonic components  $Q_k(f)$  of the corresponding acoustic volume velocity as

$$P_n(f) = \sum_{k=1}^{\infty} Z_{nk}(f, |P_1(f)|) Q_k(f) \text{ for } n = 1, 2, \dots \quad (4)$$

where the impedance term  $Z_{nk}(f, |P_1(f)|)$  represents the opposition at the frequency  $nf$  that the acoustic load (the DUT) presents to the acoustic flow at the frequency  $kf$  from the fundamental excitation frequency  $f$ . It is assumed that  $Z_{nk}(f, |P_1(f)|)$  depends only on the excitation frequency  $f$  and the amplitude level of the fundamental

component  $|P_1(f)|$ . Eqn. (4) corresponds to an extension of the linear formulation involving  $Z_T$ .

Nonlinearities can also be equivalently characterized by a scattering matrix approach as

$$P_n^-(f) = \sum_{k=1}^{\infty} S_{nk}(f, |P_1(f)|) P_k^+(f) \text{ for } n = 1, 2, \dots \quad (5)$$

where  $P_n^-(f)$  (respectively  $P_k^+(f)$ ) denotes the  $n^{\text{th}}$  harmonic of the reflected (respectively incident) acoustic wave  $p^-(t)$  (respectively  $p^+(t)$ ). The term  $S_{nk}(f, |P_1(f)|)$  represents the transfer of energy from the harmonic term ( $P_n^+(f)$ ) of the acoustic pressure at the microphone position to the harmonic term ( $P_k^-(f)$ ).

A simplified formulation of Eqns. (4) and (5) can be obtained assuming that :

- energy exchange can occur only from low to high frequency ( $S_{nk}(f, |P_1(f)|) = 0$  for  $n < k$ )
- the DUT satisfies the harmonic superposition principle [6] ( $S_{nk}(f, |P_1(f)|) = 0$  for  $1 < k < n$ ).

which takes, for example for Eqn. (5), the following form in matrix formulation

$$\begin{pmatrix} P_1^-(f) \\ P_2^-(f) \\ P_3^-(f) \\ \vdots \end{pmatrix} = \begin{pmatrix} S_{11}(f) & 0 & 0 & \dots \\ S_{21}(f) & S_{22}(f) & 0 & \dots \\ S_{31}(f) & 0 & S_{33}(f) & \dots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix} \begin{pmatrix} P_1^+(f) \\ P_2^+(f) \\ P_3^+(f) \\ \vdots \end{pmatrix} \quad (6)$$

To simplify the notations, we have omitted the dependence on  $|P_1(f)|$  on the notations of  $S_{nk}(f, |P_1(f)|)$ .

Note that when the acoustic system is linear, only the terms  $Z_{nn}(f)$  and  $S_{nn}(f)$  are not equal to zero (where  $k = n$ ). They correspond respectively to the impedance and reflection coefficient of the acoustic load as introduced in Section 3. We supposed that  $Z_{nn}(nf) = Z_{11}(f)$  and  $S_{nn}(nf) = S_{11}(f)$ .

We will now explain how to obtain the non zero terms of the scattering matrix (6).

### 4.1.2 Measurement procedure

As in the linear method, a calibration of SKT source is necessary to obtain its impedance  $Z_s(f)$  and its transfer function  $H_{ae}(f)$ .

The measurement procedure is based on different runs during which it is measured the pressure  $p(t)$  inside the tube by the microphone and the loudspeaker voltage response  $u(t)$ .

The scattering matrix (6) is then obtained row by row. Each row needs a measurement run. Basically, we measure  $S_{11}$  from a first run excited at the fundamental frequency, which gives also responses at harmonic frequencies. With a second run we measure  $S_{22}$  at a proper pressure level, which permits to get  $S_{21}$  from the first measurement.

