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# MODELLING OF A MEMS TRANSDUCER WITH A MOVING ELECTRODE IN FORM OF PERFORATED SQUARE PLATE

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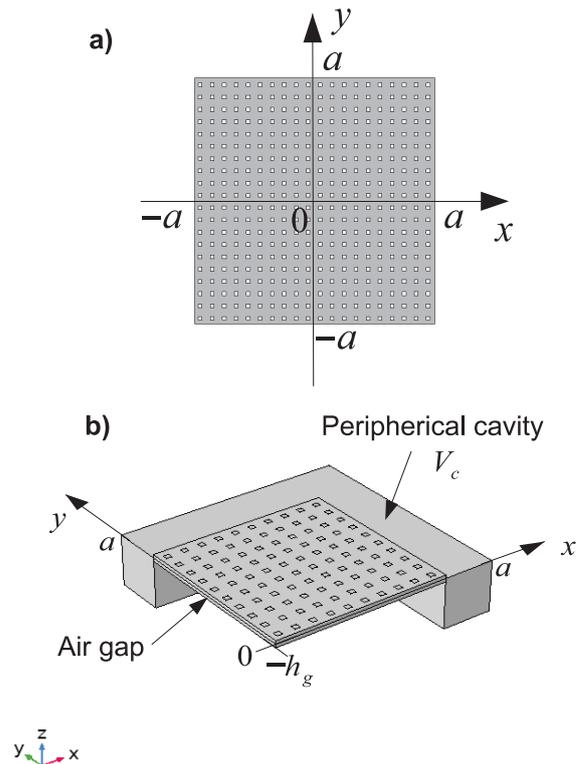
## ABSTRACT

Several realizations of electroacoustic MEMS transducers where the moving electrode is perforated for technological reasons have been published recently. Since the presence of the holes changes the vibration characteristics of the plate (in terms of eigenmodes) and the coupling between the incident acoustic pressure and the pressure inside the transducer through these holes influences significantly the behaviour of the transducer, a precise modelling of such devices is of interest. The modelling approach proposed herein employs an approximated expression of the eigenfunctions of the perforated clamped plate in form of two-dimensional series expansion over the system of functions related to the solution for one-dimensional beam clamped at both ends, the coefficients of the series being calculated from the numerical solution for the eigenfunctions of the perforated clamped plate. The convergence issues and the dependence of the approximation error on the number of terms in the series are discussed. An integral formulation providing the acoustic pressure field inside the air gap between the moving electrode and the fixed one is then proposed. It takes into account the strong coupling between the acoustic pressure field in the air gap and the displacement field of the plate, the thermal and viscous boundary layer effects and the acoustic short circuit between both sides of the perforated plate through the holes.

## 1. INTRODUCTION

The current progress in the domain of the techniques of microfabrication enables to focus on new structures of MEMS devices. The fact that the realisations of electroacoustic MEMS transducers with moving electrode in form of perforated rectangular plate clamped at all edges have appeared recently [1, 2] rises the attention paid on the theoretical modelling of such devices.

The modelling approach presented herein employs an integral formulation for the acoustic pressure in the fluid gap between the perforated plate and fixed backplate [3, 4]. Such formulation requires the use of analytically expressed eigenfunctions of the moving electrode. Since it is a very hard task (to our knowledge) to find the analytical expression for the eigenfunctions of the perforated rectangular clamped plates, an approximated solution based on the ex-



**Figure 1.** Geometry of the transducer: a) the dimensions of the perforated plate, b) 3D cut view of the transducer in the 1<sup>st</sup> quadrant

pressions of the eigenfunctions in the form of truncated series expansion calculated from the numerical solution has been used herein. This method has been previously verified on the non-perforated rectangular clamped plates [5, 6].

## 2. DESCRIPTION OF THE DEVICE

The Fig. 1 shows the geometry of the transducer, which consists of a moving electrode in form of a perforated square clamped plate with a halfside  $a$  and square holes, a backplate being separated from the perforated electrode by a thin fluid-gap of thickness  $h_g$  and a peripheral cavity of volume  $V_c$ .

### 3. EIGENFUNCTIONS OF PERFORATED SQUARE CLAMPED PLATE

This section deals with the approximate expression of the eigenfunctions of the perforated plate. Such an expression is obtained as a truncated series expansion of a numerical (FEM) solution. The numerical results have been calculated using Comsol Multiphysics software, version 5.4. with help of the Structural Mechanics Module for 3D geometry [7]. This module offers the eigenfunction study, the results of which are used below.

#### 3.1 Analytical approximation of the eigenfunctions

Present analytical expression of eigenfunctions  $\psi_{mn}$  is based on an approximation employing the known symmetric eigenfunctions of the 1D beam as the basis of the series in the following manner:

$$\psi_{mn}(x, y) = \sum_{q=1}^N \sum_{r=1}^N c_{(qr),(mn)} \phi_q(x) \phi_r(y), \quad (1)$$

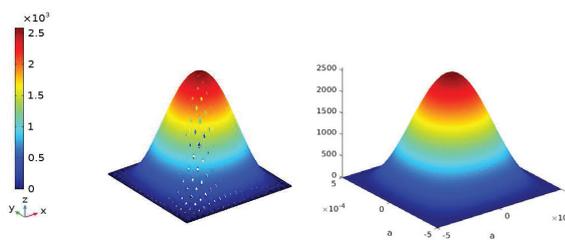
where

$$\phi_q(x) = \frac{1}{\sqrt{2a}} \left[ \frac{\cos(\alpha_q^s x)}{\cos(\alpha_q^s a)} - \frac{\cosh(\alpha_q^s x)}{\cosh(\alpha_q^s a)} \right], \quad (2)$$

$\phi_r(y)$  having the same form as  $\phi_q(x)$ . The coefficients  $c_{(qr),(mn)}$  have been calculated from the numerically calculated eigenfunctions  ${}^n\psi_{mn}$  as follows

$$c_{(qr),(mn)} = \frac{1}{2a^2} \int_{-a}^a \int_{-a}^a {}^n\psi_{mn} \phi_q(x) \phi_r(y) dx dy. \quad (3)$$

The analytical approximation of the eigenfunction has been compared with numerically calculated solution for the 1st mode obtained from FEM model as shown in Fig. 2, leading to findings on accuracy of proposed method.

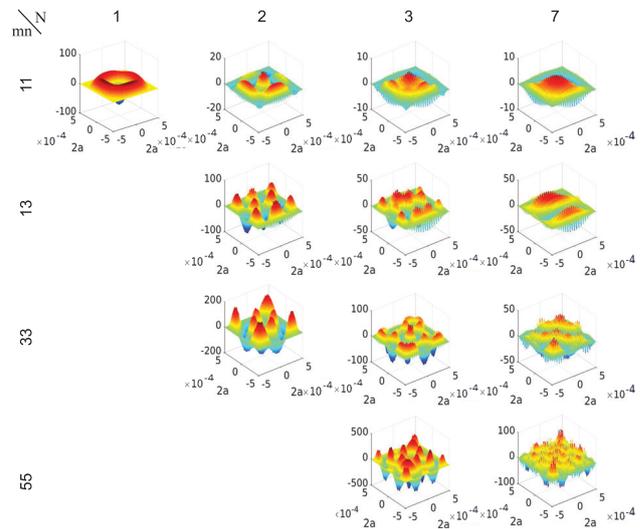


**Figure 2.** a) Numerical solution and b) analytical approximation of  $\psi_{11}(x, y)$ .

Fig. 3 shows the differences between numerical solutions and present analytical approximations for an example of 4 different modes in regard to the summation limit  $N$  in Eqn. (1) (the number of terms in the series being  $N^2$ ).

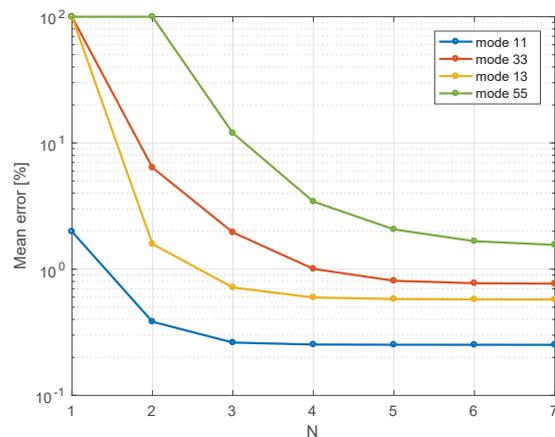
To further analyze and verify the solution proposed herein the mean error of the approximation was calculated as follows:

$$Err = \sqrt{\frac{\sum_{i=1}^M (\psi_{mni} - {}^n\psi_{mni})^2}{\sum_{i=1}^M {}^n\psi_{mni}^2}} \cdot 100\% \quad (4)$$