Row 1: A run is carried out choosing a range of frequency from  $f_1$  to  $f_2$  and an excitation level  $a = A$ . Using

the first harmonic terms  $U_1(f, A)$  and  $P_1(f, A)$  of  $u(t)$  and  $p(t)$  respectively,  $S_{11}(f, A)$ ) can be obtained from (3) as

$$S_{11}(f, A) = \frac{H_m(f)Z_s(f) - Z_c(f)(Z_s(f)H_{ae}(f) - H_m(f))}{H_m(f)Z_s(f) + Z_c(f)(Z_s(f)H_{ae}(f) - H_m(f))} \quad (7)$$

where

$$H_m(f) = \frac{P_1(f, A)}{U_1(f, A)}. \quad (8)$$

Moreover, combining the following two relations

$$P_1(f, A) = P_1^-(f, A) + P_1^+(f, A) \quad (9)$$

and

$$S_{11}(f, A) = \frac{P_1^-(f, A)}{P_1^+(f, A)} \quad (10)$$

give the reflected,  $P_1^-(f, A)$ , and incident,  $P_1^+(f, A)$ , contributions which are used in the next steps.

We need also to compute for each harmonic  $n$ ,  $n > 1$ ,  $P_n^-(f, A)$  and  $P_n^+(f, A)$  from  $P_n(f, A)$ . To do this the two following equations have to be solved

$$P_n(f, A) = P_n^-(f, A) + P_n^+(f, A) \quad (11)$$

$$-\frac{P_n(f, A)}{Z_s(nf)} = \frac{1}{\rho c}(P_n^-(f, A) - P_n^+(f, A)). \quad (12)$$

The second equation expresses the  $n^{th}$  harmonic term  $Q_{Tn}(f, A)$  of the acoustic volume velocity with respect to  $P_n^-(f, A)$  and  $P_n^+(f, A)$ . The left hand side of Eqn. (12) is obtained applying the equivalent electroacoustic circuit (Fig. 2) at harmonics  $n$  and assuming  $Q_{an}(f, A) = 0$  giving  $Q_{Tn}(f, A) = -Q_{sn}(f, A) = -P_n(f, A)/Z_s(nf)$ .

Row 2: A run is carried out choosing a range of frequency from  $nh_{min} \times f_{min}$  to  $nh_{max} \times f_{max}$  and a lowest excitation level  $a = A/10$ .  $nh_{min}$  corresponds to the first harmonic studied and  $nh_{max}$  to the last harmonic studied.

Using the first harmonic terms,  $U_1(2f, a)$  and  $P_1(nf, A_2)$ ,  $S_{22}(f, A_2)$  can be obtained from (3) as

$$S_{22}(f, A) = \frac{H_m(2f)Z_s(2f) - Z_c(2f)(Z_s(2f)H_{ae}(2f) - H_m(2f))}{H_m(2f)Z_s(2f) + Z_c(2f)(Z_s(2f)H_{ae}(2f) - H_m(2f))} \quad (13)$$

where

$$H_m(nf) = \frac{P_1(nf, A_2)}{U_1(nf, A_2)}. \quad (14)$$

Finally the second term  $S_{21}(f, |P_1(f, A)|)$  is obtained from the second equation of (6) as

$$S_{21}(f, A) = \frac{P_2^-(f, A) - S_{22}(f, A/10)P_2^-(f, A)}{P_1^-(f, A)}. \quad (15)$$

The formulas can be extended to the other rows (harmonics).

Different excitation signals can be used in this procedure.

**Excitation by stepped-sine** In this first case, the system is excited by a stepped-sine, defined as

$$e(t) = a \sin(2\pi ft + \phi) \quad (16)$$

where  $f$  is the excitation frequency and  $a$  is the associated amplitude. The phase  $\phi$  is introduced arbitrarily by the signal generator. Each experiment is done with constant values for  $a$  and  $f$ . The harmonics components of measured  $p(t)$  and  $u(t)$  are defined by

$$p(t) = \text{Re}\left(\sum_{k=1}^{nh} P_k(f, a) e^{-j2\pi kft}\right) \quad (17)$$

$$u(t) = \text{Re}\left(\sum_{k=1}^{nh} U_k(f, a) e^{-j2\pi kft}\right) \quad (18)$$

where  $nh$  represents the number of harmonics taken into account and extracted by signal processing. The terms  $U_k(f, a)$  for  $k > 1$  are insignificant and have been neglected.