**Figure 3.** Differences between the numerical solution and the present analytical approximation.

where  $\psi_{mni}$  is the value of the approximated eigenfunction  $\psi_{mn}$  at the  $i$ -th node of the mesh used to obtain the numerical eigenfunctions  ${}^n\psi_{mn}$  and  $M$  is the total number of the mesh nodes. Fig. 4 shows clearly that the mean error is decreasing with increasing number of terms of the series, but the value of error is different for different modes.



**Figure 4.** Mean error of the approximation of  $\psi_{mn}(x, y)$ .

### 4. COUPLING BETWEEN THE PLATE DISPLACEMENT AND THE ACOUSTIC PRESSURE IN THE THIN FLUID LAYER

The displacement field of the rectangular clamped plate can be written as an expansion over the orthonormal eigenfunctions [4–6]

$$\xi(x, y) = \sum_{mn} \xi_{mn} \psi_{mn}(x, y), \quad (5)$$

where the approximated eigenfunctions from the section 3.1 can be used in case of the perforated plate. The modal

coefficients  $\xi_{mn}$  are given by

$$\xi_{mn} = \frac{1}{D \left[ (k_{xm}^2 + k_{yn}^2)^2 - k_p^4 \right]} \cdot \int_{-a}^a \int_{-a}^a \psi_{mn}(x, y) [p(x, y) - p_{inc}] dx dy, \quad (6)$$

where  $D = \frac{Eh_p^3}{12(1-\nu^2)}$  is the flexural rigidity,  $\nu$  being the Poisson's ratio,  $E$  the Young's modulus,  $h_p$  is the thickness of the plate,  $k_p^4 = \frac{M_s}{D}\omega^2$ ,  $M_s = h_p\rho_p$  is the mass per unit area,  $\rho_p$  being the density of the plate.

The wave equation governing the propagation of the acoustic pressure in the air gap is expressed as

$$(\partial_{xx}^2 + \partial_{yy}^2 + \chi^2) p(x, y) = -U(x, y), \quad (7)$$

where the complex wavenumber  $\chi$  accounts for the thermo-viscous effects in the air gap, the source term  $U(x, y)$  contains the plate displacement  $\xi(x, y)$  and the incident acoustic pressure  $p_{inc}$  (since the acoustic pressure in the air gap is coupled with the incident acoustic pressure through the holes in the perforated plate) and both of them include the perforation ratio  $R = \frac{N_h S_h}{4a^2}$  that reflects how much of the plate surface is occupied by holes ( $N_h$  being the number of holes and  $S_h$  the cross-section of one hole).

The acoustic pressure field in the small cavity  $p_c$  is assumed to be uniform. The solution of the Eqn. (7) can be expressed as follows:

$$p(x, y) = \int_{-a}^a \int_{-a}^a G(x, x_0; y, y_0) U(x_0, y_0) dx_0 dy_0 + p_c I_G(x, y), \quad (8)$$

and where the Green function can be chosen as

$$G(x, x_0; y, y_0) = g(x, x_0; y, y_0) + g(x, -x_0; y, y_0) + g(x, x_0; y, -y_0) + g(x, -x_0; y, -y_0), \quad (9)$$

with

$$g(x, x_0; y, y_0) = -iH_0^- \left( \chi \sqrt{(x-x_0)^2 + (y-y_0)^2} \right), \quad (10)$$

$H_0^-$  denoting the Hankel function of the second kind of order 0.

Substituting the acoustic pressure from Eqn. (8) to Eqn. (5), the latter can be then reformulated in a matrix form

$$[-\mathbb{A} + \mathbb{B}] (\Xi) = (C), \quad (11)$$

where  $(\Xi)$  and  $(C)$  are the column vectors of elements  $\xi_{mn}$  and  $c_{mn} = -p_{inc} \int_{-a}^a \int_{-a}^a \psi_{mn}(x, y) dx dy$  respectively,  $\mathbb{B}$  is a diagonal matrix of elements  $D \left[ (k_{xm}^2 + k_{yn}^2)^2 - k_p^4 \right]$  and the elements of the matrix  $\mathbb{A}$  can be found by substituting expression for pressure Eqn. (8) into the integral  $\int_{-a}^a \int_{-a}^a \psi_{mn}(x, y) p(x, y) dx dy$ .

## 5. RESULTS AND DISCUSSION

In this section the results of the analytical procedure described herein are compared with the results of the full 3D FEM simulation of the transducer behaviour. The dimensions and other parameters of the transducer are summarized in Tab. 1, the properties of the air are given in Tab. 2.

Parameter	Value	Unit
Plate halfside $a$	$0.5 \times 10^{-3}$	m
Plate thickness $h_p$	$10 \times 10^{-6}$	m
Square hole side $a_h$	$5 \times 10^{-6}$	m
Number of holes $N_h$	100	-
Cavity volume $V_c$	$1 \times 10^{-10}$	m <sup>3</sup>
Airgap thickness $h_g$	$10 \times 10^{-6}$	m
Density (silicon) $\rho_b$	2329	kg/m <sup>3</sup>
Plate Young's modulus $E$	$160 \times 10^9$	Pa
Poisson's ratio $\nu$	0.27	-

**Table 1.** Dimensions and other parameters of the transducer.

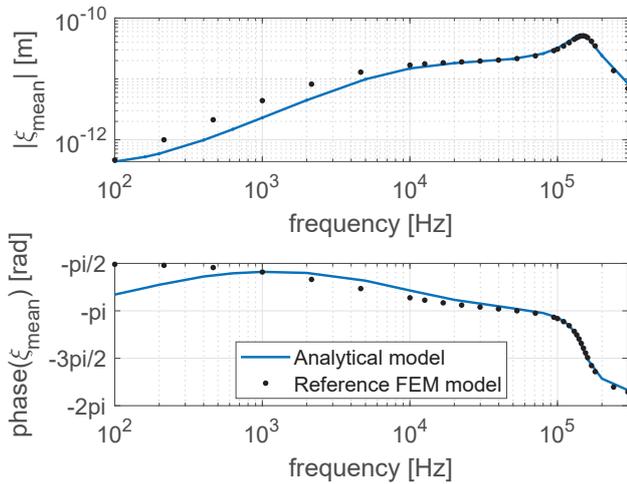
Parameter	Value	Unit
Static pressure $P_0$	101330	Pa
Static temperature $T_0$	293.15	K
Density $\rho_0$	1.204	kg/m <sup>3</sup>
Adiabatic speed of sound $c_0$	343.2	m/s
Shear dynamic viscosity $\mu$	$1.814 \times 10^{-5}$	Pa s
Bulk dynamic viscosity $\mu_B$	$1.088 \times 10^{-5}$	Pa s
Thermal conductivity $\lambda_h$	$25.77 \times 10^{-3}$	W/(mK)
Ratio of specific heats $\gamma$	1.400	-
Specific heat coefficient at constant pressure per unit of mass $C_P$	1005	J/(kgK)

**Table 2.** Parameters of the thermo-viscous fluid (air).

Fig. 5 shows the frequency dependency of the mean displacement  $\xi_{mean} = \iint_{a \times a} \xi(x, y) dx dy / a^2$ , which is the variable of interest in the domain of acoustic receivers. A good agreement can be noted between the result of the present model (blue curve) and the reference numerical (FEM) one (black points) except for very low frequencies where the discrepancy is likely due to the lack of precision of the numerical integration when calculating the members of the matrix  $\mathbb{A}$  (the pressure difference at both sides of the plate being very low at low frequencies hence the accuracy of the numerical integration becoming more important).

## 6. CONCLUSIONS

An analytical model of the miniaturized transducer with moving electrode in form of perforated rectangular



**Figure 5.** Magnitude (upper figure) and phase (lower figure) of the mean displacement of the perforated plate as a moving electrode of the MEMS transducer: Comparison between the present model (blue curve) and the reference FEM model (black dots).

clamped plate has been developed. The approximate analytical expression of the eigenfunctions of the perforated plate in form of truncated series expansion has been found and used in an analytical procedure employing an integral formulation for the acoustic pressure in the air gap loading the moving plate. A good agreement between the results of the presented model and the reference FEM solution has been found, the discrepancies at low frequency range being likely related to the precision of the numerical integration.

## 7. ACKNOWLEDGEMENT

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