**Excitation by swept-sine** The source is excited by a synchronized swept-sine defined in [7] as

$$e(t) = a \sin(\phi(t)) \quad (19)$$

where  $a$  is the excitation amplitude and  $\phi(t) = 2\pi f_1 e^{t/L}$  the phase with  $f_1$  the initial instantaneous frequency.

$L = \frac{1}{f_1} \text{round}\left(\frac{\bar{T}f_1}{\ln(\frac{f_2}{f_1})}\right)$  represents the rate of frequency increase with  $\text{round}$  the rounding to the nearest integer,  $f_2$  the final instantaneous frequency and  $\bar{T}$  the length of the signal  $e(t)$ .

The response of a nonlinear system excited with a synchronized sweep excitation is considered as a sum of harmonics of the input signal  $e(t)$  convoluted by the impulse responses  $h_n(t)$ . During a measurement run, the loudspeaker voltage response  $u(t)$  and the acoustic pressure  $p(t)$  are recorded simultaneously. Their response is expressed as

$$\begin{cases} p(t) = \sum_{n=1}^{\infty} h_n^p(t + \Delta t_n) * e(t) \\ u(t) = \sum_{n=1}^{\infty} h_n^u(t + \Delta t_n) * e(t) \end{cases} \quad (20)$$

The total impulse response  $h(t)$  of the non-linear system is obtained from the input  $e(t)$  and the output  $p(t)$  as

$$\begin{cases} h^p(t) = p(t) * \tilde{e}(t) \\ h^u(t) = u * \tilde{e}(t) \end{cases} \quad (21)$$

with  $\tilde{e}(t)$  denotes the inverse filter of the input signal. In the frequency domain, the total impulse response  $h(t)$  is expressed as

$$\begin{cases} h^p(t) = \mathcal{F}^{-1} \left[ \mathcal{F}[p(t)] \tilde{E}(f) \right] \\ h^u(t) = \mathcal{F}^{-1} \left[ \mathcal{F}[u(t)] \tilde{E}(f) \right] \end{cases} \quad (22)$$

with  $\mathcal{F}$  and  $\mathcal{F}^{-1}$  respectively the transform and the inverse Fourier transform.  $\tilde{E}(f)$  is the analytical expression of the inverse filter of the input signal.

A temporal windowing will allow to select each harmonic  $h_n^p(t)$  and  $h_n^u(t)$  in the total impulse response  $h^p(t)$  and  $h^u(t)$ . These harmonics are directly linked to the harmonics of the pressure  $P_n(f)$  present in the SKT as

$$\begin{cases} P_n(f) = a \times H_n^p(f) \\ U_n(f) = a \times H_n^u(f) \end{cases} \quad (23)$$

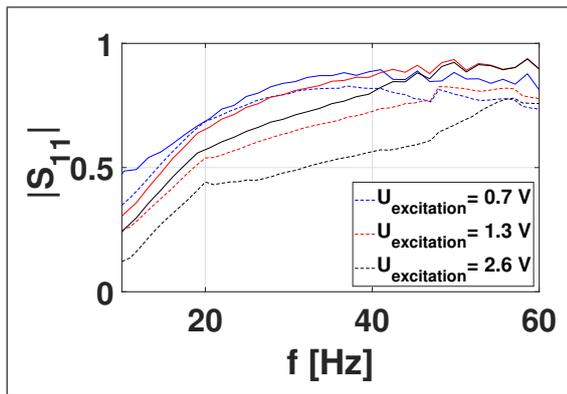
and the coefficients of the scattering matrix can be calculated.

#### 4.2 Application to a passive nonlinear absorber

This method is applied to an adjustable nonlinear acoustic absorber, made of a loudspeaker membrane described in [1]. Three excitation levels are considered in a frequency range from  $f = 10$  Hz to  $f = 60$  Hz and we limit the study to the 7 first harmonics. The absorber is excited with two different signals : a stepped-sine as described by Eqn. (16) and a swept-sine as described by Eqn. (19). Note that these two signals are different and the absorbers will not have the same behavior depending on the excitation signal chosen.

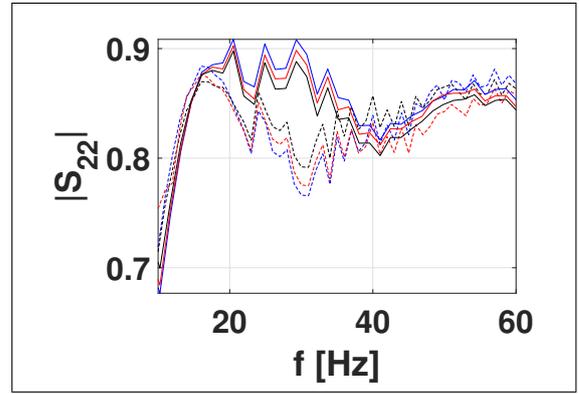
The first coefficients of the scattering matrix are reported Fig. 9, 10 and 11 .

The term  $|S_{11}|$  presents a characteristic curve of reflection coefficient (see Fig. 9) for the three excitations, with  $|S_{22}|$  its continuation at  $nf$  and at the lowest level (see Fig. 10). Its behaviour is similar to the linear estimation of  $R_T$  in Fig. 8.  $|S_{21}|$  show the presence of energy transfer between the fundamental and the second harmonics, which increases with the excitation level (see Fig. 11).

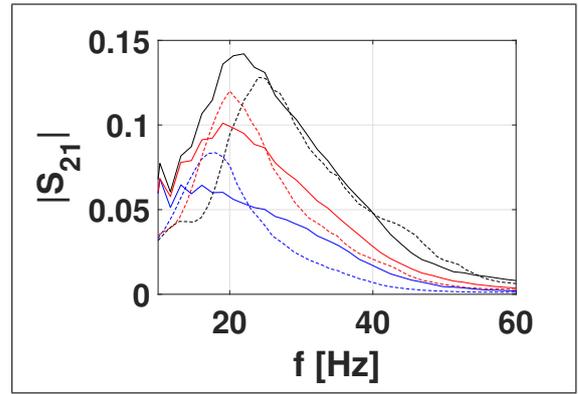


**Figure 9.** Estimation of the term  $S_{11}$  of the scattering matrix for an excitation stepped-sine (dash line) and swept-sine (full line) at same command level  $a$

Focusing on the excitation by swept-sine, it is possible to plot the harmonic distortion to evaluate the energy intake of each harmonics. This indicator is defined as



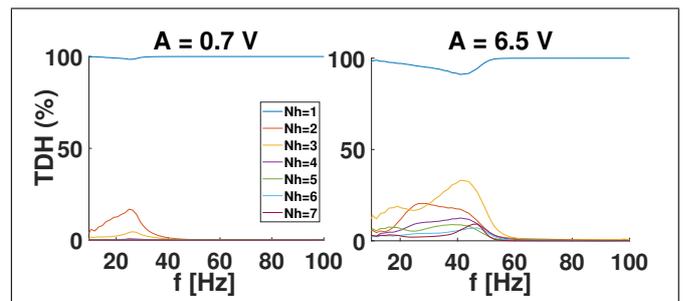
**Figure 10.** Estimation of the term  $S_{22}$  of the scattering matrix for an excitation stepped-sine (dash line) and swept-sine (full line) same command level  $a$



**Figure 11.** Estimation of the term  $S_{21}$  of the scattering matrix for an excitation stepped-sine (dash line) and swept-sine (full line) same command level  $a$

$$DH_h = 100 \times \frac{|P_n(f)|}{\sqrt{\sum_{n=1}^{nh} |P_n(f)|^2}} \quad (24)$$

Fig. 12 represents the contribution of  $P_n(f)$  in the total measured signal, considering seven harmonics. We see that the contribution of the harmonics is higher when the excitation level is important : for  $A = 0.7$  V only the two first harmonics have an impact against all the harmonics for  $A = 6.5$  V. We also see that, for the higher excitation level, the third harmonic have more impact that the second one, due to the cubic nonlinearity of the absorber.

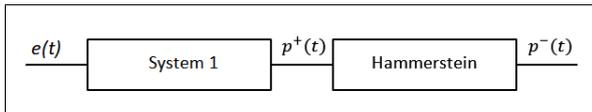


**Figure 12.** Harmonic distortion calculated for two different excitation level

## 5. ESTIMATION BY HAMMERSTEIN MODEL

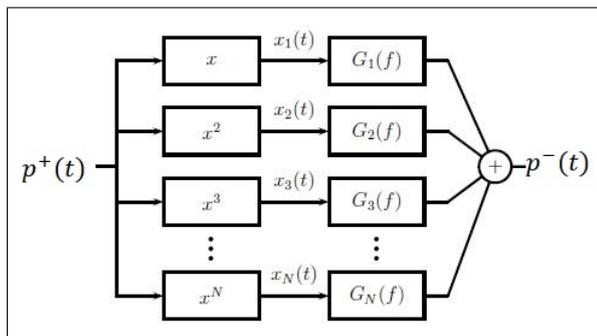
The first step of a third approach is now discussed. We consider here impedance as the response of a particular nonlinear system to a harmonic excitation and we try to identify it in time domain. A cascade of Hammerstein models is used [8]. In a cascade of  $N$  Hammerstein models, each branch is composed of one non-linear static polynomial element  $x^k$  (with zero memory) followed by a linear system representing the memory of the entire studied system  $G_k(f)$  (see Fig. 14 for example).

In our case, the coupled system SKT+DUT can be decomposed as shown Fig. 13 where the subsystem between the incident pressure wave  $p^+(t)$  and the reflected pressure wave  $p^-(t)$  is modelled using a cascade of Hammerstein models.



**Figure 13.** System visualization by Hammerstein model

The cascade of Hammerstein model is represented by the diagram in Fig. 14



**Figure 14.** Block diagram of Hammerstein model

As in [8], the model is estimated using a synchronized swept-sine excitation.

The identification of the model consists to estimate the  $N$  linear filters  $G_n(f)$  from the measurement of  $p^+(t)$  and  $p^-(t)$ , knowing  $e(t)$ . In the same way as  $p(t)$  and  $u(t)$  are given in Eqn. (22),  $p^+(t)$  and  $p^-(t)$  are written from their impulse responses as

$$\begin{cases} p^+(t) = \sum_{n=1}^{\infty} h_n^{p^+}(t + \Delta t_n) * u(t) \\ p^-(t) = \sum_{n=1}^{\infty} h_n^{p^-}(t + \Delta t_n) * u(t) \end{cases} \quad (25)$$

Recalling that incident and reflected waves are linked, in plane waves hypothesis, to the pressure and the acoustic flow as

$$\begin{cases} p(t) = p^+(t) + p^-(t) \\ q(t) = \frac{S}{\rho c} (p^+(t) - p^-(t)) \end{cases} \quad (26)$$

and combining Eqns. (22), (25) and (26), we obtained

$$\begin{cases} h_n^p(t) = h_n^{p^+}(t) + h_n^{p^-}(t) \\ h_n^q(t) = \frac{S}{\rho c} (h_n^{p^+}(t) - h_n^{p^-}(t)) \end{cases} \quad (27)$$

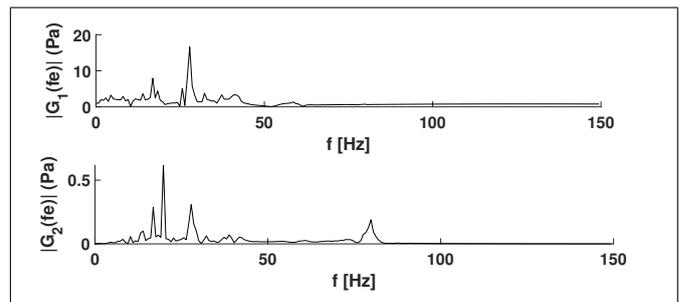
where  $h_n^q(t) = \tilde{z}(t)h_n^p(t)$  with  $\tilde{z}(t)$  the inverse Fourier of  $Z_s(nf)^{-1}$  (see Eqn. (12)). Finally solving Eqns (27), we deduce  $h_n^{p^+}(t)$  and  $h_n^{p^-}(t)$ .

Writing Eqn. (22) for  $p^+(t)$  and  $p^-(t)$ , it is possible as in [7] to obtain from the Hammerstein model the following relation written in matrix form

$$\begin{pmatrix} H_1^{p^-}(f) \\ H_2^{p^-}(f) \\ \vdots \end{pmatrix} = \begin{pmatrix} H_1^{p^+}(f) & H_1^{p^+2}(f) & \dots \\ H_2^{p^+}(f) & H_2^{p^+2}(f) & \dots \\ \vdots & \vdots & \vdots \end{pmatrix} \begin{pmatrix} G_1(f) \\ G_2(f) \\ \vdots \end{pmatrix} \quad (28)$$

where  $H_n^{p^+}(f)$  (respectively  $H_n^{p^-}(f)$ ) is the Fourier transform of  $p_n^+(t)$  (respectively  $p_n^-(t)$ ). This equation can be solved with respect to the  $G_n(f)$  characterizing thus the Hammerstein model.

The estimation of  $G_n(f)$  for the fundamental and the second harmonic are given in Fig. 15.



**Figure 15.** Estimated linear filters  $G_n(f)$

To verify the reliability of the results, it was plotted on Fig. 16 a comparison between the measured reflected pressure, obtained by (27) in frequency domain, and the reconstructed reflected pressure with  $G_n(f)$  filters as

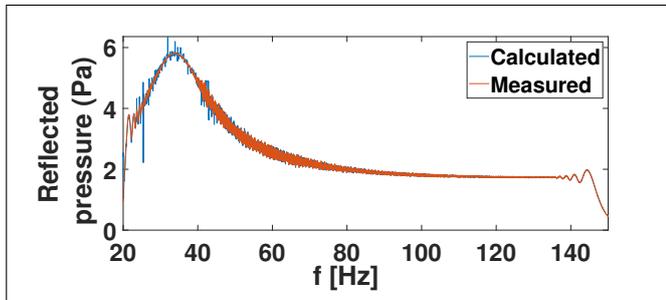
$$P^-(f) = \sum_{n=1}^{Nh} P^+(f)^n G_n(f) \quad (29)$$

The two curves are fitting well in terms of level and frequencies.

This reconstruction of  $P^-(f)$  knowing  $P^+(f)$  using the Hammerstein model will permit in a future work to obtain directly the scattering matrix with a single run test. This will be a part of future work.

## 6. CONCLUSION

We were able here to obtain the acoustic characterization of a nonlinear vibroacoustic absorber, for low frequency and high sound level using a short impedance tube named "Short Kundt's Tube" and with various approaches. The first one is based on the measure of the apparent



**Figure 16.** Comparison between the measured reflected pressure and the reconstructed reflected pressure thanks to  $G_n(f)$  filters

impedance. Impedance and reflection coefficients can be obtained by using a pressure measurement inside the tube and a prior characterization of the source. The second one permits to take into account the nonlinear behaviour of absorbers. It gives information about nonlinear energy transfer to higher harmonics under the form of a scattering matrix, coupling the harmonic terms of the incident and reflected acoustic pressures. This matrix can be obtained for a given excitation level by two different excitations: a stepped-sine treated by signal processing and a synchronized swept-sine whose impulse responses are processed. The results show a nonlinear behaviour of the different absorbers and the nonlinear representation of the system shows an energy transfer to higher harmonics. An other method based on Hammerstein model permits to have access to components of incident and reflected waves regardless of the excitation level.

In futur works, the scattering matrix will be obtain from the Hammersteinn model and compared to the other results. Tests will be realized on other absorbers. Numerical simulations will be performed to validate these experimental results.

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